2069 Bhadra (Regular)

1. Forward difference table :

x	1	2	3	4	5	6
$f(x)$	2	9	28	65	126	217

x	$f(x)$	F.F.D.	S.F.D	T.F.D.	F.F.D.
1	2				
		7			
2	9		12		
		19		6	
3	28		18		0
		37		6	
4	65		24		0
		61		6	
5	126		30		
		91			
6	217				

2. Explain the mechanism of finding a real root of a non-linear equation using secant method.

Ans: This method is an improvement over the method of false position as it does not require the condition $f(x_0)f(x_1) < 0$ of that method. Here, also the graph of the function $y = f(x)$ is approximated by a secant line but at each iteration, two most recent approximations to the root are used to find the next approximation. Also it is not necessary that the interval must contain the root.

Taking x_0, x_1 as the initial limits of the interval, we write the equation of the chord joining these as,

$$y - f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1)$$

Then the abscissa of the pt. where it crosses the x-axis $(y=0)$ is given by

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1)$$

which is an approximation to the root. The general formula for successive approximation is given by,

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \times f(x_n), \quad n \geq 1.$$

3. Find a root of $e^x = 3x$ using bisection method and Newton's Raphson method correct upto 3 decimal places.

Sol: Let, $f(x) = e^x - 3x$ (Bisection Method)

x	0	1
$f(x)$	1	-0.281

It lies between 0 & 1.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.8052	0.6214	0.4499	0.2918	0.1487	0.0221	-0.0862

$\therefore f(0.6)$ is +ve & $f(0.7)$ is -ve.

Let the initial interval $(a, b) = (0.6, 0.7)$

$a^{(+)}$	$b^{(-)}$	$c = a+b/2$	$f(c)$
0.6	0.7	0.65	-0.0345
0.6	0.65	0.6250	-0.0068
0.6	0.625	0.6125	0.0075
0.6125	0.625	0.6188	0.0004
0.6188	0.625	0.6219	-0.0032
0.6188	0.6219	0.6204	-0.0015
0.6188	0.6204	0.6196	-0.0006
0.6188	0.6196	0.6192	-0.0002
0.6188	0.6192	0.6190	0.0001
0.6190	0.6192	0.6191	0.0000

The required real root of $e^x - 3x$ is 0.6191

4.

Newton Raphson Method

$f(x) = e^x - 3x$

$f'(x) = e^x - 3$

Iteration formula,

$$x_{new} = x_0 - \frac{f(x)}{f'(x)}$$

$$= x_0 - \frac{e^x - 3x}{e^x - 3}$$

Determination of initial guess

x	0	1
$f(x)$	1	-0.281

Let $x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 0.5$

$x_1 = 0.6160$

$x_2 = 0.6190$

$x_3 = 0.6191$

$x_4 \simeq 0.6191$

Hence, the required real root of $e^x - 3x$ is 0.6191

4. Solve using Gauss Elimination Method:

$x + 2y + 3z = 6$

$2x + 3y + 5z = 10$

$2x - y + 3z = 4$

Sol^n: The given system is $AX = B$
where, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 2 & -1 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 10 \\ 4 \end{bmatrix}$

The augmented matrix form is $[A/B]$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 3 & 5 & 10 \\ 2 & -1 & 3 & 4 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -1 & -2 \\ 0 & -5 & -3 & -8 \end{array}\right]$$

$R_2 \to R_2/-1$;

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & -5 & -3 & -8 \end{array}\right]$$

$R_3 \to R_3 + 5R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$R_3 \to R_3/2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Here, we can write,

$z = 1$

$y + z = 2$

& $x + 2y + 3z = 6$

So, $y = 1$

& $x = 6 - 2y - 3z$

$= 6 - 2 - 3$

$= 1$

Hence, $x = 1, y = 1, z = 1$.

5. Write pseudocode to solve a system of linear equation of 'N' unknowns using Gauss-Jordan method.

Ans: Pseudocode:

1. Normalize the first equation by dividing it by its pivot element.
2. Eliminate x_1 term from all the other equations.
3. Now, normalize the second equation by dividing it by its pivot element
4. Eliminate x_2 from all the equations, above and below the normalized pivotal equation.
5. Repeat this process until x_n is eliminated from all but the last equation.
6. The resultant b vector is the solution vector.

6. Use Lagrange method to find f(2.5) from the following data.

x	1	2	4	5	7
f(x)	1	1.414	1.732	2.00	2.6

Soln, $f_4(x) = f_0 l_0(x) + f_1 l_1(x) + f_2 l_2(x) + f_3 l_3(x) + f_4 l_4(x)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$l_0(2.5) = \frac{(2.5-2)(2.5-4)(2.5-5)(2.5-7)}{(1-2)(1-4)(1-5)(1-7)}$$

$$= \frac{-15}{128}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$l_1(2.5) = \frac{(2.5-1)(2.5-4)(2.5-5)(2.5-7)}{(2-1)(2-4)(2-5)(2-7)} = \frac{27}{32}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$l_2(2.5) = \frac{(2.5-1)(2.5-2)(2.5-5)(2.5-7)}{(4-1)(4-2)(4-5)(4-7)}$$

$$= \frac{15}{32}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$l_3(2.5) = \frac{(2.5-1)(2.5-2)(2.5-4)(2.5-7)}{(5-1)(5-2)(5-4)(5-7)}$$

$$= \frac{-27}{128}$$

$$l_4(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$l_4(2.5) = \frac{(2.5-1)(2.5-2)(2.5-4)(2.5-5)}{(7-1)(7-2)(7-4)(7-5)}$$

$$= \frac{1}{64}$$

$$\therefore f_4(2.5) = f_0 l_0(2.5) + f_1 l_1(2.5) + f_2 l_2(2.5) + f_3 l_3(2.5) + f_4 l_4(2.5)$$

$$= \left(1 * \frac{-15}{128}\right) + \left(1.414 * \frac{17}{32}\right) + \left(1.732 * \frac{15}{32}\right) + \left(2 * \frac{-27}{128}\right) + \left(2.6 * \frac{1}{64}\right)$$

$$= 1.5065$$

7. Fit the following set of data to a curve of the form $y = ae^{bx}$ from the following observation by least square method.

x	1	2	3	4	5	6
y	5.5	6.5	9.4	15.2	30.6	49.8

Sol^n:

x_i	y_i	x_i^2	$x_i y_i$
1	5.5	1	5.5
2	6.5	4	13
3	9.4	9	28.2
4	15.2	16	60.8
5	30.6	25	153
6	49.8	36	298.8
$\Sigma x_i = 21$	$\Sigma y_i = 117$	$\Sigma x_i^2 = 91$	$\Sigma x_i y_i = 559.3$

Now,

$$\begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

$$\begin{bmatrix} 6 & 21 \\ 21 & 91 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 117 \\ 559.3 \end{bmatrix}$$

$$6a + 21b = 117 \quad \text{— (1)}$$

$$21a + 91b = 559.3 \quad \text{— (2)}$$

On solving, $a = -10.46$; $b = 8.56$

$$y = ae^{bx} = -10.46\, e^{8.56x}$$

$$\therefore y = -10.46\, e^{8.56x}$$

8. Derive the expression of Simpson's 1/3 rule for integration.

Ans: Here the total integral is further divided into n equal interval and n must be even.

