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Applied Data Science Program

RANDOM FOREST

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Massachusetts Institute of Technology

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Review

- Learning a Decision Tree
- Feature selection
  - Information Gain
- Statistical learning

Empirical Estimates

Data Set: $\{(x_i, y_i) \in \mathcal{X} \times \mathcal{Y},\ i = 1, \ldots N\}$

Empirical Error of a decision Rule f :

$$R(f) = \frac{1}{N} \sum_i^N \mathbf{I}(f(x_i) \neq y_i)$$

$\mathbf{I}(x) = 1\ if\ x \neq 0, otherwise\ it\ is\ 0$

Outline

- Overfitting: Bias-Variance Tradeoff
  - Titanic example
  - Pruning

- Bagging to reduce variance

- Random Forest

Part I: Bias-Variance Tradeoff

Titanic Data Set

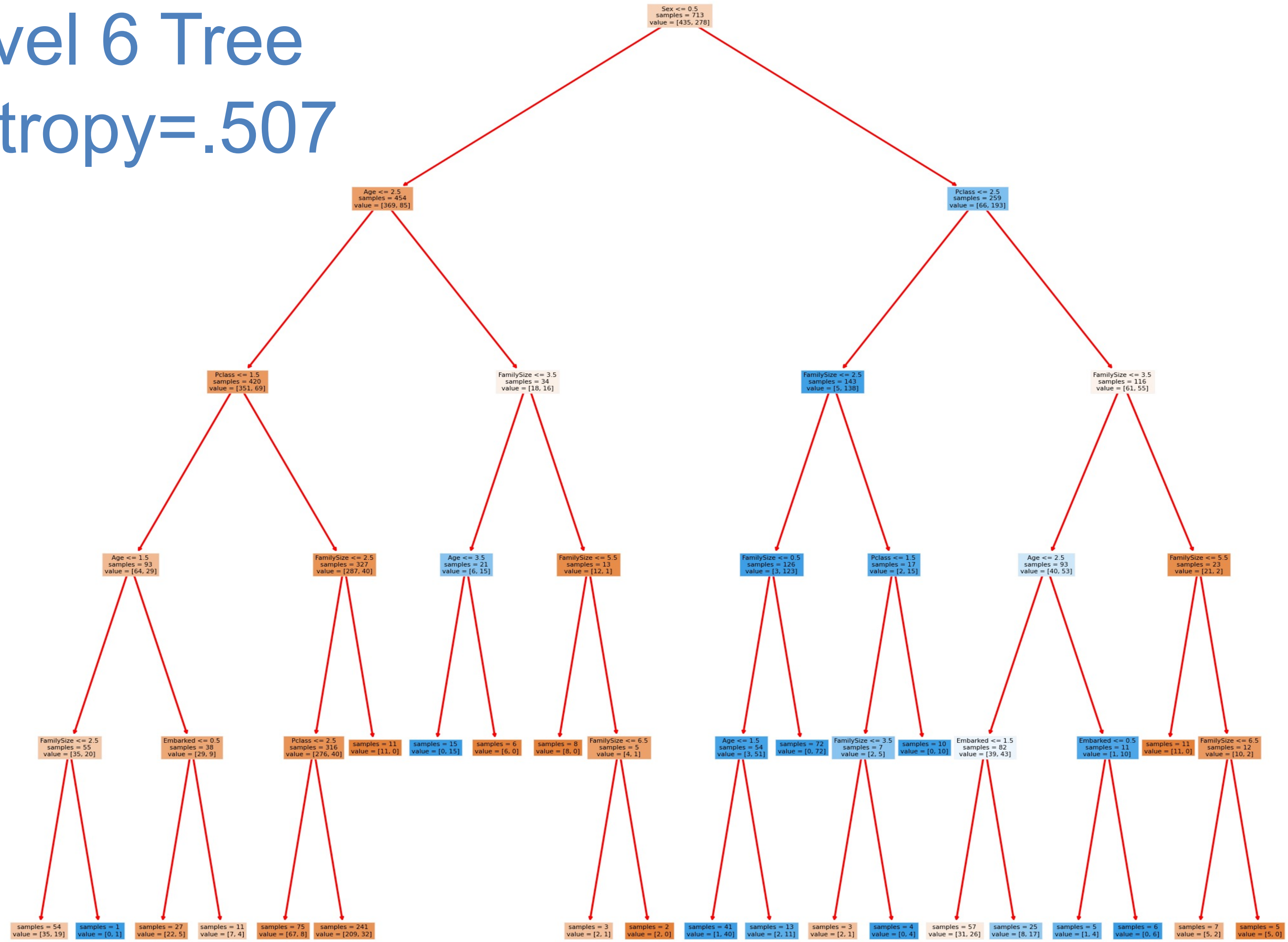
891 data point
38% Survival rate
Data Split 8:2

- **Survived**: indicator variable describing whether the person survived the shipwreck.
- **Pclass**: Passenger class. Takes values from $\{1, 2, 3\}$.
- **Sex**: The sex of the passenger.
- **Age**: The age group of the passenger. Takes values from $\{< 13, 13-25, 25-40, 40-65, 65+\}$.
- **Embarked**: The port from which the passenger embarked on the ship. Takes values from { Cherbourg, Southampton, Queenstown}.
- **FamilySize**: Size of the passenger's family (excluding the passenger) on board.

Titanic Example

	Survived	Pclass	Sex	Age	Embarked	FamilySize
492	0	1	male	40-65	Southampton	0
141	1	3	female	13-25	Southampton	0
409	0	3	female	25-40	Southampton	4
31	1	1	female	25-40	Cherbourg	1
570	1	2	male	40-65	Southampton	0
593	0	3	female	25-40	Queenstown	2
873	0	3	male	40-65	Southampton	0
399	1	2	female	25-40	Southampton	0
406	0	3	male	40-65	Southampton	0
272	1	2	female	40-65	Southampton	1

