(2) Polynomial f^n

↳ f^n defined by relation

$$x(k) = \begin{cases} a^k & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$$

is polynomial f^n

By defn of z-transform

$$Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} a^k z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k$$

$$= 1 + \left(\frac{a}{z}\right)^1 + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \cdots$$

$$\Rightarrow \frac{1}{1 - \frac{a}{z}} \quad \text{for } \left|\frac{a}{z}\right| < 1$$

$$\Rightarrow z / z - a \quad \text{for } |z| > a$$

$\therefore Z[a^k] = \frac{z}{z-a}$ for $|z| > a$

↳ ROC lies outside of circle $|z| = a$

In particular,

$a = 1$

$Z[1^k] = Z[1] = \frac{z}{z-1}$

$a = -1$

$Z[(-1)^k] = \frac{z}{z+1}$

n & k → sequence

discrete time signal

$kT \rightarrow$ an aux

③ Exponential f^n

↳ f^n defined by $x(t) = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

is exponential f^n.

By def^n of z-transform

$Z[x(t)] = Z[x(kT)]$

$$= \sum_{k=0}^{\infty} x(kT)\, z^{-k}$$

$$= \sum_{k=0}^{\infty} e^{-akT}\, z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(e^{-aT} z^{-1}\right)^{k}$$

$$= 1 + \left(e^{-aT} z^{-1}\right)^{1} + \left(e^{-aT} z^{-1}\right)^{2} + \cdots$$

$$= \frac{1}{1 - e^{-aT} z^{-1}} = \frac{z}{z - e^{-aT}}$$

$$\therefore Z\left[e^{-at}\right] = \frac{z}{z - e^{-aT}}$$

similarly

$$Z\left[e^{at}\right] = \frac{z}{z - e^{aT}}$$

&

$$Z[\underbrace{e^{an}}_{\text{variable}}] = \frac{z}{z - e^{a}}$$

variable

ending

variable

$$Z[e^{-ak}] = \frac{z}{z-e^{-a}}$$

④ **unit-Impulse or Dirac-Delta f^n $\delta(k)$**

↳ f^n defined by $\delta(k) = \begin{cases} 1 & \text{for } k=0 \\ 0 & \text{for } k \neq 0 \end{cases}$

is Dirac-Delta f^n

By def^n,

$$Z[\delta(k)] = \sum_{k=0}^{\infty} \delta(k) z^{-k}$$

$$= \delta(0) + \delta(1) z^{-1} + \delta(2) z^{-2} + \cdots$$
$$= 1 + 0 + 0 + 0 \cdots$$
$$= 1$$

$\therefore Z[\delta(k)] = 1$

⑤ **unit-Ramp f^n**

↳ f^n defined by $x(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$ is

unit-Ramp f^n.

(6) sinusoidal f^n

↳ f^n defined by $x(t) = \begin{cases} \sin\omega t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

is sinusoidal f^n

Z-transform of t^n where 'n' is +ve integer

Soln

let $x(t) = t^n$

$x(KT) = (KT)^n$

t ayesi KT le replace

By defn of z-transform

$$Z[x(t)] = Z[x(KT)]$$

$$= \sum_{K=0}^{\infty} x(KT) z^{-K}$$

$$= X(z)$$

$$\therefore Z[t^n] = \sum_{K=0}^{\infty} (KT)^n z^{-K} = X(z) \quad \text{(i)}$$

Similarly,

$$Z[t^{n-1}] = \sum_{k=0}^{\infty} (kT)^{n-1} z^{-k}$$

Now,

diff. both sides w.r.t. 'z', we get

$$\frac{d}{dz}[Z(t^{n-1})] = \frac{d}{dz}\left[\sum_{k=0}^{\infty} (kT)^{n-1} z^{-k}\right]$$

$$= \sum_{k=0}^{\infty} (kT)^{n-1} \frac{d}{dz}(z^{-k})$$

$$= \sum_{k=0}^{\infty} (kT)^{n-1} (-k) z^{-k-1}$$

$$= \sum_{k=0}^{\infty} \frac{(kT)^{n}}{kT} (-k) \frac{z^{-k}}{z}$$

$$= \frac{-1}{Tz} \sum_{k=0}^{\infty} (kT)^{n} z^{-k}$$

$$\frac{d}{dz}[Z(t^{n-1})] = \frac{-1}{Tz} X(z) \quad [\text{from } eq^n (i)]$$

$$\Rightarrow X(z) = -Tz \frac{d}{dz} Z[t^{n-1}]$$

$$\therefore Z[t^n] = -Tz \frac{d}{dz} [Z(t^{n-1})]$$

Put $n=1$

$$Z[t] = -Tz \frac{d}{dz} [Z(t^0)]$$

$$= -Tz \frac{d}{dz} [Z(1)]$$

$$\frac{z}{z-1}$$

$$= -Tz \frac{d}{dz} \left(\frac{z}{z-1}\right)$$

$$= -Tz \frac{(z-1)\cdot 1 - z\cdot 1}{(z-1)^2}$$

$$= -Tz \left(\frac{\cancel{z} - 1 - \cancel{z}}{(z-1)^2}\right)$$

$$Z[t] = \frac{Tz}{(z-1)^2}$$

Put $n=2$

$$Z[t^2] = -Tz \frac{d}{dz}[Z(t)] = -Tz \frac{d}{dz}\left[\frac{Tz}{(z-1)^2}\right]$$

$$= -T^2 z \frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -T^2 z \frac{(z-1)^2 \cdot 1 - z \cdot 2(z-1) \cdot 1}{(z-1)^4}$$

$$= -T^2 z \frac{(z-1)^2 - 2z(z-1)}{(z-1)^4}$$

$$= -T^2 z \frac{(z-\cancel{1})[(z-1)-2z]}{(z-1)^{\cancel{4}3}}$$

$$= \frac{-T^2 z [z-1-2z]}{(z-1)^3}$$

$$= \frac{-T^2 z [-1-z]}{(z-1)^3}$$

$$\therefore Z[t^2] = \frac{T^2 z [z+1]}{(z-1)^3}$$

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similarly,

we obtain

$$Z[k^n] = -z \frac{d}{dz} Z[k^{n-1}]$$

T hatxa

$$Z[k] = \frac{z}{(z-1)^2}$$

$$Z[k^2] = \frac{z(z+1)}{(z-1)^3}$$