Delaunay Triangulation in Parallel

Adarsh Prakash

CSE 633 : Parallel Algorithms

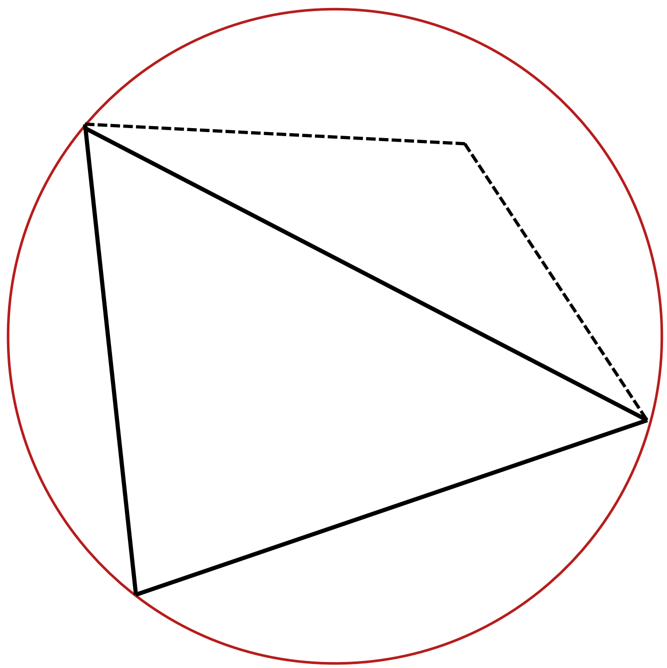
(Spring 2017)

Instructor: Dr. Russ Miller

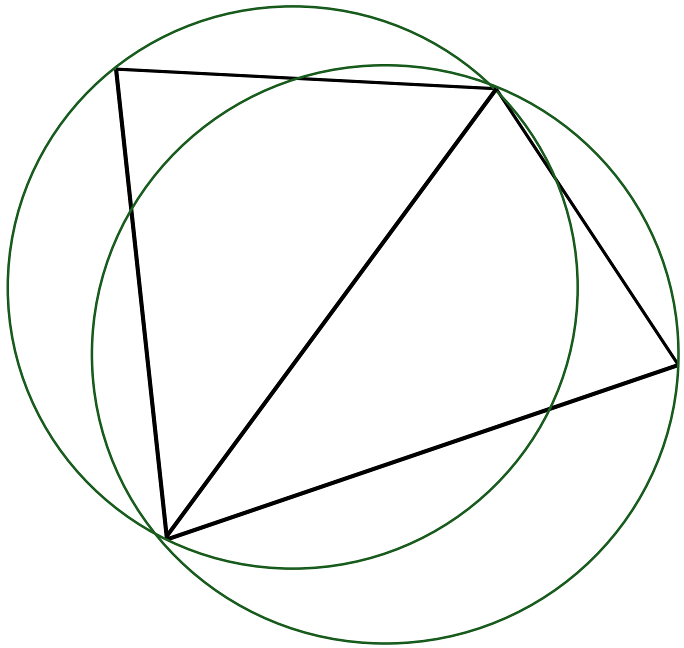
University at Buffalo

Definition

Triangle ΔABC is a **Delaunay Triangle**, if no other points lie in the circumcircle formed by ΔABC.