Level 6 Tree
Entropy=.507



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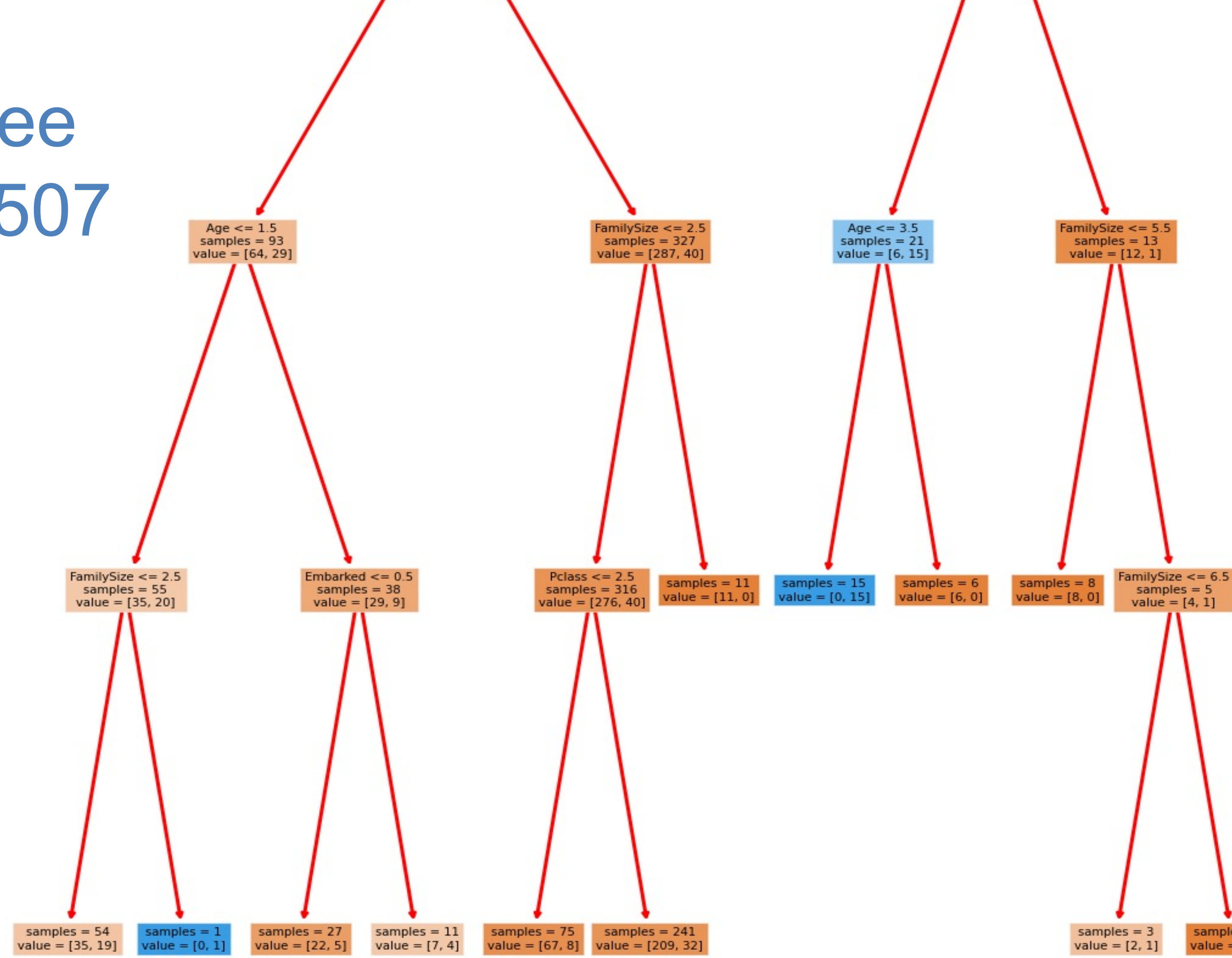
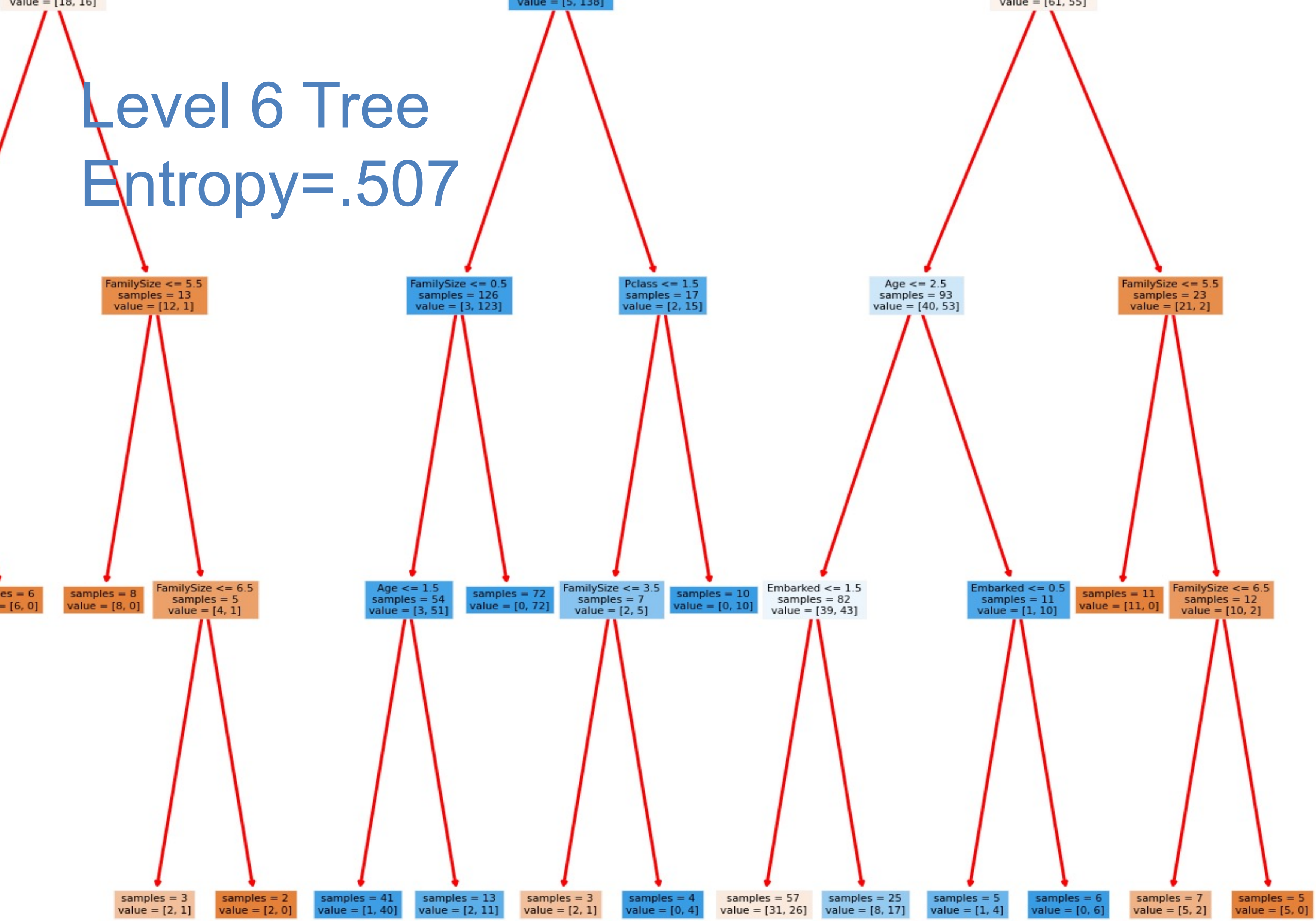
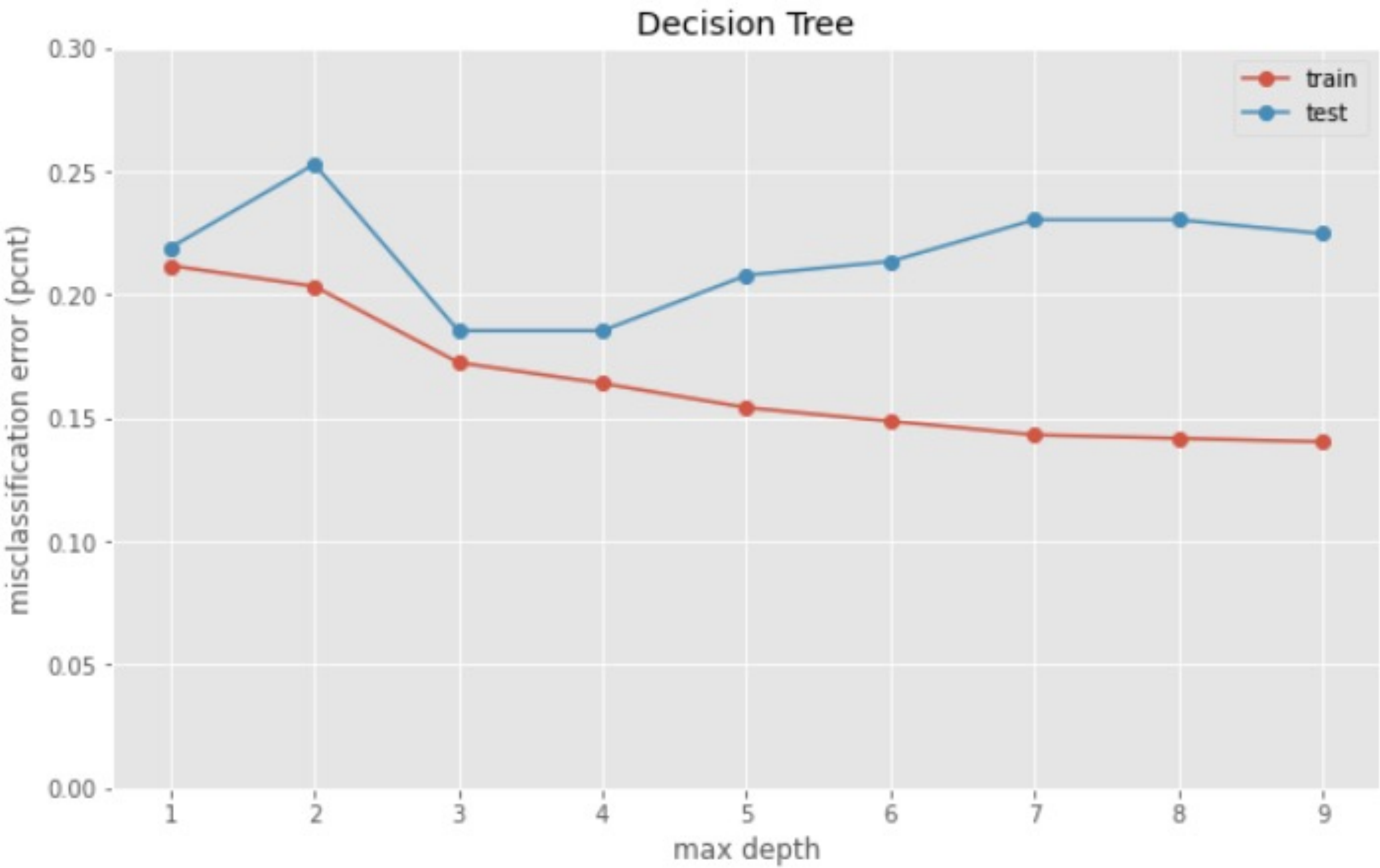


