Chapter-1

Measurement And Measurement system

Measurement:

The process of evaluating and assigning the parameter of physical body in terms of meaningful numbers is known as measurement. The measurement is meaningful if it is standardly measured and so we need standard units for comparison and standard equipments to measure.

The system used for measurement is known as instrumentation system or measurement system and by definition, instrumentation should be such that the output should be accurate and commonly accepted.

Application of Instrumentation:

In trade / Commercial purpose

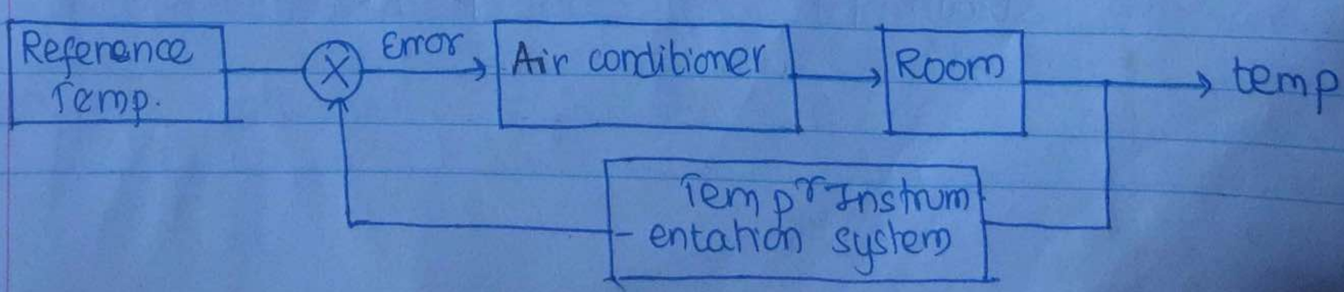
To ensure the commercial unit is producing standard commodities.

For monitoring purpose

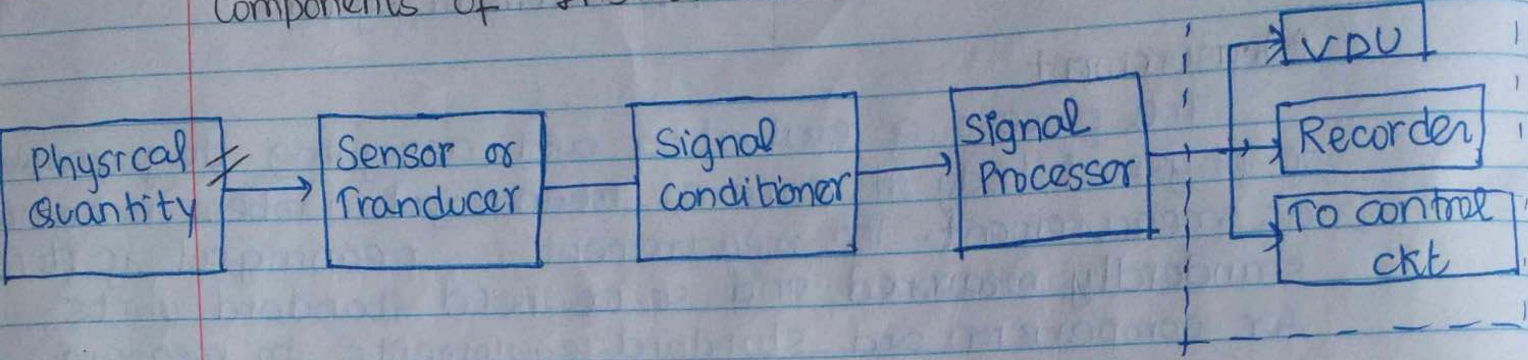
To monitor some critical requirements.

Eg: Temperature of green house
heartbeat of serious patient.

Automatic Control System:



Components of Instrumentation System.



Sensor /Transducer

(i) It converts information available at input variable into variable into the form suitable for next layer of measurement system. It is simply energy ~~converting~~ converter device.

(ii) The sensor converting non-electrical energy into electrical energy is known as transducer or viceversa is known as tranducer.

Some examples.

i/P Tempr → Thermocouple → o/P (small emf)

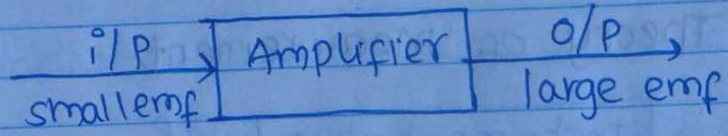
i/P Tempr → Resistance Thermometer → o/P ΔR

i/P Strain → strain guage → o/P ΔR

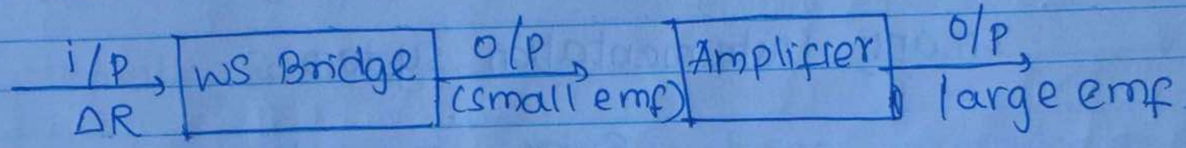
2) Signal Conditioner

The sign output of sensor is not always suitable for the rest of the system. Sensor give raw signal and conditioner implies it to make suitable for processing. For example:

Amplifier in case of thermocouple.



Wheatstone bridge followed by amplifier in case of resistance thermometer.



3) Signal Processor

The sensed signal is processed inorder to obtain suitable data for presentation. Signal processor does mathematical operations such as addition, subtraction, differentiation, integration, comparison and so on to obtain final presentable data. The common example for signal processor is ADC followed by programming.

ADC (Analog- Digital Converter)

Data Presentation

The measured value is presented in the form such that observer comfortably recognize it. ~~Adit~~ Additionally, the information may also be transferred for control operation. Eg: VDU.

True value $\longrightarrow$ $\boxed{\text{IS}}$ $\longrightarrow$ Measured value

Chapter 2

Theory of Measurement.

$$\xrightarrow[\text{True value of Qty}]{\text{i/P}} \boxed{\text{Instrumentation system}} \xrightarrow[\text{M.V of Qty.}]{\text{o/P}}$$

In any instrumentation system

$$\text{Error} = \text{T.V} - \text{M.V.}$$

$$\text{Relative Error } (\varepsilon_r) = \frac{\text{Error}}{\text{T.V of Qty}}$$

$$\%\ \text{Error} = \varepsilon_r \times 100\% = \frac{\text{Error}}{\text{TV}} \times 100\%.$$

Error

- Gross Error (human error)
- Systematic Error
  - Environmental Error
  - Instrumental Error.
- Random Error

1) Gross Error

The type of error due to human mistakes in reading, measuring, recording and calculating also may be due to wrong placement or connection of instrument. It has no mathematical way of correction.

Correction measures:

- Take reading carefully and connect carefully.
- Take atleast three readings of same quantity preferably by different observers.

2) Systematic Error

Error that is constant or change according to definite law. Also called bias. These are of two types:

a) Instrumental Errors

Errors due to shortcomings of instruments. These errors include improper tension in string, less accuracy in scale calibration.

Correcting measures:
- Use suitable instruments
- Apply corrective factors
- Recalibrate standardly

b) Environmental Errors.

All the errors due to effect of surrounding environment. These include error due to variation of temperature, pressure, humidity and effect of external fields.

Correcting Measures

- Air conditioned room
- Proper sealing and casing
- Proper shielding

Random Error

Even after solving all possible sources of error the reading varies randomly around the actual value at every measurement which is known as random error.

Remedy:

- Apply statistical analysis after minimizing gross error and systematic error.

Why random error is minimized only after minimizing gross error and systematic error?

Limiting Error or Guarantee Error

The limits of deviation of values as specified by manufacturer for the equipment is known as limiting error.

For example:

If ammeter is 1% accurate, it means the output will have maximum error of 1% of full scale. For.

i.e Ammeter 0-25A and 1% accuracy

If it measures 3A then it can measure ~~e~~ from

$3 - 0.25 = 2.75$

$3 + 0.25 = 3.25$

[$\because$ Max^m possible error = 1% of 25 = 0.25 A].

If voltmeter having accuracy of 1% and of full scale reading of 100V is used to measure

i) 80V

ii) 12V

Find max^m percentage error in both case & comment on result.

Sol^n:-

Here Accuracy = 1%

Max^m possible error = 1% of 100

$= \cancel{0.01}$. 1V.

i) For 80V,

limiting error = 1V

So, for 80V then the Measured value may be 79 to 81V.

So max^m % error $= \frac{1}{80} \times 100\%$

$= 1.25\%$

ii) For 12V

limiting error = 1V

So, for 12V then the measured value may be 11V to 13V.

So max^m % error = $\frac{1}{12} \times 100\%$

$= 8.33\%$

FA

Comment: Range selection of device plays a great role in measurement system. In 1st case Error % is low and in 2nd case it is high. So to reduce error selection the range of meter properly.

Q) The wattmeter is used to measure power in a circuit with the help of following equation:

$P = \frac{E^2}{R}$ where limiting values of voltage and resistances are: $E = 200V \pm 1\%$.

$R = 1000\Omega \pm 5\%$.

i) Calculate true nominal power consumed.

ii) Limiting error & % error of power.

Soln:-

Given $P = \frac{E^2}{R}$

P is maximum when E is maximum and R is minimum.

i.e $E = 200 + 1\%$ of 200
$= 202$

$R = 1000\Omega - 5\%$ of 1000
$= 950$.

$$P = \frac{E^2}{R} = \frac{202^2}{950} = 42.95 \text{ watt}$$

$$\text{Real } P = \frac{200^2}{1000} = 40.$$

$$\text{Limiting Error} = 42.95 - 40$$
$$= 2.95 \text{ watt.}$$

$$\% \text{ Error} = \frac{2.95}{40} \times 100\%.$$
$$= 7.375\%.$$

Q) The impedance of R-L circuit operating on AC is given by $Z = \sqrt{R^2 + \omega^2 L^2}$, The resistance R is known to be 100Ω with uncertainity of 5%. $L = 2H$ with uncertainity of 10% and ω is exactly known to be $2\pi * 50$. Calculate

i) maximum error in impedance.

ii) nominal value

iii) Maxm % limiting error.

Soln:- $Z = \sqrt{R^2 + \omega^2 L^2}$

$R = 100\Omega \pm 5\%$

$= 105.$

$L = 2H \pm 10\% = 2.02H$

$\omega = 2\pi \times 50$

$\therefore Z = \sqrt{105^2 + 314^2 \times 2.02^2} = 642.91\,\Omega$

i) Nominal value of $Z = \sqrt{100^2 + 314^2 \times 2^2} = 635.91\,\Omega$

ii) Maximum error in impedance

$= 642.91\,\Omega - 635.91\,\Omega$

$= 7\,\Omega.$

iii) Maximum % limiting Error

$= \frac{7}{635.91} \times 100\%$

$= 1.10\%.$

The voltmeter reading of 70V on it's 100V range and ammeter reading of 80mA on its 150 mA range are used to determine the power dissipation in a register. Both these instruments are guaranteed to be accurate within ±1.5% of full scale deflection. Determine power measured and limiting error of power.

Sol^n:- Measured value is $V = 70V$
$I = 80mA$

Here,

$P_{measured} = IV = 5.6$ watt.

$P_{measured} = 5.6$ watt

limiting error $= 0.28$ watt

% error $= 5\%$.

we know, According to question,

$$V = 70 \pm 1.5\% \text{ of } 100$$

Maximum $V = 70 + 1.5 = 71.5$.

$$I = 80 \pm 1.5\% \text{ of } 150$$

maximum $I = 82.25$.

So, $P = IV = 71.5 \times 82.25 \times 10^{-3} = 5.88$ watt.

Limiting error of power $= 5.88 - 5.6$
$= 0.28$ watt

% limiting error can be calculated as

$$\% \text{ error} = \frac{0.28}{5.6} \times 100\%$$
$$= \frac{1}{20} \times 100\%$$
$$= 5\%.$$

#

Statistical Analysis of Data

To minimize random errors

① Airthmetic ~~Error~~ Mean :

→ Gives approximation of TV

$$\bar{x} = \frac{x_1 + \dots x_n}{n}$$

② Deviation from mean

$d_1 = x_1 - \bar{x}$, $d_2 = x_2 - \bar{x}$ . . . $d_n = x_n - \bar{x}$

$d_1, d_2 . . . d_n$ are deviation from mean

$$\sum_{i=1}^{n} d_i = 0$$

3) Average deviation

→ Gives average accuracy of observation.

$$D = \frac{\sum_{i=1}^{n} |d_i|}{n}$$

4) standard deviation

σ (Population standard deviation) $= \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}}$ for $n > 20$

S (sample standard deviation) $= \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n-1}}$ for $n \leq 20$

5) Varience

$$V = \sigma^2 = \frac{\sum_{i=1}^{n} d_i^2}{n} \text{ for } n > 20$$

and, $\frac{\sum_{i=1}^{n} d_i^2}{n-1}$ $n \leq 20$.

Gausis[illegible]

=) Gives the coefficient of spreading.

Why random error is minimized only after minimizing gross error and systematic error?

Not satisfying

Ans:

Gross errors mainly covers human mistakes in reading instruments and recording and calculating measurement results. The responsibility of the mistake normally lies with the experimenter.

On the other hand, systematic errors mainly arises due to instrumental, environmental and observational errors.

Random errors are due to multitude of small factors which change or fluctuate from one measurement to another. The quantity being measured is affected by many happenings throughout the universe. We are aware of and account for some of the factors influencing the measurements but about the rest we are unaware & the happenings or disturbances about which we are unaware of are lumped together and are called Random errors. So, the errors which are known i.e. gross error and systematic error should be minimized first for obivous. It's causes are known so it's easier to minimize them first. After the gross error and systematic error are taken care of, then only random errors appear. For this reason, random error is minimized only after minimizing gross error and systematic error.

Gaussian Probability curve

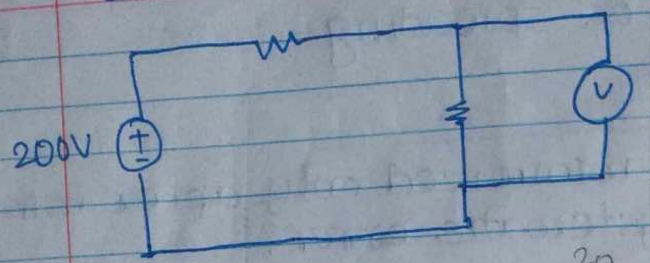
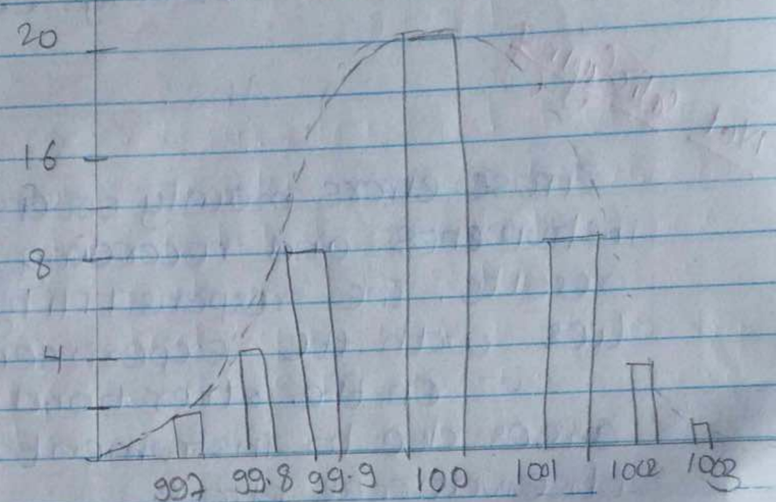


Table 1

Voltage Ratings	No of obs
99.7	1
99.8	3
99.9	12
100	19
100.1	10
100.2	4
100.3	1
	50



In the example above the voltmeter readings the voltage across the resistor (R). Reading of some voltage is iterared and listed in table-1. In the voltage readings of 50 iterations the readings are recorded to nearest 0.1V. Nominal value of voltage was 100V. But readings vary somehow above and below the true value. The result was plotted in histogram. Figure shows the largest no. of readings occur at central value, while other readings are placed more or less symmetrical about 100V on either side.

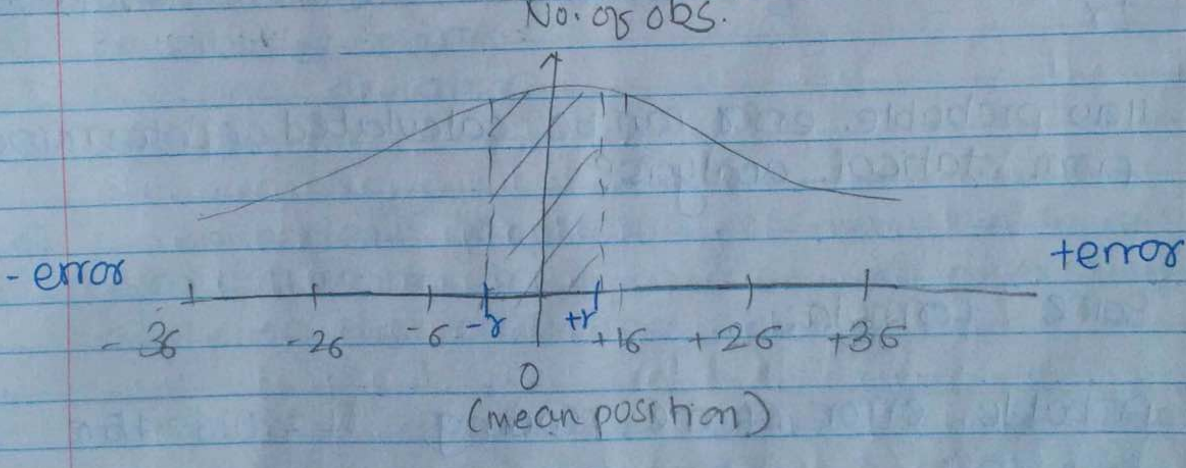
If more readings are done at very small intervals. The distribution of data becomes symmetri

-al about central value and the contour of histo-
-gram would finally become smooth curve as shown by dotted lines. This bell shaped curve is known as Gaussian Probability curve

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

The sharper is the curve, is, the more easier it is to ~~identify~~ identify the value of reading.

