7th DEC

(open method)

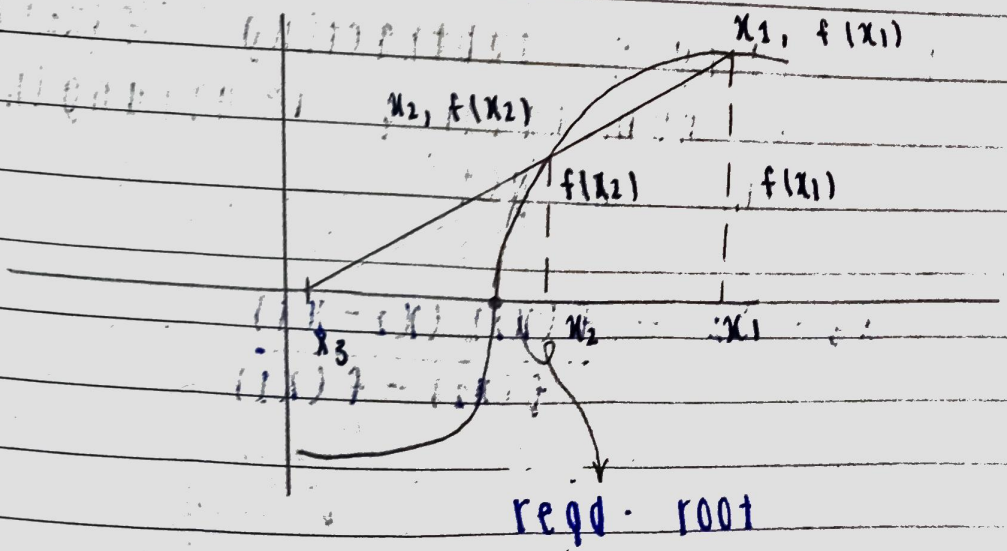
Date
Page

(iii) secant method

secant method like false position & bisection method uses two initial estimates but does not require that they must bracket the root.

- Eg:

secant method can use the points x_1 & x_2 although they don't bracket the root.



line that cuts any curve at 2 diff points;

Date
Page

slope of secant line passing through x_1 & x_2 is given by:

$$\frac{f(x_1)}{x_1 - x_3} = \frac{f(x_2)}{x_2 - x_3}$$

or, $f(x_1)(x_2 - x_3) = f(x_2)(x_1 - x_3)$

On solving,

$$x_3 = \frac{f(x_2)\,x_1 - f(x_1)\,x_2}{f(x_2) - f(x_1)}$$

By adding & subtracting $f(x_2)\,x_2$ to numerator & rearranging the terms, we get

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

↓

secant formula

1. Find root of eqn $x^3 - 2x - 5 = 0$ using secant method correct upto 4 decimal places.

Soln let $f(x) = x^3 - 2x - 5$

let
$A = B^3 - 2B - 5$;
$C = D^3 - 2D - 5$;

$$E = D - \left(((C) * (D - B)) \div (C - A) \right)$$

Iteration	X_1	X_2	$f(X_1)$	$f(X_2)$	$X_3 = X_2 - \frac{f(X_2)}{(X_2 - X_1)} \cdot f(X_2 - f(X_1$
1.	2	3	-1	16	2.05886
2.	3 (X_2)	2.05886 (X_3)	16	-0.3904	2.081277354
3.	2.05886	2.081277354	-0.3904	-0.1470	2.094823581
4.	2.081277354	2.094823581	-0.1470	0.00303	2.094549437
5.	2.094823581	2.094549437	0.00303	-0.00002	2.094551481

Note:

- x_1 ladi x_2 le replace
- x_2 ladi x_3 le replace

for every step

Thus, the reqd. root of eqn $x^3 - 2x - 5 = 0$ is 2.094551481.

(IV) Newton Raphson method

Consider a graph as in fig. We assume that x_1 is the approx. root of $f(x) = 0$.

Draw the tangent at $x = x_1$ as in fig.

Point of intersection gives the 2nd approx.

Slope of tangent is given by

$$\tan\alpha = \frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

where,

$f(x_1)$ = slope of $f(x)$ at $x = x_1$

solving for x_2, we obtain

·Note: Derivation ni sodhxa

↓

farsi, ki

Date
Page

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

↓

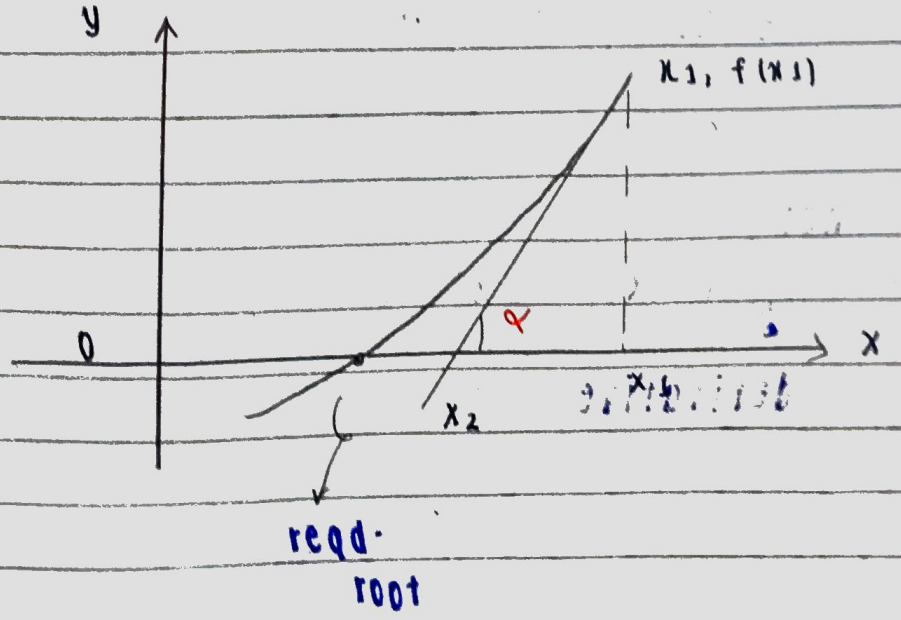
Newton-Raphson formula.

∴ Next-approx would be,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



x ko value
+ve line

Date
Page

1. Find +ve root of $x^4 - x - 10$.
correct upto 3 decimal places,
using N-R method.

Table ma f(x) - ; close by values
no. bata euta line

x	-1	2
f(x)	-10	4

yo bata euta line
(1 or 2)

Soln

let $f(x) = x^4 - x - 10$

x	1	2
f(x)	-10	4

Then,

$f'(x) = 4x^3 - 1$

derivative

• Note: • x_1 ici x_2 le replace

Let us take $x_1 = 2$ ∴ +ve root

Let

$A = B^4 - B - 10$:

$C = 4B^3 - 1$:

$D = B - (A/C)$

Iteration	x_1	$f(x_1)$	$f'(x_1)$	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$
1.	2	4	31	1·870967742
2.	1·87096 7742	0·3826	25·19	1·8557807D2
3.	1·8557 80702	0·004818	24·56	1·855584561