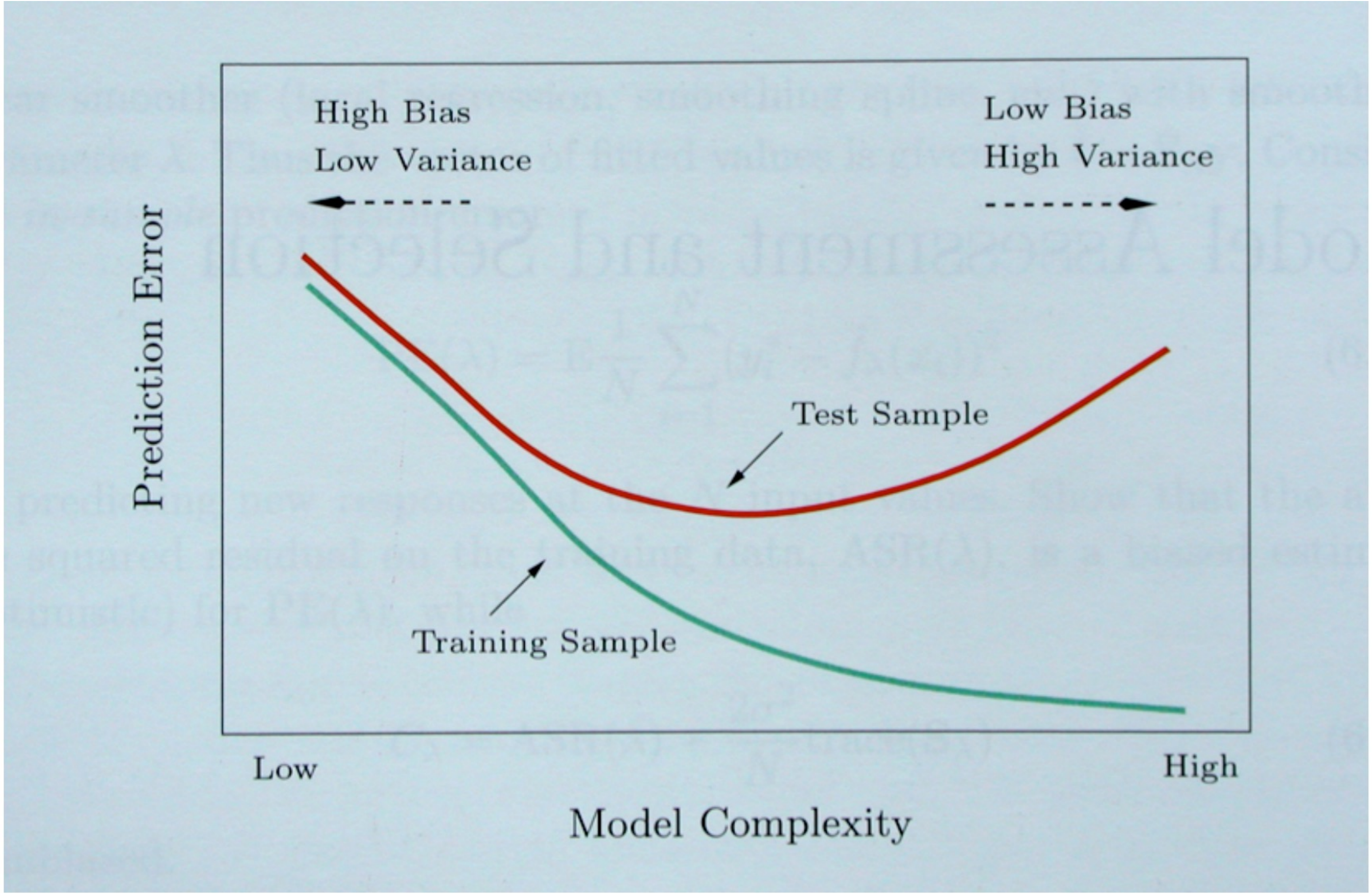
Level 6 Tree
Entropy=.507



Train vs. Test Results

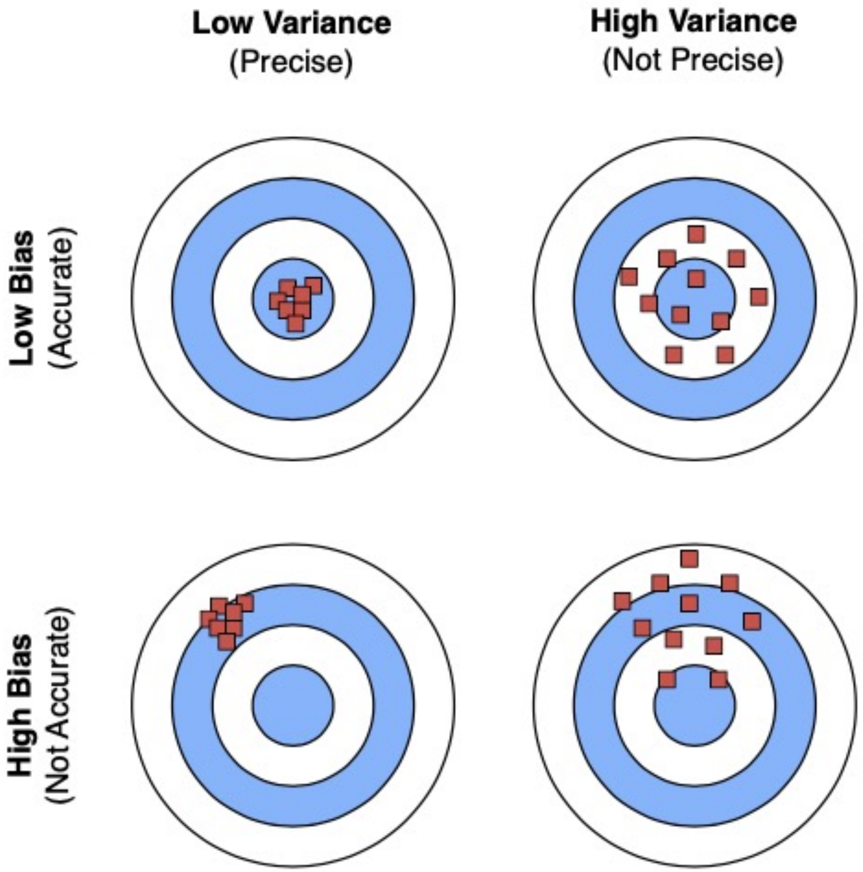


Bias-Variance Tradeoff



Hastie, Tibshirani, Friedman "Elements of Statistical Learning" 2001

Bias-Variance Tradeoff



Quick Fix: Pruning

- DT can be defined to a large level of granularity
  - Not a good way to generalize
- Pruning, Aggregation
  - Use misclassification as a guidance
  - Reduce the depth
  - Eliminate small class

Back to Example

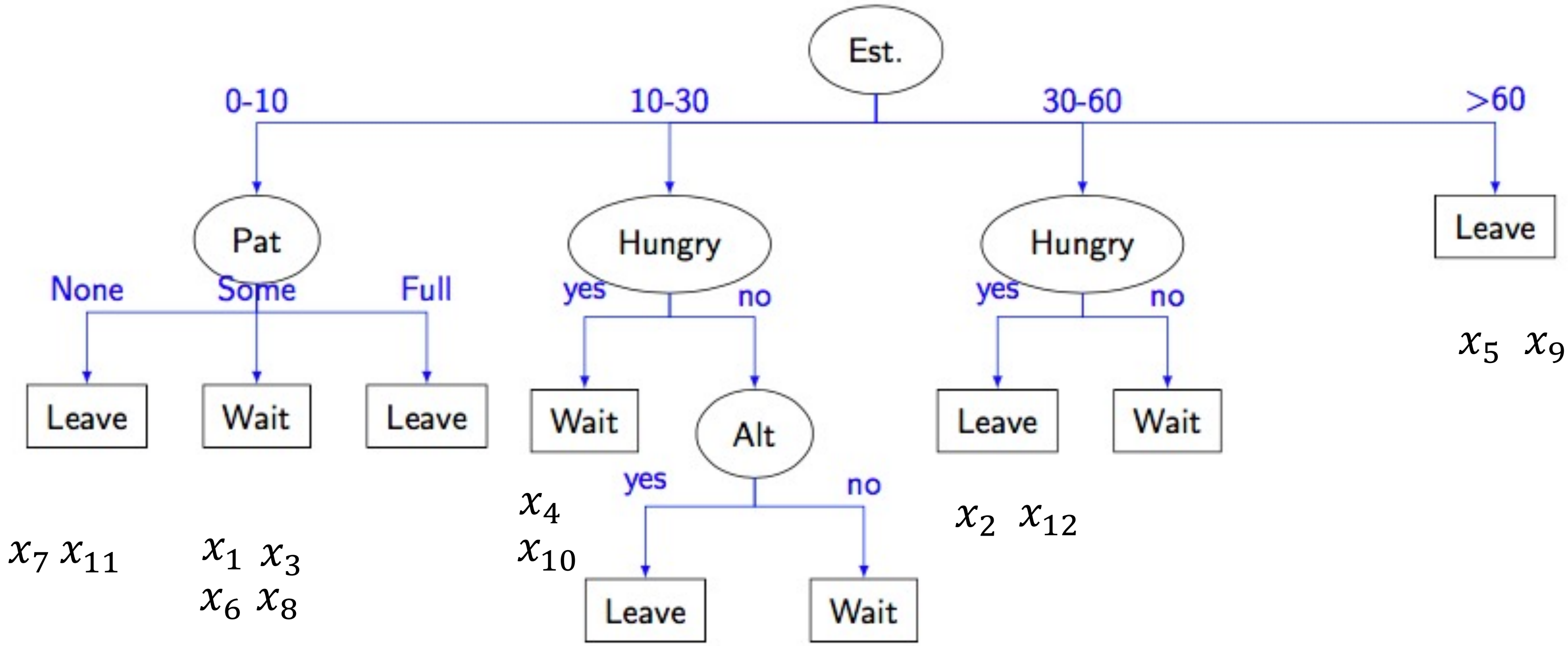
Example	Input Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
$\mathbf{x}_1$	Yes	No	No	Yes	Some	$$$	No	Yes	French	0–10	$y_1 =$ Yes
$\mathbf{x}_2$	Yes	No	No	Yes	Full	$	No	No	Thai	30–60	$y_2 =$ No
$\mathbf{x}_3$	No	Yes	No	No	Some	$	No	No	Burger	0–10	$y_3 =$ Yes
$\mathbf{x}_4$	Yes	No	Yes	Yes	Full	$	Yes	No	Thai	10–30	$y_4 =$ Yes
$\mathbf{x}_5$	Yes	No	Yes	No	Full	$$$	No	Yes	French	>60	$y_5 =$ No
$\mathbf{x}_6$	No	Yes	No	Yes	Some	$$	Yes	Yes	Italian	0–10	$y_6 =$ Yes
$\mathbf{x}_7$	No	Yes	No	No	None	$	Yes	No	Burger	0–10	$y_7 =$ No
$\mathbf{x}_8$	No	No	No	Yes	Some	$$	Yes	Yes	Thai	0–10	$y_8 =$ Yes
$\mathbf{x}_9$	No	Yes	Yes	No	Full	$	Yes	No	Burger	>60	$y_9 =$ No
$\mathbf{x}_{10}$	Yes	Yes	Yes	Yes	Full	$$$	No	Yes	Italian	10–30	$y_{10} =$ No
$\mathbf{x}_{11}$	No	No	No	No	None	$	No	No	Thai	0–10	$y_{11} =$ No
$\mathbf{x}_{12}$	Yes	Yes	Yes	Yes	Full	$	No	No	Burger	30–60	$y_{12} =$ Yes

- Attributes:

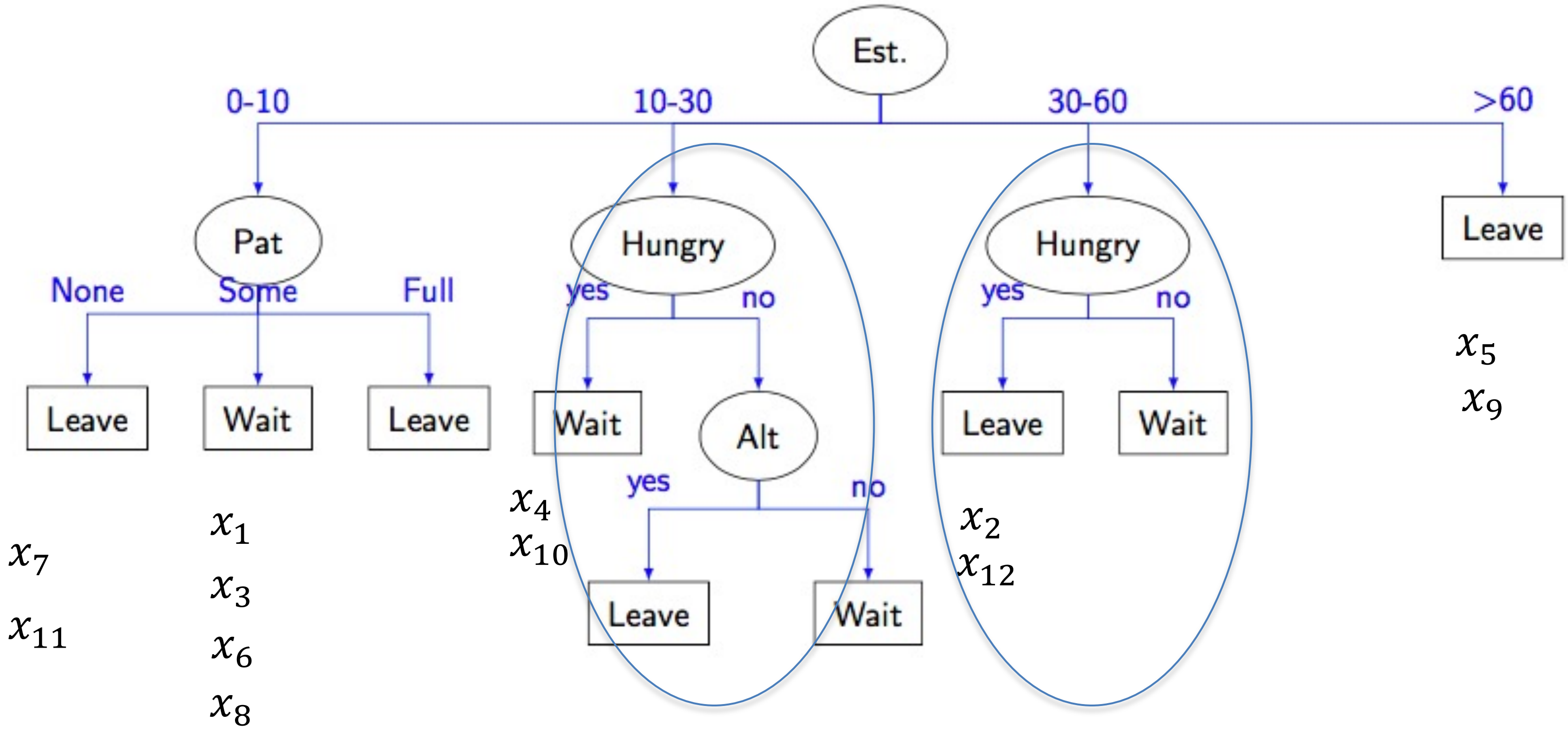
1. Alternate: whether there is a suitable alternative restaurant nearby.
2. Bar: whether the restaurant has a comfortable bar area to wait in.
3. Fri/Sat: true on Fridays and Saturdays.
4. Hungry: whether we are hungry.
5. Patrons: how many people are in the restaurant (values are None, Some, and Full).
6. Price: the restaurant's price range ($, $$, $$$).
7. Raining: whether it is raining outside.
8. Reservation: whether we made a reservation.
9. Type: the kind of restaurant (French, Italian, Thai or Burger).
10. WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60).

