(1) a) $f(x) = -2x$ en $[-\pi, \pi]$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} -2\cancel{x}\, dx = \frac{1}{\pi}\left[-x^2\right]_{-\pi}^{\pi} = \frac{1}{\pi} -\pi^2 - (-\pi)^2 = \frac{1}{\pi} -2\pi^2$$

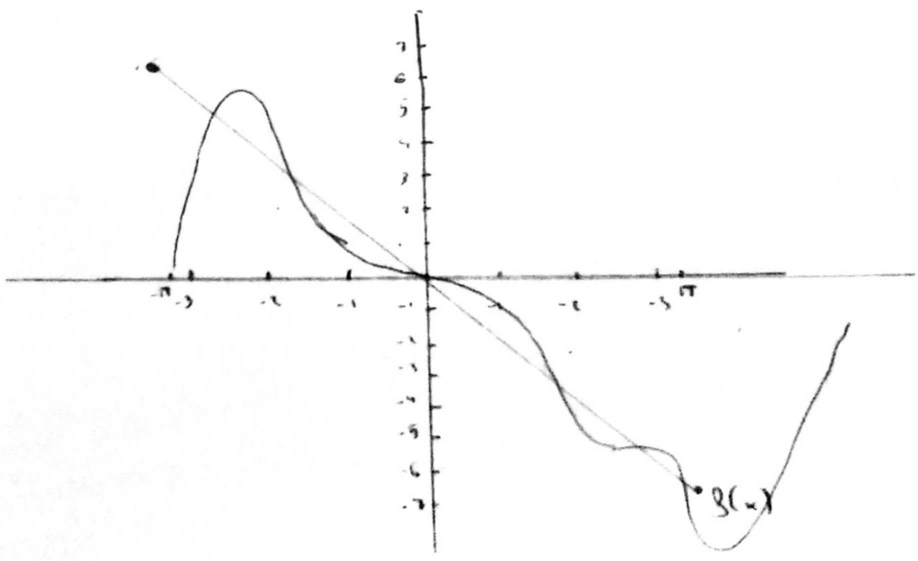
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} -2x\cos(nx)\,dx = \frac{1}{\pi}\left[-\frac{2(\overbrace{nx\sin(nx)}^{0} + \overbrace{\cos(nx)}^{-1^n})}{n^2}\right]_{-\pi}^{\pi} =$$

$$= \frac{1}{\pi}\left[\left(-\frac{2+(-1)^n}{n^2}\right) - \left(-\frac{2+(-1)^n}{n^2}\right)\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} -2x\cdot\operatorname{sen}(nx)\,dx = \frac{1}{\pi}\left[-\frac{2(\operatorname{sen}(nx) - nx\cos(nx))}{n^2}\right]_{-\pi}^{\pi} = -\frac{4(\operatorname{sen}(\pi n) - \pi n\cos(\pi n))}{n^2}$$

$$Sf(x) = \frac{\frac{1}{\pi} \cdot -2\pi^2}{2} + \sum_{n=1}^{\infty}\left(-\frac{4(\operatorname{sen}(\pi n) - \pi n\cos(\pi n))}{n^2}\cdot\operatorname{sen}(nx)\right)$$

Para $n = 3$



b) $f(x) = \begin{cases} x & -\pi \leq x \leq 0 \\ -x & 0 < x \leq \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi} -x\,dx\right) =$$

$$= \frac{1}{\pi}\left(\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \left[-\frac{x^2}{2}\right]_{0}^{\pi}\right) = \frac{1}{\pi}\left(-\frac{\pi^2}{2}\right) + \left(-\frac{\pi^2}{2}\right) = \frac{1}{\pi}\cdot -\frac{2\pi^2}{2} = \frac{1}{\pi}\cdot -\pi^2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} x\cos(nx)\,dx + \int_{0}^{\pi} -x\cos(nx)\,dx\right) =$$