∴ step size $h = \frac{b-a}{n}$

$$I = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \dots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2) + \frac{h}{3}(y_2 + 4y_3 + y_4) + \dots + \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

$$= \frac{h}{3}\left[(y_0+y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\right]$$

This is composite simpson's 1/3 rule.

9. Evaluate $\int_1^2 e^{-x^2}\,dx$ using Romberg method correct upto 3 decimal places.

Soln: Taking $h = 0.5, 0.25$ & 0.125 successively, let us evaluate the given integral by Trapezoidal rule.

i) when $h = 0.5$, the values of $y = e^{-x^2}$ are

x	1	1.5	2
y	0.3679	0.1054	0.0183

$$I_1 = \frac{0.5}{2}\left[(0.3679 + 0.0183) + (2 * 0.1054)\right]$$

$$= 0.1493$$

ii) when $h = 0.25$, the values of $y = e^{-x^2}$ are,

x	1	1.25	1.5	1.75	2
y	0.3679	0.2096	0.1054	0.0468	0.0183

$$I_2 = \frac{0.25}{2}\left[(0.3679 + 0.0183) + 2(0.2096 + 0.1054 + 0.0468)\right]$$

$$= 0.1387$$

iii) when $h = 0.125$ the values of $y = e^{-x^2}$ are,

x	1	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2
y	0.3679	0.2821	0.2096	0.1510	0.1054	0.0713	0.0468	0.0297	0.0183

$$I_3 = \frac{0.125}{2}\left[(0.3679 + 0.0183) + 2(0.2821 + 0.2096 + 0.151 + 0.1054 + 0.0713 + 0.0468 + 0.0297)\right]$$

$$= 0.1361$$

Using Romberg's formula,

$$I_1^* = I_2 + \frac{1}{3}[I_2 - I_1]$$

$$= 0.1387 + \frac{1}{3}[0.1387 - 0.1493]$$

$$= 0.1352$$

$$I_2^* = I_3 + \frac{1}{3}[I_3 - I_2]$$

$$= 0.1361 + \frac{1}{3}[0.1361 - 0.1387]$$

$$= 0.1352$$

$\therefore I_1^{**} = I_2^{*} + \frac{1}{3}[I_2^{*} - I_1^{*}]$

$= 0.1352 + \frac{1}{3}[0.1352 - 0.1352]$

$I_1^{**} = 0.1352$

10. Solve:
$y'' + xy' + y = 0$; $y(0) = 1$; $y'(0) = 0$ for $x = 0(0.1)0.2$
Using RK2 method.

Soln: Given,

$y'' + xy' + y = 0$

$y(0) = 1 = y_0$

$y'(0) = 0 = z(0) = z_0$

$x_0 = 0$ $\quad x_1 = 0.1$

$h = 0.1$ $\quad x_2 = 0.2$

Let $y' = z$ then,

$z'(x, y, z) = y''$

Here,

$f_1(x, y, z) = y' = z$ ——— (i)

$f_2(x, y, z) = z' = -(xz + y)$ ——— (ii)

Now, using RK2 method for finding y_1 & z_1

$k_1 = h * f_1(x_0, y_0, z_0)$
$= 0.1 * f_1(0, 1, 0)$
$= 0$

; $l_1 = h * f_2(x_0, y_0, z_0)$
$= 0.1 * f_2(0, 1, 0)$
$= 0.1 * [-0 * 0 - 1]$
$= -0.1$

$l_2 = h * f_2(x_0 + h, y_0 + k_1, z_0 + l_1)$
$= 0.1 * f_1(0.1, 1, -0.1)$
$= 0.1 * [-(0.1 * (-0.1) + 1)]$
$= -0.099$

$K_2 = h * f_1(x_0+h, y_0+k, z_0+l_1)$
$= 0.1 * f_1(0.1, 1, -0.1)$
$= 0.1 * (-0.1)$
$= -0.01$

$\therefore K = \frac{0 - 0.019}{2}$ $\quad l = \frac{-0.1 - 0.099}{2}$
$= -0.005$ $\quad = -0.0995$

$\therefore y_1 = y_0 + K = 1 - 0.005$ $\quad z_1 = z_0 + l$
$= 0.995$ $\quad = 0 - 0.0995$
$\quad = -0.0995$

Also, Using RK2 Method for finding y_2 & z_2
$K_1 = hf_1(0.1, 0.995, -0.0995)$; $l_1 = 0.1 f_2(0.1, 0.995, -0.0995)$
$= -0.00995$ $\quad = -0.098505$

$K_2 = hf_1(0.2, 0.98505, -0.198005)$
$= -0.198005 * 0.1$
$= -0.019801$

$l_2 = 0.1 f_2(0.2, 0.98505, -0.198005)$
$= -0.094545$

$\therefore K = \frac{K_1 + K_2}{2} = \frac{-0.00995 - 0.019801}{2}$
$= -0.005965$

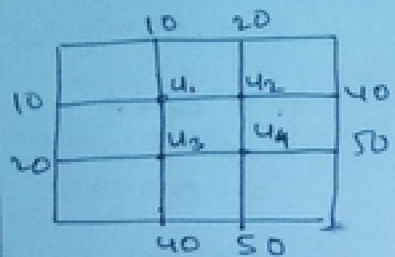
$l = \frac{l_1 + l_2}{2} = \frac{-0.098505 - 0.094545}{2}$
$= -0.096525$

$\therefore y_2 = y_1 + K$ $\quad \therefore z_2 = z_1 + l$
$= 0.995 - 0.005965$ $\quad = -0.0995 - 0.096525$
$= 0.989$ $\quad = -0.196$

Hence, $y_1 = 0.995$ $\quad$ $y_2 = 0.989$

$z_1 = -0.0995$ $\quad$ $z_2 = -0.196$

Q11) Solve the elliptical equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary conditions as shown in the figure below:



Sol: Here, To get the initial values of u_1, u_2, u_3, u_4 we assume that $u_4 = 0$ so,

$u_1 = (20+20)/4 = 40/4 = 10$ [Dia. formula]