[from: Russell & Norvig]

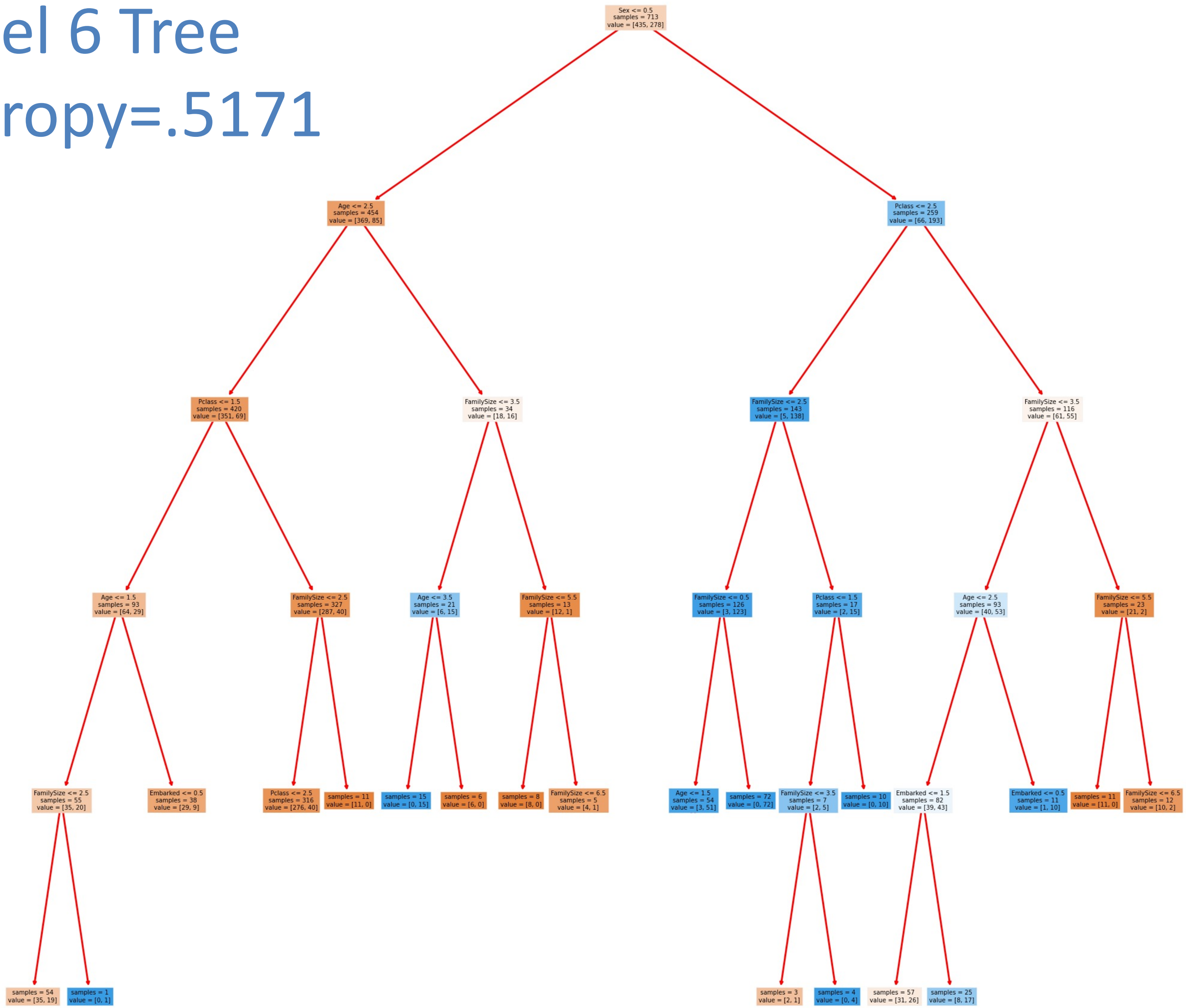
How does Pruning work here?



How does Pruning work here?



Level 6 Tree
Entropy=.5171



Summary

- Create a tree with maximum depth
  - Either until every leaf is a single data point
  - Or use all features
- Pick a subtree (a node and all leaves)
- Aggregate that leaves all the way to the node
- Compute new error
  - Misclassification
  - Entropy

Summary

- Pruning is expensive
- Pruning is counter-intuitive
  - Fixing a bad model
- How about directly building a better model

Part II: Bagging

Ensemble Learning/Random Forest

- Ensemble Methods are the key idea behind Random Forests
- Motivated by averaging techniques
- You can reduce the variance if you average a number of independent RVs
- How?

History

- Tin Kam Ho (Random Subspace Methods)

- Breiman and Cutler (Registered Random Forest as a trademark: They combine Bagging with random feature selection

- Amit and Geman: did the same independently

Ensemble Learning: Basic idea

Bagging= Bootstrap + Aggregation

Ensemble Learning: Bootstrap

- Sample data with replacement
  - Create many data sets
- Data not sampled is used for cross validation
- Data sets are correlated (or dependent)
  - Reasonably uncorrelated for large sample

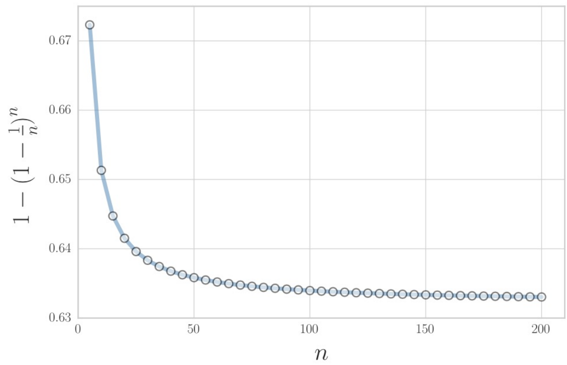
Bootstrap Sample

Size of Test Set

- Assume that we have n data points

- What's the probability that a specific data point is not selected in n samples with replacement?

$$\left(1 - \frac{1}{n}\right)^{\mathrm{n}}$$



- If n is large then this probability is:

$$\frac{1}{e} = 0.368$$

- Provides a reasonable percentage for cross validation to estimate error

Multiple Classifiers

- Build a classifier for each ”new” data set

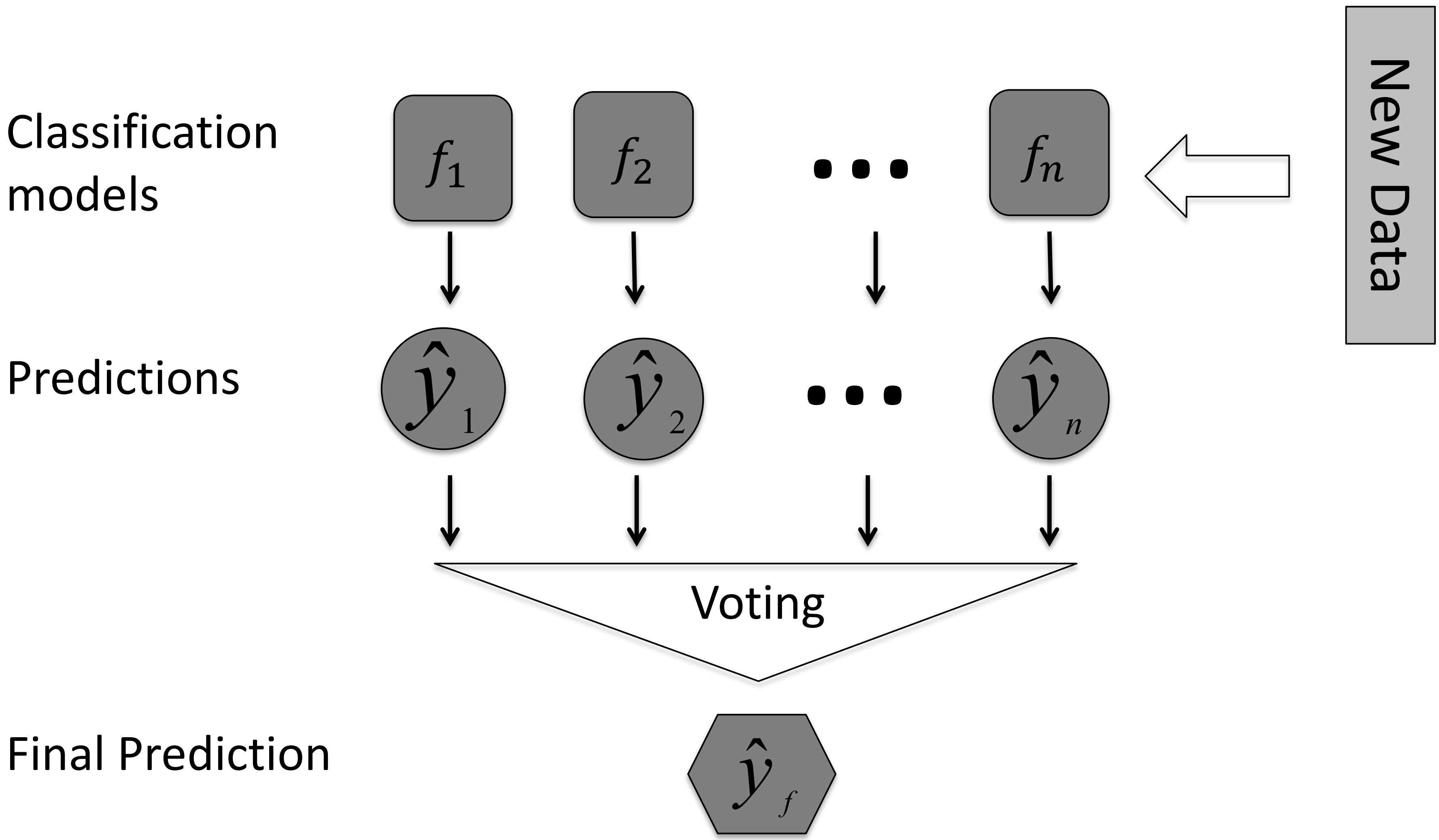
$$\hat{y}_i = f_i(x) \text{ with error } \epsilon_i$$

