Logic, Reasoning, and Persuasion, Week 9-2 Handout

1 | Calculating Probabilities with Truth Tables

Suppose we have two propositions, A and B, and the following probabilities for the possibilities:

A	B		$Prob$
T	T	$Pr(A\&B)$	0.2
T	F	$Pr(A\&\neg B)$	0.3
F	T	$Pr(\neg A\&B)$	0.4
F	F	$Pr(\neg A\&\neg B)$	0.1

To get the probabilities of these four possibilities, we can just read off that single row.

2 | Calculating Conditional Probabilities

Calculating Conditional Probability

Suppose you have the probability truth table for A and B and you want to know $Pr(B \mid A)$.

1. First, **suppose** that A is true: cross out the rows in which A is false, so only the ones in which A are *true* remain. Add all the remaining rows: this is $Pr(A)$.
2. Second, add all the remaining rows where B is true. This is $Pr(B\&A)$ (since in these rows, both A and B are true).
3. Finally, divide the number from part (2) by the number in part (1) to calculate $Pr(B \mid A)$:

$$Pr(B \mid A) = \frac{Pr(B\&A)}{Pr(A)}.$$

Example:

Step 1: $Pr(A)$

A	B	$Prob$
$\rightarrow$ **T**	T	0.2
$\rightarrow$ **T**	F	0.3
$\cancel{F}$	$\cancel{T}$	$\cancel{0.4}$
$\cancel{F}$	$\cancel{F}$	$\cancel{0.1}$

$Pr(A) = 0.2 + 0.3 = 0.5$

Step 2: $Pr(B\&A)$

A	B	$Prob$
$\rightarrow$ **T**	**T**	0.2
T	F	0.3
$\cancel{F}$	$\cancel{T}$	$\cancel{0.4}$
$\cancel{F}$	$\cancel{F}$	$\cancel{0.1}$

$Pr(B\&A) = 0.2$

Step 3: $Pr(B \mid A)$

$$Pr(B \mid A) = \frac{Pr(B\&A)}{Pr(A)}$$

$$Pr(B \mid A) = \frac{0.2}{0.5} = 0.4.$$

How would we calculate $Pr(B \mid \neg A)$, the probability of B, supposing that A is false?

1. First, suppose that A is false: cross out the rows in which A is *true*, so only the rows in which A is false remain.
2. Second, add all the remaining rows where B is true. This is $Pr(B\&\neg A)$ (since in these rows, $\neg A$ and B is true).
3. Finally, divide the number from part (2) by the number in part (1) to calculate $Pr(B \mid \neg A)$:

$$Pr(B \mid \neg A) = \frac{Pr(B\&\neg A)}{Pr(\neg A)}.$$

Step 1: $Pr(\neg A)$

A	B	$Prob$
$\cancel{T}$	$\cancel{T}$	$\cancel{0.2}$
$\cancel{T}$	$\cancel{F}$	$\cancel{0.3}$
$\rightarrow$ **F**	T	0.4
$\rightarrow$ **F**	F	0.1

$Pr(\neg A) = 0.4 + 0.1 = 0.5$

Step 2: $Pr(B\&\neg A)$

A	B	$Prob$
$\cancel{T}$	$\cancel{T}$	$\cancel{0.2}$
$\cancel{T}$	$\cancel{F}$	$\cancel{0.3}$
$\rightarrow$ **F**	T	0.4
F	F	0.1

$Pr(B\&\neg A) = 0.4$

Step 3: $Pr(B \mid A)$

$$Pr(B \mid \neg A) = \frac{Pr(B\&\neg A)}{Pr(\neg A)}$$

$$Pr(B \mid \neg A) = \frac{0.4}{0.5} = 0.8.$$

3 | CONDITIONALIZING

The standard theory of updating opinions says if you learn E, then your new opinion should be your old opinion *given that* E. This is called "Conditionalization."

Conditionalization

Suppose that you have an estimate of how likely H is, given E. Then if you get E as evidence, your new opinion should be given by

$$P_{new}(H) = P_{old}(H \mid E).$$

For example, if $P_{old}(\text{you are sick} \mid \text{you test positive}) = 0.18$, then if you learn that you tested positive, you should now have the opinion $P_{new}(\text{you are sick}) = 0.18$.

1. You think that the probability that you'll get sick, supposing that your roommate is sick, is 3/4. Yesterday, your roommate got sick. How likely do you think it is that you get sick?
2. You think that the probability that the bus comes on time, supposing that it's sunny, is 3/4, and the probability that the bus comes on time, supposing that it's raining, is 1/4. You look out the window and see that it is raining. How likely do you think it is that the bus comes on time?

