Handout for False Evidence and Good's Theorem

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1 Accounts of Evidence

The table: You need to buy a red table for your house. There's a table for sale nearby, and it looks red.

Good case: The table is red.

Bad case: The table is white, but undetectable trick lighting makes it look red.

	Good case	**bad case**
Factualism	The table is red	No evidence
Phenomenalism	Phen. state	Phen. state
False evidence	The table is red	The table is red

Table 1: Views of evidence and their verdicts

(There are other views, of course.)

1.1 How to Choose?

Possible constraints on theories of evidence:

Propositionality: evidence can bear logical relations to propositions.

Rationalization: evidence rationalizes belief and action.

Factivity: evidence is incompatible with false propositions.

Certainty: subjects can't be wrong about what the nature of their evidence.

The three views agree on *Propositionality*. Factualism and Phenomenalism agree on *Factivity*. *Certainty* and *Rationalization* are trickier.

2 Good's Theorem

2.1 An example

	Red .5	**White** .5
Buy	10	-2
Don't Buy	-5	0

Table 2: Decision table for whether to buy the table

2.2 Acting Now

$$\begin{aligned} EU(Buy) &= 10 \times .5 - 2 \times .5 = 4 \\ EU(\neg Buy) &= -5 \times .5 = -2.5 \\ EU(Act\,now) &= \max_i EU(A_i) = EU(Buy) = 4 \end{aligned}$$

2.3 Acting Later

$$\begin{aligned} EU(Buy|White) &= -2 \\ EU(Buy|Red) &= 10 \\ EU(\neg Buy|White) &= 0 \\ EU(\neg Buy|Red) &= -5 \end{aligned}$$

$$\begin{aligned} EU(Act\,Later) = \max_i EU(A_i|White) \times P(White) + \\ + \max_i EU(A_i|Red) \times P(Red) = \\ = EU(Buy|Red) \times .5 + EU(\neg Buy|White) \times .5 = \\ = 10 \times .5 = 5 > EU(Act\,Now) \end{aligned}$$

The expected utility of acting now is the maximum of a weighted average; the expected utility of acting after receiving the information is the average of the corresponding maxima.

> **Good's Lemma**: the maximum of an average can never be larger than the average of the corresponding maxima.
> **Good's theorem** is a special case of Good's Lemma.

3 Assumptions of Good's Theorem

1. You know that you are a perfect Bayesian agent (i.e., your credences are probabilistically coherent, and you update by conditionalization on your evidence).

2. Your evidence is partitional.
3. The expected utility of acting now is the expected utility of whatever act maximizes expected utility now.
4. The expected utility of acting later is a weighted average of the acts which maximize expected utility conditional on each member of the relevant partition of acts.

4 Is Good's Theorem Incompatible with False Evidence?

No—see previous section.
However: Good's theorem is incompatible with your assigning now credence > 0 to receiving false evidence. If you do that, then assumption 4 is not a good definition of the expected value of acting later.

	Red .4	Detectably White .4	Undetectably White .2
Buy	10	-2	-2
Don't Buy	-5	0	0

Table 3: Taking into account the possibility of undetectable error

You will only receive as evidence either Red or White, but you leave it open that you receive Red when it is actually White. However, when you do receive either piece of information, you ignore the possibility that you got false evidence (i.e., you ignore the Undetectably White column and decide as if your priors were as in Table 1.). If we assume that the expected utility of acting must take this into account, then (where $Red{*} = Red \vee Undetectably\, white$):

$$\begin{aligned} EU(Buy) &= 2.8 \\ EU(\neg Buy) &= -2 \\ EU(Act\, now) &= 2.8 \end{aligned}$$

$$\begin{aligned} EU(Buy|Red{*}) = 10 \times .4 - 2 * .2 &= 3.6 \\ EU(\neg Buy|Red{*}) &= -2 \\ EU(Buy|Detectably\, white) &= -2 \\ EU(\neg Buy|Detectably\, white) &= 0 \end{aligned}$$

$$EU(Act\, later) = \max_i EU(A_i|Red{*}) \times \mathbf{P}(\mathbf{Red{*}}) +$$

$$+ \max_i EU(A_i|Detectably\, white) \times \mathbf{P}(\mathbf{Detectably\ white}) =$$

$$= 3.6 \times .6 = 2.16 < EU(Actnow)$$

5 Good's Theorem and Other Theories of Evidence

If we let go of *Certainty*, then everyone must countenance the possibility that subjects will treat a false proposition as evidence. Because if you assign positive credence to being wrong about your evidence in the future (for instance, by thinking that it's true, or by taking a proposition which is not part of your evidence to be part of your evidence) but also think that in the future you will assign credence 0 to that possibility, then it will sometimes be bad to wait before acting.

1. Either we incorporate that possibility into our models; or
2. We don't.

Illustration with Factualism:

> If they take option 1, they have to model the decision as with Table 3, taking into account the possibility that they will pseudo-conditionalize, and thus violate an assumption of Good's theorem.
>
> If they take option 2, they must take it that it is rational for agents to assign credence 0 to a possibility they know might obtain.

The same two options are available to the friend of false evidence. The only difference is that instead of thinking that sometimes we will pseudo-conditionalize, they think we will sometimes conditionalize on a false proposition. Who is right about this cannot be decided by appeal to Good's theorem.

In favor of option 2: the possibilities in question are *synchronic global skeptical scenarios*. Maybe it's ok to give credence 0 to global skeptical scenarios, both synchronic and diachronic?