$u_2 = (20+10+0+40)/4 = 17.5 \simeq 18$

$u_3 = (10+20+40+0)/4 \simeq 18$

$u_4 = (18+18+50+50)/4 = 34$

Now, using Gauss-seidal for successive iteration:

$u_1 = (20 + u_2 + u_3)/4$

$u_2 = (60 + u_1 + u_2)/4$

$u_3 = (60 + u_1 + u_2)/4$

$u_4 = (100 + u_2 + u_3)/4$

u_1	u_2	u_3	u_4
14	27	27	38.5
18.5	29.250	29.250	39.625
19.625	29.813	29.813	39.906
19.906	29.953	29.953	39.977
19.977	29.988	29.988	39.994
19.994	29.997	29.997	39.999
19.999	29.999	29.999	40
20	30	30	40
20	30	30	40

Hence, $u_1 = 20$

$u_2 = 30$

$u_3 = 30$

$u_4 = 40$ //

2069 Poush (Back)

1. Write an algorithm to find a real root of a non-linear equation using the bisection method.

Ans:

1. Decide initial values for x_1 and x_2 and stopping criterion, E.
2. Compute $f_1 = f(x_1)$ and $f_2 = f(x_2)$
3. If $f_1 * f_2 > 0$, x_1 and x_2 do not bracket any root and go to step 7; otherwise continue
4. Compute $x_0 = (x_1 + x_2)/2$ and compute $f_0 = f(x_0)$
5. If $f_1 \times f_0 < 0$ then

        set $x_2 = x_0$

    else

        set $x_1 = x_0$
        set $f_1 = f_0$

6. If absolute value of $(x_2 - x_1)/x_2$ is less than error E, then

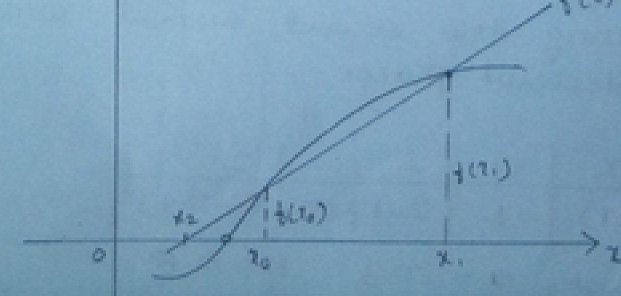
        root $= (x_1 + x_2)/2$
        write the value of root
        go to step 7

    else

        goto step 4

7. stop

2. How can you obtain a real of a non-linear equation using the secant method? Explain graphically and hence obtain the iteration formula.



Secant method, like the false position and bisection methods, uses two initial ~~state~~ estimates but does not require that they must bracket the root. Here, also the graph of the function $y = f(x)$ is approximated by a secant line but at each iteration, two most recent approximations to the root are used to find the next approximation. Also it is not necessary that the interval must contain the root.

from similar triangles,

$$\frac{f(x_0)}{x_0 - x_2} = \frac{f(x_1)}{x_1 - x_2}$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

similarly,

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$ and so on...

In general,

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$ where c = new point
a & b = two previous points.

3 Find the root of the equation $xe^x - \cos x = 0$ using the secant method correct upto 4 decimal places.

Sol^n:

x	0	1	2
$f(x)$	-1	2.1780	15.1943

Let, $a = 0.3$ $b = 0.6$

Iteration formula,

$$c = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

Thus,

a	b	c	f(c)
0.3	0.8	0.4684	−0.1440
0.3	0.4684	0.5281	0.0317
0.5281	0.4684	0.5173	−0.0013
0.5281	0.5173	0.5178	0.0000

∴ The root is 0.5178

4. Solve the following system of linear equations using the Gauss Elimination method with partial pivoting.

$$x + 2y - 12z + 8v = 27$$
$$5x + 4y + 7z - 2v = 4$$
$$-3x + 7y + 9z + 5v = 11$$
$$6x - 12y - 8z + 3v = 49$$

Soln: The augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & 2 & -12 & 8 & 27 \\ 5 & 4 & 7 & -2 & 4 \\ -3 & 7 & 9 & 5 & 11 \\ 6 & -12 & -8 & 3 & 49 \end{array}\right]$$

Interchanging R_1 and R_4

$$\sim \left[\begin{array}{cccc|c} 6 & -12 & -8 & 3 & 49 \\ 5 & 4 & 7 & -2 & 4 \\ -3 & 7 & 9 & 5 & 11 \\ 1 & 2 & -12 & 8 & 27 \end{array}\right]$$

$R_2 \to R_2 - \frac{5}{6}R_1$; $R_3 \to R_3 + \frac{1}{2}R_1$; $R_4 \to R_4 - \frac{1}{6}R_1$

$$\sim \left[\begin{array}{cccc|c} 6 & -12 & -8 & 3 & 49 \\ 0 & 14 & 41/3 & -9/2 & -221/6 \\ 0 & 1 & 5 & 13/2 & 71/2 \\ 0 & 4 & -32/3 & 15/2 & 113/6 \end{array}\right]$$

$R_3 \to R_3 - \frac{1}{14}R_2$; $R_4 \to R_4 - \frac{4}{14}R_2$

$$\sim \left[\begin{array}{cccc|c} 6 & -12 & -8 & 3 & 49 \\ 0 & 14 & 41/3 & -9/2 & -221/6 \\ 0 & 0 & 169/42 & 191/28 & 3203/84 \\ 0 & 0 & -102/7 & 123/14 & 411/14 \end{array}\right]$$

$R_4 \to R_4 + \frac{102 \times 42}{7 \times 169} \times R_3 \Rightarrow R_4 \to R_4 + \frac{612}{169} \times R_3$

$$\sim \left[\begin{array}{cccc|c} 6 & -12 & -8 & 3 & 49 \\ 0 & 14 & 41/3 & -9/2 & -221/6 \\ 0 & 0 & 169/42 & 191/28 & 3203/84 \\ 0 & 0 & 0 & 11319/338 & 167.4408 \end{array}\right]$$

so,

$$\frac{11319}{338} \times v = 167.4408$$

$$\therefore v = 5$$

$$\frac{169}{42}z + \frac{191}{28}v = \frac{3203}{84}$$

or $z = \left(\frac{3203}{84} - \frac{191 \times 5}{28}\right) \times \frac{42}{169} = 1$

