Chap : 2

27th Nov the z - transform

Defn

Let $x(t)$ be a discrete time signal function define for discrete values of 't' where

$t = KT$

$K = 0, 1, 2, \ldots$

sampling period

Then, the z - transform of fn $x(t)$ is denoted by $\mathcal{Z}[x(t)]$ & is defined by

$$\mathcal{Z}[x(t)] = \mathcal{Z}[x(KT)] = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

$$= X(z)$$

Here,

'z' is complex variable

& '$\mathcal{Z}$' is operator of z - transform

yo z ho!!

Above expression is also called one sided or unilateral z-transform.

↳ The one-sided or unilateral z-transform of sequence of $x(k)$ or $x(n)$ is given by:

$$Z[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n} = X(z)$$

$$Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

where,
'z' is complex variable

& 'Z' is operator of z-transform

1) $x(k) = \{1, 8, 4, 3, 7\}$

$x(0)$ $x(1)$ $x(2)$ $x(3)$ $x(4)$

It's z-transform is

$$\sum_{k=0}^{4} x(k) z^{-k}$$

For discrete time signal function $x(t)$ where $-\infty < t < \infty$

Then,

$$\mathcal{Z}[x(t)] = \mathcal{Z}[x(KT)] = \sum_{K=-\infty}^{\infty} x(KT)\, z^{-K} = X(z)$$

where,

'z' is complex variable.

T is sampling period

'$\mathcal{Z}$' is operator of z-transform

Above exp is called two-sided or bilateral z-transform.

Region of convergence 'ROC'

It is defined as the region where the z-transform of a f^n exists.

The z-transform of sequence $x(K)$ is

$$\mathcal{Z}[x(K)] = \sum_{K=0}^{\infty} x(K)\, z^{-K}$$

ROC is the range of 'z' for which z-transform converges.

since z-transform is a power series, it converges when $x(k)\, z^{-k}$ is absolutely summable i.e. finite sum.

stated differently $\sum_{k=0}^{\infty} x(k)\, z^{-k} < \infty$ must be satisfied for convergence.

- **Properties of ROC**

(i) ROC of z-transform is indicated with the circle in the z-plane.

(ii) ROC can not contain any pole.

(iii) If $x(k)$ is right sided sequence then ROC extend outward from outermost pole of $X(z)$.

z transform of some std f^n / sequence / signals

(i) unit-step f^n

f^n defined by $u(k) = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$

is unit step f^n.

By defn of z-transform

$$Z[u_k] = \sum_{k=0}^{\infty} u_k z^{-k} = \sum_{k=0}^{\infty} 1 \cdot z^{-k}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots$$

$$\Rightarrow \frac{1}{1-\frac{1}{z}} \quad \text{for} \quad \left|\frac{1}{z}\right| < 1$$

$$= \frac{z}{z-1} \quad \text{for} \quad |z| > 1$$

then,

$$\therefore Z[u_k] = \underbrace{\frac{z}{z-1}}_{\text{sum of geometric series}}, \quad |z| > 1$$

↳ ROC lies outside of circle $|z| = 1$