Probable error :-



Gaussian probability curve is actually presented by plotting no. of observation at any deviation from mean w.r.t magnitude of deviation from mean.

The area under gaussian probability curve betn $+\infty$ and $-\infty$ represents entire no. of observations. The area under $\pm\sigma$ represents the observation that differ

Deviation	Fraction of total area included.
$r = (\pm 0.6745\sigma)$	0.5
$\pm 1\sigma$	0.6828
$\pm 2\sigma$	0.9546
$\pm 3\sigma$	0.9974.

from mean by no more than standard deviation and this area comprise nearly 68% of total observation. Similarly, half of total obsenations are included in deviation limit of $\pm 0.6745\sigma$. So, probable error is denoted by r and is given by $r = 0.6745\sigma$. It means there is 50-50 chance that new observation will lie within deviation of $\pm r$

Q. How probable error can be calculated or determined from statical analysis?

Some formula:-

i) probable error of one reading, $r_1 = 0.6745\sigma$

ii) Probable error of mean, $r_m = \frac{r_1}{\sqrt{n-1}}$ for $n \leq 20$.

$$r_m = \frac{r_1}{\sqrt{n}} \quad \text{for } n > 20.$$

iii) standard deviation of mean, $\sigma_m = \frac{S}{\sqrt{n-1}}$ for $n \leq 20$
sample standard deviation (s)

and,

$$\sigma m = \frac{\sigma}{\sqrt{n}} \quad \text{for } n > 20$$

iv) Standard deviation of standard deviation $(\sigma\sigma) = \frac{\sigma m}{\sqrt{2}}$.

v) Range = Highest term - lowest term.

Q.N. 1) 10 measurements of resistance gave readings as 101.2, 101.7, 101.3, 101, 101.5, 101.3, 101.2, 101.4, 101.3, 101.1. Assume that only random errors are present. Calculate

i) Airthmetic mean
ii) Standard deviation of reading
iii) Probable ~~of~~ error
iv) Average deviation
v) Range.

2) In a ~~desk~~ test temperature is measured 100 times with variations in apparatus and procedure after applying the corrections. Results are :

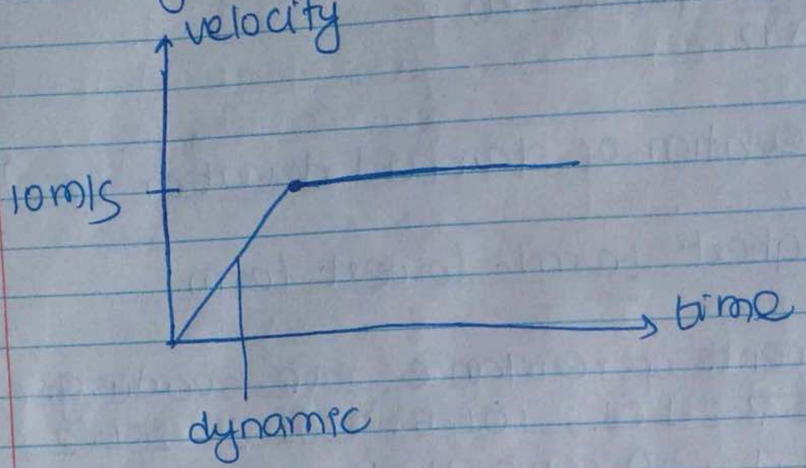
Temp(°C)	397	398	399	400	401	402	403	404	405
Frequency	1	3	12	23	37	16	4	2	2

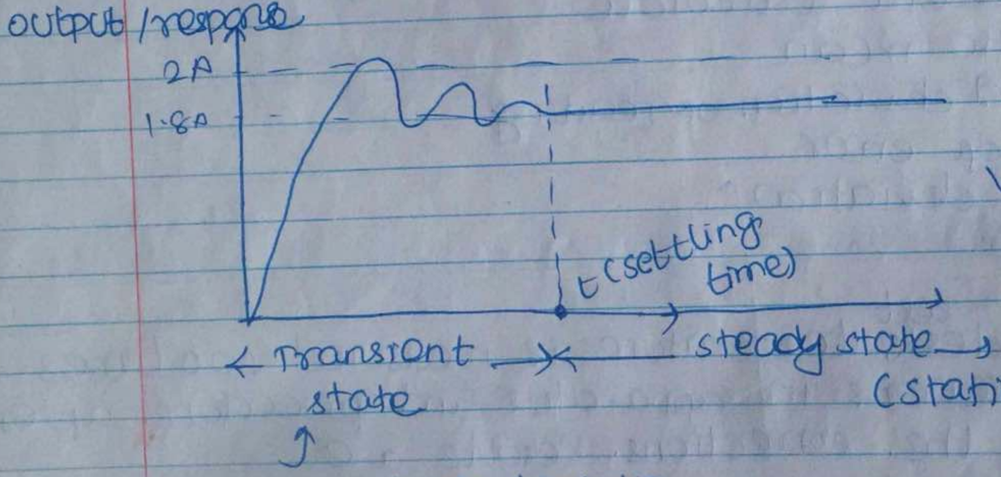
Calculate

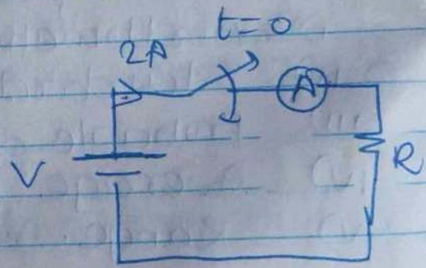
i) Mean ii) Mean deviation iii) std. deviation
iv) Probable error of reading
v) Probable error of mean.
vi) Standard deviation of mean.
vii) std. deviation of std. deviation.

Q.N.(

Static & Dynamic Characteristics of measurement system.







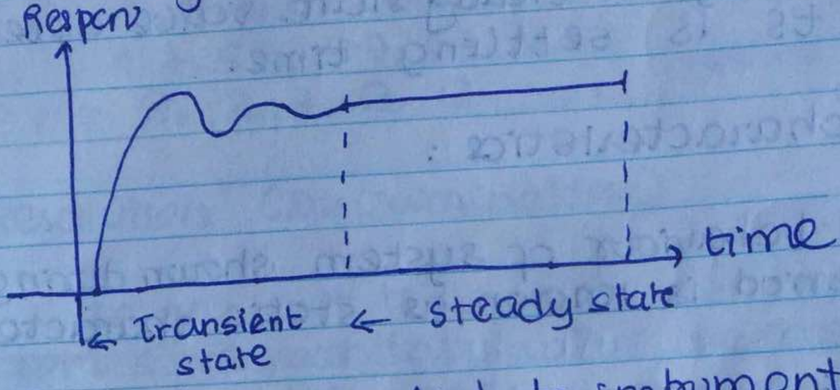
Dynamic characteristics

1) Rise time / speed of response
2) Measuring lag
3) Response time (settling time)

Static characteristics

1) Accuracy
2) Precision
3) Resolution
4) Sensitivity
5) Linearity.

Static & Dynamic Characteristics



When input is applied to instrumentation system (amm-eter) the final steady state is not reached immediately. The system goes through transient state and the behaviour shown during this transient period is known as dynamic characteristics of measurement system.

The dynamic characteristics are: are

① Speed of response: response

The rapidity with which measuring instrument responds to change in value of quantity under measurement.

2) Measuring Lag: Measuring Lag e e e e e e

Instrument doesnot immediately react to the change. Measuring lag is the delay in response of an instrument to the change in value of quantity under measurement. of of y g

③ Response time (settling time)

Time required by the instrumentation system to settle it's final steady state value after applying input. ts is settling time.

Static characteristics :

The behaviour of system shown during steady state period is known as static characteristics.

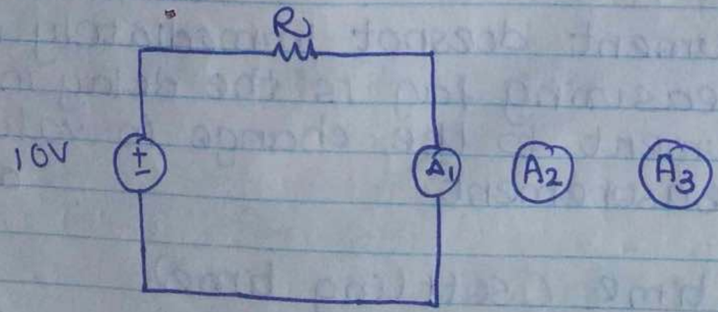
1) Accuracy :
It is the measure of closeness of measured value to the true value of quantity. It specifies difference between true value and measured value. For ckt components like resistor, inductor, capacitors

Accuracy = % of rated value

For voltmeter, ammeter,
Accuracy = % of full scale reading.

2) Precision (Repeatability)

It refers how close series of measurement values are with each other.

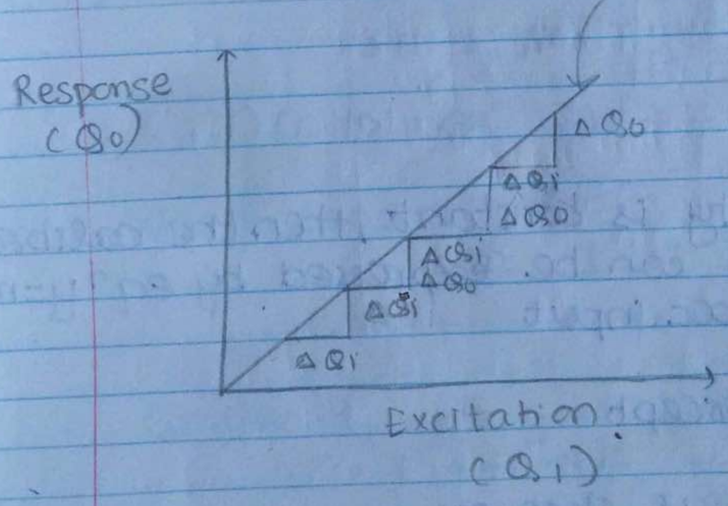


A1 = 4.6, 4.8, 5, 5.2, 5.4 . Accurate but not precise
A2 = 4.7, 4.7, 4.7, 4.7, 4.7 } Precise but not accurate
A3 = 3, 3, 3, 3, 3

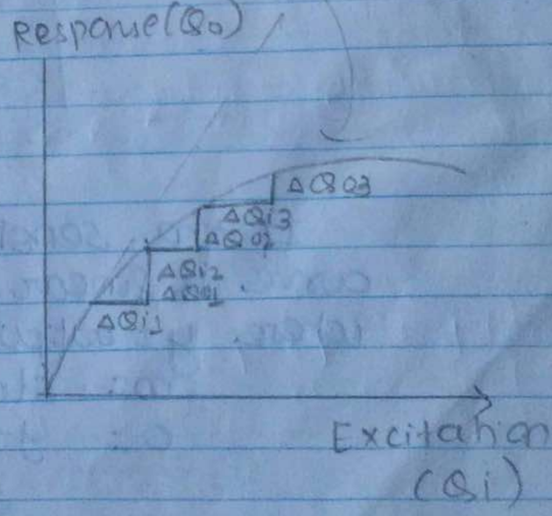
③ Resolution (Discrimination)

If an input to an instrument is slowly increased, from some arbitary value, the output, doesnot change until certain increament is excedeed. This increament is known as resolution. So, Resolution is defined as smallest change in input which results in detectable change in output. In case of analog instruments, it is significant of smallest division whereas in case of digital instrument, it is significant of LSB.

④ Sensitivity



i) Constant sensitivity



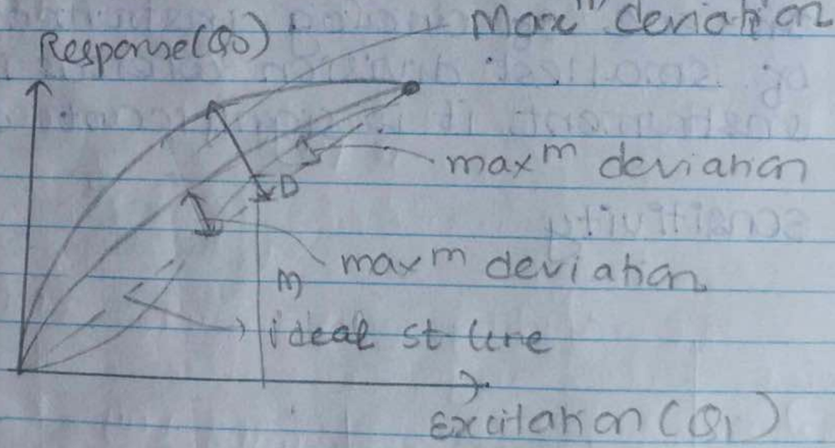
ii) Variable sensitivity

Sensitivity (S) = $\frac{\text{Small change in O/P}}{\text{Small change in I/P}}$

$$= \frac{\Delta Q_o}{\Delta Q_I}$$

The sensitivity of instrument is the ratio of magnitu of change in output to the change in input. It is the slope of calibration curve. When calibratio curve is linear, the sensitivity is constant thr -ghout whole range. However, if calibration cur is not linear sensitivity varies with excitatio

5) Linearity:



If the sensitivity is constant, then the calibrat curve is linear and can be expressed by eq^n y=mx+ where y= output, x= input
m= slope
c= y-intercept

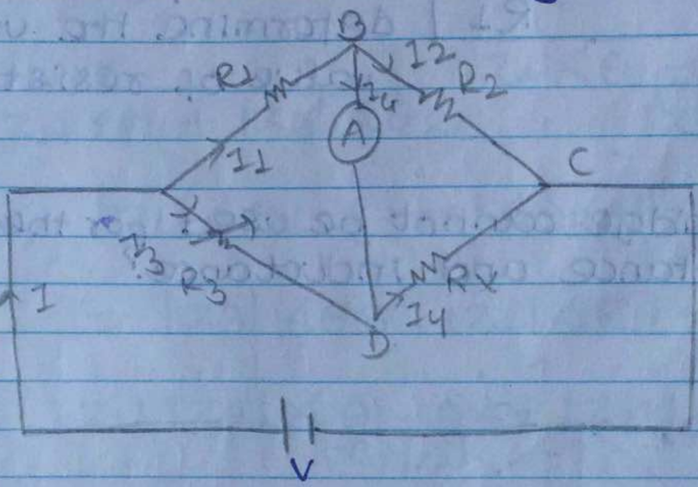
However, linearity is closeness of the calibratio curve of measuring system to the straight line It is expressed as % of deviation from strai line.

Linearity $= \frac{D}{M} \times 100\%$.

measurement of R, L, C:

Measurement of R

→ It can be measured using DC-bridge



$R_1, R_2 \rightarrow$ ratio arm
$R_3 \rightarrow$ variable decade resistance
$R_x \rightarrow$ unknown resistance.

At balanced condition : $I_G = 0$

i-e $V_B = V_D$.

or $V_{AB} = V_{AD}$

or $I_1 R_1 = I_3 R_3$ —— ①

At balanced condn:-

$$I_1 = I_2 = \frac{V}{R_1 + R_2} \quad \text{—— ②}$$

Similarly,

$$I_3 = I_4 = \frac{V}{R_3 + R_x} \quad \text{—— ③}$$

From ①, ② & ③,
$$\frac{V}{R_1 + R_2} \cdot R_1 = \frac{V}{R_3 + R_x} \cdot R_3$$

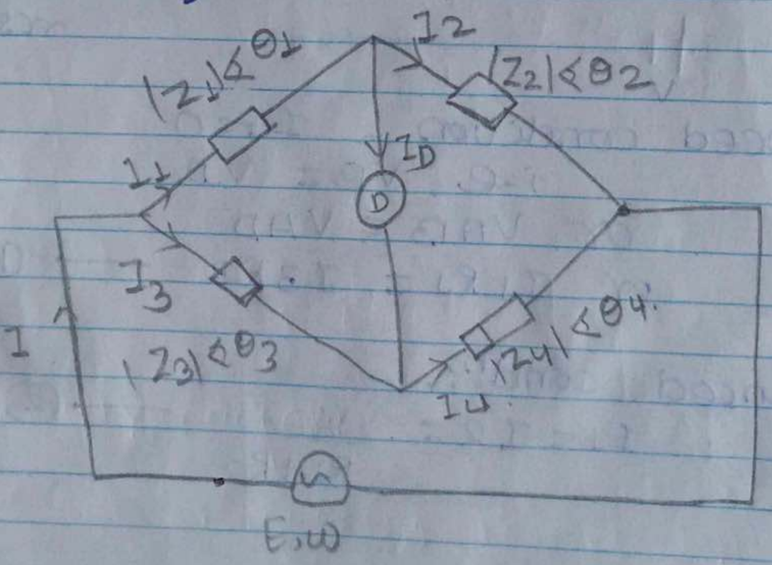
or $R_1R_3 + R_1R_x = R_1R_3 + R_2R_3$.

or $\boxed{R_1R_x = R_2R_3}$ It is required balance cond^n of DC-bridge.

So, $\boxed{R_x = \frac{R_2R_3}{R_1}}$ This eq^n can be used to determine the unknown value of resistance.

Q.N why DC bridge cannot be used for the measurement of capacitance and inductance?

AC Bridge:



For the AC bridge to be balanced,

$I_D = 0$.

i.e $V_B = V_D$

or $V_{AB} = V_{AB}$

or $I_1|Z_1|\angle\theta_1 = I_3|Z_3|\angle\theta_3$ —(1)

Similarly, $I_1 = I_2 = \dfrac{E}{|Z_1|\angle\theta_1 + |Z_2|\theta_2}$ —— (2)

And

$I_3 = I_4 = \dfrac{E}{|Z_3|\angle\theta_3 + |Z_4|\angle\theta_4}$ —— (3)

From (1), (2) & (3)

$$\frac{E}{|Z_1|\angle\theta_1 + |Z_2|\angle\theta_2}|Z_1|\angle\theta_1 = \frac{E\,|Z_3|\angle\theta_3}{(|Z_3|\angle\theta_3 + |Z_4|\angle\theta_4)}$$

or, $|Z_1|\angle\theta_1 (|Z_3|\angle\theta_3 + |Z_4|\angle\theta_4)$

$= |Z_3|\angle\theta_3 (|Z_1|\angle\theta_1 + |Z_2|\angle\theta_2)$

or, $|Z_1||Z_4| \angle\theta_1 + \theta_4 = |Z_2||Z_3| \angle\theta_2 + \theta_3$

So, for ac bridge to be balanced two cond^{n}s should be satisfied.