- If the n is large, then the outcome of these classifiers are ”not” too dependent

Aggregation

- Multiple classifiers, decision trees
- Voting between choices (aggregation)
- Reduce the variance (generalization) of classification

Majority Classifier (Aggregate)



Majority Classifier (aggregate)

- Voting:

$$\{\hat{y}_f\} = f(x) \equiv \text{ majority } (f_1(x), f_2(x), \dots . f_l(x))$$

- How does this estimator perform?
- You can also combine using weighted sum

Analysis of voting

- Assume each classifier has error

$$\epsilon_i = P(f_i(x) \neq \hat{y}_\mathrm{i}) < 0.5$$

- Assume classifiers are independent

- Error in aggregation: More than half of the classifiers are wrong

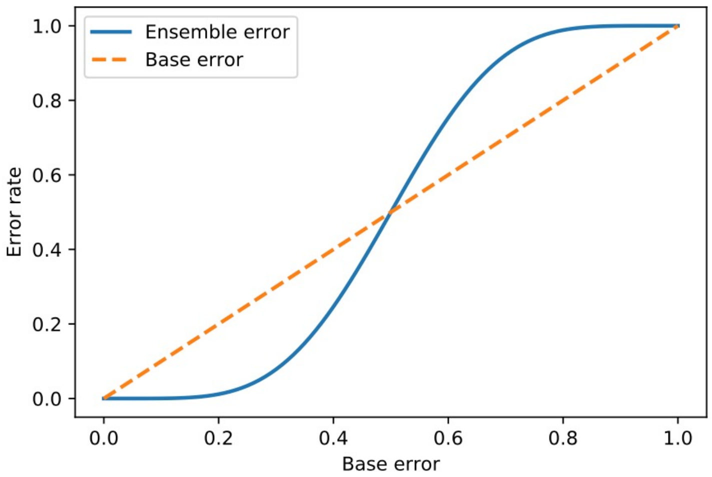
Analysis of Voting

- Uniform error

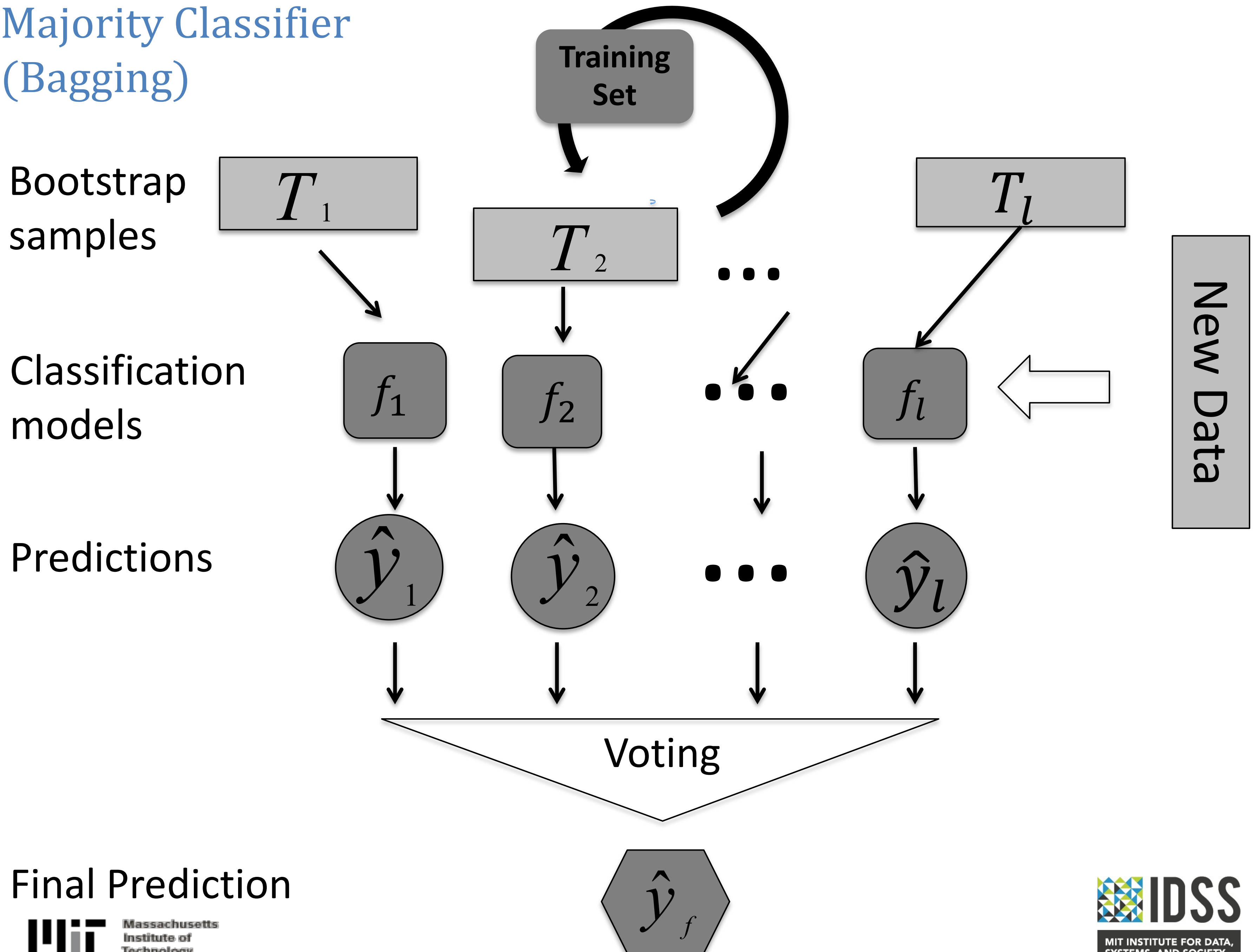
$$\epsilon = P(f_i(x) \neq \hat{y}_{\mathrm{i}}) = 0.25$$

- Then $P(f(x) \neq \hat{y}_f) = \sum_{\left\{k \geq \frac{l}{2}\right\}}^{l} \binom{l}{k} \epsilon^k (1-\epsilon)^{l-k}$

- *Improvement!*



Majority Classifier (Bagging)



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Summary: Bagging

- Bagging= Bootstrap + Aggregation
- Bootstrap: sample with replacement
- Build a tree classifier with each sample
- Aggregate through majority voting

Illustrative Example: Waiting at a restaurant

Example	Input Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
$\mathbf{x}_1$	Yes	No	No	Yes	Some	$$$	No	Yes	French	0–10	$y_1 =$ Yes
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Attributes:

1.	Alternate: whether there is a suitable alternative restaurant nearby.
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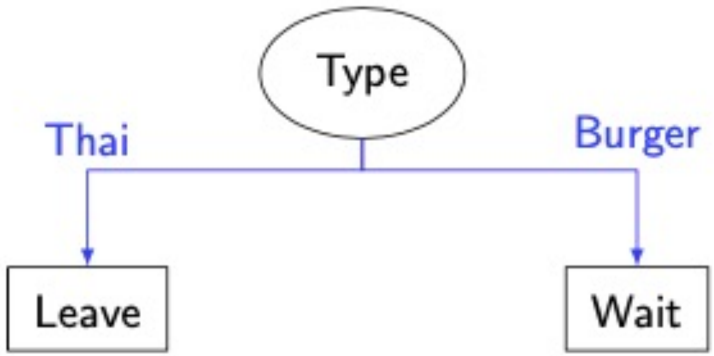
10.	WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, >60).

