

to represent quantities

more powerful than propositional logic



Predicate Logic

• Predicates & Quantifier

↳ Declarative sentence

Propositional logic ko rules + afnai 4

whose truthness / falsehood depends on one or more variables

↳ It is propositional fⁿ

• Quantifiers

① universal quantifiers

② Existential quantifiers

① universal

↳ True for all

↳ conjunction sabhi ko

↳ sabhi ko huna paryo.

L counter - eg xa yane buddina

↳
no hune case



L $\forall x P(x)$ universal quantifier
↳
FOR all

② Existential

L At least 1 ko lagi true huna
paryo

L Disjunction

L $\exists x P(x)$ existential quantifier

• Keywords

① There exists

② For at least one

③ There is

• Negating Quantified Exp.

① Every student in class has taken a course in calculus



yesko \neg "It is not the case ..."



vannele at least euta student

le calculus lexaia



yo. vaneko

existential quantification

on of \neg

i.e.

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

② There is a std in class who has ...

$$\neg \exists x P(x) \Leftrightarrow \forall x \neg P(x)$$

Nested quantifiers

- ① $\forall x \forall y (x+y = y+x)$
For every x \wedge For every y ,

$$x+y = y+x$$

- ② $\forall x \exists y (x+y=0)$
For all real no. x , there exists
 real no. y ,

$$\text{such that } x+y=0$$

- ③ For $\phi(x, y)$
 denote $(x+y=0)$

Then,

$$\exists y \forall x \phi(x, y) \rightarrow \text{False}$$



There exists a value of y , for all
 x



There exists single

y value

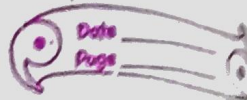
won't

satisfy $x+y=0$

$$5-5=0, 4-5 \neq 0$$

• Note : ① conjunction

$$\begin{array}{c} p \\ q \\ \hline p \wedge q \end{array}$$



② Addition

$$\begin{array}{c} p \\ \hline p \vee q \end{array}$$

③ simplification

$$\begin{array}{c} p \wedge q \\ \hline p \quad q \end{array}$$

1. Show that ^{Existential} premises
- A std. in sec A has taken course
has not read book.
 - Everyone in sec A has taken course
passed 1st exam.

Implies

someone who passed 1st exam
has not read the book

A(x): "x is in sec who has taken
the course."

B(x): "x read the book"

$P(x)$: "x passed the 1st exam."

• Hypothesis:

$$\exists x (A(x) \wedge \neg B(x))$$

$$\forall x (A(x) \rightarrow P(x))$$

• Conclusion:

$$\exists x P(x) \wedge \neg B(x)$$

statement

Reason

1 $\exists x (A(x) \wedge \neg B(x))$

Hypothesis

2 $\forall x (A(x) \rightarrow P(x))$

Hypothesis

3 $A(Q) \wedge \neg B(Q)$

Existential instantiation
on (1)

4 $A(Q) \rightarrow P(Q)$

Universal instantiation
on (2)

5 $A(Q)$

simplification of (3)

6 $P(Q)$

using modus ponens
on (4) & (5)

7 $\neg B(Q)$

simplification of (3)

8 $P(Q) \wedge \neg B(Q)$

conjunction of (6) & (7)

9. $\exists x (P(x) \wedge B(x))$

Existential generalization
- on

from 18)

2. lions are dangerous animals



safari lions are dangerous

There are lions

↳ Existential

Therefore, there are dangerous animals

so in

$L(x) : x$ is lion

$D(x) : x$ is dangerous animals

• Hypothesis:

$\forall x (L(x) \rightarrow D(x))$

$\exists x L(x)$

• conclusion:

$\exists x D(x)$

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5 construct an arg. using rules of inference to show hypothesis

• "All movies produced by J.S. are wonderful."
universal

• "J.S. produced a movie about coal miners."

Existential

implies

• "There is a wonderful movie about coal miners."

so in

$J(x)$: x is movie produced by J.S.

$C(x)$: x is movie about coal miners

$W(x)$: x is wonderful movie

• Hypothesis:

$$\forall x (J(x) \rightarrow W(x))$$

$$\exists x (J(x) \wedge C(x))$$

• conclusion:

$$\exists x (W(x) \wedge C(x))$$

statement	reason
1. $\forall x (J(x) \rightarrow W(x))$	Hypothesis
2. $\exists x (J(x) \wedge C(x))$	Hypothesis
3. $J(a) \wedge C(a)$	Existential instantiation on 2.
4. $J(a) \rightarrow W(a)$ $\neg J(a) \wedge W(a)$	uni instantiation on 1.
5. $J(a)$	simplification of (3)
6. $W(a)$	using modus ponens on (4) & (5)
7. $C(a)$	simplification on (3)
8. $W(a) \wedge C(a)$	using conjunction on (6) & (7)
9. $\exists x (W(x) \wedge C(x))$	Existential generalization on (8).