

HW 1

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Question 1

A person's muscle mass is expected to decrease with age. To explore this relationship in women, a nutritionist randomly selected 4 women from each 10-year age group, beginning with age 40 and ending with age 79, resulting in a total sample size of $n=16$. Some results follow, where X is the age, and Y is a measure of the muscle mass. Assume a simple linear regression model is appropriate.

For these data:

$$\text{Mean}(X) = 61.69 \quad \text{SD}(X) = 14.67$$

$$\text{Mean}(Y) = 52.21 \quad \text{SD}(Y) = 24.16$$

$$\hat{y} = 142.68 - 1.47 * X$$

$$\text{SE}(\text{Slope}) = 0.200 \quad \text{Root MSE} = 11.38$$

a.

Parameters: 1. $\beta_0 = 142.68$ = The predicted muscle mass in a women when her X value, age is equal to 0
1. $\beta_1 = -1.47$ = The change in muscle mass for every one unit change in age

b.

$$\hat{Y}_{63} = 142.68 - 1.47 * 63 = 50.07$$

c.

In order to calculate the correlation from the slope first multiply the slope of the regression line by the standard deviation of X and then divide by the standard deviation of Y .

$$\text{Slope: } -1.47$$

$$\text{SD}_x: 14.67$$

$$\text{SD}_y: 24.16$$

$$r = (\text{Slope} * \text{SD}_x) / \text{SD}_y = -0.8925869$$

d.

State our hypotheses:

$$H_0 : r = 0$$

$$H_a : r \neq 0$$

$$\alpha = 0.05$$

We would like to calculate the t value of the correlation. The t value's formula is: $t = \frac{r\sqrt{(n-2)}}{\sqrt{1-r^2}}$

$$t = -7.4072668$$

$$p = 3.3156464 \times 10^{-6}$$

Given this p value, we reject the null hypothesis and conclude there is a non-zero relationship between age and muscle mass.

e.

The 95% C.I. for $\beta_1 = -1.47 \pm 0.429$; [-1.899,-1.041]

I will interpret this later

f.

The 95% C.I. for a participant with an age of 63 is equal to [23.043,77.097].

I will interpret this later

g.

The residual is equal to $Y - \hat{Y} = 45 - 54.48 = -9.48$

h.

$$PRE = 0.7967114$$

$$\beta_{standardized} = \beta \frac{SD_x}{SD_y} = -0.8925869$$

Here we see that we explain roughly 0.7967 percent of the total variance in the outcome. Furthermore, it appears that the standardized coefficient between age and muscle mass is: -0.8925869 which suggests a very strong relationship between these two variables.

i.

If two women differ in age by 10 years the predicted difference in muscle mass will be -14.7 units.

j.

It is difficult to tell if the linear assumption holds with these data. However the provided model fits the data very well.

Question 2

The following SAS output contains an analysis in which 6-year graduation rates were collected over a 6-year period and analyzed for trends. The response variable Y is graduation rate (measured as a percentage). The explanatory variable X is year (beginning with year 1)

a.

$$\hat{Y} = 61.52667 + 0.95429 * year$$

b.

$$\hat{Y}_6 = 61.52667 + 0.95429 * 6 = 67.25241$$

c.

The residual is equal to $Y - \hat{Y} = 67.8 - 66.29812 = 1.50188$

Here the residual is 1.5, meaning the value was underpredicted by 1.5%.

d.

PRE = 0.7541 This PRE suggests an excellent model fit.

e.

The 95% C.I. for $\beta_1 = 0.95429 \pm 0.7565$; [0.19779, 1.71079]

I will interpret this later

f.

State our hypotheses:

$$H_0 : \beta_{year} = 0$$

$$H_a : \beta_{year} \neq 0$$

$$\alpha = 0.05$$

We reject the null hypothesis that $\beta_{year} = 0$ given the strength of the coefficient $\beta_{year} = 0.95429$ ($t(4) = 3.50, p = 0.0248$).