8th Dec

Fixed Point Iteration Method

1. Evaluate the sq. root of 5 using the eqn $x^2 - 5 = 0$ by applying fixed point iteration algorithm.

$f(x) = x^2 - 5$

Soln

$x^2 = 5$

$x = \frac{5}{x}$ → exp can be written like this.

Assume $x_0 = 1$

Then,

$x_1 = 5$ $\left[x_1 = \frac{5}{1}\right]$

$x_2 = 1$ $\left[x_2 = 5/5\right]$

$x_3 = 5$

$x_4 = 1$

∴ process does not converge to the soln.

This type of divergence is known as oscillatory divergence.

$x_4 = 2.2361$
$x_5 = 2.2361$

Hence, the sq. root of 5 is 2.2361.

2. Find real root of eq^n $x^3 + x^2 - 1 = 0$ by fixed point iteration method.. correct upto 6 decimal places.

0.7548

Sol^n

$x^3 + x^2 - 1 = 0$

or, $x^3 - 1 = -x^2$

Then,

or, $(x-1)(x^2+x+1) = -x^2$

or, $x = 1 - \frac{x^2}{x^2+x+1}$

Assume $x_0 = 1$

$x_1 = 0.6666666667$
$x_2 = 0.7894736842$
$x_3 = 0.7416762342$
$x_4 = 0.7599732662$
$x_5 = 0.7529192288$
$x_6 = 0.7556316261$
$x_7 = 0.7545875919$
$x_8 = 0.7549892953$
$x_9 = 0.7548347122$
$x_{10} = 0.7548941953$
$x_{11} = 0.7548713059$
$x_{12} = 0.7548801137$

$X_{13} = 0.7548767244$

$X_{14} = 0.7548780287$

$X_{15} = 0.754877 5268$

$X_{16} = 0.754877 7199$

$\therefore$ The reqd. root is 0.7548777199

• **Note :** Non-linear method ma

root khojne ho

curve le x-axis lai
kata meet garxa

yo chap ma only row operation
no column operation

Chap-3

Soln of linear Eqn

↳ Soln of linear Eqn can be computed two ways.

ⓘ Elimination (Direct method
- → Gauss Elimination method
- → Gauss Jordan method

ⓘⓘ Iterative method
- → Jacobi's iterative method
- → Gauss seidal method

Ⓐ i) Gauss Elimination method

↳ consider AX = B & eqn defined as:

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \to (i)$$

Then,

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

&

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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(iv) Eq^n ma lagne ani

By backward substitution we get,

$$x + 4y - z = -5 \rightarrow (i)$$
$$-3y - 5z = -7 \rightarrow (ii)$$
$$23.67z = 49.33 \rightarrow (iii)$$

solving eq^n (iii)

$$z = \frac{49.33}{23.67}$$
$$= 2.084$$

From eq^n (ii)

or, $-3y - 5(2.084) = -7$

or, $-3y = 3.42$

$\therefore y = -1.14$

From eq^n (i)

$$x + 4(-1.14) - 2.084 = -5$$

$\therefore x = 1.644$

$\therefore x = 1.644$

$y = -1.14$

$z = 2.084$

then x & y

- Note: yesma unknown variables nikalne ho

1. Apply Gauss Elimination method to solve the eqns $x+4y-z=-5$; $x+y-6z=-12$; $3x-y-z=4$.

Soln

(i) Augmented form ma lagne

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ 3 & -1 & -1 & 4 \end{array}\right]$$

(ii) R_1 use garera a_{11} ↓ ko elements lai 0 parne.

Applying row operations: → 1/1 * ↳ derivation batai

- $R_2 \to R_2 - R_1$
- $R_3 \to R_3 - 3 R_1$

$\frac{3}{1}$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & -13 & 2 & 19 \end{array}\right]$$

(iii) R_2 use garera a_{22} ↓ ko elements lai 0 parne

- $R_3 \to R_3 - \frac{(-13)}{(-3)} * R_2$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & 0 & 23.67 & 49.333 \end{array}\right]$$

$$\therefore \quad [A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix} \rightarrow \text{(ii)}$$

Reducing above matrix (ii) into upper triangular matrix

Now, applying row operation

- $R_2 \rightarrow R_2 - \dfrac{a_2}{a_1} R_1$;

- $R_3 \rightarrow R_3 - \dfrac{a_3}{a_1} R_1$;

We get,

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b_2' & c'_2 & : & d'_2 \\ 0 & b'_3 & c'_3 & : & d'_3 \end{bmatrix} \rightarrow \text{(iii)}$$

Again,

- $R_3 \rightarrow R_3 - \dfrac{b'_3}{b'_2} R_2$, we get

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ 0 & b'_2 & c'_2 & : & d'_2 \\ 0 & 0 & c''_3 & : & d''_3 \end{bmatrix}$$

yeta bata z nikalne

Again,

$$x = x^2 + x - 5$$

Assume $x_0 = 0$

$x_1 = -5$

$x_2 = 15$

$x_3 = 235$

$x_4 = 55455$

∴ It also does not converge
rather it diverges rapidly.

So, this type of divergence is known as monotone divergence.

Let's try 3rd form of g(x)

$$2x = \frac{5}{x} + x$$

or, $x = \dfrac{x + 5/x}{2}$

Now,

take $x_0 = 1$

$x_1 = 3$

$x_2 = 2.3333$

$x_3 = 2.2381$

2. Solve :

$$10x - 7y + 3z + 5u = 6$$
$$-6x + 8y - z - 4u = 5$$
$$3x + y + 4z + 11u = 2$$
$$5x - 9y - 2z + 4u = 7$$

by Gauss Elimination method.

Soln

$$[A:B] = \left[\begin{array}{cccc|c} 10 & -7 & 3 & 5 & 6 \\ -6 & 8 & -1 & -4 & 5 \\ 3 & 1 & 4 & 11 & 2 \\ 5 & -9 & -2 & 4 & 7 \end{array}\right]$$