27th Nov

For implication

$p \to q$

- converse
  - ~~is~~ p hai q
  - q hai p

$q \to p$

logical equivalence.

- Inverse
  - Dubai ma negation

$\neg p \to \neg q$

same TT

- contrapositive
  - $\neg q \to \neg p$

converse

- computed by interchanging the hypothesis & conclusion.

If we have conditional statement $p \to q$. the converse is $q \to p$.

Inverse

↳ Negation of both hypothesis & the conclusion

If we have conditional statement $p \to q$ the inverse is $\neg p \to \neg q$

contrapositive

↳ computed by interchanging hypothesis & conclusion of inverse.

If we have conditional statement $p \to q$ the contrapositive is $\neg q \to \neg p$

1. prove contrapositive is logically equivalent to $p \to q$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Among these three conditional statement formed from $p \to q$, only contrapositive is logically equivalent to $p \to q$.

e) Bi-conditional or Bi-implication ($\leftrightarrow$)

Let p & q be two propositions, the bi-conditional statement $p \leftrightarrow q$ is "p if & only if q"

- It is true when p & q have same truth values otherwise false.

• Ways to express $\leftrightarrow$

(i) p if & only if q

(ii) p iff q

(iii) p is necessary & sufficient condn for q

(iv) if p then q & conversely.

$$\boxed{p \leftrightarrow q \Leftrightarrow (p \to q) \overset{\text{AND}}{\wedge} (q \to p)}$$

• Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(1) I go to the beach whenever it is a sunny summer day.

p: It is a sunny summer day

q: I go to the beach. $\quad\quad$ $\gamma_{p \to q}$

- Converse

$q \to p$

If I go to the beach then it is a sunny ~~it is a sunny~~ summer day.

- Inverse

$\neg p \to \neg q$

I do not go to the beach whenever it is not a sunny summer day.

- Contrapositive

$\neg q \to \neg p$

If I do not go to the beach then it is not a sunny summer day.

(2) I will ski tomorrow only if it snows today.

Chap : 2

27th Nov The z - transform

Defn

Let $x(t)$ be a discrete time signal function define for discrete values of 't' where $t = KT$

$\underbrace{\quad}_{\text{sampling period}}$

$K = 0, 1, 2, \cdots$

Then, the z-transform of f^n $x(t)$ is denoted by $Z[x(t)]$ & is defined by

$$Z[x(t)] = Z[x(KT)] = \sum_{K=0}^{\infty} x(KT) z^{-K}$$

$$= X(z)$$

Here,

'z' is complex variable

& 'Z' is operator of z - transform

yo Z ho!!

Above expression is also called one sided or unilateral z-transform.

↳ The one-sided or unilateral z-transform of sequence of $x(k)$ or $x(n)$ is given by:

$$Z[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n} = X(z)$$

$$Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

where:

'z' is complex variable

& 'Z' is operator of z-transform

1) $x(k) = \{1, 8, 4, 3, 7\}$

$x(0)$ $x(1)$ $x(2)$ $x(3)$ $x(4)$

It's z-transform is

$$\sum_{k=0}^{4} x(k) z^{-k}$$

For discrete (time) signal function $x(t)$ where $-\infty < t < \infty$

Then,

$$\mathcal{Z}[x(t)] = \mathcal{Z}[x(kT)] = \sum_{k=-\infty}^{\infty} x(kT)\, z^{-k} = X(z)$$

where,

'z' is complex variable.

T is sampling period

'$\mathcal{Z}$' is operator of z-transform

Above exp is called two-sided or bilateral z-transform

Region of convergence 'ROC'

- It is defined as the region where the z-transform of a f^n exists.

The z-transform of sequence $x(k)$ is

$$\mathcal{Z}[x(k)] = \sum_{k=0}^{\infty} x(k)\, z^{-k}$$

ROC is the range of 'z' for which z-transform converges.

since z-transform is a power series, it converges when $x(k) z^{-k}$ is absolutely summable i.e. finite sum.

stated differently $\sum_{k=0}^{\infty} x(k) z^{-k} < \infty$ must be satisfied for convergence.