[from: Russell & Norvig]

Restaurant example

Tree 1: $x_{11}, x_{12}, x_7, x_2, x_{12}$

x_{11}, x_2

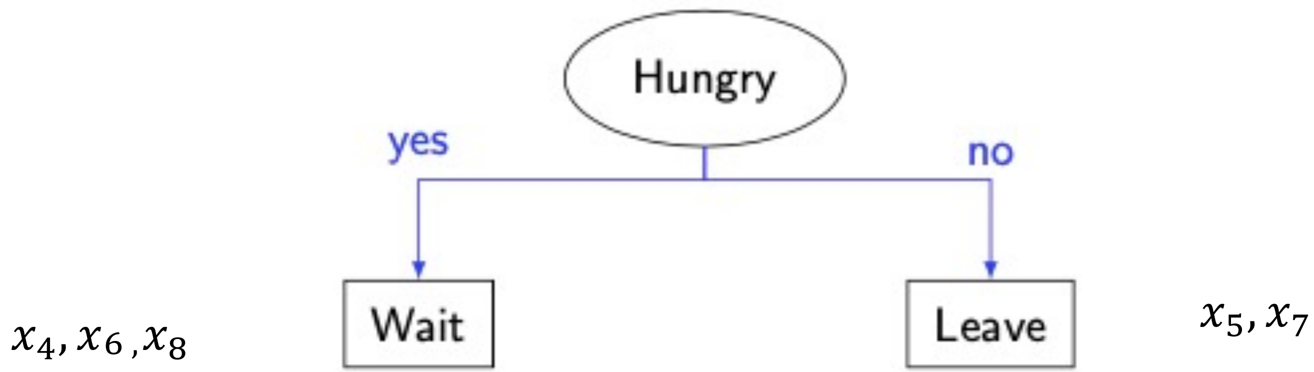


x_{12}, x_7, x_{12}

Loss: 0.55098

Restaurant example

Tree 2: x_5, x_4, x_6, x_8, x_7

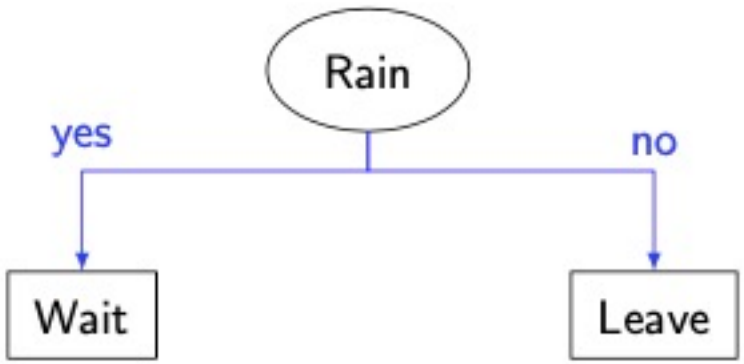


Loss: 0.

Restaurant Example

Tree 3: $x_9, x_{10}, x_4, x_2, x_{10}$

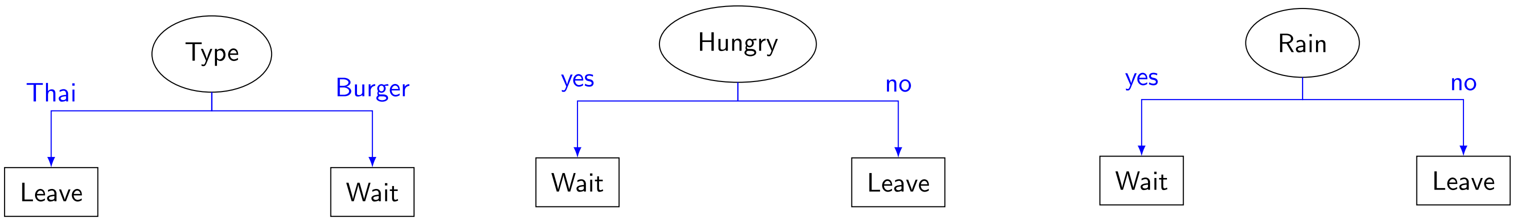
$x_9\ x_4$



$x_{10}\ x_2\ x_{10}$

Loss: 0.4

What's the Aggregate Classifier



x_1: 1, 1, 0 → 1

x_2: 0, 1, 0 → 0

x_3: 1, 0, 0 → 0

x_4: 0, 1, 1 → 1

x_5: 1, 0, 0 → 0

x_6: 0, 1, 1 → 1

x_7: 1, 0, 1 → 1

x_8: 0, 1, 1 → 1

x_9: 1, 0, 1 → 1

x_{10}: 0, 1, 0 → 0

x_{11}: 0, 0, 0 → 0

x_{12}: 1, 1, 0 → 1

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$\mathbf{x}_9$	No	Yes	Yes	No	Full	$	Yes	No	Burger	>60	y_9 = No
$\mathbf{x}_{10}$	Yes	Yes	Yes	Yes	Full	$$$	No	Yes	Italian	10–30	y_{10} = No
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[from: Russell & Norvig]

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Mis-classification error: 3/12

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Drawback: We lost the tree!

Random Forest: Part III

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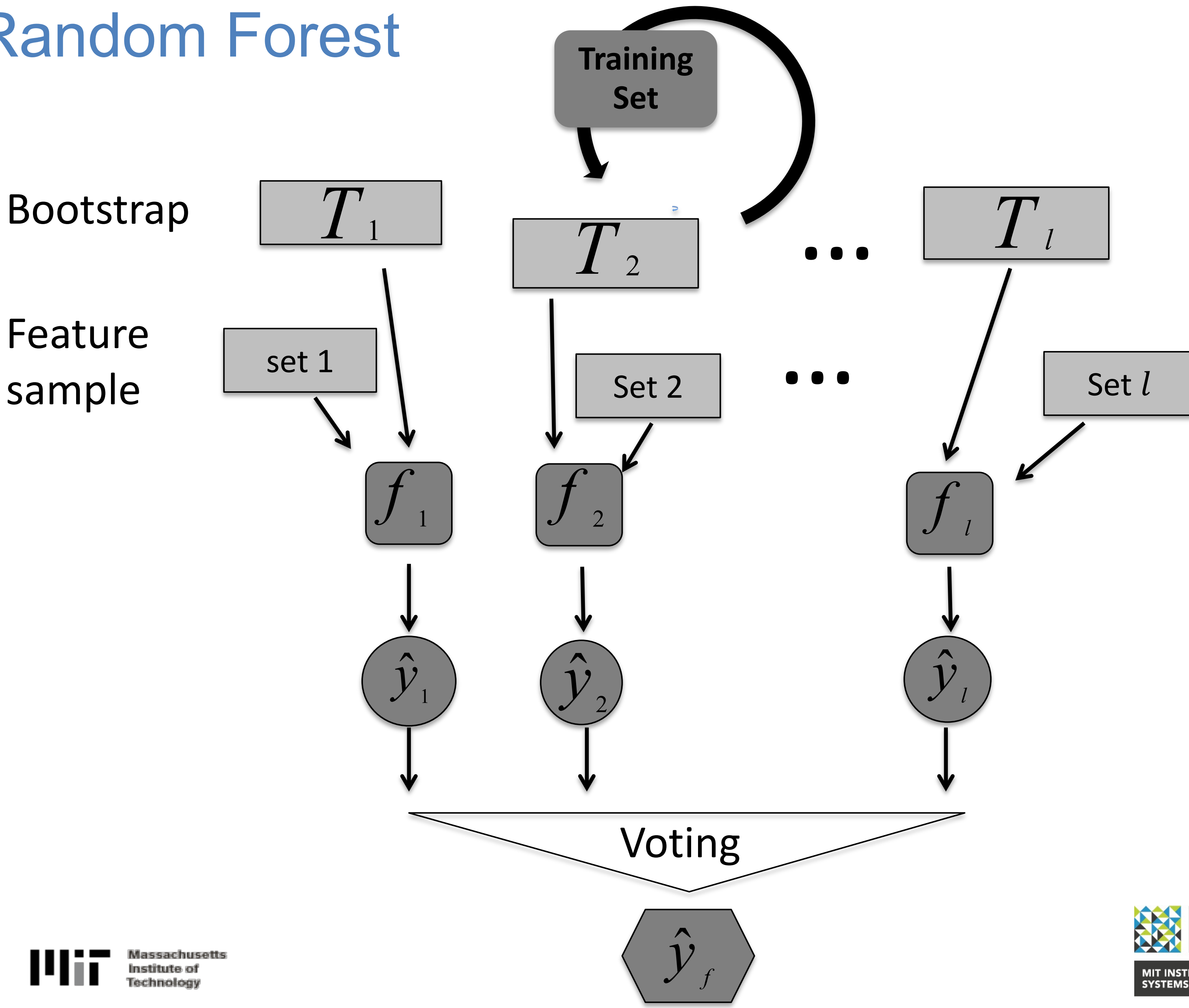
Random Forest

- What if the samples are not independent
- You can increase the independence through sampling the features at each node
- Benefit: better generalization
- *Downside: less interpretable, less powerful*