1) Magnitude cond^n

The product of magnitude of any two opposite arm impedances must be equal to that of other two

i.e $|Z_1||Z_4| = |Z_2||Z_3|$

ii) Angle cond^n

The sum of angle of any two opposite arms must be equal to that of other two.

$\angle\theta_1 + \theta_4 = \angle\theta_2 + \theta_3$

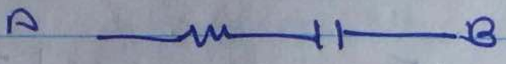
Measurement of Inductance

1) Maxwell's bridge $(1 < Q < 10)$
2) Hay's bridge $(Q > 10)$

Impedance :

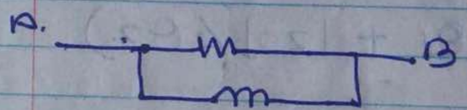
$Z_{AB} = R + jX_L$

$Z_{AB} = R + j\omega L$



$Z_{AB} = R - jX_C$

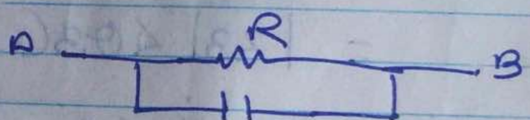
$Z_{AB} = R - \frac{j}{\omega C}$



$\frac{1}{Z_{AB}} = \frac{1}{R} + \frac{1}{jX_L}$

$\frac{1}{Z_{AB}} = \frac{1}{R} - \frac{j}{\omega L}$

$Z_{AB} = \frac{j\omega R L}{R + j\omega L}$



$\frac{1}{Z_{AB}} = \frac{1}{R} + \frac{1}{-jX_C}$

$= \frac{1}{R} + j\omega C$

$Z_{AB} = \frac{R}{1 + j\omega RC}$

Quality factor:

$Q \propto \frac{\text{Energy stored by coil in a cycle}}{\text{Energy consumed by a coil}}$

$Q = \frac{X_L}{R} = \frac{\omega L}{R}$

10 Measurement of resistances gave readings as 101.2, 101.7, 101.3, 101, 101.5, 101.3, 101.2, 101.4, 101.3, 101.1. Assume that only random errors are present. Calculate:

i) Arithmetic mean
ii) Standard deviation of reading
iii) Probable error
iv) Average deviation
v) Range.

Sol^n: Arithmetic mean = $\frac{101.2+101.7+101.3+101+101.5+101.3+101.2+101.4+101.3+101.1}{10}$

$\bar{x} = 101.3.$

x	$x-\bar{x}=d$	d^2
101.2	-0.1	0.01
101.7	0.4	0.16
101.3	0	0
101	-0.3	0.09
101.5	0.2	0.04
101.3	0	0
101.2	-0.1	0.01
101.4	0.1	0.01
101.3	0	0
101.1	-0.2	0.04
	$\Sigma d=$	$\Sigma d^2 = 0.36$

ii) Standard deviation of reading $\sigma = \sqrt{\frac{\Sigma d^2}{n}}$

$= \sqrt{\frac{0.36}{10}} = 0.18\ \Omega.$

Sample standard deviation (s) = $\sqrt{\frac{\Sigma d^2}{n-1}}$

$= \sqrt{\frac{0.36}{9}}$

$= 0.208\ \Omega.$

iii) Probable error of one reading

$r_1 = 0.6745\,s$

$= 0.6745 \times 0.208$

$= 0.14\ \Omega$

iv) Average deviation = $\frac{\Sigma|d|}{n} = \frac{1.1}{10} = 0.11$

v) Range = Highest term − lowest term

$= 101.7 - 101$

$= 0.7.$

② Soln.

Temp°C	397	398	399	400	401	402	403	404	405
Frequency	1	3	12	23	37	16	4	2	2.

To calculate:

a) Arithmetic mean b) mean deviation
c) std deviation d) Probable error of reading
e) Probable error of mean
f) std. deviation of mean
g) std. deviation of std deviation

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Tempʳ T°C	Frequency of occurence, f	T×f	Deviation d	f×d	d^2	$f\times d^2$
397	1	397	−3.78	−3.78	14.288	14.288
398	3	1194	−2.78	−8.34	7.728	23.185
399	12	4788	−1.78	−21.36	3.168	38.020
400	23	9200	−0.78	+17.94	0.608	13.993
401	37	14837	+0.22	+8.14	0.048	1.708
402	16	6432	+1.22	+19.52	1.488	23.814
403	4	1612	+2.22	+8.88	4.928	19.714
404	2	808	+3.22	+6.44	10.368	20.737
405	2	810	+4.22	+8.44	17.808	35.618
	100	40078		$\Sigma\lvert f\times d\rvert = 102.8$		$\Sigma fd^2 = 191.08$

a) Mean temperature $= \frac{40078}{100} = 400.780°C$

b) mean deviation $\bar{D} = \frac{102.8}{100} = 1.028°C$

c) Standard deviation (σ) ~~$\bar{D} = \frac{102.8}{100} = 1.028°C$~~

$$= \sqrt{\frac{191.08}{100}} = 1.380°C.$$

d) Probable error of one reading $r_1 = 0.6475\ \sigma$

$= 0.6745 \times 1.38$

$= 0.93°C.$

e) Probable error of mean $r_m = \frac{0.93}{\sqrt{100}} = 0.093°C.$

Standard deviation of mean $\sigma_m = \frac{\sigma}{\sqrt{n}}$

$= \frac{1.38}{\sqrt{100}} = 0.138°C$

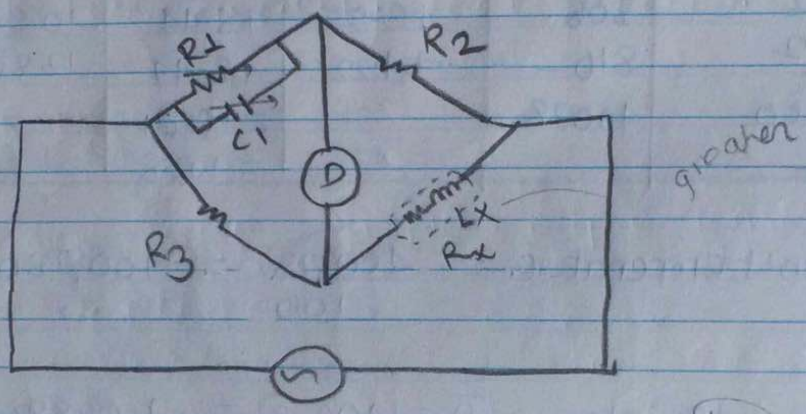
f) Standard deviation of standard deviation

$$\sigma\sigma = \frac{\sigma m}{\sqrt{2}} = \frac{0.138}{\sqrt{2}}$$

$$= 0.0796°C$$

Measurement of inductance (L)

<u>Maxwell's Bridge:-</u>



Under balanced condition,

$$Z_1 . Z_x = Z_2 . Z_3$$

Here,

$$Z_2 = R_2 \quad , \quad Z_3 = \cancel{Z_2} \; R_3$$

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{-jX_C} = \frac{1}{R_1} + j\omega C$$

$$\frac{1}{Z_x} = R_x + jX$$

or, $$Z_x = \frac{1}{Z_1} Z_2 Z_3 .$$

$$Z_x = \left(\frac{1}{R_1} + j\omega C\right) R_2 R_3$$

$$R_x + jX = \frac{R_2 R_3}{R_1} + j\omega C R_2 R_3$$

Comparing Real & Imaginary part separately,

$$\boxed{R_x = \frac{R_2 R_3}{R_1}} \quad \text{——— (1)}$$

and $X = \omega C R_2 R_3$

$\omega L_x = \omega C R_2 R_3$

Sliding effect

$$\boxed{\therefore L_x = C R_2 R_3} \quad \text{——— (2)}$$

These two eq^n s (1) & (2) can be used to obtain balanced cond^n and thus give value of unknown resistance & inductance.

Q) why maxwell's bridge (maxwell's Inductance /Capacitance bridge) cannot be used for high & low Q-factor coils?

For high Q-factor coils ($Q>10$)

The angle of unknown branch will be nearly equal to $+90°$ But the sum of angles of branch (2) & (3) $\theta_2 + \theta_3 = 0$

So, To satisfy angle cond^n

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

$$\theta_1 \approx -90°$$

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For this, the value of resistance R_1 should be very high. As the cost of decade resistance and it's inaccuracy increases with the resistance value which makes the bridge impractical for high quality factor coils.

For low quality factor coils (the bridge will be) it will be very difficult to obtain the balanced condition and so it is not used for low quality factor coils.

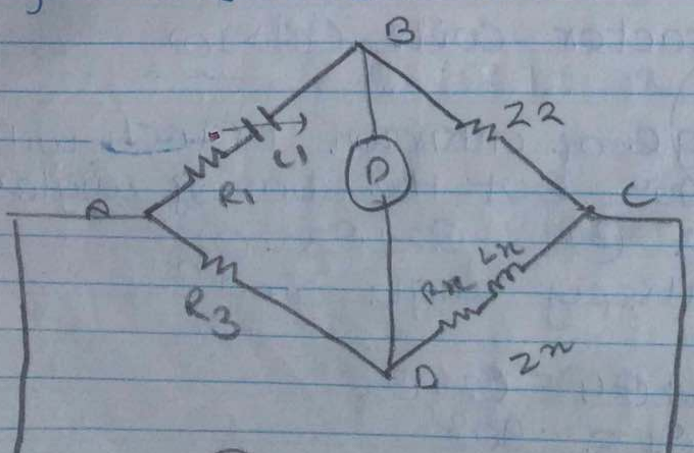
Advantages:

1. The bridge gives simple expression for unknown values of resistance and inductance in terms of known elements.

   i.e $Rx = \frac{R_2 R_3}{R_1}$ $Lx = C_1 R_2 R_3$

2. Both ~~the~~ equations are independent of frequency.

Hay's Bridge: $(Q > 10)$



Here, $Z_1 = R_1 - \frac{j}{\omega C}$

$Z_2 = R_2$, $Z_3 = R_3$

$Z_x = R_x + j\omega L_x$

And under balanced cond^n

$\cancel{Z_1 \cdot Z_x = Z_2 \cdot Z_3}$

$$Z_1 Z_x = Z_2 Z_3$$

$$\left(R_1 - \frac{j}{\omega C_1}\right)\left(R_x + j\omega L_x\right) = R_2 R_3$$

or $\left(R_1 R_x + \frac{L_x}{C_1}\right) + j\left(\omega R_1 L_x - \frac{R_x}{\omega C_1}\right) = R_2 R_3$

Equating real & imaginary parts,

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{—— (i)}$$

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0.$$

$$L_x = \frac{R_x}{\omega^2 C_1 R_1} \quad \text{—— (ii)}$$

Solving (i) x (ii), we get

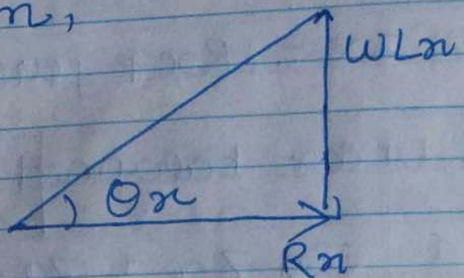
$$\left[\; R_x = \frac{\omega^2 R_1 R_2 R_3 C_1^2}{\omega^2 R_1^2 C_1^2 + 1} \;\right] \quad \text{—— (iii)}$$

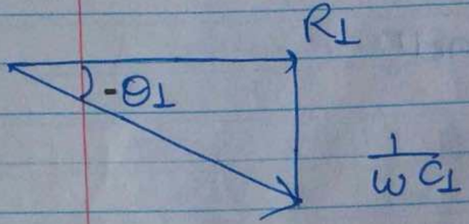
And,

$$\left[\; L_x = \frac{C_1 R_2 R_3}{\omega^2 R_1^2 C_1^2 + 1} \;\right] \quad \text{—— (iv)}$$

Eq^n (iii) & (iv), gives the expression for R_x & L_x for unknown coil.

for unknown arm,





For balanced $cond^n$.

$$\theta_2 + \theta_3 = (-\theta_1) + \theta_x$$

$$\theta_1 = +\theta x \quad [\text{as } \theta_2 + \theta_3 = 0].$$

Taking tangent on both side,

$$\tan \theta x = \tan \theta_1.$$

$$\frac{\omega L_x}{R_x} = \tan \theta_1$$

$$Q = \tan \theta_1$$

Here, $\tan \theta_1 = \frac{1}{\omega C_1 R_1}$ ——— (v)

~~For~~ From eq^n (5)

$$Q_1 = \frac{1}{\omega C_1 R_1}$$

Here for high quality factor (Q), we have to take lower value of resistance R_1.

From eq^n (4),

$$L_x = \frac{C_1 R_2 R_3}{\omega^2 R_1^2 C_1^2 + 1}$$

$$= \frac{C_1 R_2 R_3}{\left(\frac{1}{Q}\right)^2 + 1}$$

For high Q factor $Q > 10$,

$$1 >> \left(\frac{1}{Q}\right)^2$$

So,

$$L_x = C_1 R_2 R_3$$

This expression is same as the expression for maxwell's bridge and so balanced cond^n is achieved easily for high Q-coils.

Advantages:

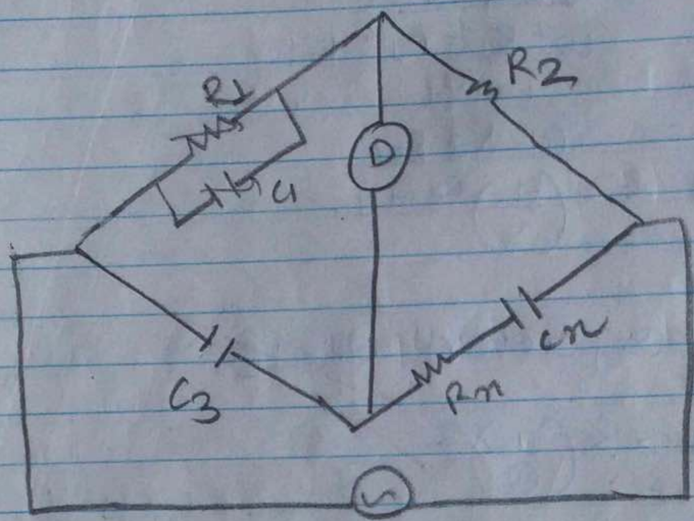
For high Q-coils, balanced cond^n can be achieved easily and the bridge will be economic.

Disadvantage:

It cannot be used for low Q-coil because $\left(\frac{1}{Q}\right)^2$ cannot be neglected if $Q < 10$ and the expression becomes complex.

Measurement of C

Schering's Bridge



$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = -\frac{1}{\omega C_3}$$

$$Z_x = R_x - \frac{j}{\omega C_x}$$

Under balanced condition,

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{1}{Z_1} Z_2 Z_3.$$

or, $Z_x = \left(\frac{1}{R_1} + j\omega C_1\right) \times R_2 . \left(\frac{-j}{\omega C_3 R_1}\right)$

or $Z_x = \frac{\omega C_1 R_2}{\omega C_3} - \frac{jR_2}{\omega C_3}$

$$R_x - \frac{j}{\omega C_x} = \frac{C_1 R_2}{C_3} - \frac{j R_2}{\omega C_3}$$

Comparing $R_x = \frac{C_1}{C_3} R_2$

And, $C_x = \frac{R_1}{R_2} C_3$ —— (ii)

eq^n (i) & (ii) give the values of capacitance & resistance is known as branch.

Q) The bridge is balanced at 1000 Hz and has following constants.

Arm AB 0.2 μf pure capacitance

Arm AD 50 Ω pure resistance

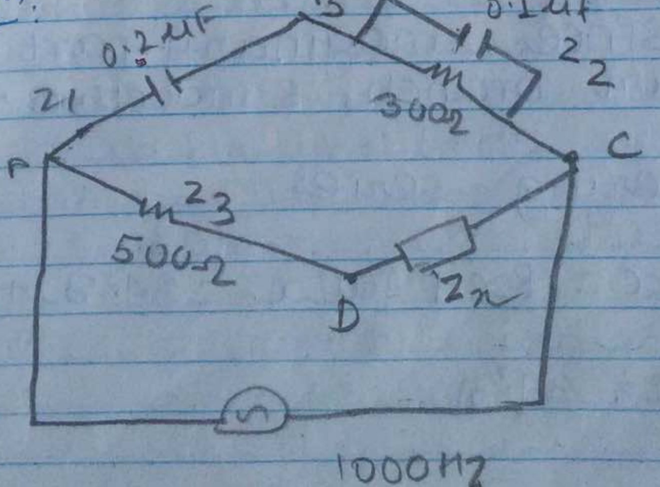
Arm BC R = 300 Ω in parallel with C = 0.1 μf

Find R & "C" or "L" constants of arm CD

i) considering as series

ii) considering as parallel

Sol^n:



Since there is balanced condⁿ

$$Z_x Z_1 = Z_2 Z_3$$

$$\text{or } Z_x = \frac{1}{Z_1} Z_2 Z_3$$

$$Z_1 = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 1000 \times 0.2 \times 10^{-6}}$$

$$= -j\,795.77\,\Omega.$$

$$Z_2 = \frac{R}{1 + j\omega CR} = \frac{300}{1 + j\,2\pi \times 1000 \times 0.1 \times 10^{-6} \times 300}$$

$$= \cancel{294.8}$$

$$= 289.7 - 54.608j$$

$$Z_3 = 500\,\Omega.$$

$$Z_x = \frac{1}{-j\,795.77} \times (289.7 - 54.608j) \times 500$$

$$\boxed{Z_x = 34.311 + j\,182.02.}$$

~~If consi~~ Since, imaginary part is +ve, the unknown branch is inductive.

