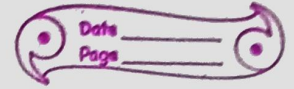


8th DEC



Fixed Point Iteration method

1. Evaluate the sq. root of 5 using the eqⁿ $x^2 - 5 = 0$ by applying fixed point iteration algorithm.

$$f(x) = x^2 - 5$$

Soln

$$x^2 = 5$$

$$x = \frac{5}{x}$$

exp can be written like this.

Assume $x_0 = 1$

Then,

$$x_1 = 5$$

$$[x_1 = \frac{5}{1}]$$

$$x_2 = 1$$

$$[x_2 = \frac{5}{5}]$$

$$x_3 = 5$$

$$x_4 = 1$$

\therefore Process does not converge to the root.

This type of divergence is known as oscillatory divergence.

$$x_4 = 2.2361$$

$$x_5 = 2.2361$$

Hence, the sq. root of 5 is 2.2361.

2. Find real root of eqⁿ $x^3 + x^2 - 1 = 0$ by fixed point iteration method. correct upto 6 decimal places.

0.7548

Solⁿ

$$x^3 + x^2 - 1 = 0$$

$$\text{or, } x^3 - 1 = -x^2$$

Then,

$$\text{or, } (x-1)(x^2 + x + 1) = -x^2$$

$$\text{or, } x = \frac{1 - x^2}{x^2 + x + 1}$$

Assume $x_0 = 1$

$$x_1 = 0.6666666667$$

$$x_2 = 0.7894736842$$

$$x_3 = 0.7416762342$$

$$x_4 = 0.7599732662$$

$$x_5 = 0.7529192288$$

$$x_6 = 0.7556316261$$

$$x_7 = 0.7545875919$$

$$x_8 = 0.7549892953$$

$$x_9 = 0.7548347122$$

$$x_{10} = 0.7548941953$$

$$x_{11} = 0.7548713059$$

$$x_{12} = 0.7548801137$$

$$x_{13} = 0.7548767244$$

$$x_{14} = 0.7548780287$$

$$x_{15} = 0.7548775268$$

$$x_{16} = 0.7548777199$$

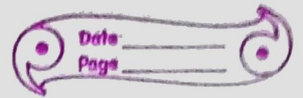
\therefore The reqd. root is 0.7548777199

• **Note :** Non-linear method ma

root \swarrow khojne ho

\swarrow
curve le x-axis lai
kata meet garxa

no chap ma only row operation
no column operation



chap-3

soln of linear Eqn

↳ soln of linear Eqn can be computed two ways,

① Elimination / direct method

② Iterative method

→ Gauss Elimination method

→ Jacobi's iterative method

→ Gauss Jordan method

→ Gauss Seidal method

① ii Gauss Elimination method

↳ consider $AX=B$ & eqn defined as:

$$\left. \begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned} \right\} \rightarrow (i)$$

Then,

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

&

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(iv) Eqⁿ ma lagne ani

By backward substitution we get,

$$x + 4y - z = -5 \quad \rightarrow (i)$$

$$-3y - 5z = -7 \quad \rightarrow (ii)$$

$$23.67z = 49.33 \quad \rightarrow (iii)$$

Solving eqⁿ (iii)

$$z = 49.33$$

$$23.67$$

$$= 2.084$$

From eqⁿ (ii)

$$\text{or, } -3y - 5(2.084) = -7$$

$$\text{or, } -3y = 3.42$$

$$\therefore y = -1.14$$

From eqⁿ (i)

$$x + 4(-1.14) - 2.084 = -5$$

$$\therefore x = 1.644$$

$$\therefore x = 1.644$$

$$y = -1.14$$

$$z = 2.084$$

then x & y

• Note: yesma unknown variables
nikalne ho

2. Apply Gauss Elimination method to solve the eqns $x+4y-z = -5$; $x+y-6z = -12$; $3x-y-z = 4$.

soln

① Augmented form ma lagne

$$[A:B] = \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 1 & 1 & -6 & : & -12 \\ 3 & -1 & -1 & : & 4 \end{bmatrix}$$

② R_1 use garera a_{11} ↓ ko elements lai 0 parne.

Applying row operations:

• $R_2 \rightarrow R_2 - R_1$

• $R_3 \rightarrow R_3 - 3R_1$

1/1 *

derivation

batai

$$[A:B] = \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 0 & -3 & -5 & : & -7 \\ 0 & -13 & 2 & : & 19 \end{bmatrix}$$

③ R_2 use garera a_{22} ↓ ko elements lai 0 parne

• $R_3 \rightarrow R_3 - \frac{(-13)}{(-3)} R_2$
 (-3)

$$[A:B] = \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 0 & -3 & -5 & : & -7 \\ 0 & 0 & 23.67 & : & 49.333 \end{bmatrix}$$

$$\therefore [A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix} \rightarrow (ii)$$

Reducing above matrix (ii) into **upper triangular matrix**

Now, applying row operation

$$R_2 \rightarrow R_2 - \frac{a_2}{a_1} R_1 ;$$

$$R_3 \rightarrow R_3 - \frac{a_3}{a_1} R_1 ;$$

We get,

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b'_2 & c'_2 & : & d'_2 \\ 0 & b'_3 & c'_3 & : & d'_3 \end{bmatrix} \rightarrow (iii)$$

Again,

$$R_3 \rightarrow R_3 - \frac{b'_3}{b'_2} R_2, \text{ we get}$$

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b'_2 & c'_2 & : & d'_2 \\ 0 & 0 & c''_3 & : & d''_3 \end{bmatrix}$$

y

yeta bata z nikalne

Again,

$$x = x^2 + x - 5$$

Assume $x_0 = 0$

$$x_1 = -5$$

$$x_2 = 15$$

$$x_3 = 235$$

$$x_4 = 55455$$

\therefore It also does not converge
rather it diverges rapidly.

So, this type of divergence is known as
monotone divergence.

Let's try 3rd form of $g(x)$

$$2x = \frac{5}{x} + x$$

$$\text{or, } x = \frac{x + 5/x}{2}$$

Now,

take $x_0 = 1$

$$x_1 = 3$$

$$x_2 = 2.3333$$

$$x_3 = 2.2381$$

2. SOLVE :

$$10x - 7y + 3z + 5u = 6$$

$$-6x + 8y - z - 4u = 5$$

$$3x + y + 4z + 11u = 2$$

$$5x - 9y - 2z + 4u = 7 \quad \text{by Gauss Elimination method.}$$

Soln

$$[A:B] = \left[\begin{array}{cccc|c} 10 & -7 & 3 & 5 & 6 \\ -6 & 8 & -1 & -4 & 5 \\ 3 & 1 & 4 & 11 & 2 \\ 5 & -9 & -2 & 4 & 7 \end{array} \right]$$