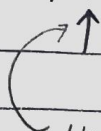


Find the Z-transform of following sequences

1) $x(k) = \{3, 5, 2, 4, 1, 0, 9, -12\}$



this symbol indicates 0th index element

i.e. $x(0) = 4$

soln,

the symbol '↑' is used to indicate the zeroth position element of sequence.

so,

$$x(0) = 4$$

$$x(1) = 1$$

$$x(2) = 0$$

$$x(3) = 9$$

$$x(4) = -12$$

and

$$x(-1) = 2$$

$$x(-2) = 5$$

$$x(-3) = 3$$

then, by definition of Z-transform,

$$Z[x(k)] = \sum_{k=-3}^4 x(k)z^{-k}$$

$$= x(-3)z^3 + x(-2)z^2 + x(-1)z^1 + x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4}$$

$$= 3z^3 + 5z^2 + 2z + 4 + \frac{1}{z} + 0 + \frac{9}{z^3} - \frac{12}{z^4}$$

2) $f(k) = \{5, 6, -3, -2, 8, 10\}$

$$3) f(k) = \begin{cases} 5^k & \text{for } k < 0 \\ 2^k & \text{for } k \geq 0 \end{cases}$$

soln,

$$Z[f(k)] = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=-\infty}^{-1} f(k) z^{-k} + \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=-\infty}^{-1} 5^k z^{-k} + \sum_{k=0}^{\infty} 2^k z^{-k}$$

$$= \sum_{k=-\infty}^{-1} \left(\frac{5}{z}\right)^k + \sum_{k=0}^{\infty} \left(\frac{2}{z}\right)^k \quad \text{f not needed}$$

$$= (\dots + 5^{-3} z^3 + 5^{-2} z^2 + 5^{-1} z^1) + [1 + 2 z^{-1} + 2^2 z^{-2} + 2^3 z^{-3} + \dots]$$

$$= \left[\left(\frac{z}{5}\right)^2 + \left(\frac{z}{5}\right)^1 + \left(\frac{z}{5}\right)^0 + \dots \right] + \left[1 + \left(\frac{2}{z}\right)^1 + \left(\frac{2}{z}\right)^2 + \dots \right]$$

$$= \frac{\frac{z}{5}}{1 - \frac{z}{5}} \quad , \quad \left| \frac{z}{5} \right| < 1 \quad + \quad \frac{1}{1 - \frac{2}{z}} \quad , \quad \left| \frac{2}{z} \right| < 1$$

$$= \frac{z}{5-z} + \frac{z}{z-2} \quad , \quad |z| < 5 \quad \text{and} \quad |z| > 2$$

Properties of Z-transform

① Linearity

If $x_1(k)$ and $x_2(k)$ are two sequences, then,

$$Z[a x_1(k) + b x_2(k)] = a Z[x_1(k)] + b Z[x_2(k)]$$

where,
 a and b are constant.

② Multiplication by a^k

If $x(k) = 0$ for $k < 0$ and $Z[x(k)] = X(z)$ for $k \geq 0$, then,

$$Z[a^k x(k)] = X\left(\frac{z}{a}\right)$$

Proof:

$$\text{let } Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

then,

$$Z[a^k x(k)] = \sum_{k=0}^{\infty} \{a^k x(k)\} z^{-k}$$

$$= \sum_{k=0}^{\infty} x(k) (a^k z^{-k})$$

$$= \sum_{k=0}^{\infty} x(k) \left(\frac{z}{a}\right)^{-k} \quad \#$$

$$= \sum_{k=0}^{\infty} x(k) z_1^{-k} \quad \text{where } z_1 = \frac{z}{a}$$

$$= x(z_1)$$

$$= x\left(\frac{z}{a}\right)$$

$$\therefore \mathcal{Z}[a^k x(k)] = x\left(\frac{z}{a}\right)$$

③ Multiplication by k

If $x(k) = 0$ for $k < 0$ and $\mathcal{Z}[x(k)] = X(z)$ for $k \geq 0$.

Then,

$$\mathcal{Z}[kx(k)] = -z \frac{d}{dz} X(z)$$

Proof:

We have,

$$\mathcal{Z}[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

then,

$$\begin{aligned} \mathcal{Z}[kx(k)] &= \sum_{k=0}^{\infty} \{kx(k)\} z^{-k} \\ &= \sum_{k=0}^{\infty} x(k) (kz^{-k}) \end{aligned}$$

$$= \sum_{k=0}^{\infty} x(k) (-k) z^{-k} (-1)$$

$$= \sum_{k=0}^{\infty} x(k) \{ (-k) z^{-k-1} \} (-z)$$

$$= -z \sum_{k=0}^{\infty} x(k) \frac{d}{dz} (z^{-k})$$

$$= -z \frac{d}{dz} \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$= -z \frac{d}{dz} Z[x(k)]$$

$\therefore Z[kx(k)] = -z \frac{d}{dz} X(z)$
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