Random Forest



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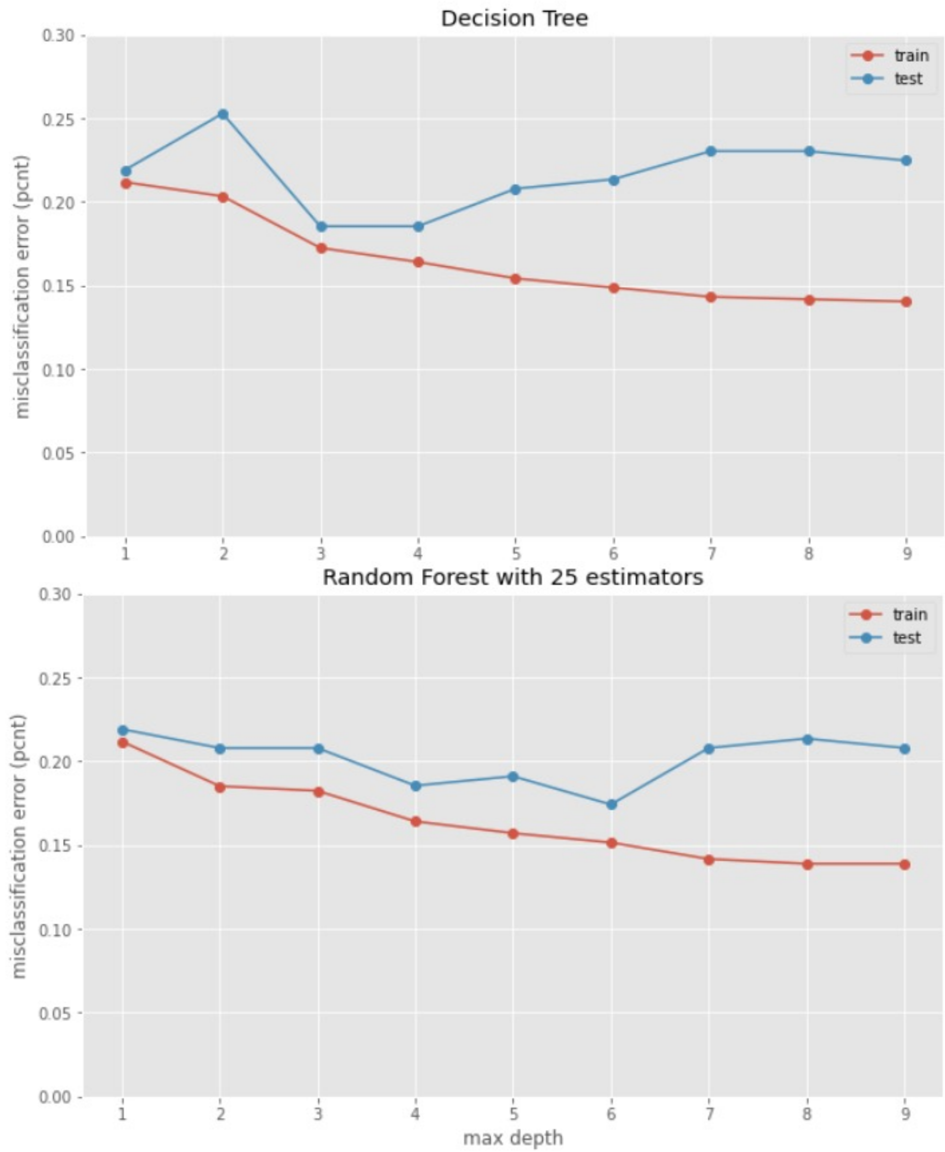
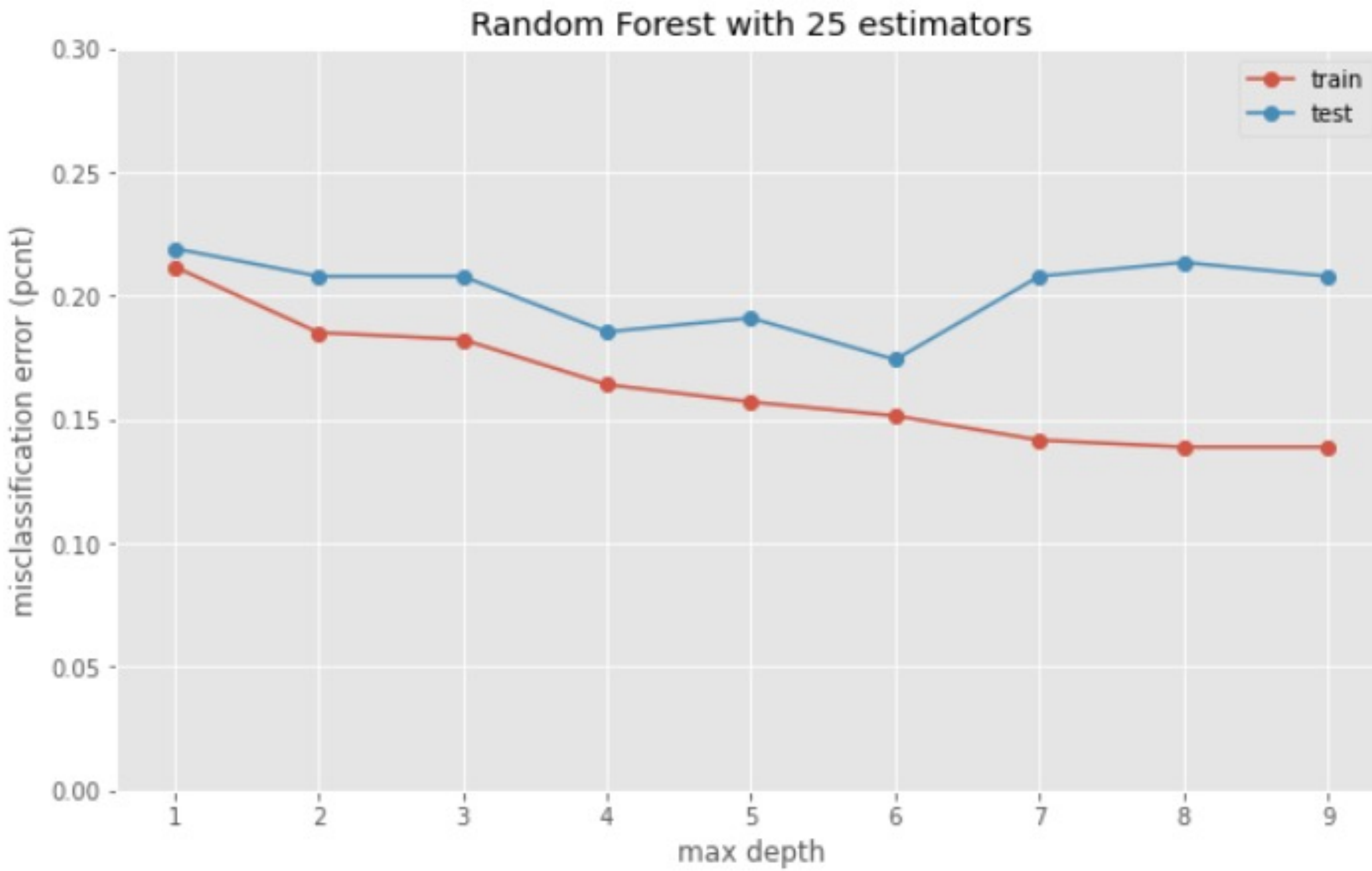
More Elaborate

- Can we diversify further?
- Select a random set of feature at each tree level
- Continue with previous process
- More diverse, difficult to interpret

DT vs Random Forest



Random Forest—Titanic data

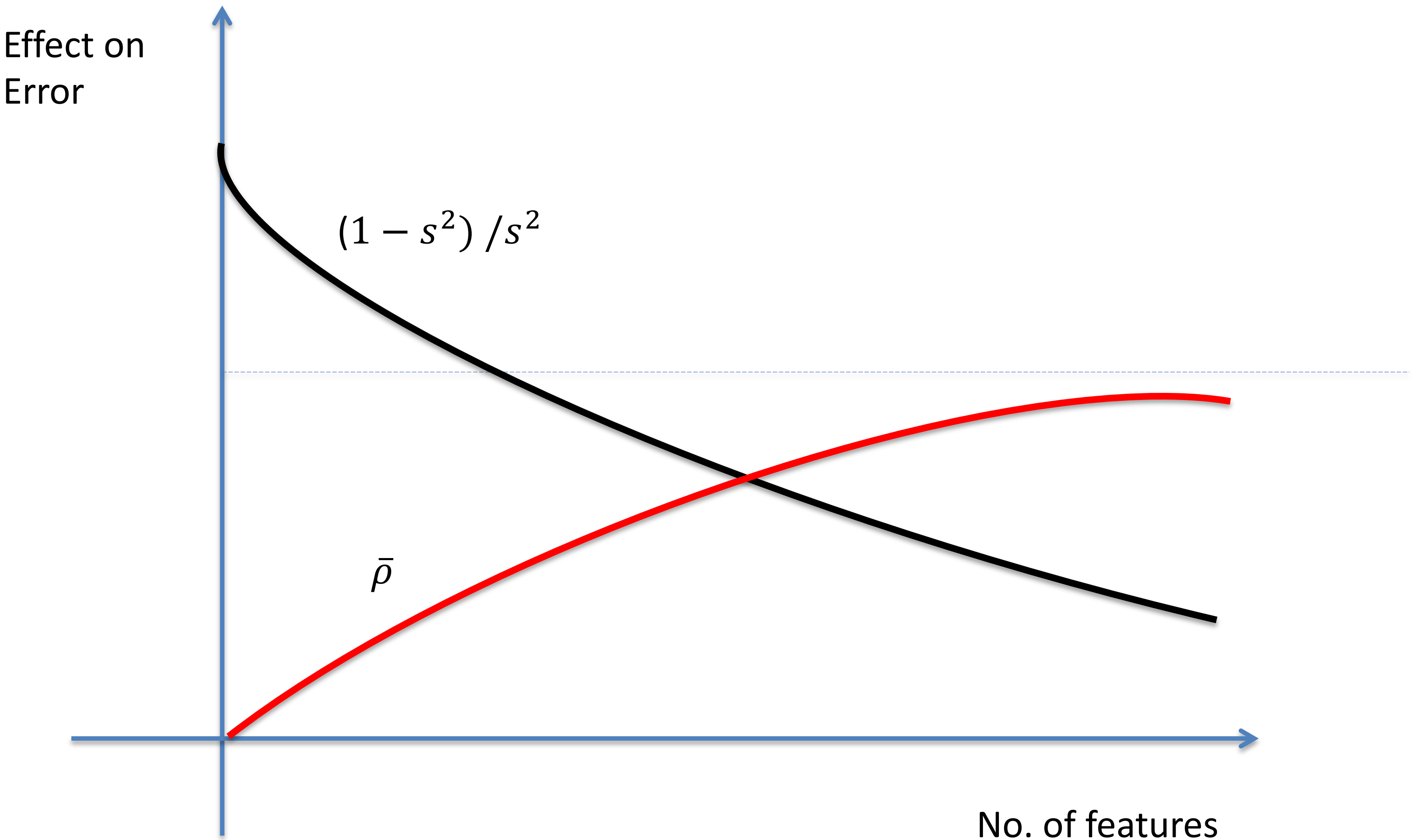


Generalization Error of Random Forests

$$Error \leq \bar{\rho}\ (1 - s^2)\ / s^2$$

- $\bar{\rho}$: Correlation between classifiers
- s: is a measure of the strength of the classifier (1-error)
- These two terms are connected!