Now, Considering series

$$Z_x = R_x + j\omega L_x = 34.31 + j\,182.02$$

So, we get,

$$R_x = 34.31$$

$$L_x = \frac{182.02}{\cancel{34.31}\ 2\times\pi\times1000} = 0.028\ H$$

$$\therefore\ L_x = 28\ mH$$

Considering parallel ckt

$$\frac{1}{Z_x} = \frac{1}{R_x} - \frac{j}{\omega L_x}$$

$$\text{or,}\quad \frac{1}{34.31 + j182.02} = \frac{1}{R_x} - \frac{j}{\omega L_x}$$

$$\text{or,}\quad 10^{-3} - j5.3\times10^{-3} = \frac{1}{R_x} - \frac{j}{\omega L_x}$$

$$R_x = 10^{3} = 1\,K\Omega \quad \text{and} \quad L_x = \frac{1}{\omega\times5.3\times10^{-3}} = \frac{1}{2\pi\times1000\times5.3\times10^{-3}}$$

$$\therefore\ L_x = 30\,mH.$$

A moving coil voltmeter have uniform scale with 100 divisions and give full scale reading of 200V. The instrument read up to 1/5 th of a scale division with fair degree of ~~accur~~ certainity. Determine resolution of instrument in volt.

2 Sol^{n}:- Full scale reading = 200V

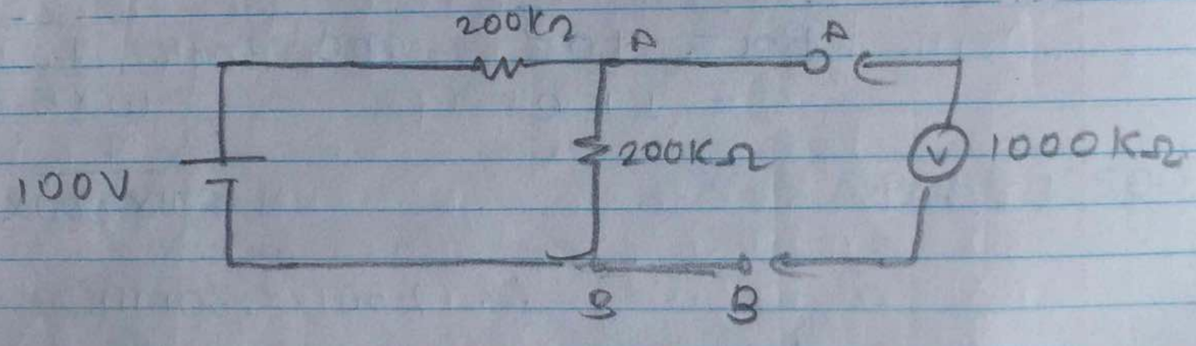
no. of division = 100

1 scale division $= \frac{200}{100} = 2V$

Since instrument can read upto $\frac{1}{5}$th of this small division

So, Resolution $= \frac{1}{5} \times 2 = 0.4$ V.

Q) A 50V range voltmeter is connected across the terminals AB of the circuit below. Find the reading of the voltmeter under open circuit and loaded condition. Find accuracy and loading error (% error). The voltmeter has resistance of 1000kΩ.



Actual reading or open circuit voltage
$= \frac{200}{200+200} \times 100$
$= 50V$

But, when voltmeter is loaded, R_{AB}
$= \frac{200 \times 1000}{200+1000} = 166.67\Omega.$

So, voltmeter reading $= \frac{166.67}{166.67+200} \times 100$
$= 45.45$ V.

$\therefore$ Voltmeter reading $= 45.45V$

$\% \text{ error} = \frac{50-45.45}{50} \times 100\%$
$= 9.1\%$.

Accuracy $= 100 - 9.1\% = 90.9\%$

So, if we wish to achieve high accuracy in voltage measurement, input resistance of voltage should be very high as compared to circuit resistance.

Classification of Sensors:

On the basis of application of external energy.

a) Passive sensor
b) Active sensor ~~Ex~~ Eg: thermocouple.

Potentiometer is position sensor (Passive sensor)

On the basis of physical principle involved:

(Potentiometer, Strain guage) Thermo couple
a) Resistive sensor d) Thermoelectric sensor LVDT
b) Inductive sensor LVDT e) Electromagnetic sensor
c) Capacitive sensor f) Piezoelectric sensor
LVC

g) Hall effect sensor.

1) Resitive sensor

a) Potentiometer (POT)

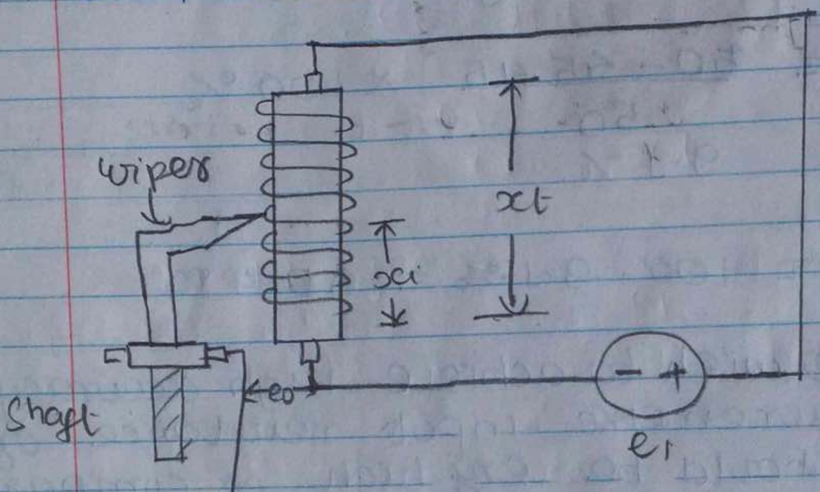


fig: Linear POT

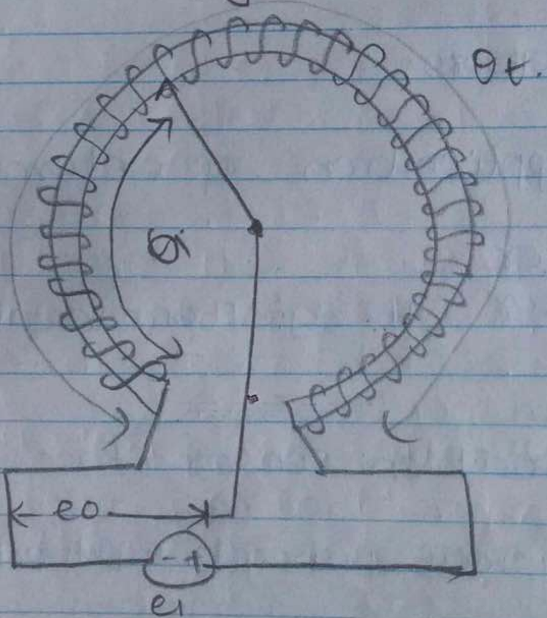


fig: Rotatory POT

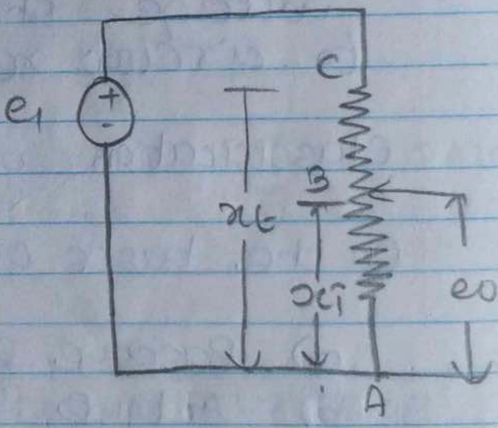


fig: Equivalent ckt of linear POT.

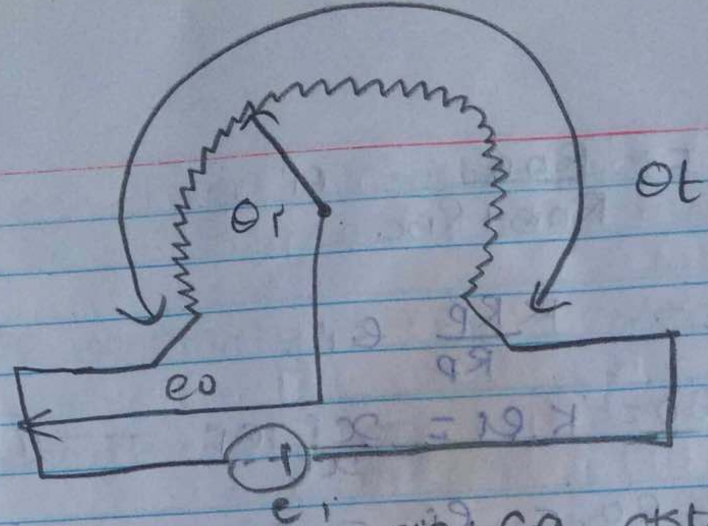


fig: Eq. ckt of Rotatary POT.

Resistive Derivation for output

Let us take linear POT,
Here,

e_i = i/p voltage
e_o = O/p voltage
x_i = Displacement of wiper with respect to zero position
x_t = Total length of POT wire
R_p = Total resistance of POT wire

Resistance per length $= \frac{R_p}{x_t}$.

Resistance of displacement $(R_{AB}) = \frac{R_p}{x_t} x_i$

or $R_{AB} = \frac{x_i}{x_t} \cdot R_p$

or $R_{AB} = K \cdot R_p$. where $[0 \le K \le 1]$.

So, The o/p voltage at ideal condn due to displacement x_1 is.

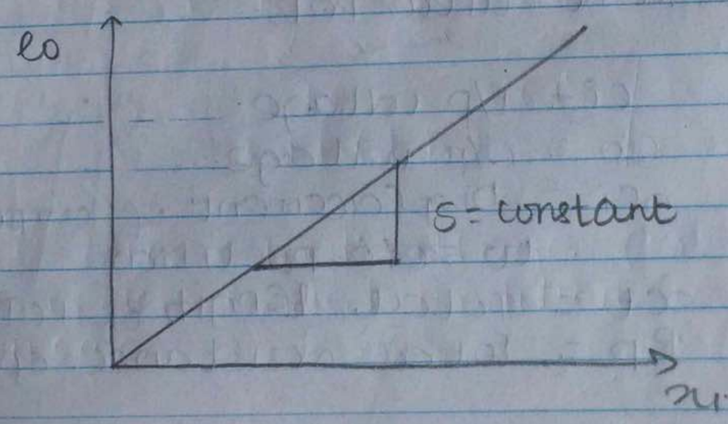
$$e_o = \frac{R_{AB}}{R_{AB}+R_{BC}} e_i$$

$$= \frac{K R_P}{R_P} e_i$$

or, $$e_o = K e_i = \frac{x_i}{x_t} e_i \quad \text{--- (1)}$$

or $$\frac{e_o}{x_i} = \frac{e_i}{x_t} = \text{constant}$$

$$\text{sensitivity } (S) = \frac{e_o}{x_i} = \text{constant}$$



It shows the sensitivity of POT at ideal condⁿ is constant & hence the input - output curve is linear.

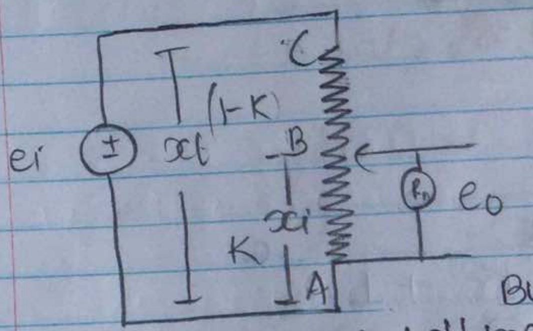
Similarly, the same steps can be applied for rotatory potentiometer & the output will be similar as eqⁿ (1) by replacing x_i & x_t by 'θ_i' & 'θ_t'

or $$e_o = K e_i = \frac{\theta_i}{\theta_t} e_i \quad \text{--- (1)}$$

$$\frac{e_o}{\theta_i} = \frac{e_t}{\theta_t} = \text{constant}$$

i.e sensitivity $= \frac{e_o}{\theta_i}$ = constant

Loading effect in POT



If the resistance of voltmeter is infinite, the output voltage e_o is given by

$$e_o = Ke_i = \frac{x_i}{x_t} e_i \quad \text{— (1)}$$

But, in actual practice, the input resi voltmeter resistance R_m is finite and so the reading indicated by voltmeter is always less than the actual value, which is known as loading effect or loading error. Due to loading error, there exist non-linear relationship between input and output voltage of potentiometer.

Here,

The resistance $R_{AB} = KR_p \parallel R_m$

$$= \frac{KR_pR_m}{KR_p + R_m}.$$

The output voltage due to loading effect is given by

$$e_o = \frac{R_{AB}}{R_{AB} + R_{BC}} e_i$$

$$= \frac{\frac{KR_pR_m}{KR_p + R_m}}{\frac{KR_pR_m}{KR_p + R_m} + (1-K)R_p} e_i$$

$$e_o = \frac{K R_p R_m}{K R_p^2 - K^2 R_p^2 + R_m R_p} e_i$$

$$\text{or } e_o = \frac{K}{K \frac{R_p}{R_m} - K^2 \frac{R_p}{R_m} + 1} e_i \quad \text{———} \ (2)$$

let $\frac{R_m}{R_p} = \alpha$

$$\text{So, } e_o = \frac{K}{\frac{K}{\alpha} - \frac{K^2}{\alpha} + 1} \cdot e_i$$

$$\boxed{e_o = \frac{\alpha K}{\alpha + K(1-K)} e_i} \quad \text{———} \ (3)$$

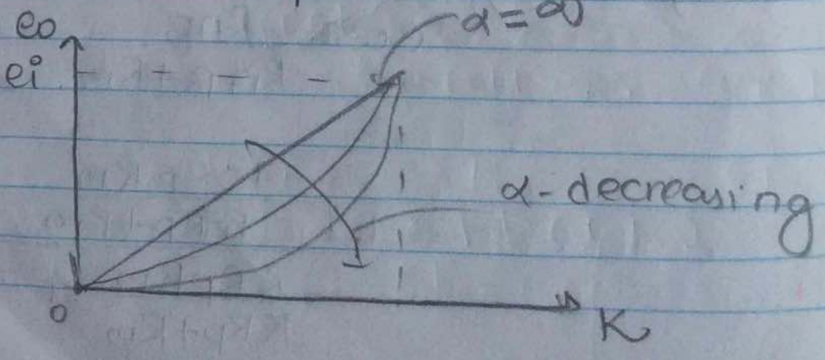
eq^n (3) gives output voltage for potentiometer sensor under loaded condition which clearly is non-linear.

If $R_m \to \infty$, $\alpha \to \infty$

$$e_o = K e_i$$

which is same as eq^n (1)

Graphically,



Loading error

① Relative error

$$E_r = \frac{T.V - M.V}{T.V.}$$

$$= \frac{\text{o/p without loading effect} - \text{o/p with loading effect}}{\text{o/p without loading effect.}}$$

$$E_r = \frac{Kei - \frac{\alpha K}{\alpha + K(1-K)} ei}{Kei}$$

$$E_r = \frac{K(1 \cancel{-} K)}{\cancel{Kei}} \quad \frac{K(1-K)}{\alpha + K(1-K)}$$

Therefore relative error depends upon value of K and α where K is the position of wiper.

To find out value of K for which Er is maxim-um, we have,

$$\frac{\partial E_r}{\partial K} = 0.$$

$$\frac{\partial}{\partial K}\left[\frac{K(1-K)}{\alpha + K(1-K)}\right] = 0.$$

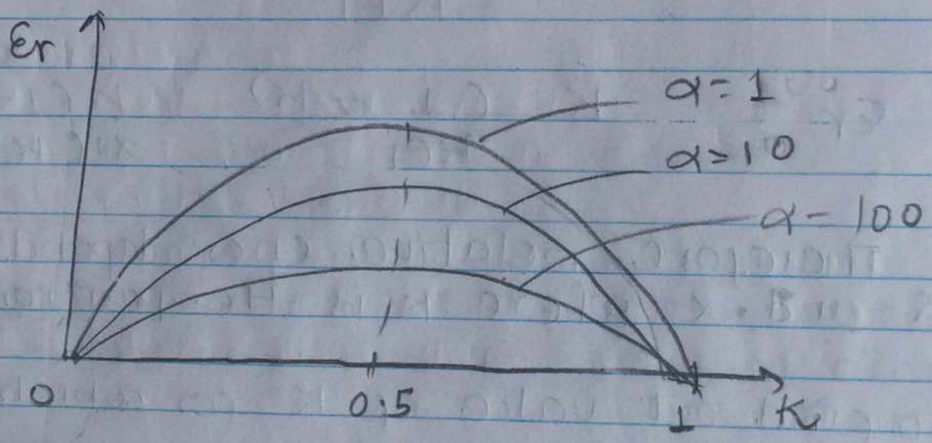
$$\text{or, } \alpha(1-2K)\left(\qquad \right) = 0.$$

since $\alpha = \frac{R_m}{R_p} > 0$, so, $1-2K$ should be equal to zero

$$1-2K=0$$
$$K=\frac{1}{2}=0.5$$
$$\text{i.e. } \frac{x_i}{x_t}=0.5$$

This, the relative error is maximum when $K=0.5$ i.e when wiper is at midpoint of ~~wiper~~ potentiometer

Plotting for different value of K & α we get:



ii) Absolute error:

$$\text{Absolute error } (\epsilon_a) = \frac{\text{o/p voltage w/o loading effect} - \text{o/p voltage with loading effect}}{\text{i/p voltage } (e_i)}$$

$$= \frac{Ke_i - \frac{\alpha K}{\alpha + K(1-K)} e_i}{e_i}$$

$$\boxed{E_a = \frac{K^2(1-K)}{\alpha + K(1-K)}} \quad \text{——— (1)}$$

$$\boxed{\therefore \quad E_a = K E_r}$$

The value of 'K' for which 'E_a' is maximum can be obtained as,

$$\frac{\partial E_a}{\partial K} = 0$$

$$\text{or } \frac{\partial}{\partial K}\left\{\frac{K^2(1-K)}{\alpha + K(1-K)}\right\} = 0$$

$$\text{or } (\alpha + (1-K)K)(\alpha K(1-K) - K^2)$$
$$- K^2(1-K)(1-\alpha K) = 0$$

For above eqn.

