

18th Dec

Proof Methods

If p then q



conclusion

hypothesis

① Qn diye ma simple way ma jane



direct method

$p \rightarrow q$



start



end

If n is odd, n^2 is odd



p



q

$$\text{let } n = 2k + 1$$

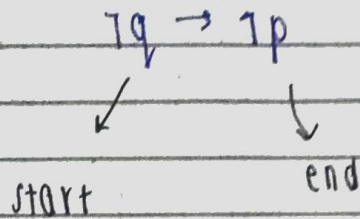
$$n^2 = (2k + 1)^2$$

$$= 4k^2 + 4k + 1$$

$$= 2(2k + 2k) + 1$$

odd

② Direct wayena vane,



If n^2 is odd, n is odd

$$\begin{aligned}
 \hookrightarrow n &= 2k \\
 n^2 &= 4k^2 \\
 &= 2(2k^2) \quad \downarrow \\
 &\quad \text{not odd}
 \end{aligned}$$

$$n^2 = (2k)^2$$

$$\begin{aligned}
 n &= 2k \\
 &\quad \downarrow \\
 &\quad \text{not odd}
 \end{aligned}$$

$$1q \rightarrow 1p.$$

$$3n + 2 = 2k + 1$$

$$n = \frac{2k-1}{3} \rightarrow \text{not clear}$$

so,
 $p \rightarrow q$ x

$$\neg q \rightarrow \neg p$$

• Proof by contraposition

1. If n is an integer & $3n+2$ is odd, then n is odd

soln

$$\neg q \Rightarrow n \text{ is not odd}$$

$$n = 2k \quad [n \text{ is even}]$$

Then,

for $\neg p$

$$\hookrightarrow 3n+2 \text{ is not odd}$$

substituting $2k$ in $(3n+2)$

$$= 3(2k) + 2$$

$$= 6k + 2$$

$$= 2(3k+1)$$

is not odd $[\because \text{is even}]$

$$\therefore \neg q \rightarrow \neg p$$

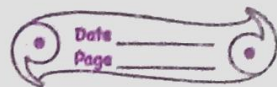
Thus,

negation of conclusion of conditional statement implies hypothesis is false.

so,

original cond. statement is true.

The contrapositive of $p \rightarrow q$
is $\neg q \rightarrow \neg p$



2. Prove that if $n = ab$ where a & b are the integers then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

soln

$\neg q$ means $(a > \sqrt{n} \vee b > \sqrt{n})$

Now,

$$ab > \sqrt{n} \sqrt{n}$$

$$\therefore ab > n$$

$ab \neq n$ which contradicts original cond. statement

$$n = ab$$

3. $\sqrt{2}$ is irrational (proof by contradiction)

↓

$\neg p$ prove false

Let,

$\sqrt{2}$ is rational

Then,

$\sqrt{2}$ can be represented as ratio of integers

a & b

s.t. $b \neq 0$ & both having no common factor

$$\sqrt{2} = \frac{a}{b}$$

b

$$\sqrt{2} b = a$$

sq. both sides,
 $2b^2 = a^2$

since,

$$a^2 = \text{even no.}$$



so, a is also even

then,

$$a = 2c \text{ [let]}$$

Again,

$$2b^2 = 4c^2$$

$$b^2 = 2c^2$$



since,

$$b^2 = \text{even no.}$$



b is also even

Both a & b are even, so common factor is 2 betⁿ a & b .

$\sqrt{2} = a/b$ where a & b have no common factor that leads to contradiction

if 2 divides both a & b

so, $\neg p$ is false. Thus, p is true.

4. PROVE if a^2 is even then a is even
(by contrapositive)

contrapositive of $p \rightarrow q$ is
 $\neg q \rightarrow \neg p$

$\neg q \rightarrow a$ is odd

$$\therefore a = 2k + 1$$

Then,

$$\begin{aligned} a^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \\ &= 2 \quad \downarrow \end{aligned}$$

odd

i.e. $\neg p$

• NOTE:

$(p \rightarrow q)$ ma $\neg p = \text{TRUE}$

vale

$p \rightarrow q$ \downarrow

always TRUE