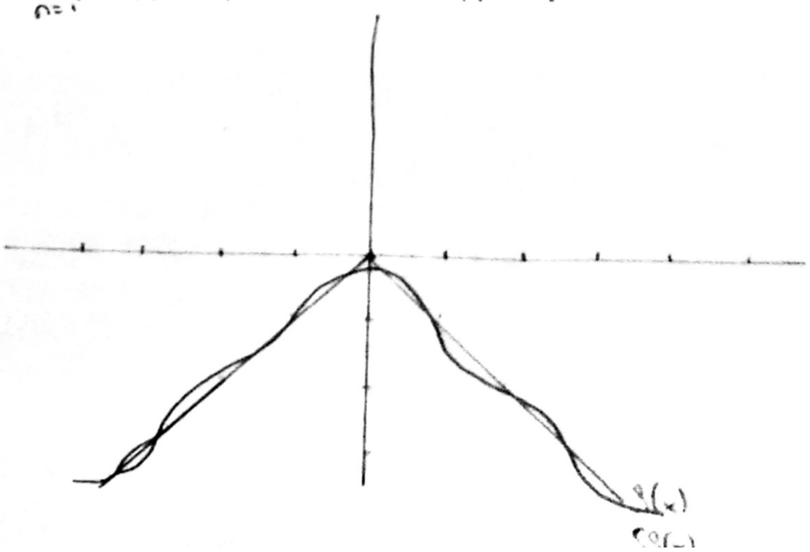
$$= \dots = -\frac{\pi n \sin(\pi n) + \cos(\pi n) - 1}{n^2} = -\frac{(-1)^n - 1}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\text{sen}(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} x\,\text{sen}(nx)\,dx + \int_{0}^{\pi} -x\cos(nx)\,dx\right) =$$

$$= \dots = -\frac{\sin(\pi n) - \pi n\cos(\pi n)}{n^2} = -\frac{-\pi n(-1)^n}{n^2}$$

$$Sf(x) = \frac{\frac{1}{\pi}\cdot -\pi^2}{2} + \sum_{n=1}^{\infty}\left(-\frac{(-1)^n - 1}{n^2}\right)\cdot\cos(nx) + \left(-\frac{\pi n(-1)^n}{n^2}\right)\cdot\text{sen}(nx)$$

para $n=3$



1° c) $f(x) = x^2$ en $[-\pi, \pi]$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{\pi^3}{3} - \frac{(-\pi)^3}{3}\right) = \frac{1}{\pi}\,\frac{2\pi^3}{3} = \frac{2\pi^3}{3\pi}$$

$$= \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)\,dx = \dots = \frac{1}{\pi}\left(\frac{2((\pi n^2-2)\overbrace{\operatorname{sen}(\pi n)}^{0} + 2\pi n\cos(\pi n))}{n^3}\right) =$$

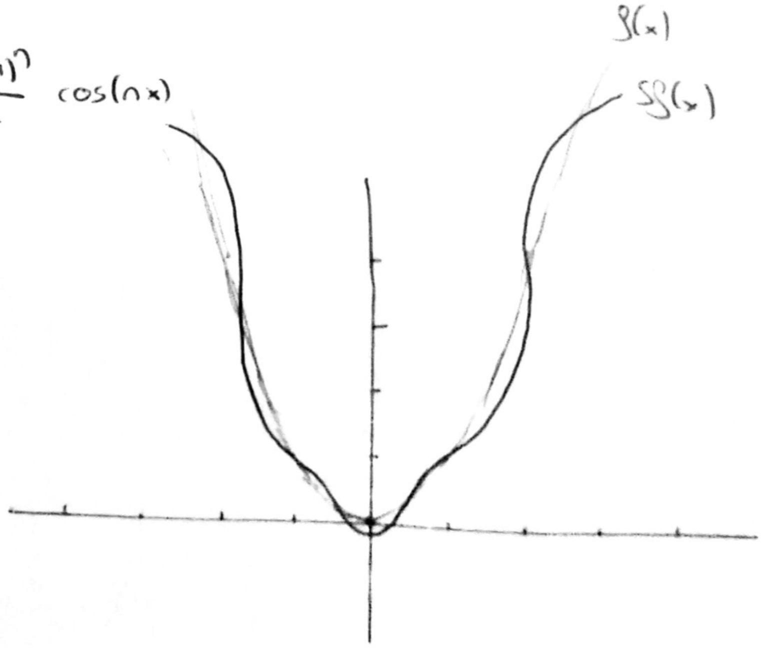
$$= \frac{1}{\pi}\left(\frac{4\pi n\cos(\pi n)}{n^3}\right) = \frac{1}{\pi}\left(\frac{4\pi(-1)^n}{n^2}\right) = \frac{4(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\operatorname{sen}(nx)\,dx = \dots = \frac{1}{\pi}\left(\frac{\overbrace{2nx\operatorname{sen}(nx)}^{0} + (2-n^2x^2)\cos(nx)}{n^3}\right) =$$