$14y + \frac{41}{3}z - \frac{9}{2}v = -\frac{221}{6}$

or $14y = -\frac{221}{6} + \frac{9 \times 5}{2} - \frac{41 \times 1}{3}$

or $y = -2$

Also,

$6x - 12y - 8z + 3v = 49$

or $6x = 49 + 12 \times (-2) + 8 \times 1 ~~+~~ 3 \times 5$

$x = 3$

Hence, $x = 3;\ y = -2;\ z = 1;\ v = 5$

5. Find dominant Eigen value and the corresponding vector of the following matrix using power method.

$$\begin{bmatrix} 1 & 4 & -1 \\ 4 & 2 & 5 \\ -1 & 5 & 10 \end{bmatrix}$$

Soln: Let the initial guess vector be,

$$x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

And, let, $A = \begin{bmatrix} 1 & 4 & -1 \\ 4 & 2 & 5 \\ -1 & 5 & 10 \end{bmatrix}$

$$A x_0 = \begin{bmatrix} 4 \\ 11 \\ 14 \end{bmatrix} = 14 \begin{bmatrix} 0.286 \\ 0.786 \\ 1 \end{bmatrix}$$

$$A x_1 = \begin{bmatrix} 2.43 \\ 7.716 \\ 13.644 \end{bmatrix} = 13.644 \begin{bmatrix} 0.178 \\ 0.566 \\ 1 \end{bmatrix}$$

$$AX_2 = \begin{bmatrix} 1.442 \\ 6.844 \\ 12.652 \end{bmatrix} = 12.652 \begin{bmatrix} 0.114 \\ 0.541 \\ 1 \end{bmatrix}$$

$$AX_3 = \begin{bmatrix} 1.278 \\ 6.538 \\ 12.591 \end{bmatrix} = 12.591 \begin{bmatrix} 0.102 \\ 0.519 \\ 1 \end{bmatrix}$$

$$AX_4 = \begin{bmatrix} 1.178 \\ 6.446 \\ 12.493 \end{bmatrix} = 12.493 \begin{bmatrix} 0.094 \\ 0.516 \\ 1 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 1.158 \\ 6.408 \\ 12.486 \end{bmatrix} = 12.486 \begin{bmatrix} 0.093 \\ 0.513 \\ 1 \end{bmatrix}$$

$$AX_6 = \begin{bmatrix} 1.145 \\ 6.398 \\ 12.472 \end{bmatrix} = 12.472 \begin{bmatrix} 0.092 \\ 0.513 \\ 1 \end{bmatrix}$$

$$AX_7 = \begin{bmatrix} 1.144 \\ 6.394 \\ 12.473 \end{bmatrix} = 12.473 \begin{bmatrix} 0.092 \\ 0.513 \\ 1 \end{bmatrix}$$

Hence, dominant eigen value is 12.473 and corresponding vector is $\begin{bmatrix} 0.092 \\ 0.513 \\ 1 \end{bmatrix}$

6 Using Lagrange's interpolation formula evaluate $f(27.5)$ from the table.

x	26	27	28	29	30
$f(x)$	3.846	3.704	3.571	3.448	3.333

Sol^n: $f_4(x) = f_0 l_0(x) + f_1 l_1(x) + f_2 l_2(x) + f_3 l_3(x) + f_4 l_4(x)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$\therefore l_0(27.5) = \frac{(27.5-27)(27.5-28)(27.5-29)(27.5-30)}{(26-27)(26-28)(26-29)(26-30)}$$

$$= \frac{-5}{128}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$\therefore l_1(27.5) = \frac{(27.5-26)(27.5-28)(27.5-29)(27.5-30)}{(27-26)(27-28)(27-29)(27-30)}$$

$$= \frac{15}{32}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$\therefore l_2(27.5) = \frac{(27.5-26)(27.5-27)(27.5-29)(27.5-30)}{(28-26)(28-27)(28-29)(28-30)}$$

$$= \frac{45}{64}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$\therefore l_3(27.5) = \frac{(27.5-26)(27.5-27)(27.5-28)(27.5-30)}{(29-26)(29-27)(29-28)(29-30)}$$

$$= \frac{-5}{32}$$

$$l_4(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$\therefore l_4(27.5) = \frac{(27.5-26)(27.5-27)(27.5-28)(27.5-29)}{(30-26)(30-27)(30-28)(30-29)}$$

$$= \frac{3}{128}$$

$$\therefore f_4(27.5) = f_0 l_0(27.5) + f_1 l_1(27.5) + f_2 l_2(27.5) + f_3 l_3(27.5) + f_4 l_4(27.5)$$

$$= \left(3.846 \times \frac{-5}{128}\right) + \left(3.704 \times \frac{15}{32}\right) + \left(3.571 \times \frac{45}{64}\right) + \left(3.448 \times \frac{-5}{32}\right) + \left(3.333 \times \frac{3}{128}\right)$$

$$= 3.636$$

7. Using natural cubic spline interpolation technique, estimate the value of $y(0.5)$ from the following data.

x	0	1	2	3
y	2.0	2.2	1.0	0.5

Sol^n: Here x is at equal interval thus we can write,

$$k_{i-1} + 4k_i + k_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

we have $h = 1$

8) The distance $y(t)$ traversed in time t by an object moving in a straight line is given below:

t (in seconds)	0.0	0.1	0.2	0.3	0.4	0.5	0.6
y (in times)	0.0	1.5	7.1	14.3	24.5	36.7	50.0

approximate the velocity and acceleration at 0.2 seconds.

Soln,

Here,

Forward difference table is:-

t	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0.0	0.0						
0.1	1.5	1.5					
0.2	7.1	5.6	4.1				
0.3	14.3	7.2	1.6	-2.5			
0.4	24.5	10.2	3	1.4	3.9		
0.5	36.7	12.2	2	-1	-2.4	-6.3	
0.6	50.0	13.3.	1.1	-0.9	0.1	2.5	8.8

At $t = 0.2$ seconds

$h = 0.1 \quad \therefore P = \frac{t - t_0}{h} = \frac{t - 0.2}{0.1} \qquad t_0 = 0.3\,sec.$

$t = 0.2$ seconds.