$\alpha = 1, \quad K = 0.688$
$\alpha = 10, \quad K = 0.669$
$\alpha = 100, \quad K = 0.6669.$

Hence, for $\frac{R_m}{R_p} > 10$. the maximum error occur at $K = 0.67 = 2/3$

And the error value is:

$$E = \frac{K^2(1-K)}{\alpha + K(1-K)}$$

$$E \simeq \frac{K^2(1-K)}{\alpha} \quad \text{Since } \alpha > 10 \text{ \& } K < 1$$

And $E_{a\,max} = \frac{0.67^2 (1-0.67)}{\alpha}$

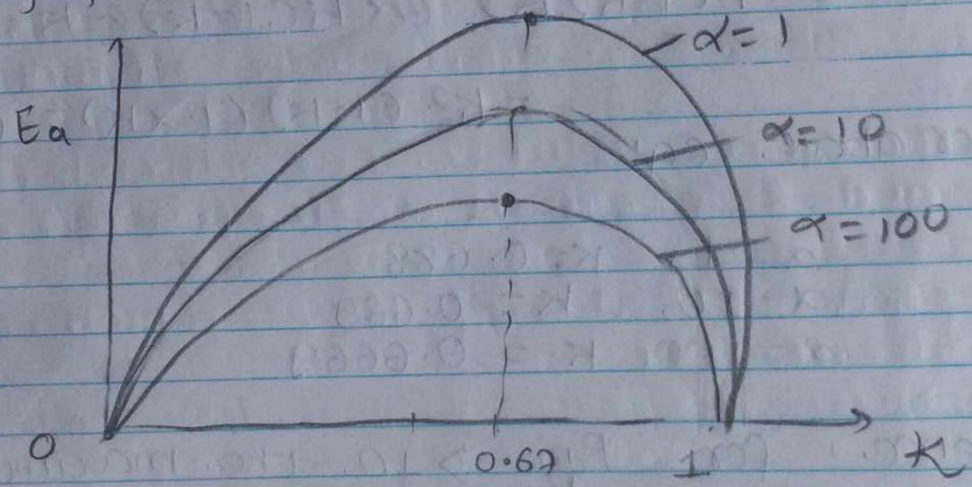
In percentage

$= \frac{\cancel{0.3649}\ 0.148}{\alpha} \times 100\%$

$= \frac{14.8}{\alpha} \%$

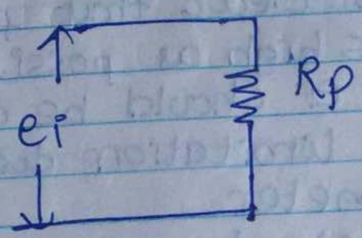
$E_{a\,max} \simeq \frac{15}{\alpha} \%$

In graph,



Power rating of POT

The power rating of POT is the ~~set~~ safe value of it heat which can be (separately) dissipated by potentiometer without any damage.



As we know,

$$P = \frac{e_i^2}{R_p}$$

For max^m power

$$P_{max} = \frac{e_{i\,max}^2}{R_p}$$

$$e_{i\,max} = \sqrt{P_{max} R_p}$$

$e_{i\,max}$ is the max^m value of input voltage that can be applied to potentiometer.

Q) Linearity and sensitivity of potentiometer are two conflicting requirements. Justify this statement.

As, we know,

o/p eq^n of potentiometer

$$e_o = \frac{K}{\frac{K}{\alpha} - \frac{K^2}{\alpha} + 1} e_i$$

For linearity, $\alpha \to \infty$

$\frac{R_m}{R_p} \to \infty$ $\quad R_m \to$ Resistance of meter

$R_p \to 0$, i.e R_p should be as small as possible ($R_p << R_m$)

तलको loading effect due to non-linearity

But sensitivity is given as,

$$\text{sensitivity} = \frac{\text{magnitude of o/p}}{\text{magnitude of i/p}} = \frac{e_o}{x_i}$$

From this, it is clear that for high sensitivity e_o should be as high as possible. for this, supply voltage e_i should be as high as possible. But, e_i has limitation due to heat dissipation in potentiometer.

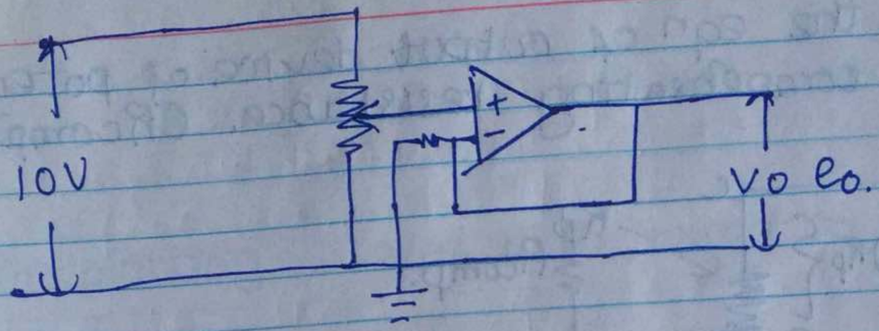
So, heat dissipation limit $(p) = \frac{e^2_{max}}{R_p}$

So, to keep sensitivity high and power dissipation unchanged we have to increase R_p along with excitation voltage e_i.

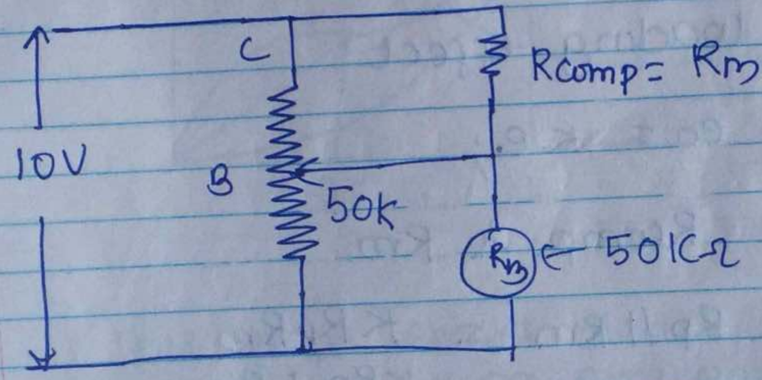
So, from above analysis, to improve linearity, we have to decrease R_p whereas to improve sensitivity we have to increase R_p. So, to achieve one we have to compromise another and ~~ther~~ thus, they are conflicting requirements.

Methods to reduce loading effect

1) use Digital voltmeter instead of analog
2) use Buffer amplifier



3) Using compensating Resistance

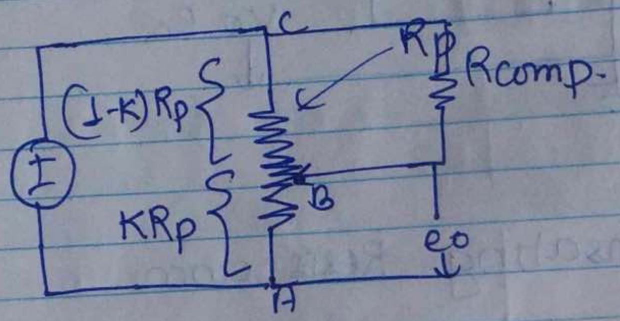


Truth value $= 10 \times \frac{50}{100} = 5V$

M. V (with meter) $= 10 \times \frac{25}{100} = \cancel{5V} = 3.33V$

with compensate
Resistance $(e_o) = \frac{25}{25+25} \times 10 = 5V.$

Derive the eqn of output device of potentiometer using compensating resistance (Rcomp).



without loading effect

$$e_0 = K e_i$$

But with Rcomp & Rm

$$R_{AB} = K R_p \parallel R_m = \frac{K R_p R_m}{K R_p + R_m}$$

$$R_{BC} = (1-K) R_p \parallel R_{comp} = \frac{(1-K) R_p R_m}{(1-K) R_p + R_m}$$

Now,

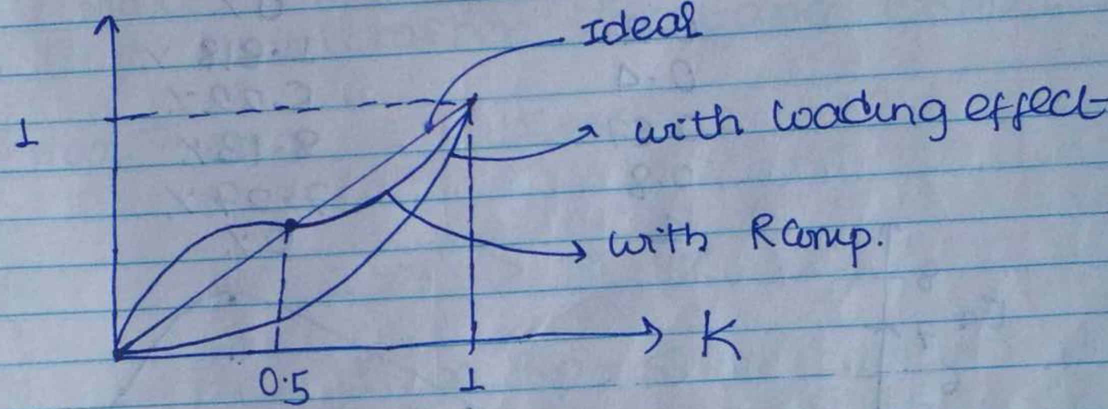
$$e_0 = \frac{R_{AB}}{R_{AB} + R_{BC}} \times e_i$$

$$= \frac{\frac{K R_p R_m}{K R_p + R_m}}{\frac{(1-K) R_p R_m}{K R_p + R_m} + \frac{(1-K)(R_p R_m)}{(1-K) R_p + R_m}} e_i$$

$$\frac{R_m}{R_p} = \alpha$$

$$e_0 = \frac{K(1-K+\alpha)}{K(1-K+\alpha) + (1-K)(K+\alpha)} e_i \quad \text{--- (1)}$$

On plotting e_0/e_i with respect to K for $\alpha = 1$



This graph shows clearly that loading effect decreases on using compensating resistance and is zero at midpoint of potentiometer.

Numerical :

A 5000Ω voltage dividing potentiometer feeds load of 8000Ω. Calculate the % error based on full scale. of (slider position at) POT at slider position 0.02, 0.4, 0.67, 0.8, 1 per unit of total travel. Plot the graph between per unit travel and % error.

Sol^n $R_p = 5000$ Here, $\alpha = \frac{R_m}{R_p} = \frac{8000}{5000}$

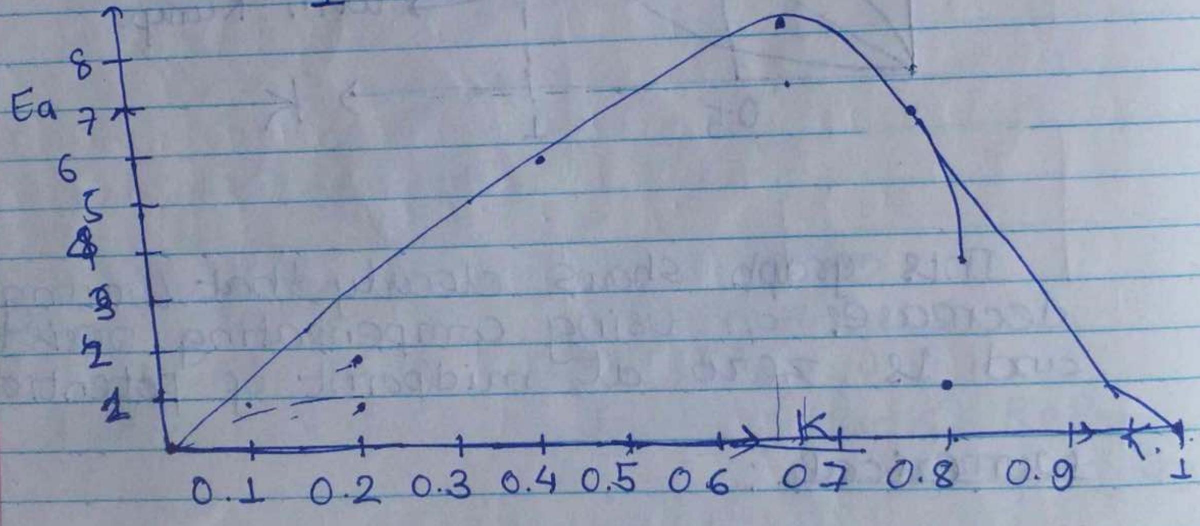
$R_m = 8000$

$= 1.6$

we know,

$$E_a = \frac{K^2 (1-K)}{\alpha + K(1-K)} \times 100\ \%$$

K	Ea
0	0 %
0.2	1.818 %
0.4	6.22 %
0.67	8.13 %
0.8	7.27 %
1	0 %



Q. The output of potentiometer is to be read by a recorder of 10 KΩ resistance. The non-linearity ~~of~~ must be held at 1 %. A family of potentiometer having thermal rating of 5W and resistance ranging from 100Ω to 10 KΩ in 100Ω steps are available. Choose from the potentiometers family that has the greatest possible sensitivity and meets the

non-linearity requirement. Find the maximum excitati-
-on voltage permissible with this potentiometer.
what is the sensitivity if potentiometer is
single turn (360° units).

Soln:

Maxm non-linearity = 1% {Error}
voltmeter resistance (Rm) = 10KΩ

$$\text{max}^m \text{ error} = \frac{15}{\alpha} \%$$

$$\text{or } 1\% > \frac{15}{\alpha} \%$$

$$\text{or } \frac{15}{R_m} \times R_p < 1$$

$$\text{or } R_p < \frac{1 \times R_m}{15}$$

$$\text{or } R_p < \frac{1 \times 10000}{15}$$

$$\text{So, } R_p < 666.67\,\Omega$$

For sensitivity, we chose Rp as high as possible.

But, we have resistances,

100, 200, ... 600, 700, 800; . . . 10KΩ

So, To achieve best sensitivity,
we chose, $R_p = 600\,\Omega$

$$P = \frac{e_i^2}{R_p}$$

$$5 = \frac{e_i^2}{600} \Rightarrow e_i = 54.77$$

Power rating (P) = ~~600 Ω~~ 5 watt

$$\text{sensitivity} = \frac{\text{output } (e_o)}{\text{Input } (\theta_i)} = \frac{e_i}{\theta_t}$$

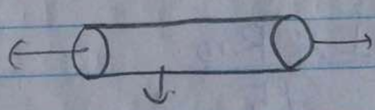
$$= \frac{54.77}{360}$$

$$= 0.0152 \text{ V/degree}$$

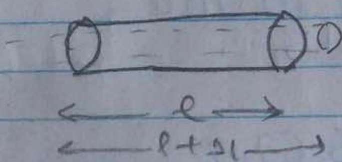
So, ~~$R_p < 666.67\,\Omega$~~

And For sensitivity we chose R_p as high as possible

Strain Gauge



$\rightarrow R = \frac{\rho l}{A}$ —(1)



$\downarrow A$ $2\Delta A + \Delta A$ & $\Delta A = -ve$)

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{original dimension}}$$

$$\frac{\Delta R}{\text{strain}} = ?$$

When stress S is applied, there will be change in resistance not only due to change in length and change in area (change in physical dimension but also due to change in resistivity. This effect is known as piezo-resistive effect and the small change in resistivity that occurs due to stress is known as piezo-resistivity

$$R = \frac{\rho l}{A} \quad \text{—— (1)}$$

Diff (1) with respect to stress 'S'.

$$\frac{\partial R}{\partial S} = \frac{\rho}{A}\frac{\partial l}{\partial S} - \frac{\rho l}{A^2}\frac{\partial A}{\partial S} + \frac{l}{A}\frac{\partial \rho}{\partial S} \quad \text{—— (II)}$$

Now dividing (II) by (1), we get

$$\frac{1}{R}\frac{\partial R}{\partial S} = \frac{1}{l}\frac{\partial l}{\partial S} - \frac{1}{A}\frac{\partial A}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \quad \text{—— (3)}$$

If variation is very small.

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho} \quad \text{—— (4)}$$

So, per unit change in resistance depends upon

i) per unit change in length
ii) per unit change in area
iii) per unit change in resistivity.

But, $A = \frac{\pi D^2}{4}$ —— (5)

Diff. with respect to S,

$$\frac{\partial A}{\partial S} = \frac{\pi}{4} 2D \cdot \frac{\partial D}{\partial S} \quad \text{—— (6)}$$

Dividing 6 by 5

$$\frac{1}{A}\frac{\partial A}{\partial S} = \frac{2}{D}\frac{\partial D}{\partial S} \quad \text{— } 7.$$

Substituting 7 in eqn 3

$$\frac{1}{R}\frac{\partial R}{\partial S} = \frac{1}{l}\frac{\partial l}{\partial S} - \frac{2}{D}\frac{\partial D}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S}$$

Again for small variation:

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{2\Delta D}{D} + \frac{\Delta \rho}{\rho} \quad \text{—— } 8$$

Now, Poisson's ratio is defined as the ratio of lateral strain to the longitudinal strain.

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain } (\varepsilon)}$$

$$= \frac{-(\Delta D/D)}{(\Delta l/l)}$$

$$\text{or, } -\frac{\Delta D}{D} = \mu\frac{\Delta l}{l} = \mu\varepsilon \quad \text{—— } 9$$

From 8 & 9.

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + 2\mu\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{R} = (1+2\mu)\varepsilon + \frac{\Delta \rho}{\rho} \quad \text{—— } 10$$

Gauge factor (Strain coeff of resistance)

It is defined as the ratio of per unit change in resistance to the per unit change in length.

i.e $G = \frac{\Delta R/R}{\Delta l/l}$

$\frac{\Delta R}{R} = G\frac{\Delta l}{l} = G\epsilon$ —— (11)

From eqn (10) & (11),

$G\epsilon = (1+2\mu)\,\epsilon + \frac{\Delta \rho}{\rho}$

or $G = [1+2\mu] + \frac{\Delta \rho/\rho}{\epsilon}$

$G = 1 + 2\mu + \frac{\Delta \rho}{\rho} \times \epsilon$ (Gauge factor due to piezo resistivity) —— (12)

1 → Gauge factor due to change in length

2μ → Gauge factor due to change in diameter

But practically piezoresistivity is negligible in comparision to strain.

So $G = 1 + 2\mu$ —— (13)

And the change in resistance from eqn (11)

$\Delta R = GR\epsilon$ —— (14)

It is similar to $\Delta R = \alpha \Delta t\, R$
where $\alpha \rightarrow$ temp. coefficient of resistivity.

