18th Dec

Proof Methods

If p then q

hypothesis

conclusion

① Qn diye ma simple way ma jane

direct method

$p \to q$

start

end

If n is odd, n^2 is odd

p

q

let $n = 2k+1$

$n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k + 2k) + 1$

odd

② Direct vayena vane,

$\neg q \rightarrow \neg p$

start   end

If n^2 is odd, $\therefore$ n is odd

$n = 2k$

$n^2 = 4k^2$

$= 2(2k^2)$

not odd

$n^2 = (2k)^2$

$n = 2k$

not odd

$\neg q \rightarrow \neg p$.

$\cdot\, 3n+2 = 2k+1$

$n = \dfrac{2k-1}{3}$ → not clear

so,

$p \to q$ ✗

$\neg q \to \neg p$

• Proof by contraposition

1. If n is an integer & '$3n+2$' is odd, then n is odd

Sol^n

$\neg q \Rightarrow n$ is not odd

$n = 2k$ [n is even]

Then,

for $\cdot\,\neg p$

↳ $3n+2$ is not odd

substituting $2k$ in $(3n+2)$

$\Rightarrow 3(2k)+2$

$\Rightarrow 6k+2$

$\Rightarrow 2(3k+1)$

is not odd [∵ is even]

$\therefore \neg q \to \neg p$

Thus,

negation of conclusion of conditional statement implies hypothesis is false.

so,

original cond. statement is true.

The contrapositive of $p \to q$
is $\neg q \to \neg p$

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2 Prove that if $n = ab$ where a & b are +ve integers then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

soln

$\neg q$ means $(a > \sqrt{n} \vee b > \sqrt{n})$

Now,

$ab > \sqrt{n}\sqrt{n}$

$\therefore ab > n$

$ab \neq n$ which contradicts original cond. statement $n = ab$.

3 $\sqrt{2}$ is irrational (proof by contradication)

$\neg p$ prove garne

Let,

$\sqrt{2}$ is rational

Then,

$\sqrt{2}$ can be represented as ratio of integers a & b

s.t. $b \neq 0$ & both having no common factor

$\sqrt{2} = \frac{a}{b}$

$\sqrt{2}b = a$

sq. both sides,

$$2b^2 = a^2$$

since,

a^2 = even no.

↓

so, a is also even

Then,

$a = 2c$ [let]

Again,

$$2b^2 = 4c^2$$
$$b^2 = 2c^2$$

↓

since,

b^2 = even no.

↓

b is also even

Both a & b are even, so common factor is 2 betn a & b.

$\sqrt{2} = a/b$ where a & b have no common factor that leads to contradiction of 2 divides both a & b.

so, $\neg p$ is false. Thus, p is true.

4. prove if a^2 is even then a is even (by contrapositive)

contrapositive of $p \to q$ is
$\neg q \to \neg p$

$\neg q \to$ a is odd

$\therefore a = 2k + 1$

Then,

$a^2 = (2k+1)^2$
$= 4k^2 + 4k + 1$
$= 2(2k^2 + 2k) + 1$
$\cancel{= 2}$

odd

i.e. $\neg p$

· Note:

$p \to q$ ma $\neg p$ = TRUE

vane

$p \to q$

always TRUE