- Properties of ROC

(i) ROC of z-transform is indicated with the circle in the z-plane.

(ii) ROC can not contain any pole.

(iii) If $x(k)$ is right sided sequence then ROC extend outward from outermost pole of $X(z)$.

Z transform of some std f^n / sequence / signals

(i) unit-step f^n

f^n defined by $u(k) = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$

is unit step f^n.

By defn of z-transform

$$Z[U_k] = \sum_{k=0}^{\infty} U_k z^{-k} = \sum_{k=0}^{\infty} 1 \cdot z^{-k}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots$$

$$\Rightarrow \frac{1}{1 - \frac{1}{z}} \quad \text{for} \quad \left|\frac{1}{z}\right| < 1$$

$$= \frac{z}{z-1} \quad \text{for} \quad |z| > 1$$

Then,

$$\therefore \underbrace{Z[U_k] = \frac{z}{z-1}}_{\text{sum of geometric series}}, \quad |z| > 1$$

↳ ROC lies outside of circle $|z| = 1$

② Polynomial f^n

f^n defined by relation

$$x(k) = \begin{cases} a^k & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$$

is polynomial f^n

By defn of z-transform

$$Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} a^k z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k$$

$$= 1 + \left(\frac{a}{z}\right)^1 + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \cdots$$

$$\Rightarrow \frac{1}{1-\frac{a}{z}} \quad \text{for } \left|\frac{a}{z}\right| < 1$$

$$\Rightarrow z / z - a \quad \text{for } |z| > a$$

$\therefore Z[a^k] = \dfrac{z}{z-a}$ for $|z| > a$

↳ ROC lies outside of circle $|z| = a$

In particular,

$a = 1$

$$Z[1^k] = Z[1] = \frac{z}{z-1}$$

$a = -1$

$$Z[(-1)^k] = \frac{z}{z+1}$$

n & k → sequence

③ Exponential f^n

discrete time signal

kT → anuxa

↳ f^n defined by $x(t) = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

is exponential f^n

By def^n of z-transform

$$Z[x(t)] = Z[x(kT)]$$

$$= \sum_{k=0}^{\infty} x(kT)\, z^{-k}$$

$$= \sum_{k=0}^{\infty} e^{-akT} z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(e^{-aT} z^{-1}\right)^{k}$$

$$= 1 + \left(e^{-aT} z^{-1}\right)^{1} + \left(e^{-aT} z^{-1}\right)^{2} + \cdots$$

$$= \frac{1}{1 - e^{-aT} z^{-1}} \qquad = \frac{z}{z - e^{-aT}}$$

$$\therefore\ Z\left[e^{-at}\right] = \frac{z}{z - e^{-aT}}$$

similarly

$$Z\left[e^{at}\right] = \frac{z}{z - e^{aT}}$$

&

$$Z\big[e^{a\underset{\text{variable}}{\underbrace{n}}}\big] = \frac{z}{z - e^{a}}$$

variable

ending

variable

$$z[e^{-ak}] = \frac{z}{z-e^{-a}}$$

(4) Unit-Impulse or Dirac-Delta fn $\delta(k)$

↳ fn defined by $\delta(k) = \begin{cases} 1 & \text{for } k=0 \\ 0 & \text{for } k \neq 0 \end{cases}$

is Dirac-Delta fn

By defn,

$$z[\delta(k)] = \sum_{k=0}^{\infty} \delta(k) z^{-k}$$

$$= \delta(0) + \delta(1) z^{-1} + \delta(2) z^{-2} + \cdots$$

$$= 1 + 0 + 0 + 0 \cdots$$

$$= 1$$

$\therefore z[\delta(k)] = 1$

(5) Unit-Ramp fn

↳ fn defined by $x(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$ is

unit-Ramp fn.

