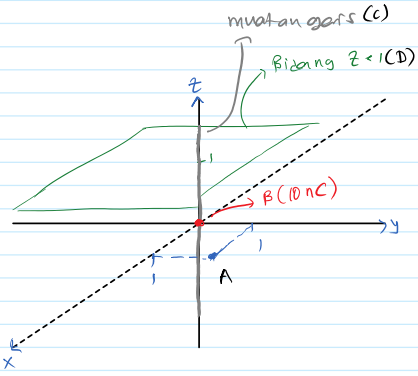
PR

17 January 2022

1.

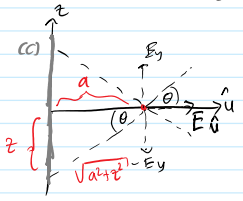


misal titik A : (1,1,0), $k = \frac{1}{4\pi\varepsilon_0}$

(1) oleh muatan titik, $r_A - r_B = (1,1,0)$

$$E_{BA} = k \cdot \frac{1\times10^{-8}}{2^{3/2}}\hat{i} + k \cdot \frac{1\times10^{-8}}{2^{3/2}}\hat{j}$$

(2) oleh muatan garis (c), misal $\hat{u} = \frac{1}{2}\sqrt{2}\hat{i} + \frac{1}{2}\sqrt{2}\hat{j}$



$dQ = \rho_\ell\, dz$

$d\theta = 5\times10^{-9}\, dz$

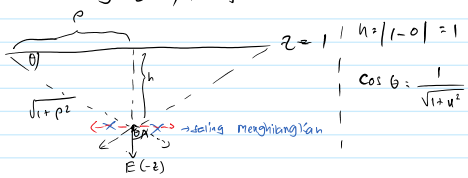
$$dE_{\hat{u}} = \frac{k\,dQ}{r^2}\cos\theta = \frac{k a\, dQ}{(a^2+z^2)^{3/2}}, \quad a = \sqrt{2}$$

$$dE_{\hat{u}} = \frac{\sqrt{2}k\,dQ}{(z^2+2)^{3/2}}$$

$$E_{\hat{u}} = 5\sqrt{2}k\times10^{-9}\int_{-\infty}^{\infty}\frac{dz}{(z^2+2)^{3/2}} = 5k\times10^{-9}\hat{u}$$

$$= \left(\frac{5}{2}\sqrt{2}k\hat{i} + \frac{5}{2}\sqrt{2}k\hat{j}\right)\times10^{-9}$$

(3) Bidang $z = 1$, tinjau bidang $x = y$



$\cos\theta = \frac{1}{\sqrt{1+u^2}}$

$dQ = \rho_s\, dS = 2\times10^{-9}\, dS$

$$dE_{-z} = k\cdot\frac{dQ}{(1+\rho^2)}\cdot\cos\theta = \frac{2\times10^{-9}k}{(1+\rho^2)^{3/2}}dS, \quad dS = \rho\, d\rho\, d\phi$$

$$dE_{-z} = 2\pi\times10^{-9}k\int_0^{2\pi}\int_0^{\infty}\frac{\rho}{(1+\rho^2)^{3/2}}\,d\rho\,d\phi$$

misal $u = (1+\rho^2)$, $du = 2\rho\, d\rho$

$$= \frac{1}{2}\int_0^{2\pi}\int_1^{\infty} u^{-3/2}\,du\,d\phi = \frac{1}{2}\int_0^{2\pi}\left[-2\frac{1}{\sqrt{u}}\right]_1^{\infty}d\phi \quad \rightarrow = [0-(-2)] = 2$$

$$= \int_0^{2\pi} d\phi = 2\pi$$

$$dE_z = -dE_{(-z)} = -4\pi\times10^{-9}k\,\hat{k}$$

maka, medan di titik A adalah

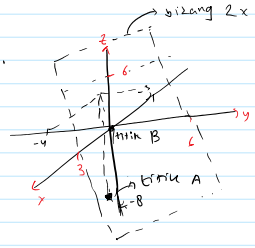
$$E = E_{(3)} + E_{(1)} + E_{(3)}$$

$$= k\left[\left(\frac{1\times10^{-8}}{2^{3/2}} + \frac{5}{2}\sqrt{2}\times10^{-9}\right)\hat{i} + \left(\frac{1\times10^{-8}}{2^{3/2}} + \frac{5}{2}\sqrt{2}\times10^{-9}\right)\hat{j} - 4\pi\times10^{-9}\hat{k}\right]$$

$$= 10^{-9}k\,(7{,}071\,\hat{i} + 7{,}071\,\hat{j} - 4\pi\hat{k}), \quad k \approx 9\times10^{9}\ Nm^2/c$$

$$= 63{,}639\hat{i} + 63{,}639\hat{j} - 36\pi\hat{k}\ N/c$$

2.