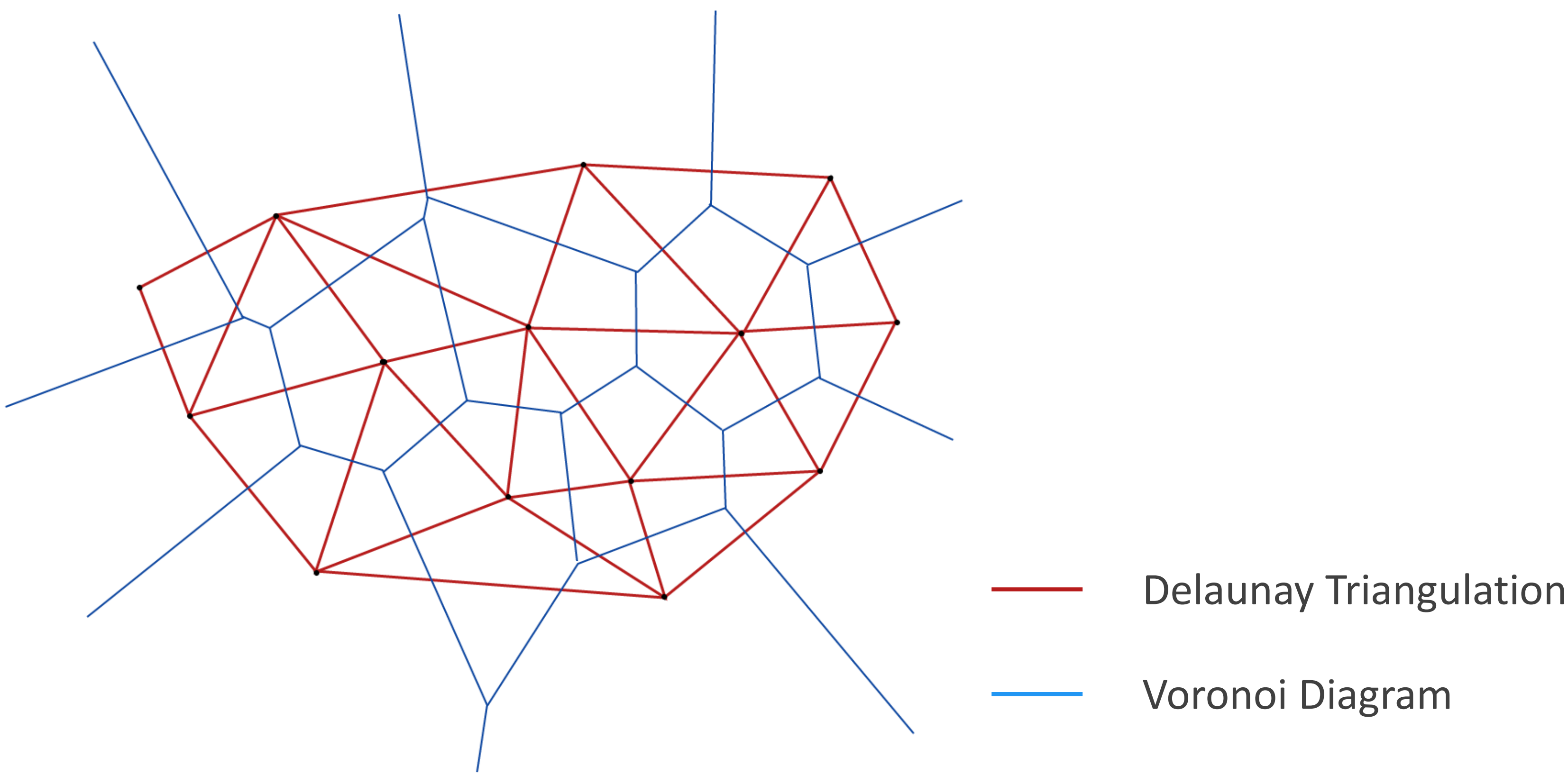
Invalid Delaunay Δ



Valid Delaunay Δs

Delaunay – Voronoi: Duality

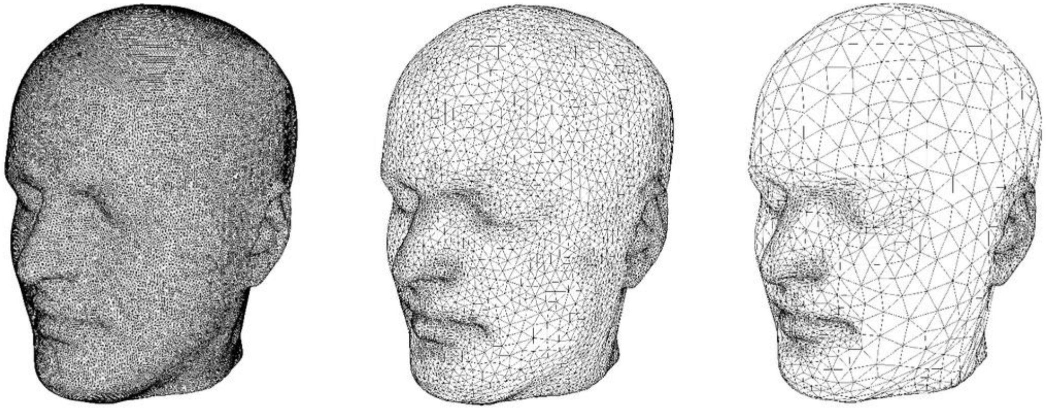
A **Voronoi diagram** is constructed by connecting centers of all the circumcircles formed by the Delaunay Triangles in a graph.

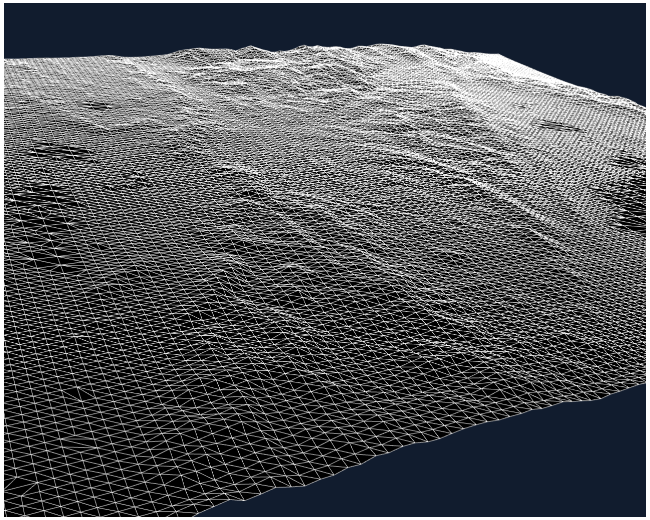


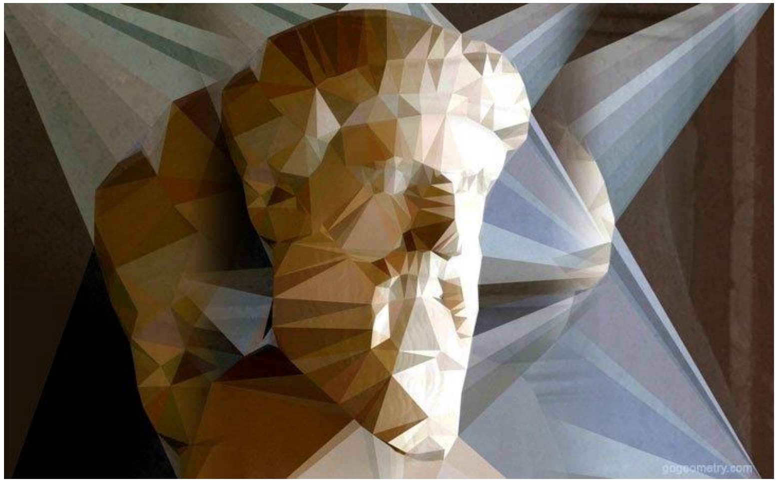
Direct Applications

- Nearest Neighbor
- Graph Locality / Point Location
- Surface Mapping / Reconstruction
- Game Development
- Motion Capture
- Path Planning (Autonomous Navigation)
- Physics – studying forces..
- Chemistry – atomic charges..
- Biology, Astrophysics and so on.