So, G is also termed as strain coeff of resistance

strain is unitless quantity but expressed in terms of microstrain.

$$1 \text{ micro strain} = \frac{1 \text{ micrometer}}{1 \text{ meter}}$$

$$= 10^{-6}$$

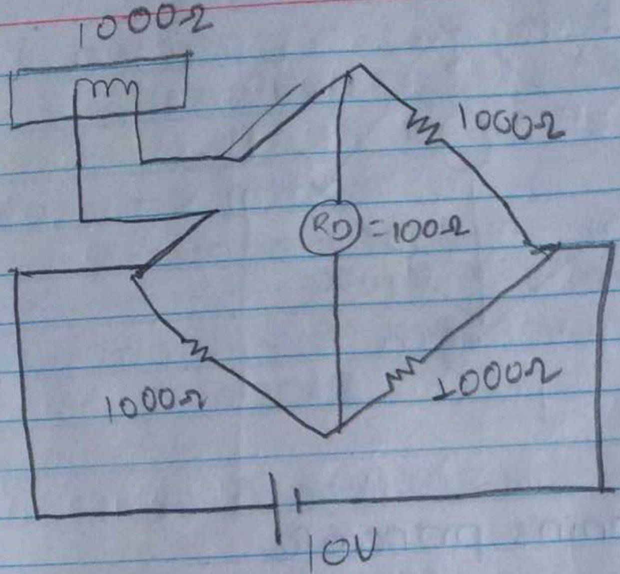
$$5\ \mu \text{strain} = 5 \times 10^{-6}$$

Tensile → +ve
Compression → −ve

Numerical

Q) In order to measure the strain in cantilever beam, the single strain guage of resistance 1 KΩ and gauss factor 2 and temp^r coeff 10×10^{-6} per °C, is mounted on a beam and ~~cont~~ connected to 1 arm of wheatstone bridge. The other arms have resistances 1 KΩ each. The bridge dectector is 100 Ω and it's sensitivity is 10mm per μ Ampere. Supply is 10V.
Calculate

Sol:-
i) The detector deflection for 0.1% strain
ii) The change in effective strain indicated when ~~increases~~ the room temperature increases by 10°C



Here, Gauge resistance = 1000Ω

$G = 2$

temp. coeff (α) = $10 \times 10^{-6}/°C$

sensitivity = 10mm/μA

Deflection = ?

i) For 0.1% strain, $\varepsilon = 0.1\% = \frac{0.1}{100} = 0.001$

So, $\Delta R = G R \varepsilon$

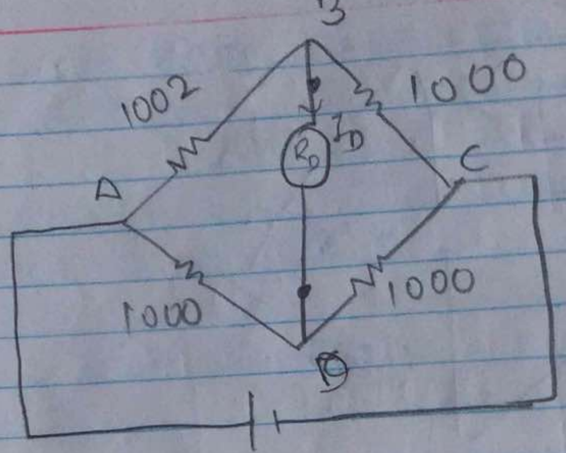
$= 2 \times 1000 \times 0.001$

$= 2\Omega$

So new Gauge resistance (R_G) = $R_G + \Delta R$

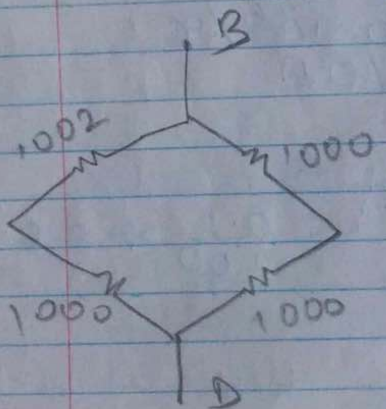
$= 1002\ \Omega$.

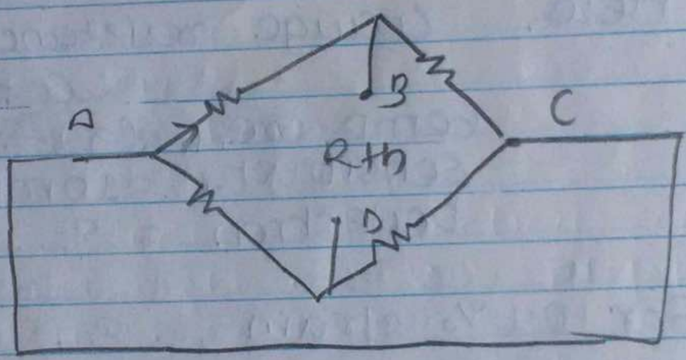
So, ckt becomes,



Using Thevenin's principle

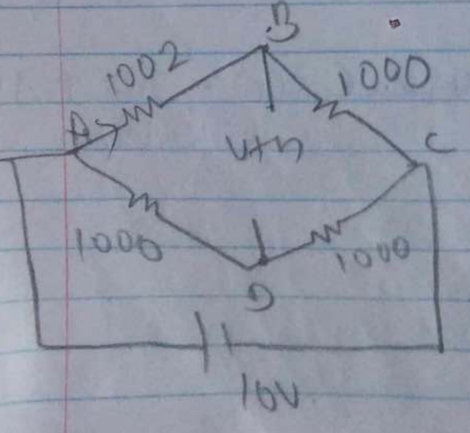
For Rth





So $R_{th} = 1002 \parallel 1000 + 1000 \parallel 1000$

$= 1000.4995\,\Omega$



$V_{th} = V_{BA} + V_{AD}$

$= V_{AD} - V_{AB}$

$= I_2 \times 1000 - I_1 \times 1002$

$= \cancel{5 \times 10^{-3} \times 1000 - 4.99 \times 10^{-3} \times 1002}$

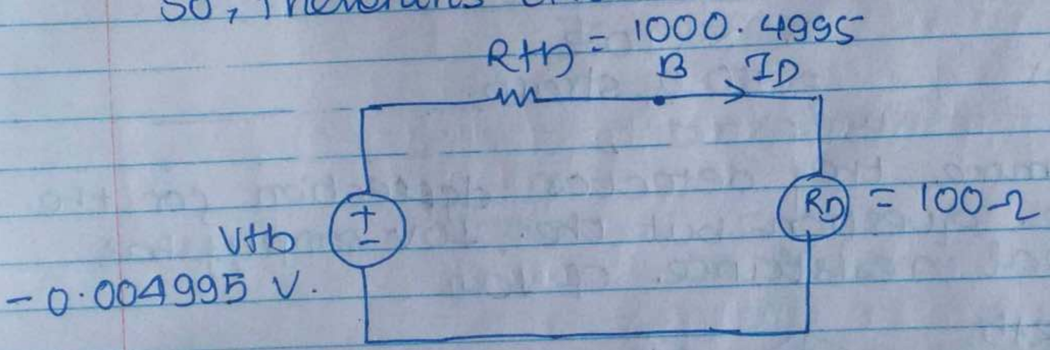
Here, $I_1 = \frac{10}{1002 + 1000} = \cancel{4.99}\ \frac{10}{2002}$

$I_2 = \frac{10}{1000 + 1000} = \frac{10}{2000}$.

$\therefore V_{th} = \frac{10}{2000} \times 1000 - \frac{10}{2002} \times 1002$

$= -0.004995 V$.

So, Thevenin's ckt becomes.



So, $I_D = -\frac{0.004995}{1000.4995 + 100}$

$= -4.5338\ \mu A$.

$= 4.5338\ \mu A$ along DB.

ii) Detector Deflection $= S \times I_D$

$= 10 \times 4.5338$

$= 45.388$ mm.

For 10°C change in temperature

$$\Delta R = \alpha R \Delta t$$
$$= 10 \times 10^{-6} \times 1000 \times 10$$
$$= 0.1\,\Omega.$$

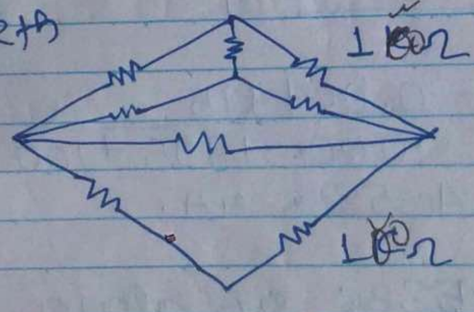
So, For equivalent strain

$$\Delta R = G R \varepsilon$$
$$\text{or } \varepsilon = \frac{\Delta R}{G R}$$
$$= \frac{0.1}{2 \times 1000}$$
$$= 5 \times 10^{-5}$$
$$= 50\,\mu \text{ strain.}$$

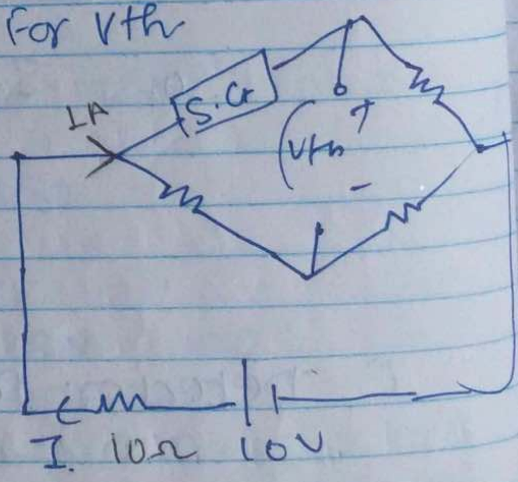
Determine the detector deflection for the above question but the 10V supply has internal resistance of 10Ω

H/W

For R_{th}



For V_{th}



Resistance Thermometer:

$$\text{Temp}\ (\Delta\theta) \longrightarrow \boxed{R \cdot T} \longrightarrow \Delta R$$

Q) 1) Find the strain that resolves from the tensile force of 100 N applied to a 1m long aluminium bar having cross-sectional area of $A = 4 \times 10^{-4}\ m^2$ Modulus of elasticity $(Y) = 69\ GN/m^2$.

Sol^n:
We know, $Y = \frac{\text{stress}}{\text{strain}}$

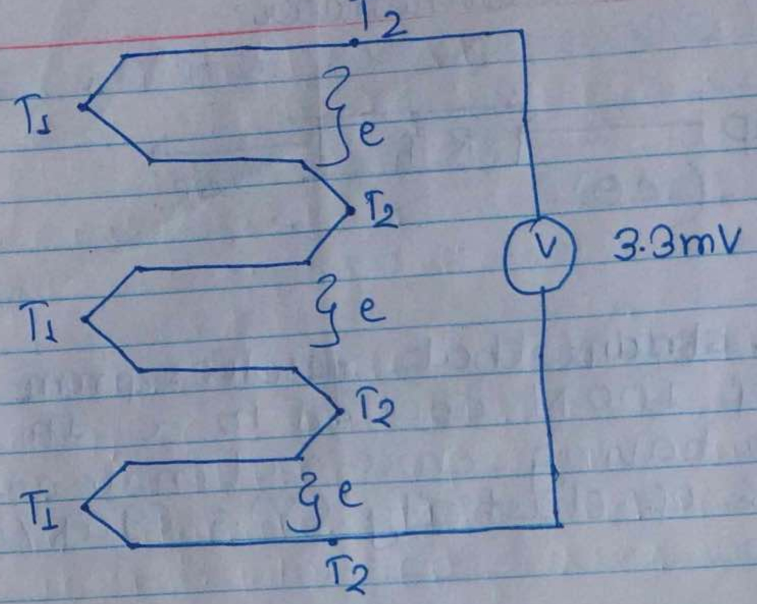
$= \frac{F}{A \times (\text{strain})}$

$\text{strain} = \frac{F}{A \times Y}$

$= \frac{100}{4 \times 10^{-4} \times 69 \times 10^{9}}$

$\therefore \text{strain} = 3.623 \times 10^{-6}$.

2) A series connected thermopile is made up of copper - constantan thermocouple with T_1 at 150°C and it's net output emf is 3.3 mV for the arrangement of 3 junction as shown below. Calculate the value of temperature T_2, taking the sensitivity of each junction 50 µV/°C,



Sol^n :-

Given $T_1 = 150°C$

Emf at ~~eq~~ each junction $(e) = \frac{3.3}{3} mV$

$= 1.1 mV$

$e =$ sensitivity $\times \Delta T$

$1.1 \times 10^{-3} = 50 \times 10^{-6} \times \Delta T$

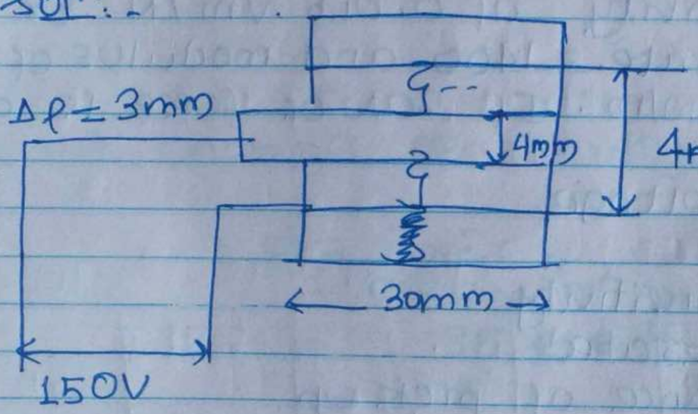
$\Delta T = 22°C$

$T_2 = 128°C$ or $172°C$

$150 - 22$ or $150 + 22$

A capacitive transducer is made up of two concentric cylindrical electrodes. The outer diameter of inner cylinder is 4mm and the dielectric medium is air. The inner diameter of outer electrode is 4.1 mm. Calculate the dielectric stress when the voltage of 150V is applied across the electrodes. Is it within the safe limit? The length of electrode is 30mm. Calculate the sensitivity and the change in capacitance if the inner electrode is moved through a distance of 3mm. The breakdown voltage of air is 30 KV/mm.

Soln:-



Here, Air gap = $\frac{4.1-4}{2}$ = 0.05mm

So, Dielectric stress = $\frac{\text{Voltage}}{\text{Air gap}}$

$= \frac{150}{0.05}$

Safe limit = 8 KV/mm.

sensitivity = $\frac{\partial C}{\partial l} = \frac{\partial}{\partial l}\left(\frac{2\pi\varepsilon_0 l}{\ln D/d}\right)$

$= \frac{2\pi\varepsilon_0}{\ln(D/d)} = \frac{2\times\pi\times 8.85\times10^{-12}}{\ln\left(\frac{4.1}{4}\right)}$

$= 2.25$ nf/m.

$\Delta C =$ sensitivity $\times \Delta l$
$= 2.25 \times 10^{-9} \times 3 \times 10^{-3}$

$= 6.75$ pf.

Q7) A barium titanate piezoelectric pickup has dimension of $12mm * 12mm * 3mm$ [A] and a voltage sensitivity of 0.015 Vm/N. ε_r for barium titanate $= 1400$ and modulus of elasticity $= 10 \times 10^{10}$ N/m. If 20N of force is applied calculate:

1) output voltage
2) Strain
3) charge sensitivity
4) Charge generated
5) capacitance of pick up.

soln: We have,
$V_{out} = gpt$.

where $g = \frac{d}{\varepsilon_0 \varepsilon_r} = \cancel{\frac{3 \times 10^{-3}}{8.85 \times 10^{-12} \times 1400}}$
$=$

Here $g = 0.015$ Vm/N.
$p = \frac{20N}{12 * 12 * 10^{-6}}$

$t = 3 \times 10^{-3}$.

$\therefore V_{out} = gpt = 0.015 \times \frac{20}{12 * 12 \times 10^{-6}} \times 3 \times 10^{-3}$
$= 6.25V$

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

$$= \frac{F}{A \times Y} = \frac{20}{12 \times 12 \times 10^{-6} \times 10 \times 10^{10}}$$

$$\text{Strain} = 1.38 \times 10^{-6}$$
$$= 1.38 \ \mu\text{strain}$$

iii) Charge sensitivity

$$d = \varepsilon_0 \varepsilon_r g$$
$$= 8.85 \times 10^{-12} \times 1400 \times 0.015$$
$$= 1.85 \times 10^{-10} \ C/N.$$

iv) Charge generated

$$Q = d \times F$$
$$= 1.85 \times 10^{-10} \times 20$$
$$= 3.72 \ nC$$

5) $$C = \frac{\varepsilon_0 \varepsilon_r A}{t}$$

$$= \frac{8.85 \times 10^{-12} \times 1400 \times 12 \times 12 \times 10^{-6}}{3 \times 10^{-3}}$$

$$= 0.59 \ nF.$$

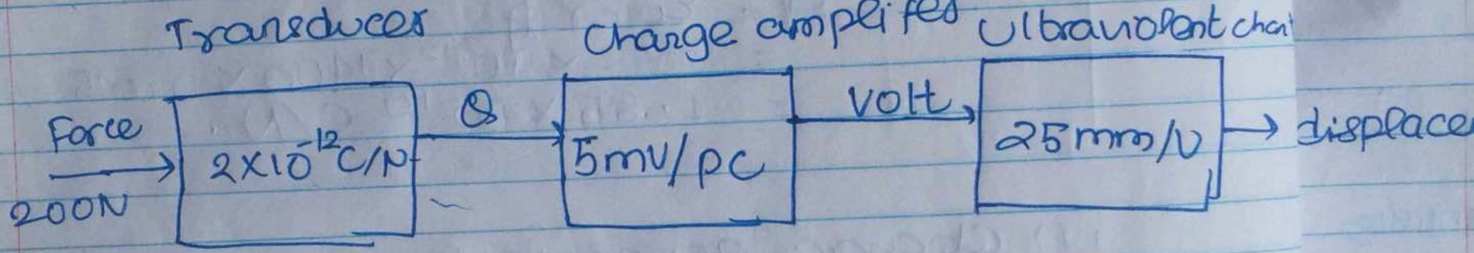
A piezo electric pressure transducer having a charge sensitivity of $2*10^{-12}$ C/N is connected to a charge amplifier of gain 5mV/pC. The amplifier output is connected to an ultra-violent ~~charg~~ chart recorder whose sensitivity is set to ~~20~~ 25mm/V. Determine the overall sensitivity and deflection of chart due to force of 200N.

4)

Sol^n:-

charge sensitivity $= 2\times10^{-12}$ C/N.