$$= \dots = 0$$

$$Sf(x) = \frac{4\pi^2}{3}\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos(nx)$$

para $n=3$



d) $g(x) = \begin{cases} \cos(2x) & x \in [-\pi, 0) \\ \cos(10x) & x \in [0, \pi) \end{cases}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x) = \frac{1}{\pi}\left(\int_{-\pi}^{0}\cos(2x)dx + \int_{0}^{\pi}\cos(10x)dx\right) = \frac{1}{\pi}\left(\underbrace{\left[\frac{\sin(2x)}{2}\right]_{-\pi}^{0}}_{0} + \underbrace{\left[\frac{\sin(10x)}{10}\right]_{0}^{\pi}}_{0}\right)$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x)\cos(nx) = \frac{1}{\pi}\left(\int_{-\pi}^{0}\cos(2x)\cdot\cos(nx) + \int_{0}^{\pi}\cos(10x)\cos(nx)\right) = \dots =$$

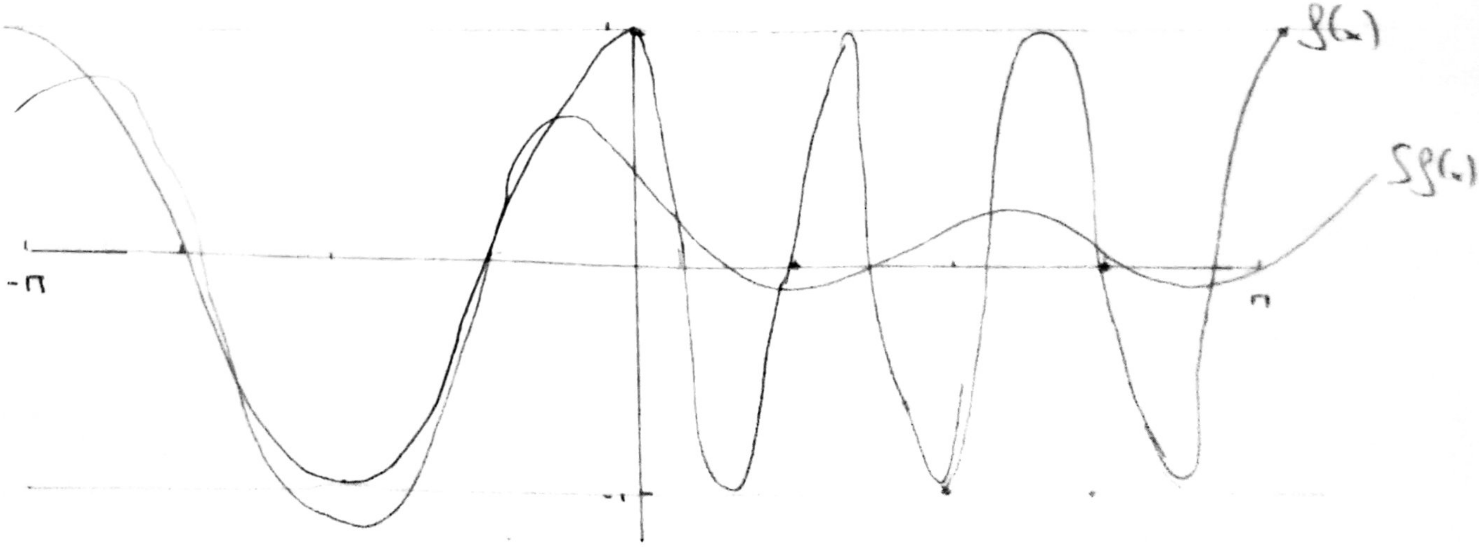
$$= \frac{1}{\pi}\left(\frac{n\sin(\pi n)}{n^2-4} + \frac{n\sin(\pi n)}{n^2-100}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x)\cdot \operatorname{sen}(nx) = \frac{1}{\pi}\left(\int_{-\pi}^{0}\cos(2x)\cdot\operatorname{sen}(nx) + \int_{0}^{\pi}\cos(10x)\operatorname{sen}(nx)\right) = \dots =$$