Applications



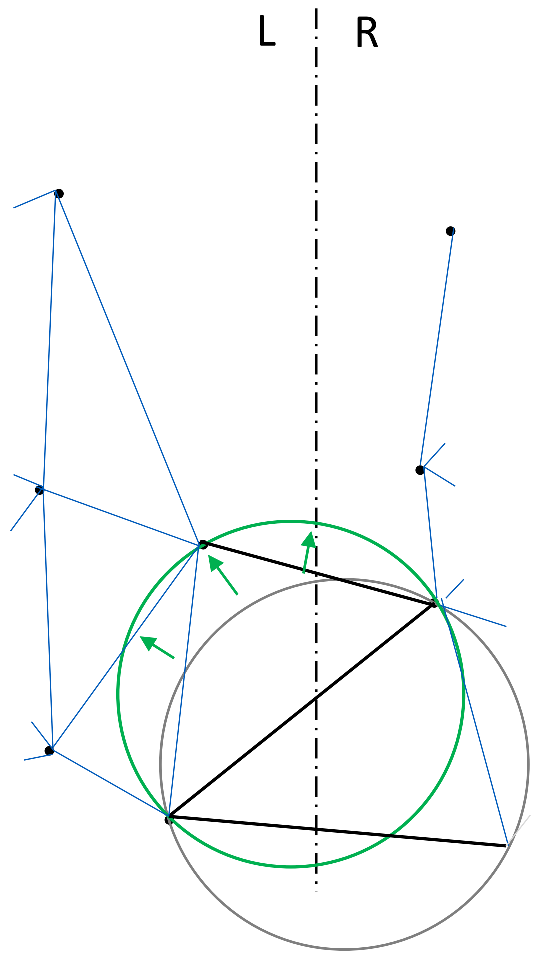




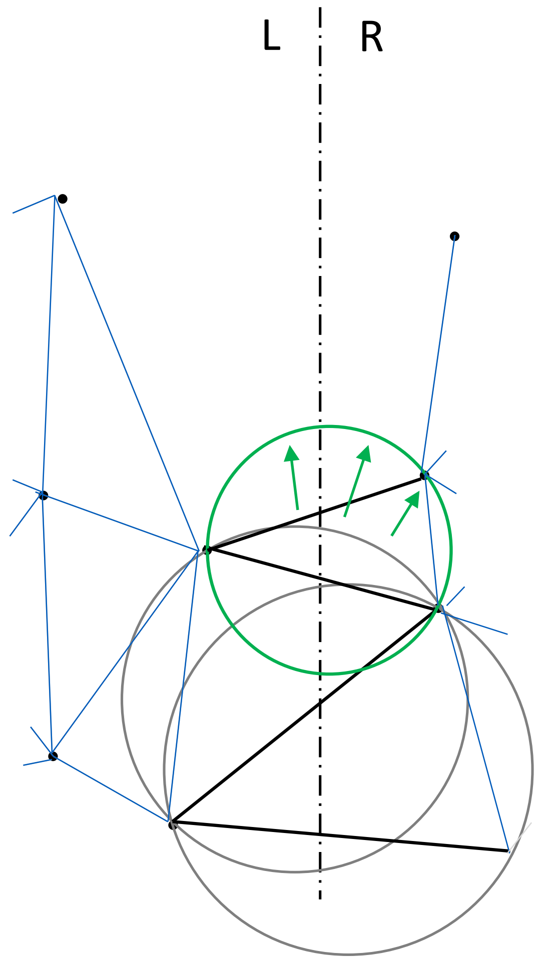
Algorithm

- **Divide-and-conquer** algorithm proposed by **Leonidas Guibas** and **Jorge Stolfi** [1].
- Follows closely the Voronoi construction algorithm from **Shamos** and **Hoey** [2].
- Difference is it clearly describes how to make use of quad-edge data structure to avoid computation of complete hull.
- Properties:
  - A quad-edge knows its direction (origin-destination NOT point-point)
  - A quad-edge maintains pointers to all edges leaving from and terminating at their origin and destination. (4-8 pointers depending on implementation)
- Objective is to parallelize this algorithm.