Karena kerapatan muatan di bidang merata, maka dapat digunakan rumus hasil penurunan untuk medan listrik pada bidang

•) Akibat Bidang

$E = 2\pi k\rho_s \hat{a}_n$, normal bidang adalah $n = \langle 2,1,1\rangle$

maka, $\hat{a}_n = \left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle$

perhatikan bahwa karena koordinat z titik yg ditinjau < z bidang, digunakan $\hat{a}_n$ kebalikannya yaitu $\hat{a}_n = \left\langle -\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right\rangle$

Maka $E = 2\pi k\times10^{-9}\left(-\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}\right)$

•) Akibat muatan titik 100 nC di (-3,-4,-8), misal titik A dan titik yang ditinjau adalah titik B

akan dicari terlebih dahulu $r_{AB} = (0,0,0) - (-3,-4,-8) = \langle 3,4,8\rangle$

$$|r_{AB}| = \sqrt{9+16+64} = \sqrt{89}$$

maka, diperoleh $E_{AB} = k\frac{Q\, r_{AB}}{|r_{AB}|^3} = 100\times10^{-9}k\left(\frac{3}{89^{3/2}}\hat{i} + \frac{4}{89^{3/2}}\hat{j} + \frac{8}{89^{3/2}}\hat{k}\right)$

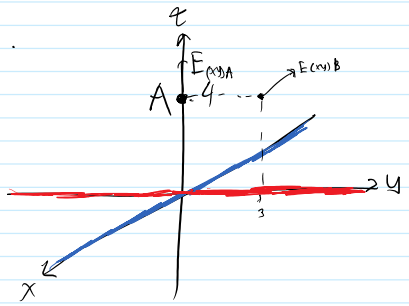
Maka, $E = E_{titik} + E_{permukaan}$

$$= 10^{-9}k\left[\left(-\frac{4\pi}{\sqrt{6}} + \frac{3}{89^{3/2}}\right)\hat{i} + \left(-\frac{2\pi}{\sqrt{6}} + \frac{4}{89^{3/2}}\right)\hat{j} + \left(-\frac{2\pi}{\sqrt{6}} + \frac{8}{89^{3/2}}\right)\hat{k}\right], \quad k \approx 9\times10^{9}\ Nm^2/c$$

$$= 10^{-9} k \left[\left(-\frac{4\pi}{\sqrt{2}} + \frac{3}{8g^{3/2}} \right) \hat{i} + \left(-\frac{2\pi}{\sqrt{6}} + \frac{4}{8g^{3/2}} \right) \hat{j} + \left(-\frac{2\pi}{\sqrt{6}} + \frac{8}{8g^{3/2}} \right) \hat{k} \right], k \approx 9 \times 10^{-9} \ Nm^2/c$$

$$= -46{,}139\,\hat{i} - 23{,}043\,\hat{j} - 23\,\hat{k} \ N/c$$ //

3.



karena muatan garis tersebar secara merata, maka dapat digunakan rumus hasil penurunan untuk muatan garis.

•) untuk titik A

Akibat muatan garis di sb-x $\vec{E} = 2k \frac{\rho_L}{\rho} \hat{a}_z = 2.9 \times 10^{-9} \cdot \frac{5 \times 10^{-9}}{4} = \frac{90}{4} \hat{k}$

di sb-y $\vec{E} = 2k \frac{\rho_L}{\rho} \hat{a}_z = \frac{90}{4} \hat{k}$

maka, $E_{(xy)A} = 2 \cdot \frac{90}{4} = 45\,\hat{k} \ N/c$ //

•) untuk titik B

Akibat muatan garis di sb. y dapat dilihat vektor posisi B adalah $\vec{OB} = 3\hat{j} + 4\hat{k}$, $|\vec{OB}| = 5 = \rho$

maka, $\hat{a}_\rho = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

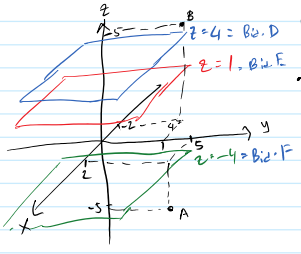
$$E = 2k \frac{\rho_L}{\rho} \hat{a}_\rho = 2.9 \times 10^{-9} \cdot \frac{5 \times 10^{-9}}{5} \cdot \left(\frac{3}{5}\hat{j} + \frac{4}{5}\hat{k} \right) = 10{,}8\,\hat{j} + 14{,}4\,\hat{k} \ N/c$$

Akibat muatan garis di sb-y (arah resultan medan listrinnya ada di $\hat{a}_z$)

$$E = 2k \frac{\rho_L}{\rho} \hat{a}_z = 2.9 \times 10^{-9} \cdot \frac{5 \times 10^{-9}}{4} \hat{k} = 22{,}5\,\hat{k} \ N/c$$

maka, $\vec{E}_{(xy)B} = 10{,}8\,\hat{j} + 36{,}9\,\hat{k} \ N/c$ //

4.