Using Newtons forward interpolation formula,

$$y_t = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 14.3 + \left(\frac{t-0.3}{0.1}\right)10.2 + \frac{1}{2}\left(\frac{t-0.3}{0.1}\right)\left(\frac{t-0.3-0.1}{0.1}\right)\times 2 +$$

$$\frac{1}{6}\left(\frac{t-0.3}{0.1}\right)\left(\frac{t-0.4}{0.1}\right)\left(\frac{t-0.5}{0.1}\right)\times(-0.9)$$

$$or,\ y_t = 14.3 + 102(t-0.3) + 100(t-0.3)(t-0.4)$$
$$-150(t-0.3)(t-0.4)(t-0.5)$$
$$= 14.3 + 102(t-0.3) + 100(t^2 - 0.7t + 0.120)$$
$$-150\left[t^3 - 0.5t^2 - 0.7t^2 + 0.35t + 0.120t - 0.060\right]$$
$$= 14.3 + 102t - 30.6 + 100t^2 - 70t + 12$$
$$-150t^3 + 180t^2 - 70.5t + 9$$
$$= 4.7 - 38.5t + 280t^2 - 150t^3.$$

Now,
Velocity at $t = 0.2\,sec$.

$$\Rightarrow \frac{\partial y_t}{\partial t} = -38.5 + 2\times 280t - 3\times 150t^2$$
$$= -38.5 + 2\times 280\times 0.2 - 3\times 150\times 0.2^2$$
$$= -38.5 + 112 - 18$$
$$= 55.5$$

Also, acceleration at $t = 0.2\,sec$.

$$\frac{\partial^2 y_t}{\partial t^2} = 2\times 280 - 3\times 2\times 150\times t$$
$$= 560 - 180$$
$$= 380$$

Q.9) Evaluate the following using Gaussian 3 point formula.

$$\int_0^2 e^{-x/3}\,dx$$

Soln,

Here,

Changing the limit of the integration from (0, 2) to (-1, 1),

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$$

$$= \frac{1}{2}(2-0)u + \frac{1}{2}(b+a)$$

$$= u+1.$$

$\therefore f(0) = 0.717$

$\therefore f(u) = e^{-\left(\frac{u+1}{3}\right)}$

$dx = du$

Then, Using Gaussian 3 point formula, we get

$$I = \int_{-1}^{1} e^{-\left(\frac{u+1}{3}\right)} du$$

$$= \frac{8}{9} f(0) + \frac{5}{9}\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{3/5}\right)\right]$$

$$= \frac{8}{9} \times 0.717 + \frac{5}{9}\left[f\left(-\sqrt{3/5}\right) + f\left(\sqrt{3/5}\right)\right]$$

$$= 0.637 + \frac{5}{9}\left[0.928 + 0.553\right]$$

$$= 1.460.$$ //

Q.10) Solve the ordinary differential equations $y'' = xy'^2 - y^2$ for $x = 0.6$ with initial conditions $y(0) = 1$, $y'(0) = 0$ by using RK-2 Method (Take $h = 0.3$).

Soln

Given,

$y'' = xy'^2 - y^2$

initial conditions,

$y(0) = 1$

$y'(0) = 0$

$h = 0.3$.

Here, let $y' = z$ then $y'' = z'$ Now,

$y' = z = f_1(x, y, z)$ —— (i)

$y'' = z' = xz^2 - y^2 = f_2(x, y, z)$ —— (ii).

so,

$y(0) = y_0 = 1$. ; $x_0 = 0$

$y'(0) = z(0) = z_0 = 0$.

$h = 0.3$.

Now using RK-2 Method for finding y, & z,

$k_1 = hf_1(x_0, y_0, z_0)$
$= 0$

$l_1 = hf_2(x_0, y_0, z_0)$
$= 0.3 \times f_2(0, 1, 0)$
$= -0.3$.

$k_2 = hf_1(x_0+h, y_0+k_1, z_0+l_1)$
$= hf_1(0.3, 1, -0.3)$
$= -0.090$

$l_2 = hf_2(0.3, 1, -0.3)$
$= 0.3 \times (-0.973)$
$= -0.292$

$\therefore k = \frac{k_1 + k_2}{2} = -0.045$

$\therefore l = \frac{l_1 + l_2}{2} = -0.296$.

$y_1 = y_0 + k$
$= 1 - 0.045$
$= 0.955$

$z_1 = z_0 + l_1$
$= -0.296.$

Again, for, $x_2 = 0.6$.

$k_1 = hf_1(x_1, y_1, z_1)$
$= hf_1(0.3, 0.955, -0.296)$
$= -0.089$

$l_1 = hf_2(0.3, 0.955, -0.296)$
$= 0.3 \times -0.886$
$= -0.266.$

$k_2 = hf_1(0.6, 0.866, -0.562)$
$= -0.169$

$l_2 = hf_2(0.6, 0.866, -0.562)$
$= 0.3 \times -0.560$
$= -0.168$

$\therefore k = \frac{k_1 + k_2}{2} = -0.129$

$\therefore l_1 = \frac{l_1 + l_2}{2} = -0.217$

$\therefore y_2 = y_1 + k$
$= 0.955 - 0.129$
$= 0.738$

$\therefore z_2 = z_1 + l$
$= -0.296 - 0.217$
$= -0.513.$

Hence,
for $x = 0.6$.
$y = 0.738$
$y'' = -0.513.$

Q.11) Write Pseudo-Code to solve an initial value problem (first order differential equation) using the Runge-Kutta fourth order Method.

Ans:- R-K 4th ordered Method is most commonly used and is often referred to as Runge-Kutta method only.

Working Rule:-

For finding the increment k of y corresponding to an increment h of x by Runge-Kutta Method from $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ is as follows:

<u>Steps</u> → Calculate successively

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

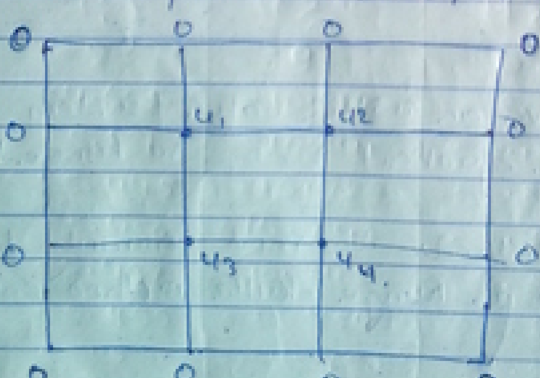
$k_4 = hf(x_0 + h, y_0 + k_3)$

→ Compute $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

→ Approximate values of $y_1 = y_0 + k$

Here, k is the weighted mean of k_1, k_2, k_3 & k_4.

Q.12) Solve the Poissons equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2x^2y^2$ over the square domain $0 \le y \le 3$ with $h = k = 1$ and Boundary Conditions are:- $u(0,0) = 0$, $u(3,0) = 0$, $u(0,3) = 0$, & $u(3,3) = 0$



Here,
Given,
The, eq

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2x^2y^2$.

$h = 1$

∴ The standard 5-point formula for given eqⁿ is:-

$u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = \cancel{2x(i)^2 h(j)^2}$

$h^2 f(ih, jh)$.

$\Rightarrow u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 1 f(i,j)$
$= 2 i^2 j^2$

For, ~~to~~ $f(i,j) = f(1,2)$

~~$u_{1,2}$~~ $u_{0,2} + u_{2,2} + u_{1,3} + u_{1,1} - 4u_{1,2} = 2 \times 1 \times 4$

or, $0 + u_2 + 0 + u_3 - 4u_1 = 8$

or, $u_1 = (u_2 + u_3 - 8)/4$ —— ①.