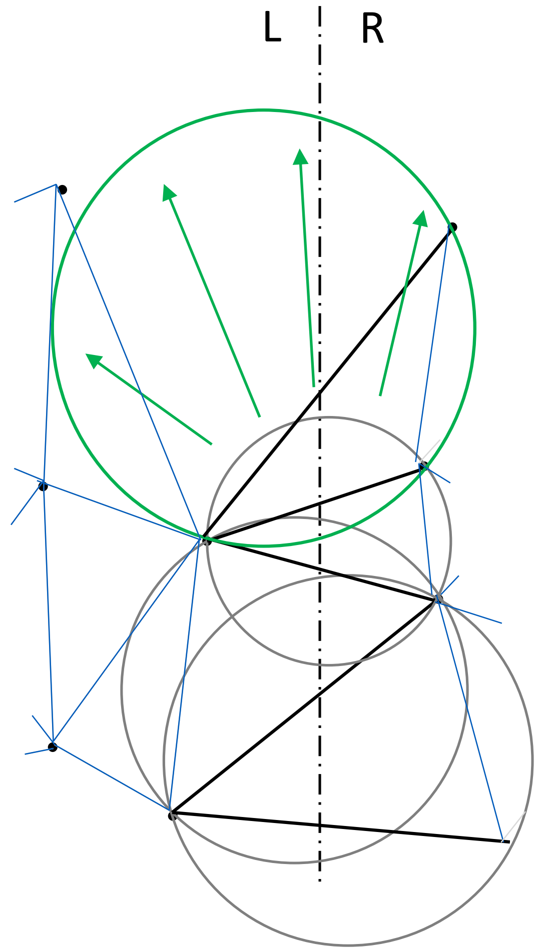
Algorithm: Merge Step



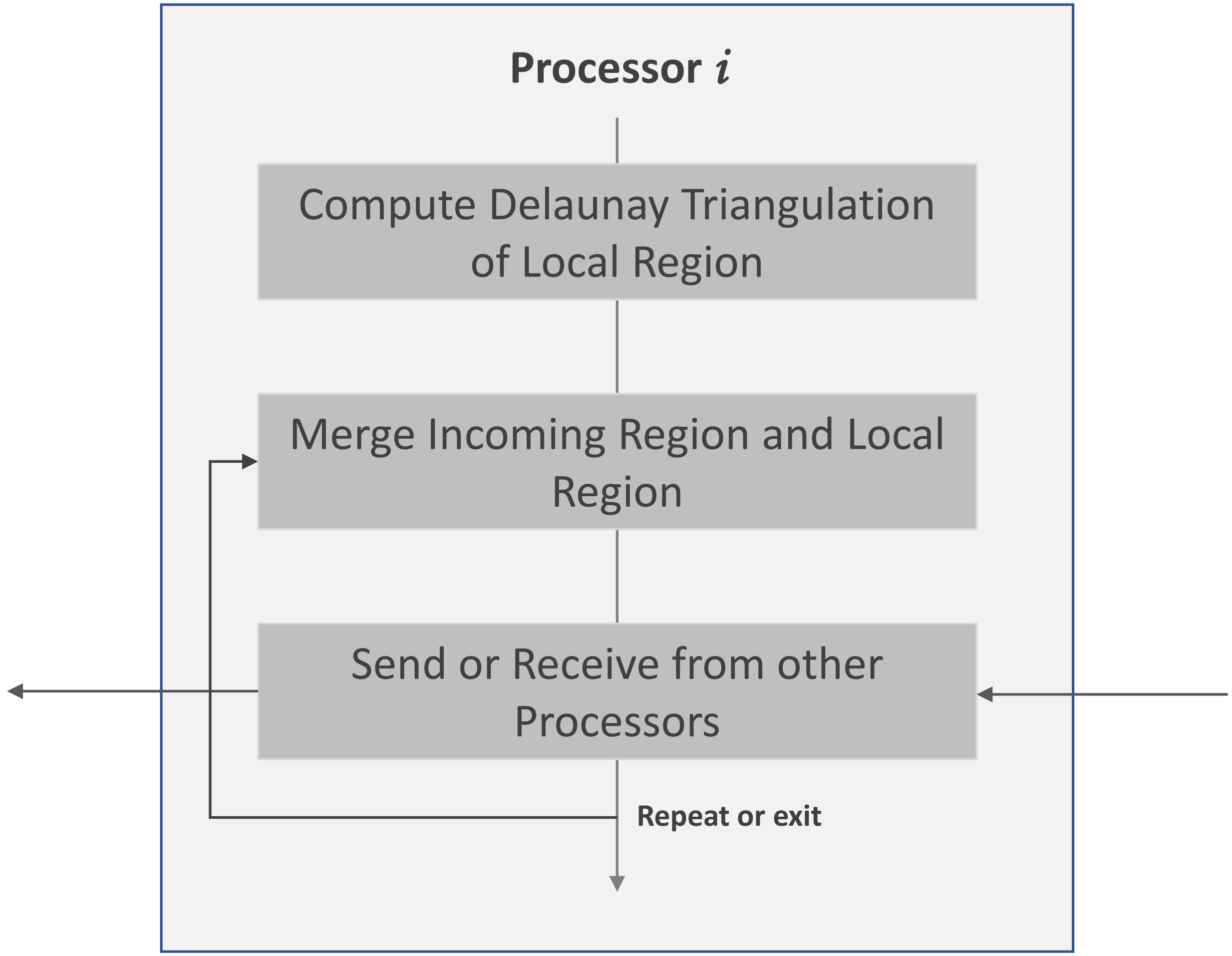
Algorithm: Merge Step



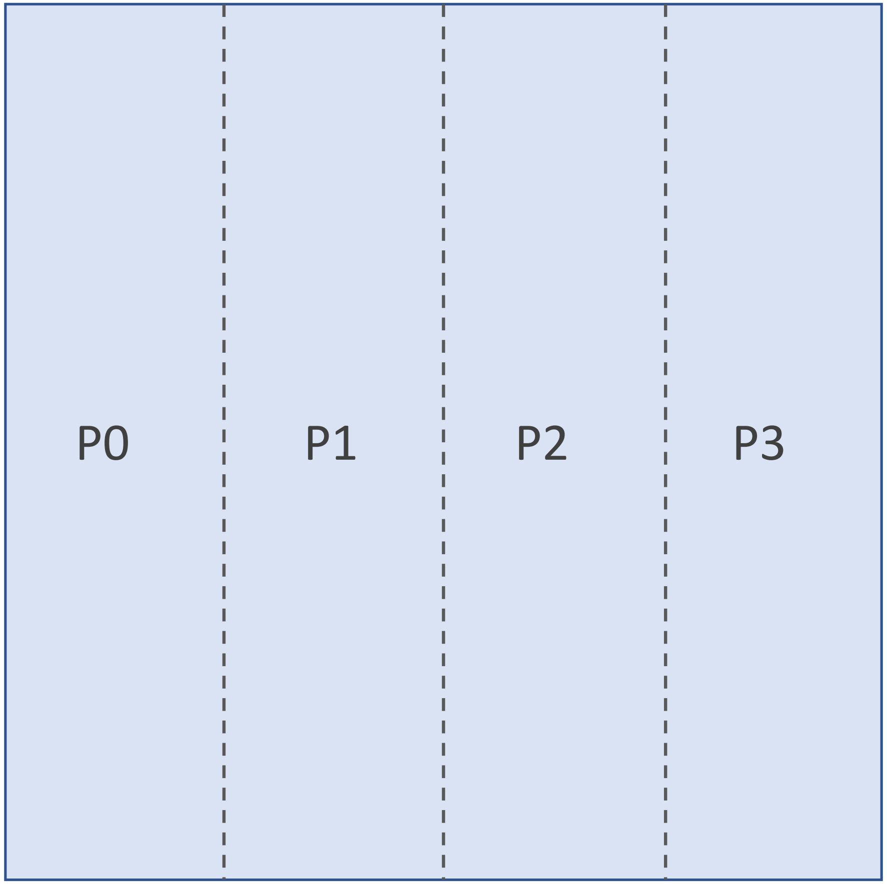
Algorithm: Merge Step



Algorithm: Parallel Overview



Domain Decomposition



Input space divided equally by X-Coordinate among Processors

Implementation

- Implementation in C and MPI
- Pseudo code from paper for serial version of merge – made life easier
- Jobs were run on ***general-compute*** and ***largemem*** partitions of CCR
- All communications are point-to-point: **MPI_Send** and **MPI_Recv**
- Data send/receive happens in a single block (as many as 31 million edges ~ 700mb)
- Approx. 500 jobs to cluster

Implementation

- **Input**:
  - Randomly generated points – **Bivariate Uniform Distribution** using Python **numpy** package
  - Equal range and density across both the axes
  - No duplicates and **pre-sorted** by **X-Coordinate**
  - Each coordinate is "**double**" precision ∈ **[0, 200*n]**
- **Output**:
  - Edge endpoints as indices

```
0, 162.422299106, 626335123.072
1, 235.609542392, 21674347.1286
2, 348.128895741, 545885503.786
3, 388.434040826, 160544722.935
...
```