Muatan tersebar merata pada permukaan, maka dapat digunakan rumus hasil penurunan

•) Pada A (2,5,-5)

$E_{DA} = -2k\pi\rho_{SD}\hat{k}$ (karena $z_A < z_D$)
$= -2k\pi(-8) = 16k\pi \times 10^{-9}\,\hat{k}$

$E_{EA} = -2k\pi\rho_{SE}\hat{k}$ (karena $z_A < z_E$)
$= -2k\pi(6) = -12k\pi \times 10^{-9}\hat{k}$

$E_{FA} = -2k\pi\rho_{SF}\hat{k}$ $(z_A < z_F)$
$= -2k\pi(3) = -6k\pi \times 10^{-9}\hat{k}$

$\vec{E}_A = k\pi \times 10^{-9}(16 - 12 - 6)\,\hat{k}$
$= 9\pi(-2)\,\hat{k} = -18\pi\hat{k} \ N/c$ //

•) Pada B (-2,4,5)

$E_{DB} = 2k\pi\rho_{SD}\hat{k}$ (karena $z_B > z_D$)
$= 2k\pi(-8) = -16k\pi \times 10^{-9}\,\hat{k}$

$\vec{E}_B = k\pi(-16 + 12 + 6)$
$= 18\pi\hat{k} \ N/c$

$E_{DB} = 2k\pi\rho_{SD}\hat{k}$ (karena $z_B > z_D$)

$= 2k\pi(-8) = -16k\pi \times 10^{-9}\hat{k}$

$E_{EB} = 2k\pi\rho_{SE}\hat{k}$ (karena $z_B > z_E$)

$= 2k\pi(6) = 12k\pi \times 10^{-9}\hat{k}$

$E_{FB} = 2k\pi\rho_{SF}\hat{k}$ ($z_B > z_F$)

$= 2k\pi(3) = 6k\pi \times 10^{-9}\hat{k}$

$E_B = k\pi(-16+12+6)$

$= 18\pi\hat{k}$ N/C //

5. titik P(1,4,-2)

$\underline{E} = 2e^{5x}\left[y(5x+1)\hat{a}_x + x\hat{a}_y\right]$

Persamaan garis medan adalah

$\frac{E_y}{E_x} = \frac{dy}{dx}$, E_x dan E_y adalah komponen x dan y dari $\underline{E}$

$\frac{dy}{dx} = \frac{x}{y(5x+1)} \Leftrightarrow y\,dy = \frac{x\,dx}{5x+1}$

integralkan kedua sisi,

$\int y\,dy = \frac{1}{2}y^2 + C_1$

$\int \frac{x\,dx}{5x+1}$, misal $u = 5x+1 \rightarrow du = 5dx$, $x = \frac{1}{5}(u-1)$

$= \frac{1}{5}\int \frac{\frac{1}{5}(u-1)}{u}du = \frac{1}{25}\int 1 - \frac{1}{u}\,du$

$= \frac{1}{25}u - \frac{1}{25}\ln(u) + C_2$

$= \frac{1}{25}(5x+1) - \frac{1}{25}\ln(5x+1) + C_2$

Diperoleh

$\frac{1}{2}y^2 = \frac{1}{25}(5x+1) - \frac{1}{25}\ln(5x+1) + C$

$y^2 = \frac{2}{25}(5x+1) - \frac{2}{25}\ln(5x+1) + C$

Substitusikan P(1,4,-2), diperoleh

$16 = \frac{12}{25} - \frac{2}{25}\ln 6 + C \rightarrow C = \frac{2}{25}\ln 6 + \frac{388}{25}$

Maka, persamaan garis medannya adalah

$y^2 = \frac{2}{25}(5x+1) - \frac{2}{25}\ln(5x+1) + \frac{2}{25}\ln 6 + \frac{388}{25}$

$y^2 = 0{,}4x - 0{,}08\ln(5x+1) + \frac{2}{25}(1 + \ln 6 + 194)$

$y^2 = 0{,}4x - 0{,}08\ln(5x+1) + 15{,}7433$ //