For $f(i,j) = f(2,2)$

$u_{1,2} + u_{3,2} + u_{2,3} + u_{2,1} - 4u_{2,2} = 2 \times 4 \times 4$

or, $u_1 + 0 + 0 + u_4 - 4u_2 = 32$

or, $u_2 = (u_1 + u_4 - 32)/4$ —— ②

For $f(i,j) = f(1,1)$

$u_{0,1} + u_{2,1} + u_{1,2} + u_{1,0} - 4u_{1,1} = 2$

or, $0 + u_4 + u_1 + 0 - 4u_3 = 2$

or, $u_3 = (u_1 + u_4 - 2)/4$ —— ③

for $f(i,j) = f(2,1)$

$u_{1,1} + u_{3,1} + u_{2,2} + u_{2,0} - 4u_4 = 2\times4\times1$

or, $u_3 + 0 + u_2 + 0 - 4u_4 = 8$

or, $u_4 = (u_2+u_3-8)/4$ ——— (v)

From eqn (ii) & (iv), we found,

$u_1 = u_4 = (u_2+u_3-8)/4$.

Thus, above eqns reduces to:-

$u_1 = (u_2+u_3-8)/4$
$u_2 = (2u_1 - 32)/4 = (u_1-16)/2$
$u_3 = (2u_1 - 2)/4 = (u_1-1)/2$
$u_4 = u_1$

$u_2 = u_3 = 0,\ u_1 = -2$

Let the initial values for Gauss-Seidal iteration Method be $u_2 = 0$, $u_3 = 0$, [illegible]

u_1	u_2	u_3
-2	-9	-1.5
-4.625	-10.313	-2.813
-5.281	-10.641	-3.141
-5.445	-10.723	-3.223
-5.486	-10.743	-3.243
-5.497	-10.748	-3.248
-5.499	-10.750	-3.250
-5.5	-10.750	-3.250
-5.5	-10.750	-3.250

Hence,

$u_1 = -5.5$

$u_2 = -10.750$

$u_3 = -3250$

$u_4 = -5.5$''

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not done :- 3, 8, 11

Q.1) Find a real root of $x^5 - 3x^3 - 1 = 0$ correct up to four decimal places using the Secand Method.

Soln,

Here, Let $f(x) = x^5 - 3x^3 - 1$ so that $f(0) = -1$ & $f(2) = 7$

Taking initial approximation $x_0 = 0$ & $x_1 = 2$, we have.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	-1	-1.343	-3	-3.531	7	49.781	161

Now,

Iteration formula:

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Initially let $a = 0.3$ $b = 0.9$

a	b	c
0.3	0.9	-0.1263
0.9	-0.1263	-0.7629
-0.1263	-0.7629	-0.7190
-0.7629	-0.7190	-0.7415
-0.7190	-0.7415	-0.7418
-0.7415	-0.7418	-0.7418
-0.7418	-0.7418	-0.7418

Hence, the root is -0.7418 //.

Q.2) Write a Pseudo-Code to find a root of a non-linear using Bisection Method.

Pseudo-Code:

1. Decide initial values for x_1 & x_2 & stopping criterion E
2. Compute $f_1 = f(x_1)$ & $f_2 = f(x_2)$.
3. If $f_1 \times f_2 > 0$, x_1 and x_2 do not bracket any root and go to step 7.
4. Compute $x_0 = (x_1 + x_2)/2$ and compute $f_0 = f(x_0)$.
5. If $f_1 \times f_0 < 0$ then.

   set $x_2 = x_0$

   else.

   set $x_1 = x_0$

   set $f_1 = f_0$
6. If absolute value of $(x_2 - x_1)/x_2$ is less than error E, then

   root $= (x_1 + x_2)/2$

   write the value of root.

   go to step-7.

   else

   go to step 4.
7. stop..

Q.4) Solve the following set of linear equation using a suitable iterative method.

$2x + y + z - 2w = -10$

$4x + 2z + w = 8$

$3x + 2y + 2z = 7$

$x + 3y + 2z - w = -5.$

Sol

Given Equations are:-

$2x+y+z-2w=-10$

$4x+2z+w=8$

$3x+2y+2z=7$

$x+3y+2z-w=-5$.

Here,

Adjusting the equations in the form of partial pivoting as:-

$4x+2z+w=8$ ——— (i)

$x+3y+2z-w=-5$ ——— (ii)

$3x+2y+2z=7$ ——— (iii)

$2x+y+z-2w=-10$ ——— (iv)

Now, in order to solve these equations, use Gauss-Seidal Method as:-

$x=(8-2z-w)/4$

$y=(-5-x-2z+w)/3$

$z=(7-3x-2y)/2$

$w=(2x+y+z+10)/2$

Now, let the initial values of z & w be 0 i.e. $z=w=0$,

x	y	z	w
2	-2.333	-1.833	4.917
1.688	0.632	1.601	7.804
-0.751	0.118	4.745	6.690
-2.042	-1.922	4.641	4.317
-1.400	-2.866	2.745	3.545
-0.259	-2.229	1.659	4.457
0.056	-1.306	2.110	5.458

x	y	z	ω
-0.410	-1.44	3.015	5.531
-0.890	-1.536	3.299	4.991
-0.837	-1.903	2.943	4.622
-0.627	-1.879	2.562	4.714
-0.459	-1.650	2.539	4.985
-0.516	-1.520	2.748	5.095
-0.648	-1.584	2.857	5.004
-0.695	-1.692	2.850	4.889
-0.646	-1.723	2.746	4.865
-0.589	-1.679	2.705	4.924
-0.593	-1.634	2.741	4.970
-0.613	-1.633	2.786	4.964
-0.634	-1.658	2.793	4.933
-0.615	-1.674	2.770	4.918
-0.609	-1.669	2.753	4.927
-0.613	-1.657	2.755	4.941
-0.600	-1.652	2.767	4.944
-0.621	-1.657	2.773	4.938
-0.681	-1.662	2.769	4.933
-0.615	-1.663	2.764	4.933
-0.615	-1.660	2.763	4.938
-0.617	-1.658	2.765	4.938
-0.613	-1.658	2.767	4.937
-0.618	-1.660	2.767	4.936
-0.617	-1.660	2.766	4.935
-0.617	-1.659	2.768	4.936
-0.617	-1.659	2.766	4.936

Hence,

$x = -0.617$; $y = -1.659$; $z = 2.766$; $\omega = 4.936$.