Sample Input

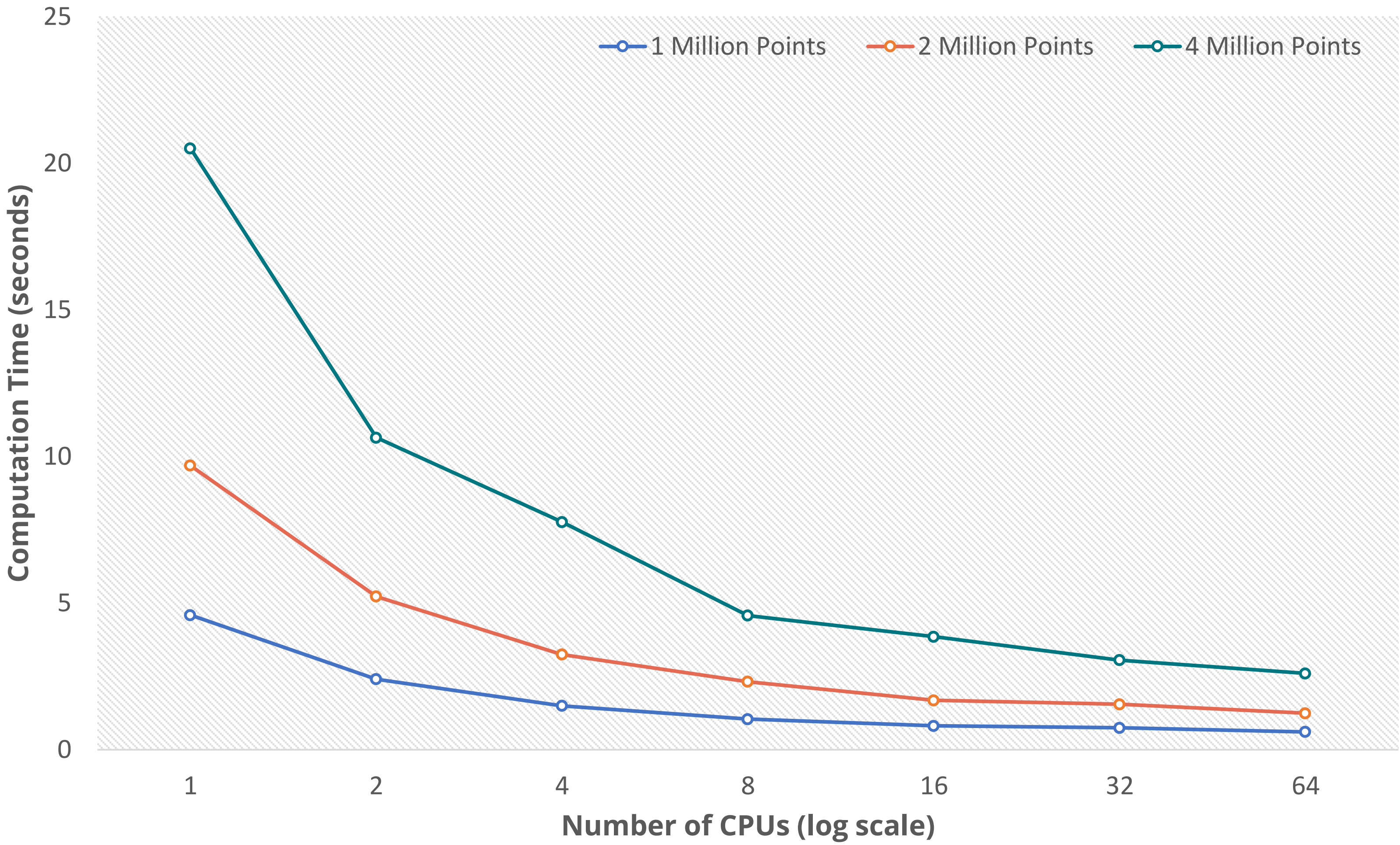
```
0 1
0 3
0 4
0 2
...
```

Sample Output

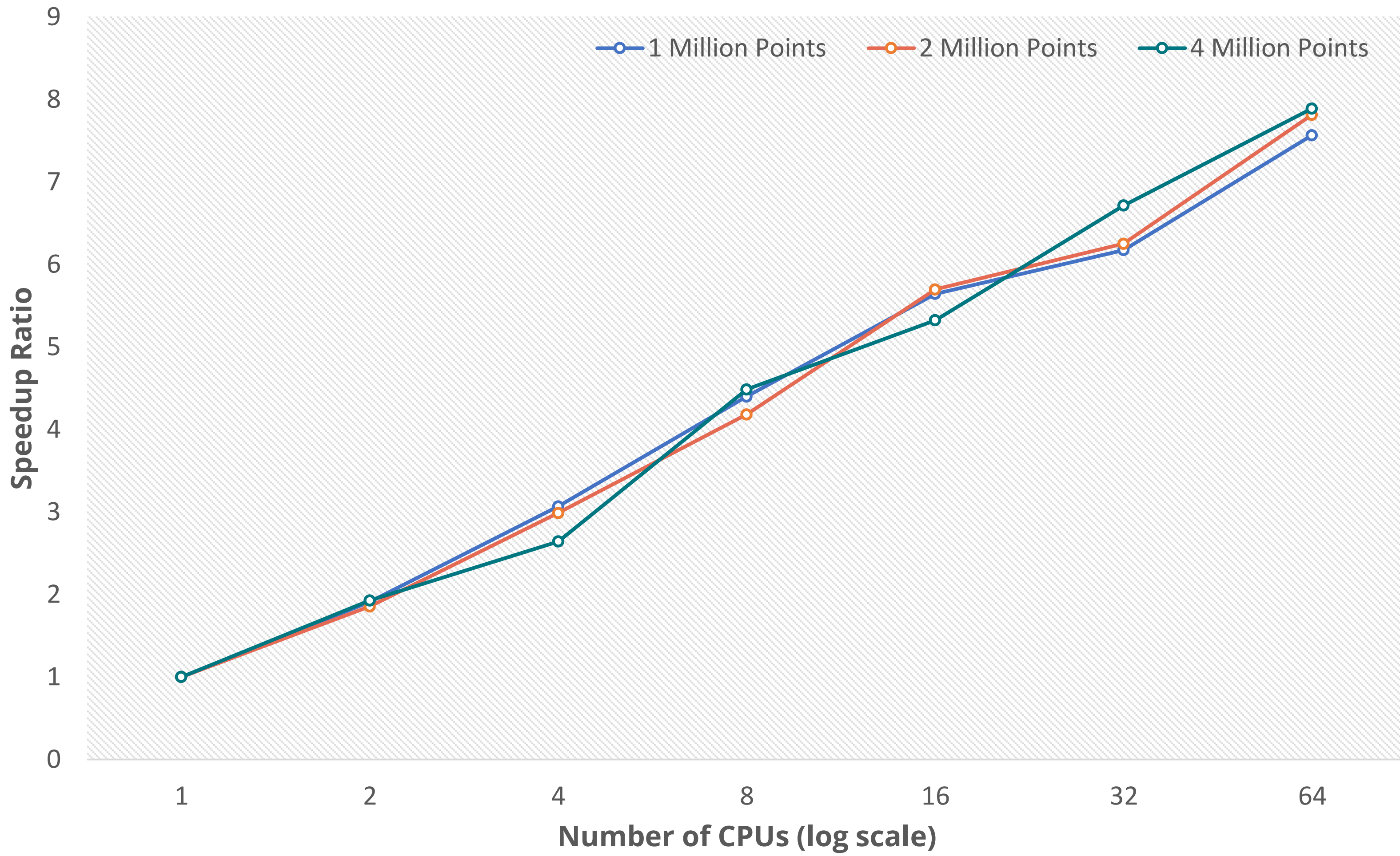
Results

- Run-times averaged over 3-5 jobs/runs
- Tried for several core-node combinations:
  - 2 CPUs per node with **shm** (intranode) and **tmi** (internode)
  - 1 CPU per node with **dapl** (internode)
  - 1 CPU per node with **tmi** (internode)
  - Upto 32 CPUs per node with **tcp** (intranode) and **tcp** (internode) – **I_MPI_FABRICS** and **I_MPI_FALLBACK** to the rescue!
- All results **validated** against results from standard packages:
  - Python (scipy.spatial.Delaunay) - faster
  - Matlab (triangulation)