$$= \frac{1}{\pi}\left(\frac{n(\cos(\pi n)-1}{n^2-4} - \frac{n(\cos(\pi n)-1)}{n^2-100}\right)$$

$$Sg(x) = \sum_{n=1}^{\infty}\frac{1}{\pi}\left(\frac{n\sin(\pi n)}{n^2-4} + \frac{n\sin(\pi n)}{n^2-100}\right)\cdot\cos(xn) + \frac{1}{\pi}\left(\frac{n(\cos(\pi n)-1}{n^2-4} - \frac{n(\cos(\pi n)-1)}{n^2-100}\right)$$

para $n=3$



(2) a) $g(t) = t \quad t \in [0,1]$

$$a_{jk} = \int_0^1 t \cdot \Psi_{jk}(t)\,dt = \int_0^{1/2} t \cdot 1 + \int_{1/2}^1 t \cdot -1 = \left[\frac{t^2}{2}\right]_0^{1/2} + \left[-\frac{t^2}{2}\right]_{1/2}^1 =$$

$$= \frac{1}{8} + \left(-\frac{1}{2} + \frac{1}{8}\right) = \frac{4}{8} = \frac{1}{2}$$

$$b = \int_0^1 t\,\Phi(t)\,dt = \int_0^1 t \cdot t\,dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\left[Sg(t) = \sum_{j=0}^{\infty} \sum_{k=0}^{2^{j-1}} \frac{1}{2} \Psi_{jk}(t) + \frac{1}{3}\Phi(t) \right]$$ donde

$$\Psi_{jk}(t) = \begin{cases} 1 & t \in [0, 1/2) \\ -1 & t \in [1/2, 1) \end{cases}$$

$$\Phi(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & \text{en otro caso} \end{cases}$$

b) $g(t) = t^2 \quad t \in [0,1]$

$$a_{jk} = \int_0^1 t^2 \Psi_{jk}(t)\,dt = \int_0^{1/2} t^2 \cdot 1 + \int_{1/2}^1 t^2 \cdot -1 = \left[\frac{t^3}{3}\right]_0^{1/2} + \left[-\frac{t^3}{3}\right]_{1/2}^2 =$$

$$= \frac{1}{24} + \left(-\frac{8}{3}\right) - \left(-\frac{1}{24}\right) = -\frac{31}{12}$$

$$b = \int_0^1 t^2\,\Phi(t)\,dt = \int_0^1 t^3\,dt = \left[\frac{t^4}{4}\right]_0^1 = \frac{1}{4}$$

$$\left[Sg(t) = \sum_{j=0}^{\infty} \sum_{k=0}^{2^{j-1}} -\frac{31}{12} \Psi_{jk}(t) + \frac{1}{4}\Phi(t) \right]$$