4 | DEFINITELY NOT CLUE

Your dear teacher Adrian has been murdered. Your job is to figure out (1) who the murderer is, (2) where the murder took place, and (3) what the weapon was.

4.1 | Location

The murder either happened in the Kitchen or the Living Room. Let

1. K = "The murder happened in the Kitchen."
2. L = "The murder happened in the Living Room."

You also know that the murder either happened in the day or in the night. Let D = "The murder happened in the day," so that $\neg D$ is "The murder did *not* happen in the day" (it happened in the night). Finally, you know these probabilities:

D	K	*Prob*
T	T	0.4
T	F	0.2
F	T	0.1
F	F	0.3

D	L	*Prob*
T	T	0.2
T	F	0.4
F	T	0.3
F	F	0.1

1. Before you learn whether the murder happened in the day or at night, should you think its more likely the murder happened in the kitchen or in the living room? That is, is $Pr(K)$ or $Pr(L)$ higher?
2. Supposing that the murder happened in the day, which room do you think the murder more likely happened in? That is, is $Pr(K \mid D)$ or $Pr(L \mid D)$ higher?
3. Supposing that the murder happened in the night, which room do you think it more likely happened in? That is, is $Pr(K \mid \neg D)$ or $Pr(L \mid \neg D)$ higher?

You learn that the murder happened during the night. You conditionalize, so that

$$Pr_1(K) = Pr_0(K \mid \neg D) \quad \text{and} \quad Pr_1(L) = Pr_0(L \mid \neg D).$$

And you decide that the murder most likely happened in the ______________.

4.2 | Murder Weapon

You know the murder was either committed by poisoning or strangulation. Let P = "The murder was a poisoning", so that $\neg P$ is "The murder was not a poisoning," which we know is equivalent to "The murder was a strangulation.". You also know:

P	K	*Prob*
T	T	0.3
T	F	0.1
F	T	0.1
F	F	0.5

P	L	*Prob*
T	T	0.1
T	F	0.3
F	T	0.5
F	F	0.1

1. Supposing that the murder happened in the kitchen, was it more likely by strangulation or poisoning? That is, is $Pr(P \mid K)$ or $Pr(\neg P \mid K)$ more likely?
2. Supposing that the murder happened in the living room, was it more likely by strangulation or poisoning? That is, is $Pr(P \mid L)$ or $Pr(\neg P \mid L)$ more likely?

Because you learned that the murder happened during the night, you think it more likely that the murder happened in the ___________. You learn that you were correct! The murder indeed happened there – the forensics leave no doubt. You conditionalize on the fact that the murder happened in the ___________, so that

$$Pr_2(P) = Pr_1(P \mid \_\_)$$
$$Pr_2(\neg P) = Pr_1(\neg P \mid \_\_).$$

And you decide that the murder most likely occurred via ____________.

4.3 | Prime Suspect

Your suspects are Ann, Bob, and Cal. You know that the oil lamp in the room where the murder happened was either on or off when the murder occurred. Let O = "The lamp was On". You also know these probabilities:

O	A	*Prob*
T	T	0.1
T	F	0.5
F	T	0.3
F	F	0.1

O	B	*Prob*
T	T	0.4
T	F	0.2
F	T	0.1
F	F	0.3

O	C	*Prob*
T	T	0.1
T	F	0.5
F	T	0.0
F	F	0.4

1. Who is most likely to be the murderer, (supposing you don't know whether the lamp was on or off)?
2. Supposing the lamp was on, who is most likely to be the murderer?
3. Supposing the lamp was off, who is most likely to be the murderer?

You learn that **the lamp in the kitchen was on, and the lamp in the living room was off.** Because you know where the murder occurred, you know whether the light was on or off where the murder occurred. You conditionalize, so that

$$Pr_3(A) = Pr_2(A \mid \_\_)$$
$$Pr_3(B) = Pr_2(B \mid \_\_)$$
$$Pr_3(C) = Pr_2(C \mid \_\_)$$

And you conclude that ________ is most likely to be the murderer, and they most likely used ________ to murder their victim in the ________. □

Bonus: The suspect fled, either toward New York or Philadelphia, at 6am or 11am, in a car or in a train. You know the probabilities given in the table.

You learn that *either* they took a car at 6am *or* they took a train at 11am. Which city are they more likely to have fled to?

Mode	*Time*	*Dest.*	*Prob*
Car	*6am*	*NYC*	0.10
Car	*6am*	*Philly*	0.05
Car	*11am*	*NYC*	0.15
Car	*11am*	*Philly*	0.10
Train	*6am*	*NYC*	0.30
Train	*6am*	*Philly*	0.00
Train	*11am*	*NYC*	0.05
Train	*11am*	*Philly*	0.25