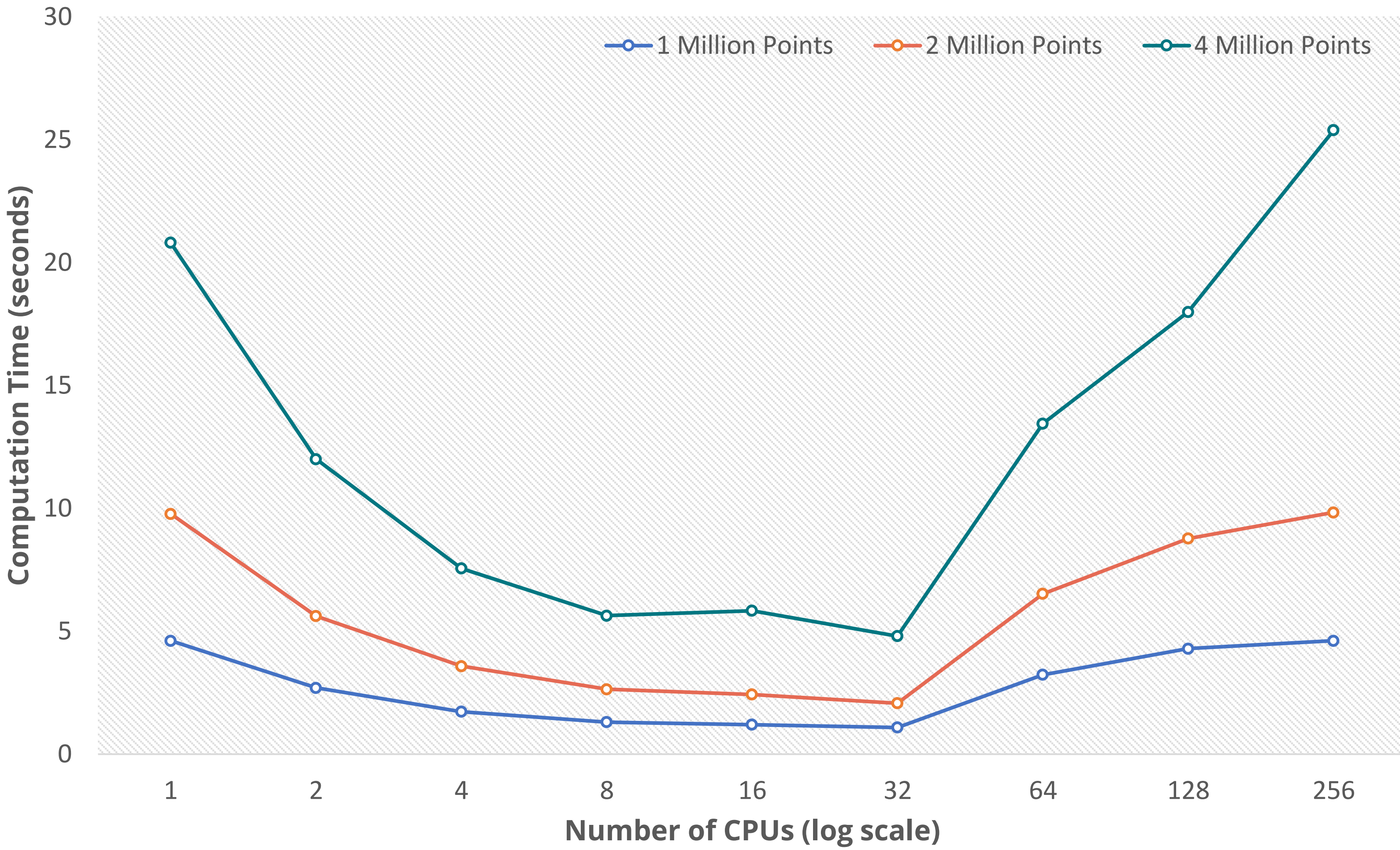
Time v/s CPU (1 CPU per node – TMI)



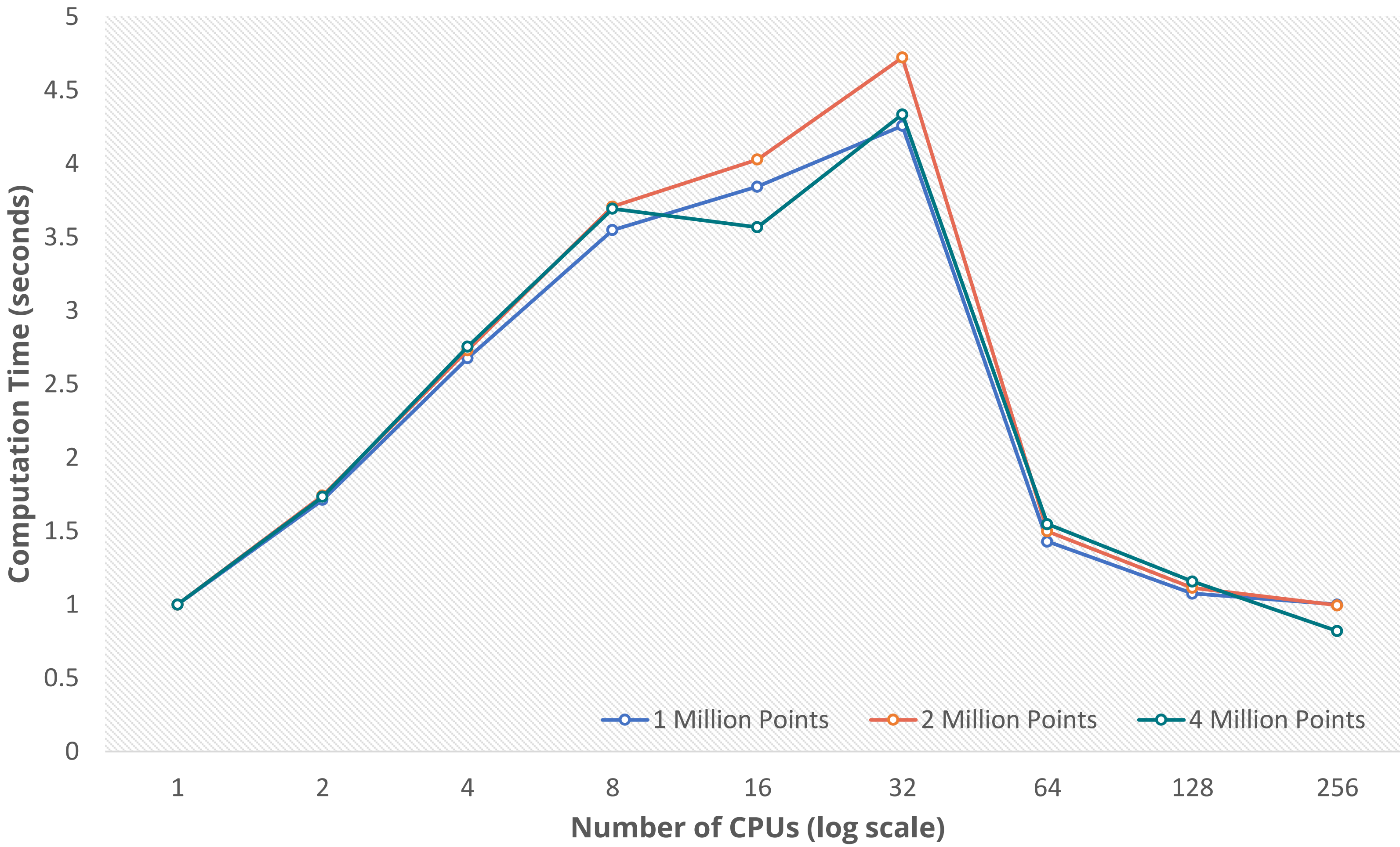
Speedup v/s CPU (1 CPU per node – TMI)