Gain = 5mV/pC



overall sensitivity $= \dfrac{\text{displacement}}{\text{force}}$

$= \dfrac{\text{charge}}{\text{force}} \times \dfrac{\text{Voltage}}{\text{charge}} \times \dfrac{\text{displacement}}{\text{voltage}}$

$= 2\times10^{-12} * \dfrac{5\times}{10^{-12}} * \dfrac{25}{10^{3}}$

$= 0.25$ mm/N

Deflection = S x Force

= 50 mm.

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4) A linear potentiometer is 50 mm long and is uniformly wound with a wire having resistance of 10 kΩ; under normal condition slider is at center of potentiometer. Find the linear displacement when the resistance of POT is measured ~~by~~ for two cases are:

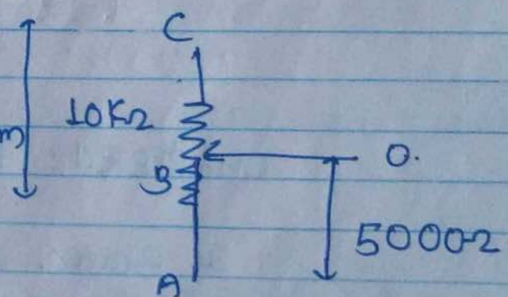
a) 3850 Ω

b) 7560 Ω. Are the displacement in same direction? If it is possible to measure a minimum value of 10 Ω with the above arrangement. Find the resolution of potentiometer in mm.

Soln:

Given $l = 50\,mm$

displacement $R_{wire} = 10\,k\Omega$



Resistance per unit length $= \dfrac{10 \times 10^3}{50}$

$= 200\,\Omega/mm.$

Here, change in resistance $= 5000 - 3850$

$\Delta R = 1150$

Now, Resistance per unit length

$\cancel{200\,\Omega/mm} \times \dfrac{\Delta R}{\Delta l}.$

$\Delta R = 1150.$ so $\Delta l = \dfrac{1150}{200} = 5.75\,mm$ along $\overrightarrow{BA}$

2) 7560Ω.

$\Delta R = 2560\Omega$

$\Delta l = \frac{\Delta R}{200} = \frac{2560}{200} = 12.8$ m along $\overrightarrow{BC}$

(not in same direction)

iii) Resolution = ?

Resolution = Input equivalent to minimum detectable o/p

$= \frac{10}{200}$

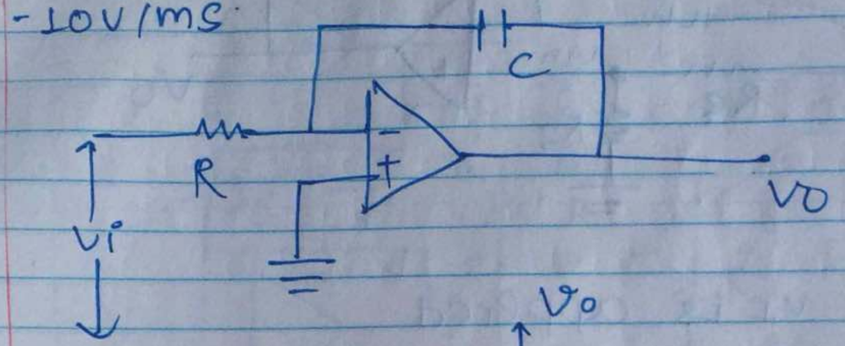
$= 0.05$ mm.

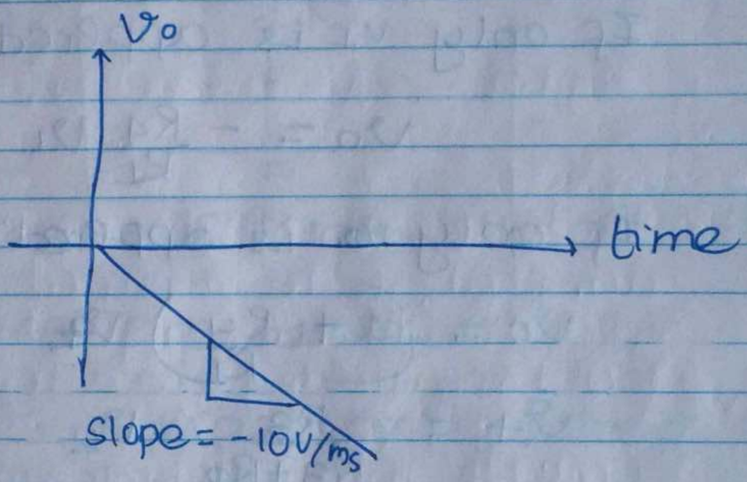
CHAPTER4: Electrical Signal Processing & Transmission

self study!

* Basic op-amp characteristics
* op-amp mode of operation
  → Inverting
  → Non-inverting
* Application of op-amp
  → Inverter
  → voltage follower / Buffer
  → Adder
  → Subtractor / differential amplifier
  → multiplier / divider
  → Differentiator / Integrator.

Design an integrator which gives a RAMP voltage of -10V/ms
(ramp)





We know,

O/p of integrator

$$= -\frac{1}{RC}\int V_{in}\,dt$$

Since o/p voltage is ramp, input must be constant dc

let $v_{in} = 10V(dc)$

$$v_o = \left(-\frac{10}{RC}\right)t$$

So, desired output slope $= -10V/ms$

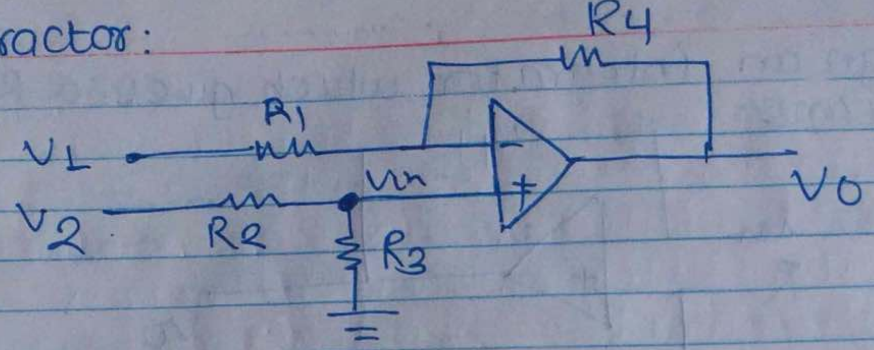
$$= -\frac{10}{RC} = -\frac{10}{10^{-3}}\ V/S.$$

So $RC = 10^{-3}$

let $R = 1k\Omega$

$$C = \frac{10^{-3}}{10^{3}} = -10^{-6}F = 1\mu F.$$

Subtractor:



If only v_1 is applied

$$v_0 = -\frac{R_4}{R_1} v_1 \qquad \text{inverting}$$

If only v_2 is applied

$$v_0 = \left(1 + \frac{R_4}{R_1}\right) \cancel{v_+} \qquad \text{(non-inverting)}$$

$$v_+ = \frac{R_3}{R_3+R_2} \cdot v_2$$

$$v_{02} = \left(1 + \frac{R_4}{R_1}\right)\left(\frac{R_3}{R_2+R_3}\right) v_2$$

when both input are applied

$$v_0 = v_{01} + v_{02}$$

$$= \left(1 + \frac{R_4}{R_1}\right)\left(\frac{R_3}{R_2+R_3}\right) v_2 - \frac{R_4}{R_1} v_1.$$

If all resistances are equal

$$v_0 = v_2 - v_1$$

Instrumentation Amplifier

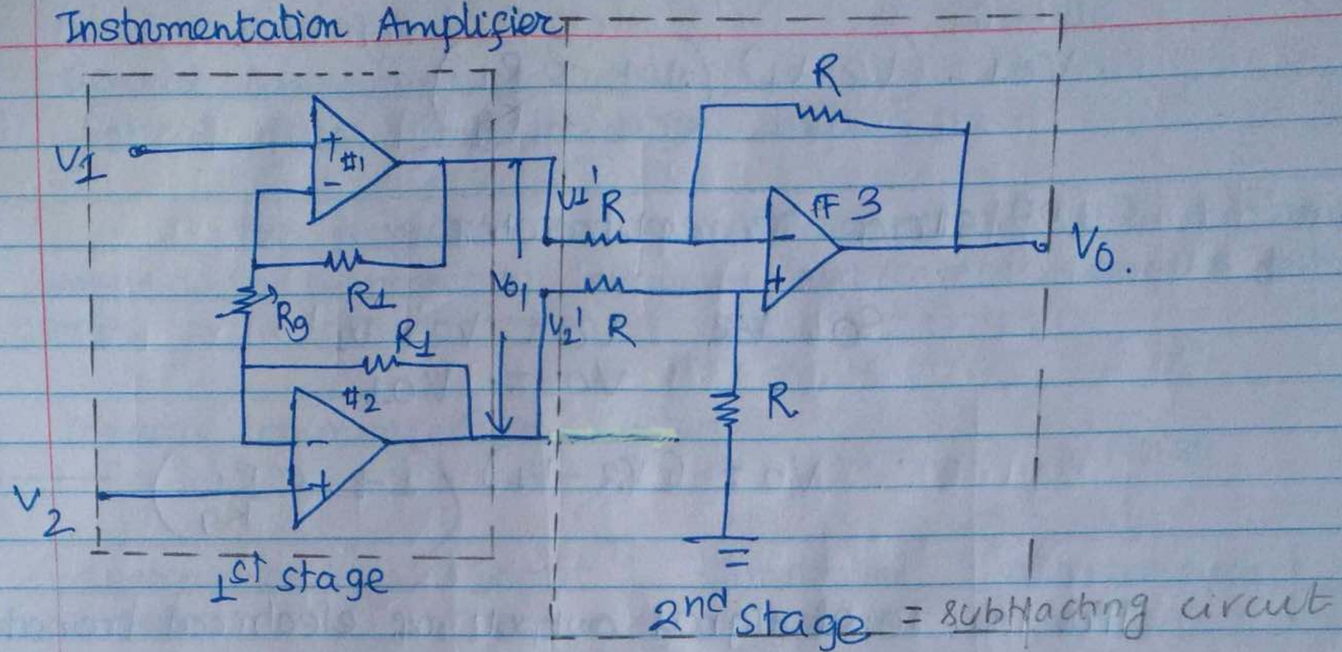


Fig: Instrumentation Amplifier

(2 stage- 3 amplifier configuration)

For #1

$$V_{+1} = V_{-1} = V_1$$

for #2, $V_{+2} = V_{-2} = V_2$

So, $I_g = \frac{V_{-2} - V_{-1}}{R_g}$

$$I_g = \frac{V_2 - V_1}{R_g}$$

Since no current flows inside opamp,

$$V_{o1} = I_g (R_1 + R_g + R_1)$$
$$V_{o1} = I_g (2R_1 + R_g) = \frac{V_2 - V_1}{R_g} (2R_1 + R_g)$$

$$V_{o1} = (V_2 - V_1)\left(1 + \frac{2R_1}{R_g}\right)$$

2^{nd} stage is subtractor

So, ~~V2~~ $V_o = V_2' - V_1'$

$$V_o = V_{o1}$$

$$\therefore V_o = (V_2 - V_1)\left(1 + \frac{2R_1}{R_g}\right) \quad \text{——} ①$$

The low level signal output of electrical transducer often needs to be amplified before further processing and this is done by using instrumentation amplifier which has following characteristics:

a) Selectable gain with high gain accuracy and stability
b) Low drift and high noise rejection capability ~~due~~

Due to these characteristics, it is highly useful in instrumentation purpose where transduced signal are often buried in noise.

It consists of two stages: 1^{st} stage has two op-amps where 2 inputs V_1 & V_2 are applied to the non-inverting terminals. The output of 1^{st} stage is taken through string of registers R_1, R_g & R_1. The two resistances R_1 are inbuilt to IC whereas R_g is placed externally. By changing value of R_g, we can change the gain of instrumentation amplifier i.e from eq^n ①.

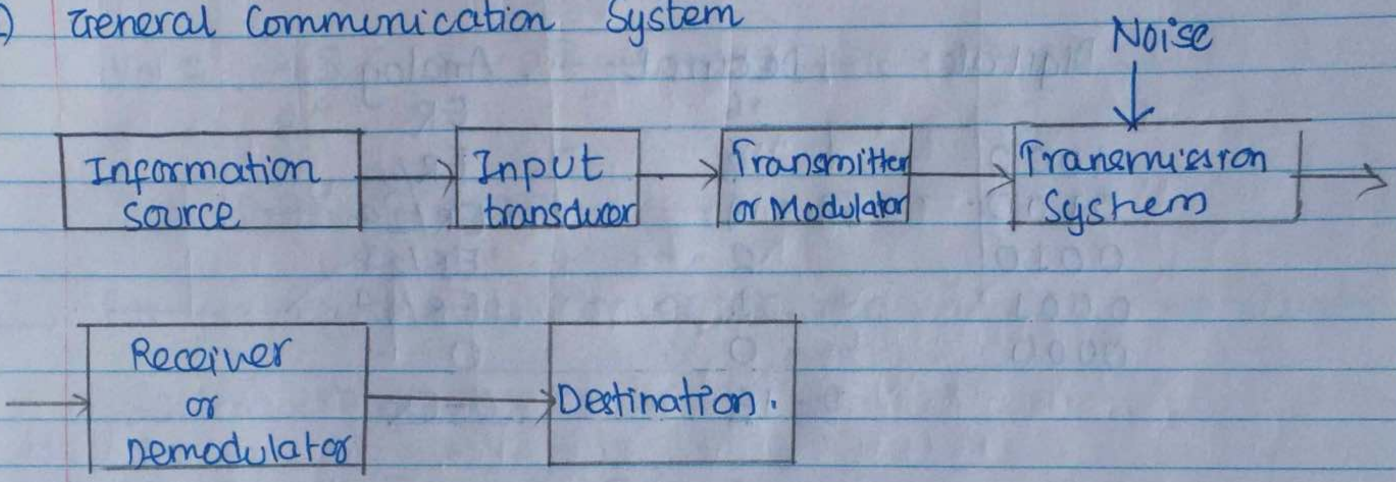
$$\frac{V_o}{V_2 - V_1} = 1 + \frac{2R_1}{R_g}$$

Communication System:

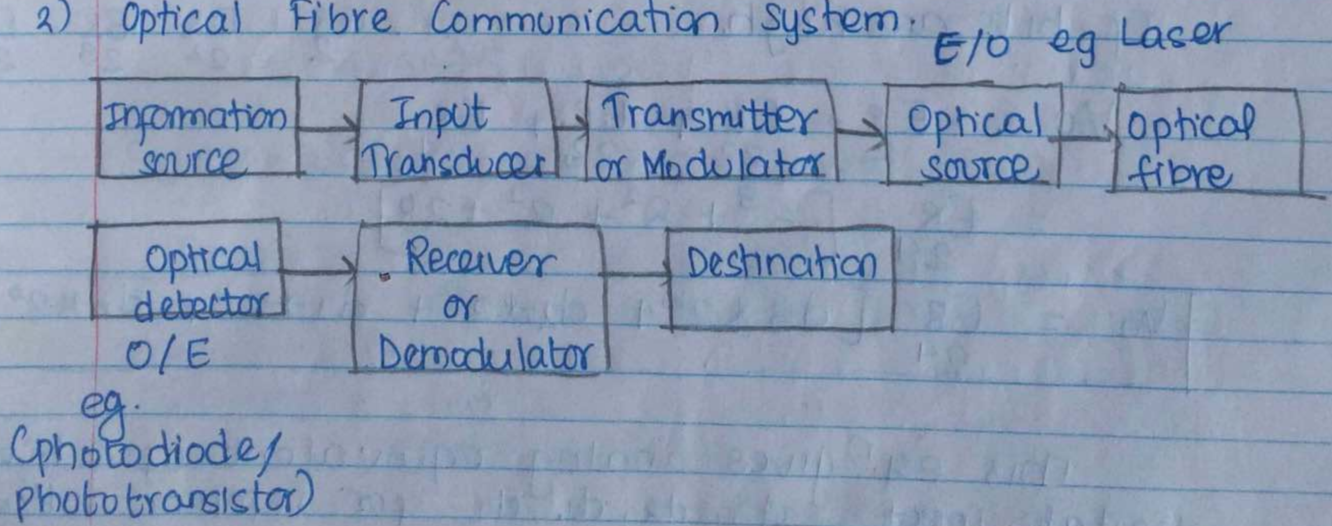
1) General Communication system
2) Optical Fibre Communication system

The purpose of communication system is to transmit message or information between two points : source & destination.

1) General Communication System



2) Optical Fibre Communication system.



Chapter 5- A/D and D/A conversion.

self-study

Analog signal vs Digital signal

the form of bits

Digital to Analog Conversion (number)

$$\text{MSB} \leftarrow \begin{matrix} 1 & 0 & 1 & 1 \\ 2^3 & 2^2 & 2^1 & 2^0 \end{matrix} \rightarrow \text{LSB}$$

$N = 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 11.$

Digital	Decimal	Analog
	16	ER
1000	8	ER/2
0100	4	$ER/2^2$
0010	2	$ER/2^3$
0001	1	$ER/2^4$.
0000	0	0
	0 - 16	0 - ER

If bits are high

d_3	d_2	d_1	d_0
$\frac{ER}{2}$	$\frac{ER}{2^2}$	$\frac{ER}{2^3}$	$\frac{ER}{2^4}$

$$V_o = \frac{ER}{2} + \frac{ER}{2^2} + \frac{ER}{2^3} + \frac{ER}{2^4}$$

$$= \frac{ER}{2^4}\left[2^3 + 2^2 + 2^1 + 2^0\right]$$

$$\boxed{V_o = \frac{ER}{2^4}\left[d_3 * 2^3 + d_2 * 2^2 + d_1 * 2^1 + d_0 * 2^0\right]}$$

This eq^n gives analog equivalent of 4 bit digital input d_3, d_2, d_1, d_0 for reference voltage ER

For n-bit system

$$V_o = \frac{ER}{2^n}\left[d_{n-1} 2^{n-1} + d_{n-2} 2^{n-2} + \dots + d_0 2^0\right]$$

Find the equivalent analog output for a digital input of 1011, if it's full range is 5V. Also find the actual range of converter.