Q-5) Find the largest Eigen value & corresponding Eigen Vector of the following Matrix, using power Method.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2. \end{bmatrix}$$

Soln,

Given;

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ and initial Guess vector be $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ then,

Using Power Method.

$$AX_0 \xrightarrow{*} Z_1 \longrightarrow \lambda_1 X_1$$

We have,

$$AX_0 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 1. \end{bmatrix} = \lambda_1 X_1$$

$$AX_1 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \lambda_2 X_2$$

$$AX_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = -3 \begin{bmatrix} -0.75 \\ 1 \\ -0.750 \end{bmatrix} = \lambda_3 X_3$$

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.750 \\ 1 \\ -0.750 \end{bmatrix} = 3.5 \begin{bmatrix} -0.714 \\ 1 \\ -0.714 \end{bmatrix} = \lambda_4 X_4$$

$$AX_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.714 \\ 0 \\ -0.714 \end{bmatrix} = 3.4290 \begin{bmatrix} -0.708 \\ 1 \\ -0.708 \end{bmatrix} = \lambda_5 X_5$$

$$AX_5 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.708 \\ 1 \\ -0.708 \end{bmatrix} = 3.416 \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = \lambda_6 X_6$$

$$AX_6 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = 3.415 \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = \lambda_7 X_7$$

$$AX_7 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = 3.414 \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = \lambda_8 X_8$$

$$AX_8 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = 3.414 \begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix} = \lambda_9 X_9.$$

Since, $X_n \simeq X_{n+1}$

Thus, largest Eigen value is 3.414 and largest Eigen Vector is $\begin{bmatrix} -0.707 \\ 1 \\ -0.707 \end{bmatrix}$.

Q.6) Find the values of y at $x=1.6$ & $x=4.8$ from the following points using Newton's interpolation technique.

x	1	2	3	4	5
y	4	7.5	4	8.5	9.6.

Soln,

Forward Difference Table is:

x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	4				
2	7.5	3.5			
3	4	-3.5	-7		
4	8.5	4.5	8	15	
5	9.6.	1.1	-3.4	-11.5	-26.5.

We take $x_0=0$ and $p=\frac{x-0}{h}=x$.

∴ Using Newton's forward interpolation formula, we get

$f(x)=f(0)+\frac{x}{1}\Delta f(0)+\frac{x(x-1)}{1\times 2}$

(i) at $x=1.6$.

$$p=\frac{x-x_0}{h}=1.6-1=0.6$$

$$y_p=y(x)=y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0+\ldots\ldots$$

$$\therefore y(1.6)=4+0.6\times 3.5+\frac{0.6(0.6-1)(-7)}{2}+\frac{0.6(0.6-1)(0.6-2)}{3!}\times 15$$

$$+\frac{0.6(0.6-1)(0.6-2)(0.6-3)}{4!}\times(-26.5)$$

$= 4 + 2.1 + 0.840 + -3.360 + 0.990$

$= 4.470$

(ii) at $x = 4.8$.

$p = \frac{x - x_0}{h} = \frac{4.8 - 1}{1} = +3.8$.

$\therefore y(4.8) = 4 + 3.8 \times 3.5 + \frac{3.8(3.8-1)}{2!} \times (-7) + \frac{3.8(3.8-1)(3.8-2)}{3!} \times 15$

$+ \frac{3.8(3.8-1)(3.8-2)(3.8-3)}{4!} \times (-26.5)$

$= 4 + 13.3 + -37.240 + -16.918 + 47.880$.

$= 11.022$.

Hence, $y(1.6) = 4.470$
$y(4.8) = 11.022$.

Q.7) Find the curve of the form $y = ax^b$ that fits the following set of observations using least square method?

x	1	2	3	4	5
y	1.2	2.5	6.25	15.75	28.65

Here,

x_i	y_i	$\ln x_i$	$\ln y_i$	$(\ln x_i)^2$	$(\ln x_i)(\ln y_i)$
1	1.2	0	0.182	0	0
2	2.5	0.693	0.916	0.480	0.635
3	6.25	1.099	1.833	1.207	2.0313
4	15.75	1.386	2.757	1.922	3.822
5	28.65.	1.609	3.355	2.590.	5.400
Sum		4.787	9.043	6.199	11.870.

Then,

$$b = \frac{n\sum \ln x_i \ln y_i - \sum \ln x_i \sum \ln y_i}{n\sum(\ln x_i)^2 - (\sum \ln x_i)^2}$$

$$= \frac{5 \times 11.870 - 4.787 \times 9.043}{5 \times 6.199 - (4.787)^2}$$

$$= 1.999.$$

Also,

$$\ln a = \frac{1}{n}\left(\sum \ln y_i - b\sum \ln x_i\right)$$

or, $\ln a = \frac{1}{5}\left(9.043 - 1.999 \times 4.787\right)$

$\ln a = -0.105$

$\therefore a = 0.9$

Hence,

$a = 0.9$ and $b = 1.999$ for, we obtain power function as:- $y = 0.9x^{1.999}$

Q.8) The following table gives the angle in radians (θ) through which a rotating rod has turned for various values of time in seconds (t). Find the angular velocity and angular acceleration at $t = 0.2$.

t	0	0.2	0.4	0.6	0.8
θ	0	0.122	0.493	0.123	2.022

Sol?

here,

Q9) Evaluate the integral $I = \int_{0.2}^{1.2} (\log(x+1) + \sin 2x)\, dx$, using Gaussian 2-point & 3-point formulae.

Soln

Given,

$$I = \int_{0.2}^{1.2} (\log(x+1) + \sin 2x)\, dx .$$

i) Gaussian 2-point formula:- $(n=2)$

$$I = \int_{0.2}^{1.2} [\log(x+1) + \sin 2x]\, dx$$

$$= f(-1/\sqrt{3}) + f(1/\sqrt{3})$$ [after changing the limits into $(-1, 1)$].

For,

changing the limits of integration, from $(0.2, 1.2) \rightarrow (-1, 1)$, by,

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$$

$$= \frac{1}{2}(1.2-0.2)u + \frac{1}{2}(1.2+0.2)$$

$$= \frac{1}{2}u + 0.7 \Rightarrow 0.5u + 0.7.$$

Also,

$dx = 0.5\, du.$

so that,

$$f(u) = \log(0.5u + 0.7) + \sin 2(0.5u + 0.7)$$

Then,

Gaussian 2point formula is given by:-

$I = \int_{-1}^{1} 0.5 f(u)\, du$

$= \int_{-1}^{1} 0.5 \left[\log(0.5u+0.7) + \sin 2(0.5u+0.7)\right] du$

$= 0.5\left[f(-1/\sqrt{3}) + f(1/\sqrt{3})\right]$

$= 0.5\left[\{\log(0.5\times -1/\sqrt{3}+0.7) + \sin 2(0.5\times -1/\sqrt{3}+0.7)\} + \right.$