(6) sinusoidal f^n

↳ f^n defined by $x(t) = \begin{cases} \sin \omega t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

is sinusoidal f^n

Z-transform of t^n where 'n' is +ve integer

soln

let $x(t) = t^n$

$x(KT) = (KT)^n$

t ayesi KT le replace

By defn of Z-transform

$$Z[x(t)] = Z[x(KT)]$$

$$= \sum_{K=0}^{\infty} x(KT) z^{-K}$$

$$= X(z)$$

$$\therefore Z[t^n] = \sum_{K=0}^{\infty} (KT)^n z^{-K} = X(z)$$ ↳ (i)

Similarly,

$$Z[t^{n-1}] = \sum_{k=0}^{\infty} (kT)^{n-1} z^{-k}$$

Now,

diff. both sides w.r.t. 'z', we get

$$\frac{d}{dz}[Z(t^{n-1})] = \frac{d}{dz}\left[\sum_{k=0}^{\infty} (kT)^{n-1} z^{-k}\right]$$

$$= \sum_{k=0}^{\infty} (kT)^{n-1} \frac{d}{dz}(z^{-k})$$

$$= \sum_{k=0}^{\infty} (kT)^{n-1} (-k) z^{-k-1}$$

$$= \sum_{k=0}^{\infty} \frac{(kT)^{n}}{kT} (-k) \frac{z^{-k}}{z}$$

$$= \frac{-1}{Tz} \sum_{k=0}^{\infty} (kT)^{n} z^{-k}$$

$$\frac{d}{dz}\left[Z\left(t^{n-1}\right)\right] = \frac{-1}{Tz} X(z) \quad \text{[from eq}^{n}\text{ (i)]}$$

$$\Rightarrow X(z) = -Tz \frac{d}{dz} Z\left[t^{n-1}\right]$$

$$\therefore Z\left[t^{n}\right] = -Tz \frac{d}{dz}\left[Z\left(t^{n-1}\right)\right]$$

Put $n = 1$

$$Z[t] = -Tz \frac{d}{dz}\left[Z\left(t^{0}\right)\right]$$

$$= -Tz \frac{d}{dz}\left[Z(1)\right]$$

$$\frac{z}{z-1}$$

$$= -Tz \frac{d}{dz}\left(\frac{z}{z-1}\right)$$

$$= -Tz \frac{(z-1)\cdot 1 - z\cdot 1}{(z-1)^2}$$

$$= -Tz \left(\frac{\cancel{z} - 1 - \cancel{z}}{(z-1)^2}\right)$$

$$Z[t] = \frac{Tz}{(z-1)^2}$$

Put $n=2$

$$Z[t^2] = -Tz \frac{d}{dz}[Z(t)] = -Tz \frac{d}{dz}\left[\frac{Tz}{(z-1)^2}\right]$$

$$= -T^2 z \frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -T^2 z \frac{(z-1)^2 \cdot 1 - z \cdot 2(z-1) \cdot 1}{(z-1)^4}$$

$$= -T^2 z \frac{(z-1)^2 - 2z(z-1)}{(z-1)^4}$$

$$= -T^2 z \frac{\cancel{(z-1)}[(z-1) - 2z]}{(z-1)^{\cancel{4}\,3}}$$

$$= \frac{-T^2 z[z-1-2z]}{(z-1)^3}$$

$$= \frac{-T^2 z[-1-z]}{(z-1)^3}$$

$$\therefore Z[t^2] = \frac{T^2 z[z+1]}{(z-1)^3}$$

→ 3 diy →

similarly,

we obtain

$$Z[k^n] = -z \frac{d}{dz} Z[k^{n-1}]$$

T hatxa

$$Z[k] = \frac{z}{(z-1)^2}$$

$$Z[k^2] = \frac{z(z+1)}{(z-1)^3}$$