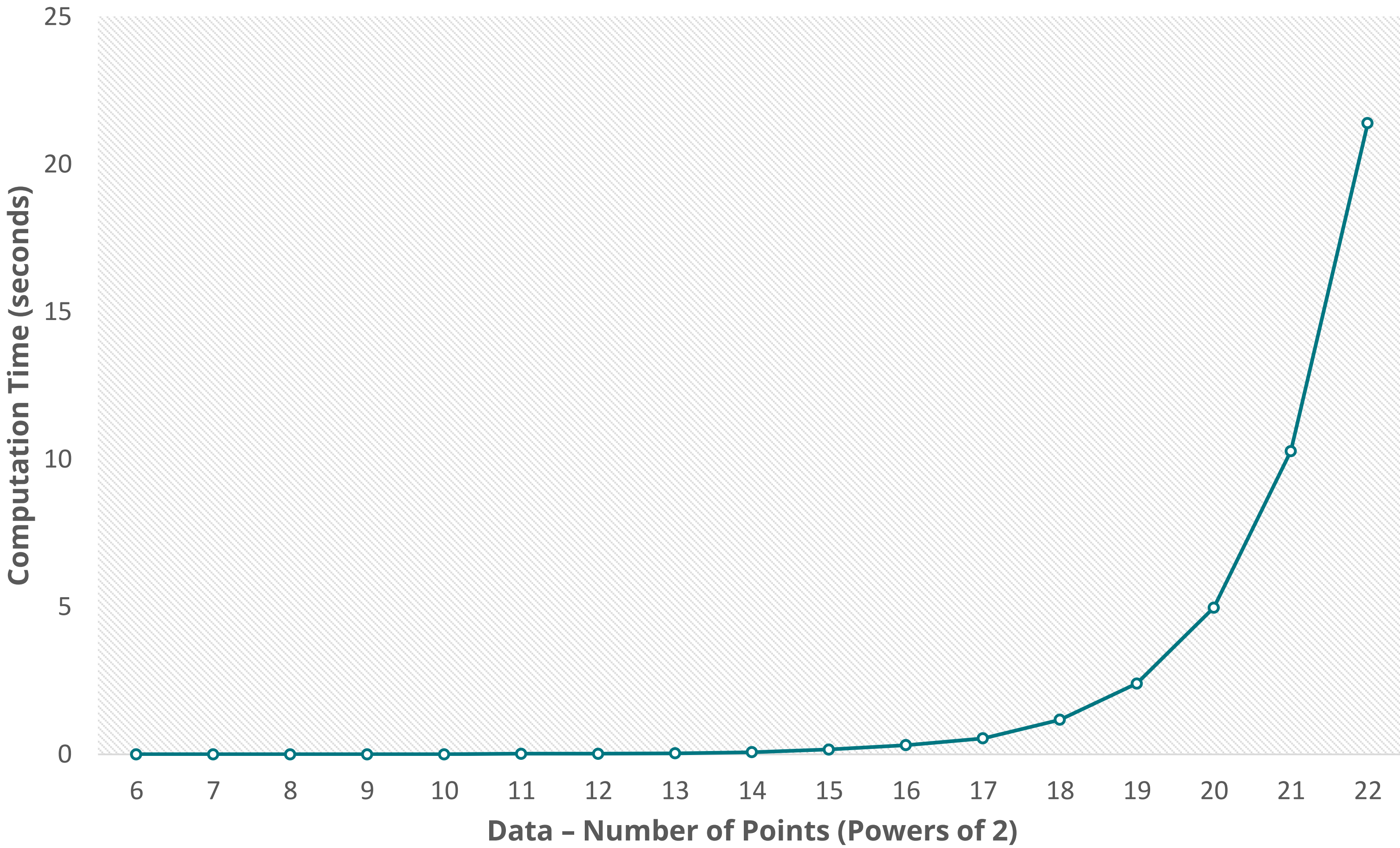
Time v/s CPU (32 CPUs per node – TCP – no shm)



Speedup v/s CPU (32 CPUs per node – TCP – no shm)



Asymptotic Growth (8 CPUs with 1 CPU per node)



Conclusion

- That drop in speedup for 32-cpus-per-node?
  - **Communication Cost: Intranode < Internode**
  - Difference is significant for TCP and hence the sudden drop
- **Hard Merge – High Communication Costs – No Linear Speedup**
- But, there **is** gain
- Data still needs to fit into a single machine!

References

- *Primitives for the Manipulation of General Subdivisions and the Computation of Voronoi Diagrams* – **Guibas, L.** and **Stolfi, J.**
- *Closest-Point Problems* – **Shamos, M.I.** and **Hoey, D.**
- *On computing Voronoi diagrams by divide-prune-and-conquer* – **Amato, N.M.** and **Ramos, E.A.**
- *Chapter 10: Computational Geometry, Algorithms – Sequential and Parallel* **– Miller, R.** and **Boxer, L.**

Thank You

adarshpr@buffalo.edu

Binaries, scripts, code and results available at: https://github.com/adrsh18/parallel

Thanks to Dr. M Jones and CCR @ UB

Backup: Analysis

- Sequential runtime: ***O(n * logn)*** [*T(n) = 2 * T(n/2) + O(n)*]
- "Heavy" merge step with *O(n)*. Parallelization possible?!!
- Analysis with ***p*** processors:
  - Each processor locally and simultaneously computes DT on $\frac{n}{p}$ points → $O\{\frac{n}{p} * log\,(\frac{n}{p})\}$
  - DTs from each processor is stitched together (happens *logp* times) → $O(n * logp)$
  - So, total runtime = $O\,\{\frac{n}{p} * log\,(\frac{n}{p}) \;\; + n * logp\}$
- If *p = logn*, runtime = ***O(n log(logn))***