Tradeoffs



Classification and Regression Trees (CART)

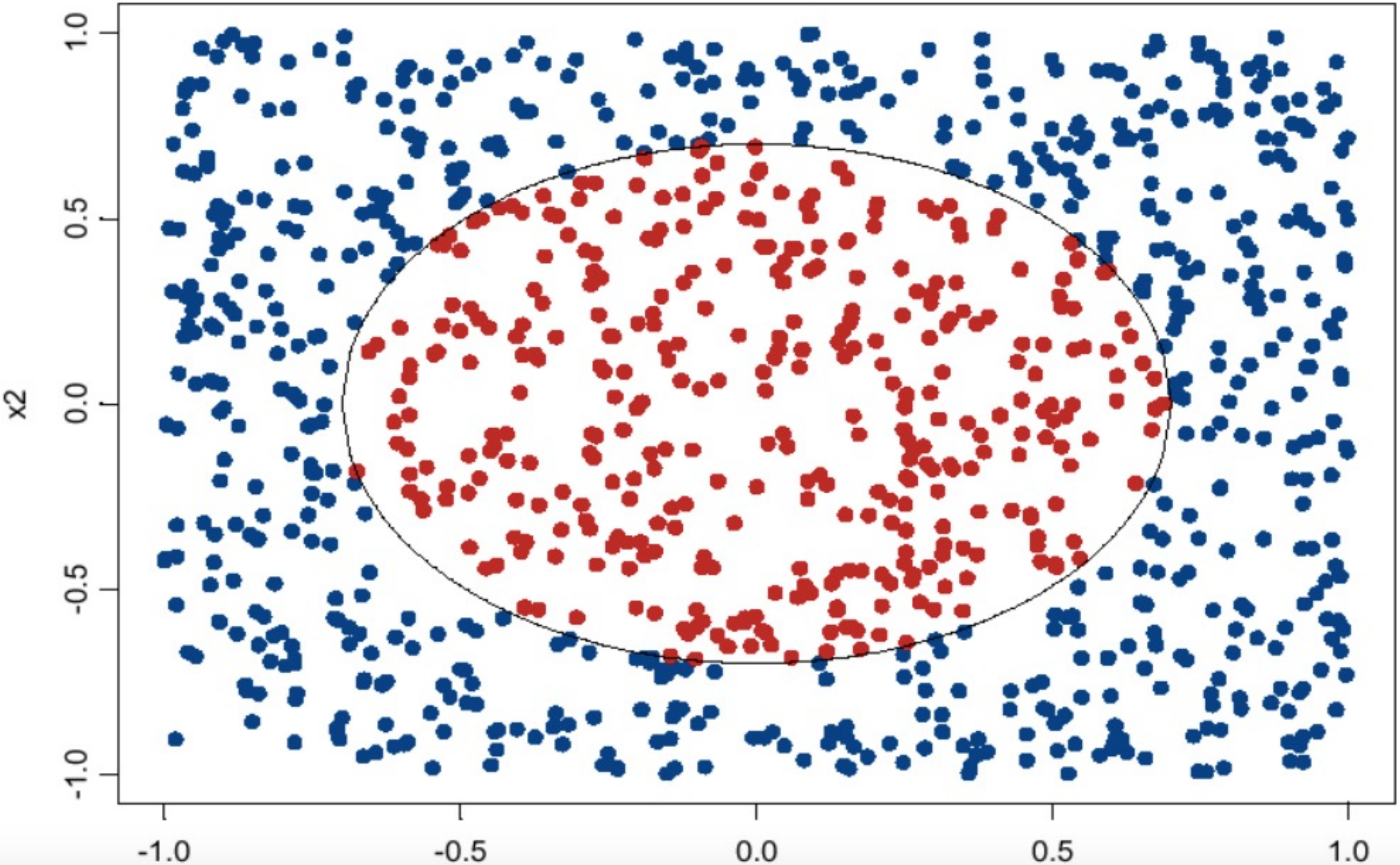
- What if the features are numerical?

- At each level, you construct a linear classifier of the form:

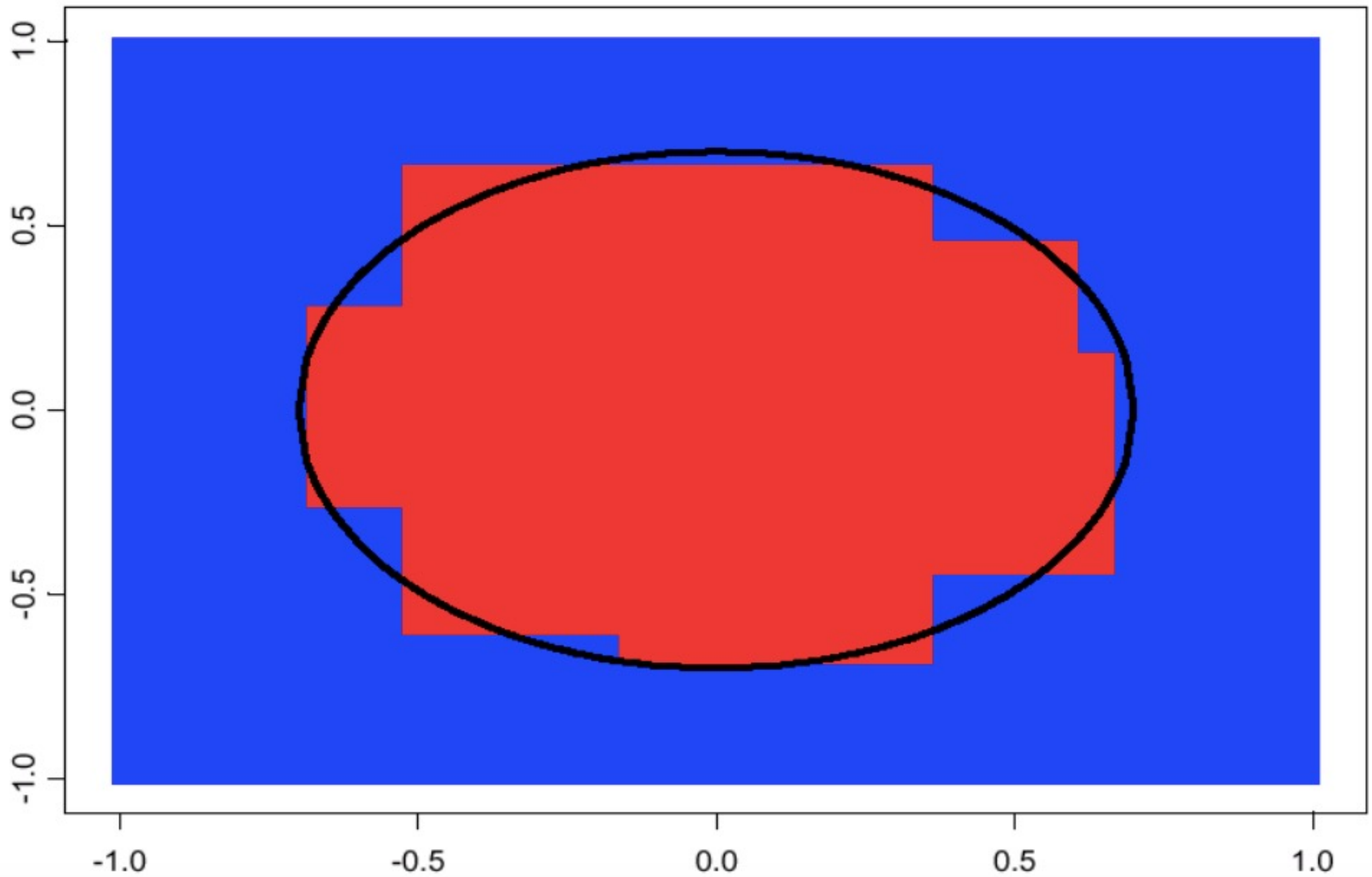
$$a' \, x + b \geq 0$$

- Continue the same way!

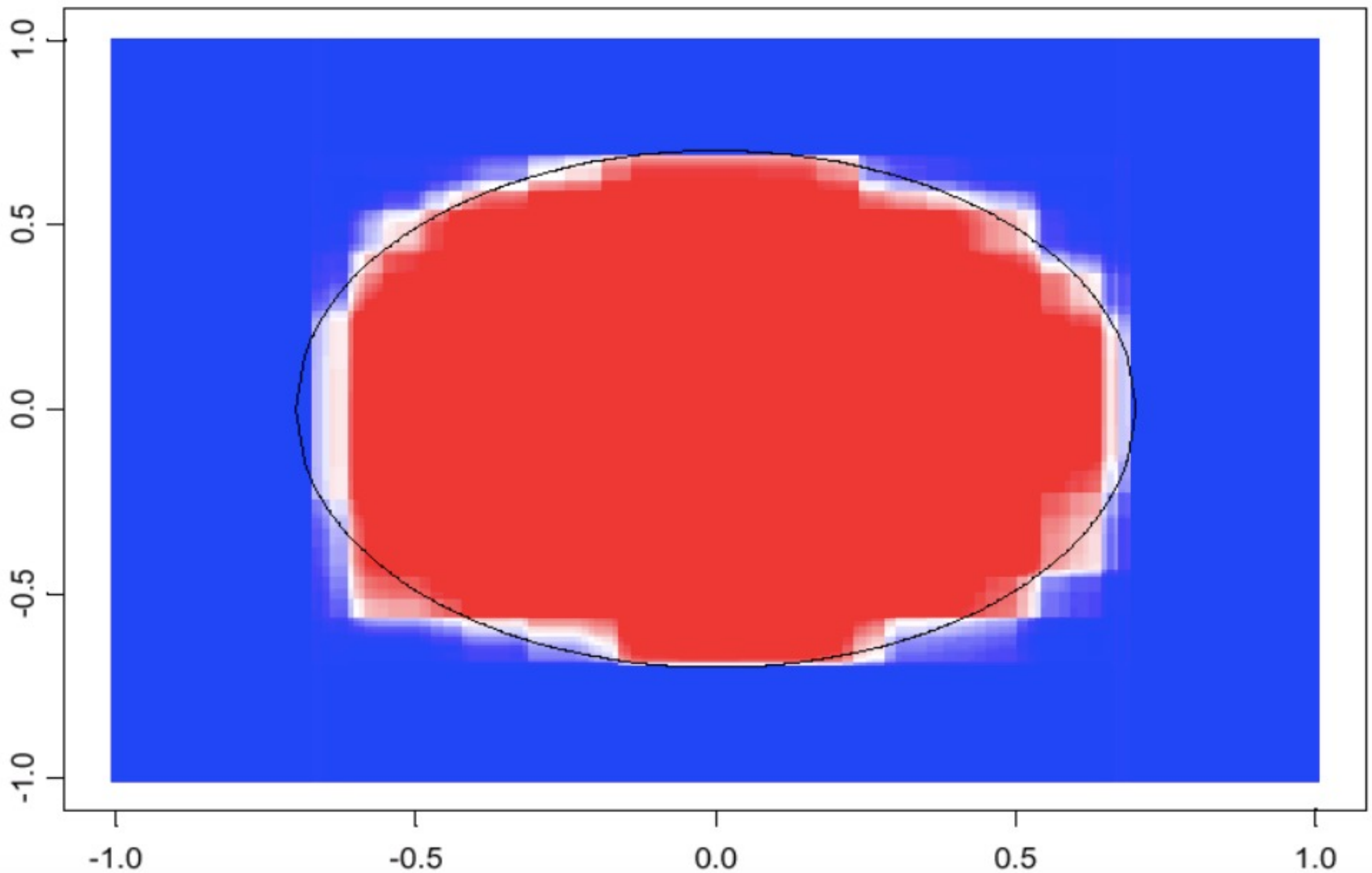
Random Forest



One outcome



Effect of Bagging



Summary

- Overfitting increases variance on test data

- Reduce overfitting by
  - Pruning
  - Bagging
  - Random Forests

- Regression trees

Discussion

Thank You