$\left.\{\log(0.5\times(1/\sqrt{3})+0.7) + \sin 2(0.5\times 1/\sqrt{3}) + 0.7\}\right]$

$= 0.5[0.347 + 0.914]$

$= 0.631.$

Gaussian 3-point formula is given by:-

$f(0) = \log(0+0.7) + \sin 2(0.7)$

$= 0.831$

$I = \int_{-1}^{1} 0.5 f(u)\, du = 0.5\left[\frac{8}{9} f(0) + \frac{5}{9}\left[f(-\sqrt{3/5}) + f(\sqrt{3/5})\right]\right]$

$= 0.5\left[0.738 + \frac{5}{9}(0.081 + 0.860)\right]$

$= 0.630.$

Q.10) Solve the differential equation,

$\frac{dy}{dx} = (1+x^2)y$, within $x \le 0(0.2)\,0.4$ & $y(0)=1$

using RK 4th order Method.

sol^n,

Given,

$\frac{dy}{dx} = (1+x^2)y.$

$y(0) = 1 = y_0$; $x_0 = 0$, $x_n = 0.4$, $h = 0.2$.

x	0	0.2	0.4
y	1	?	?

$\therefore y(0.2) = ?$ and $y(0.4)$

Now, Applying RK-4th order Method, we have,

for, $y(0.2) = y_1$

$K_1 = h \times f_1(x_0, y_0)$

$= 0.2$

$K_2 = hf(x_0 + 0.5h, y_0 + 0.5K_1)$

$= 0.2f(0.1, 1.1) = 0.222$

$K_3 = hf(x_0 + 0.5h, y_0 + 0.5K_2)$

$= 0.2f(0.1, 0.222) = 0.045.$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2f(0.2, 1.045) = 0.217.$

$\therefore k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$= \frac{0.2 + 2 \times 0.222 + 2 \times 0.045 + 0.217}{6}$

$= 0.159.$

$\therefore y(0.2) = y_0 + k = 1 + 0.159 = 1.159. = y_1$ //

Again,
for $y_2 = y(0.4)$.

$k_1 = hf(x_1, y_1)$
$= 0.2 f(0.2, 1.159) = 0.241$

$k_2 = hf(x_1 + 0.5h, y_1 + 0.5k_1)$
$= 0.2 f(0.3, 1.280) = 0.279$

$k_3 = hf(x_1 + 0.5h, y_1 + 0.5k_2)$
$= 0.2 f(0.3, 1.299) = 0.283.$

$k_4 = hf(x_1 + h, y_1 + k_3)$
$= 0.2 f(0.4, 1.442) = 0.335.$

$\therefore k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$= \frac{0.241 + 2 \times 0.279 + 2 \times 0.283 + 0.335}{6}$

$= 0.283.$

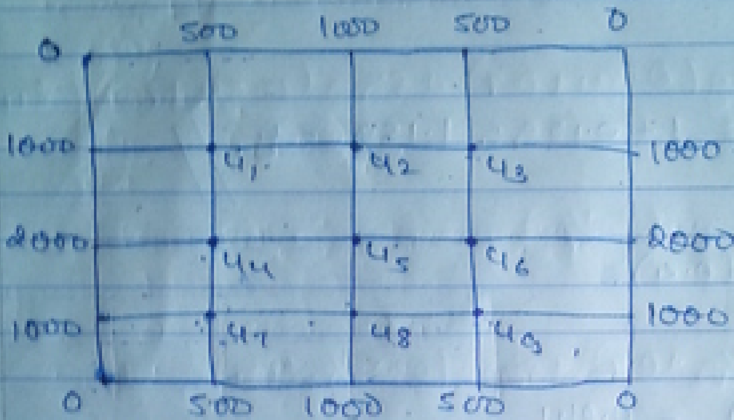
$\therefore y(0.4) = y_1 + k = 1.159 + 0.283 = 1.442$

Hence, $y_1 = 1.159$ and $y_2 = 1.442$.

$u_1, u_3, u_7, u_9 =$.

$u_2 = u_8$ $\quad u_4 = u_6$.

13) Solve the elliptical equation $u_{xx} + u_{yy} = 0$ for the following square mesh with the boundary values as shown.



Soln,

Here, First finding the initial values in the following order, $u_9 = 0$, $u_7 = 0$

$u_5 = \frac{1}{4}[2000 + 1000 + 2000 + 1000]$ [Std. formula].

$= 1500$

$u_1 = \frac{1}{4}[0 + 1500 + 1000 + 2000] = \overset{1125}{\cancel{2000}}$ [Diag. formula]

$u_3 = \frac{1}{4}[0 + 1500 + 2000 + 1000] = 1125$ [Diag. formula]

$u_7 = \frac{1}{4}[0 + 1500 + 1125 + 0] = 656.25$. [Diag. formula]

$u_2 = \frac{1}{4}[1000 + 1125 + 1500 + 1125] = 1188$. [Std. formula].

$u_4 = \frac{1}{4}[2000 + 1188 + \cancel{5}1125 + 1125 + 1500] = 1438$ [Std. formula].

Since, the values of u being symmetrical about x-axis & y-axis.

$\therefore u_1 = u_3 = u_7 = u_9 = 1125$

$\therefore u_2 = u_8 = 1188$

$\therefore u_4 = u_6 = 1438$

$\therefore u_5 = 1500$.

Now, Applying Gauss-Seidal Iteration process using the standard formula as:-

$$u_1^{(n+1)} = \frac{1}{4}\left[1000 + u_2^n + 500 + u_4^n\right]$$

$$u_2^{n+1} = \frac{1}{4}\left[u_1^{n+1} + u_1^n + 1000 + u_5^n\right]$$

$$u_4^{n+1} = \frac{1}{4}\left[2000 + u_5^n + u_1^{n+1} + u_1^n\right]$$

$$u_5^{(n+1)} = \frac{1}{4}\left[u_4^{n+1} + u_4^n + u_2^{n+1} + u_2^n\right].$$

u_1	u_2	u_4	u_5
1032	1164	1414	1301
1020	1088	1338	1251
982	1063	1313	1201
969	1038	1288	1176
957	1026	1276	1157
951	1016	1266	1146
946	1011	1260	1138
943	1007	1257	1134
941	1005	1255	1131
940	1003	1253	1129
939	1002	1252	1128
939	1001	1251	1126

Since, there is negligible difference between the value obtained in the last two iterations.

So,

$u_1 = u_3 = u_7 = u_9 = 939$

$u_2 = u_8 = 1001$

$u_4 = u_6 = 1251$

$u_5 = 1126$

Q.11) Solve the following boundary value problem using the finite difference Method, by dividing the interval into four sub-intervals.

$$\frac{d^2y}{dx^2} = x + y, \quad y(0) = y(1) = 0.$$