We have, $Er = 5$

For 4-bit system

$$V_0 = \frac{\cancel{4}5}{2^4}\left[1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0\right]$$

$$= \cancel{2.75} \; 3.4375$$

For full range of converter, the range of total value is $0000 - 1111_2$.

So max^m value is

$$\frac{5}{2^4}\left[1\times2^3 + 2^2 + 2^1 + 2^0\right]$$

$$= 4.6875$$

So, Actual range = {0 − 4.6875V}

$$\text{Error} = \frac{\cancel{Er}}{\cancel{2^4}} = 5 - 4.6875$$

$$= 0.3125\text{V} \; \{\text{wt. of LSB}\}$$

$$\%\text{ error} = \frac{\text{Error}}{\text{Rated full scale}} \times 100\%$$

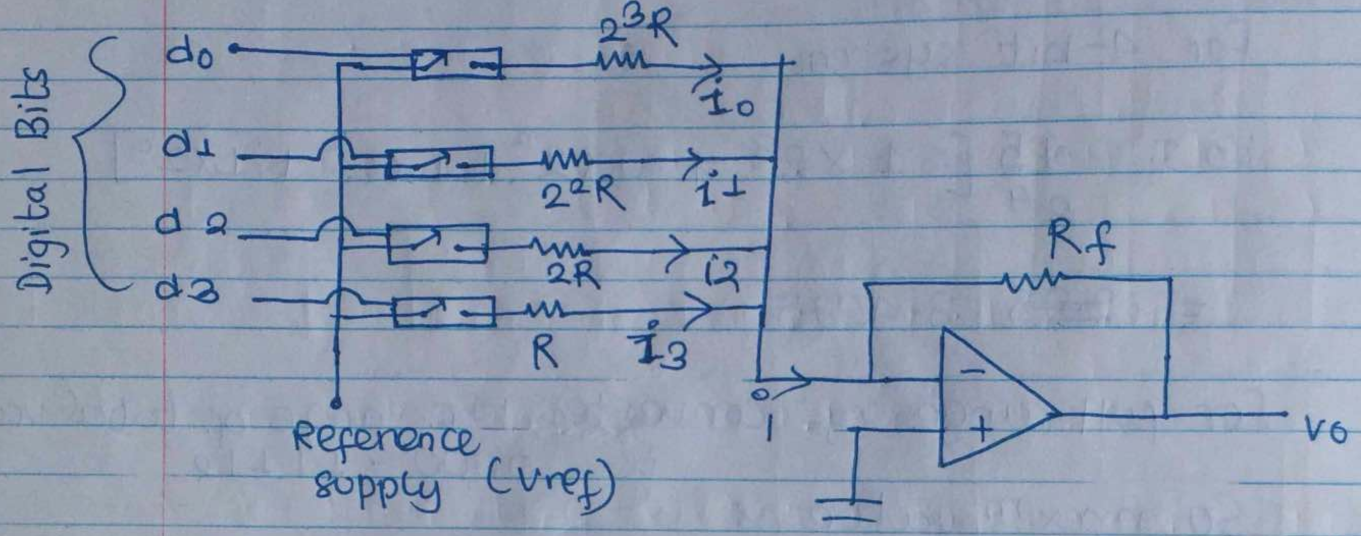
$$= \frac{0.3125}{5} \times 100\%$$

$$= 6.25\%$$

Digital ~~An~~ to Analog Converter (Techniques)

i) Weighted Resistor Network (WRN)
ii) R-2R Ladder Network.

i) WRN DAC



In this DAC resistors are weighted, reverse of binary system i.e resistance value increases as we move from MSB to LSB by factor of 2 as shown in figure.

Bits	d_3	d_2	d_1	d_0
Binary wt	2^3	2^2	2^1	2^0
Resistor used	2^0R	2^1R	2^2R	2^3R

So,
for n bit converter,

Resistance for MSB bit = R
" " LSB bit = $2^{n-1}R$

For different combination of Digital bits the respective current in the resistor branch are :

Digital input	Analog o/p
1 0 0 0	$i = i_3 = \frac{V_{ref}}{R}$
0 1 0 0	$i = i_2 = \frac{V_{ref}}{2R}$
0 0 1 0	$i = i_1 = \frac{V_{ref}}{2^2 R}$
0 0 0 1	$i = i_0 = \frac{V_{ref}}{2^3 R}$
1 1 1 1	$i = i_3 + i_2 + i_1 + i_0$.

(If all bits are high)

$$i = \frac{V_{ref}}{R} + \frac{V_{ref}}{2R} + \frac{V_{ref}}{2^2 R} + \frac{V_{ref}}{2^3 R}$$

$$= \frac{V_{ref}}{2^3 R} (2^3 + 2^2 + 2^1 + 2^0)$$

If input is $d_3 d_2 d_1 d_0$

$$i = \frac{V_{ref}}{2^3 R} [d_3 \times 2^3 + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0]$$

And now, the final analog o/p voltage is given by

$$V_0 = -i R_f$$

$$= -\frac{V_{ref}}{2^3 R} R_f [d_3 \times 2^3 + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0]$$

For n-bit system,

$$V_0 = -\frac{V_{ref}}{2^{n-1} R} R_f \left[d_{n-1} 2^{n-1} + d_{n-2} 2^{n-2} + \ldots d_0 \times 2^0 \right]$$

Drawbacks of WRN DAC

For 8 bit system,

Resistance at MSB = R (say 1 KΩ)

" at LSB = 2^7 R (128 KΩ)

Range of resistor = 1 - 128 KΩ

If tolerance of resistor = ±5%.

So, limiting error at MSB = 5% of 128

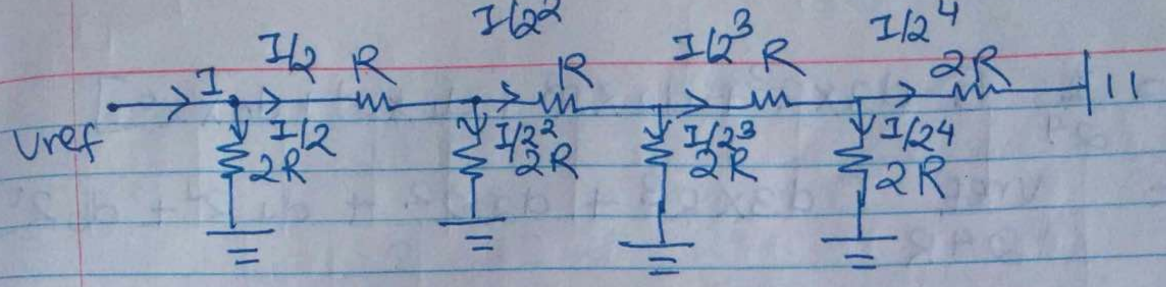
= 6.4 KΩ > resistor at MSB

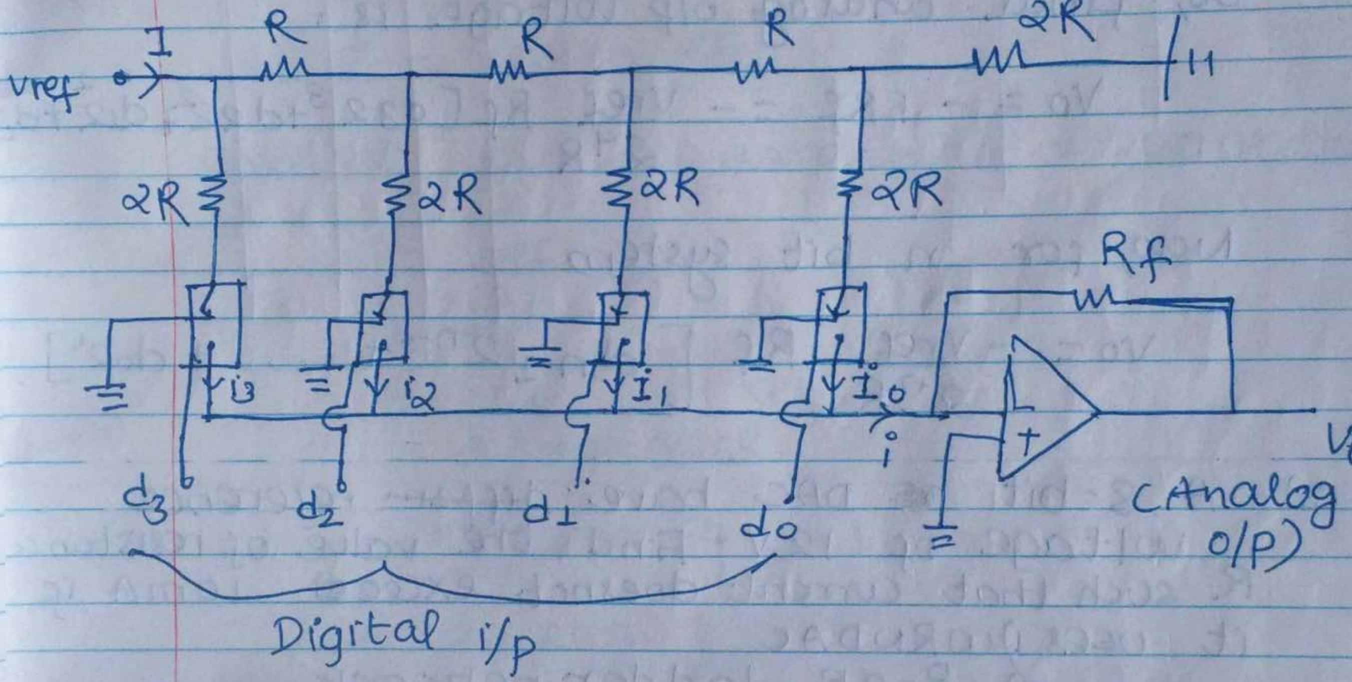
Drawbacks

As no. of bits increases, the value of resistance also increases and for higher bit converter, there will be very large difference in the value of resistance associated with MSB and LSB & even the tolerance value of resistance in LSB becomes much greater than resistance in MSB which is not suitable for ~~IC~~ IC fabrication

2) R-2R ladder network.

vref





In the right side of each node there are two equal resistors 2R and 2R connected in parallel so the current leaving any node will be divided ~~in~~ equally.

When all bits are high,

$$i = i_0 + i_1 + i_2 + i_3$$

$$= \frac{I}{2^4} + \frac{I}{2^3} + \frac{I}{2^2} + \frac{I}{2}$$

$$= \frac{I}{2^4}\left[1 + 2^1 + 2^2 + 2^3\right].$$

$$= \frac{1}{2^4} [d_3 \times 2^3 + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0]$$

$$= \frac{V_{ref}}{2^4 R} [d_3 \times 2^3 + d_2 2^2 + d_1 2^1 + d_0 2^0].$$

So, final analog o/p voltage is :

$$V_0 = - i R_f = - \frac{V_{ref}}{2^4 R} R_f [d_3 2^3 + d_2 2^2 + d_1 2^1 + d_0 2^0].$$

Now, for n bit system

$$\boxed{V_0 = - \frac{V_{ref}}{2^n R} R_f \left[d_{n-1} 2^{n-1} + \dots + d_0 2^0\right].}$$

Q1 A 8-bit ~~DS~~ DAC have ~~differ~~ reference voltage of 12V. Find the value of resistance R such that current doesnot exceed 10mA, if it uses 1) WRNDAC
2) R-2R ladder network
Also find the smallest value of quantized current in both cases.

Soln: We have

1) Using WRNDAC

$$i = \frac{V_{ref}}{2^{n-1} R} [d_{n-1} 2^{n-1} + \dots + d_0 2^0]$$

For 8-bits,
for i to be maximum, $d_7 \rightarrow d_0$ all bits should be high.

$$i = \frac{V_{ref}}{2^7 R} [d_7 2^7 \ldots + d_0 2^0].$$

$$10 \times 10^{-3} = \frac{12}{2^7 R} [2^7 + 2^6 + \ldots + 2^0]$$

$$\Rightarrow R = 2390.65 \Omega$$

LSB

Smallest value of current, is = 00000001

$$i = \frac{V_{ref}}{2^7 R} \times 2^0$$

$$= \frac{12}{2^7 \times 2390.65} \times 1$$

$$= 3.92 \times 10^{-5}$$

$$= 39 \mu A$$

Analog to Digital Converters

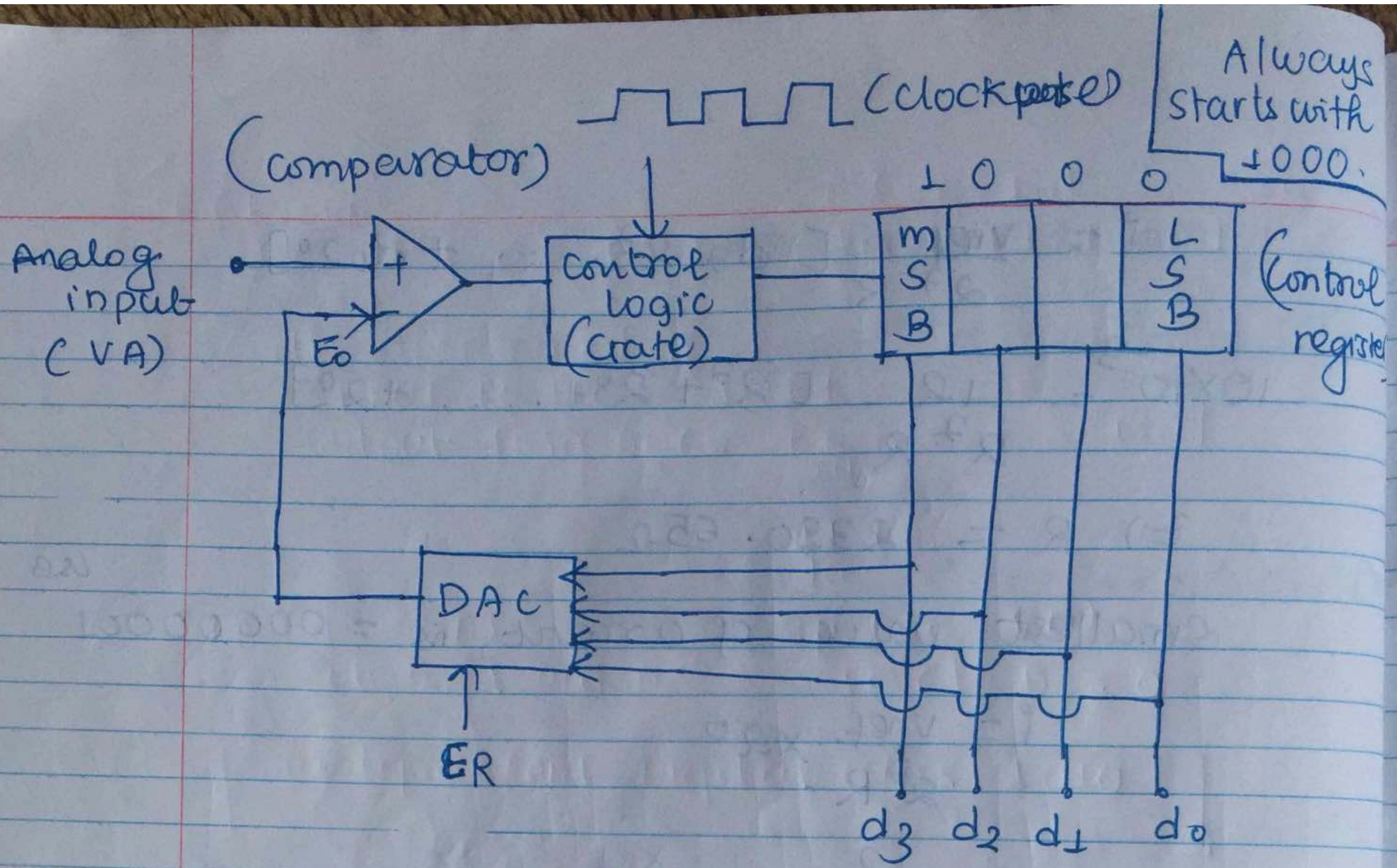
i) Sucessive approximation ADC

ii) Ramp ADC

iii) Dual Ramp ADC

iv) Flash ADC

i) Successive approximation ADC



O/p of DAC

$$E_0 = \frac{E_R}{2^n}\left[d_{n-1}2^{n-1} + \ldots + d_0 2^0\right]$$

Control Register

It is most widely used method of ADC. It has complex circuitory than other type of ADC (normally Ramp ADC) but has much shorter conversion time than Ramp ADC. It's conversion time is fixed for any analog input.

Here the comparator compares the analog input voltage and the reference variable voltage generated by DAC. The output of comparator is fed to control logic which modifies the contents of register bit by bit until the register data are the digital equivalent of analog input voltage VA.

If (VA > Eo) If (VA < Eo)

Note : To increase the bit: To decrease the bit:

- set the current bit as it is
- set the next bit equals to 1

- set the current bit as 0
- Set the next bit equals to 1

For example:

① First approximation to DAC = 1000

Output from DAC = $\frac{ER}{2^4}[2^3] = \frac{+ER}{2}$

If (Eo < VA)
set the D3 as it is (i.e D3 = 1) and set the next bit i.e D2 = 1

If Eo > VA
Reset the D3 ie D3 = 0 and set the next bit i.e $D_2 = 1$

(ii) Second Approximation:
Input to DAC = 1100
Output from DAC = $\frac{ER}{2^4}[2^3 \cancel{+} + 2^2]$

$= \frac{3}{4} ER$

If (Eo < VA)
set D2 as it is (i.e D2 = 1) and set the next bit $D_1 = 1$

If (Eo > VA)
Reset D2 = 0 and Set D1 = 1

(iii) Third Approximation
Input to DAC = 1010
Output from DAC = $\frac{ER}{2^4}(2^3 + 2^1) = \frac{10}{16} ER$

If $(E_0 < V_A)$
set $D_1 = 1$ and $D_0 = 1$

If $(V_A < E_0)$
Reset $D_1 = 0$ and set $D_0 = 1$.

(iv) Fourth approximation
Input to DAC = 1011
Output from DAC = $\frac{E_R}{2^4}[2^3 + 2^1 + 2^0]$

$= \frac{11}{16} E_R$

If $(E_0 < V_A)$
set $D_0 = 1$ ~~and D~~

If $(E_0 > V_A)$
reset $D_0 = 0$.

$\boxed{\text{So, equivalent Digital o/p} = 1010_2}$