11th Dec

to represent quantities

more powerful than propositional logic

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Predicate Logic

propositional logic ko rules + afnai 4

· Predicates & Quantifier

- Declarative sentence
  whose truthness / falsehood depends on one or more variables.
- It is propositional f^n

· Quantifiers

(i) Universal quantifiers

(ii) Existential quantifiers

(i) Universal

- True for all
- conjuction aauxa.
- sabai ko huna paryo.

↳ counter-eg xa yane budaina

↓

no hune case

↳ $\forall x P(x)$ universal quantifier

↓

For all

(ii) Existential

↳ At least 1 ko lagi true huna paryo

↳ Disjunction

↳ $\exists x P(x)$ existential quantifier

• Keywords

(i) There exists

(ii) For at least one

(iii) There is

• Negating quantified exp.

① Every student in class has taken a course in calculus

yesko ¬ "It is not the case ..."

vannale at least euta student le calculus lexaina

yo vaneko existential quantification of ¬

i.e.

$$\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$$

② There is a std in class who has ...

$$\neg \exists x\, P(x) \Leftrightarrow \forall x\, \neg P(x)$$

Nested quantifiers

① $\forall x \forall y \ (x+y = y+x)$

For every x & For every y,

$x+y = y+x$

② $\forall x \exists y \ (x+y=0)$

For all real no. x, there exists real no. y,

such that $x+y=0$

③ For $Q(x,y)$

denote $(x+y=0)$

Then,

$\exists y \forall x \ Q(x,y) \rightarrow$ False

There exists a value of y, for all x

Tara euta single y value won't satisfy $x+y=0$

$5-5=0$, $4-5 \neq 0$

... Inference for Quanti...

	Name
[illegible]	Universal instantiation
[illegible]	Universal generalization
[illegible]	Existential instantiation
[illegible]	Existential generalization

Note : (i) conjuction

$$\frac{\begin{array}{c} p \\ q \end{array}}{p \wedge q}$$

(ii) Addition

$$\frac{p}{p \vee q}$$

(iii) simplification

$$\frac{p \wedge q}{p \mid q}$$

Existential instn

1. Show that premises
   - A std in sec A has taken course has not read book.
   - Everyone in sec A has taken course passed 1st exam.

Implies
someone who passed 1st exam has not read the book.

A (x) : "x is in sec who has taken the course."

B (x) : "x read the book"

P(X) : "X passed the 1st exam."

- Hypothesis:

$$\exists x(A(x) \wedge \neg B(x))$$

$$\forall x(A(x) \longrightarrow P(x))$$

- Conclusion:

$$\exists x\, P(x) \wedge \neg B(x)$$

	Statement	Reason
1.	$\exists x(A(x) \wedge \neg B(x))$	Hypothesis
2.	$\forall x(A(x) \rightarrow P(x))$	Hypothesis
3.	$A(a) \wedge \neg B(a)$	Existential instantiation on (1)
4.	$A(a) \rightarrow P(a)$	Universal instantiation on (2)
5.	$A(a)$	Simplification of (3)
6.	$P(a)$	Using modus pones on (4) & (5)
7.	$\neg B(a)$	Simplification of (3)
8.	$P(a) \wedge \neg B(a)$	Conjuction of (6) & (7)

9. $\exists x\, (P(x) \wedge \neg B(x))$ Existential generalization from 8)

2. lions are dangerous animals

sabai lions are dangerous.

There are lions

Existential

Therefore, there are dangerous animals

soln

$L(x)$: x is lion

$D(x)$: x is dangerous animals

- Hypothesis:

$\forall x\, (L(x) \rightarrow D(x))$

$\exists x\, L(x)$

- Conclusion:

$\exists x\, D(x)$

Pg. 66

5 construct an arg. using rules of inference to show hypothesis

- "All movies produced by J.S. are wonderful."

  universal

- "J.S. produced a movie about coal miners."

  Existential

implies

- "There is a wonderful movie about coal miners."

soln

J(x) : x is movie produced by J.S.

C(x) : x is movie about coal miners.

W(x) : x is wonderful movie

- Hypothesis :

$$\forall x (J(x) \rightarrow W(x))$$

$$\exists x (J(x) \wedge C(x))$$

- conclusion :

$$\exists x (W(x) \wedge C(x))$$

	Statement	Reason
1.	$\forall x\,(J(x) \to W(x))$	Hypothesis
2.	$\exists x\,(J(x) \wedge C(x))$	Hypothesis
3.	$J(a) \wedge C(a)$	Existential instantiation on 2.
4.	$J(a) \to W(a)$ $\hookrightarrow \neg J(a) \wedge W(a)$	Uni instantiation on 1.
5.	$J(a)$	simplification of (3)
6.	$W(a)$	using modus ponens on (4) & (5)
7.	$C(a)$	simplification on (3)
8.	$W(a) \wedge C(a)$	using conjuction on (6) & (7)
9.	$\exists x\,(W(x) \wedge C(x))$	Existential generalization on (8).