14 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2075 Baisakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	BGE, BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function. Is V = arg(z) is harmonic? If yes, find a corresponding harmonic conjugate. [1+1+3]

   b) Define conformal mapping. Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. [1+4]

2. a) Distinguish between Cauchy integral Theorem and Cauchy integral formula. Using Cauchy integral formula evaluate $\int_c \frac{e^z}{(z+1)(z-2)}dz$ where C is the circle $|z-1| = 3$. [1+4]

   b) State and Prove Taylor's series for function of complex variable. [5]

3. a) Define an isolated pole. Using Cauchy's residue theorem evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C is the circle $|z-i| = 2$. [5]

   b) Evaluate the integral by contour integration: [5]

$$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)}dx$$

4. a) Obtain the z-transform of $(1-e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [2+3]

   b) Obtain the inverse z-transform of:

$$X(z) = \frac{2z^3+z}{(z-2)^2(z-1)}$$ by using partial fraction method. [5]

5. a) Define z-transform of function f(t). Find the z-transform of following sequences: [1+2+2]

(i) $f(k) = \left\{ \begin{matrix} 15,10,7,4,1,-1,3,6 \\ \uparrow \end{matrix} \right\}$

(ii) $f(k) = \begin{cases} 5^k & ; \quad k < 0 \\ 2^k & ; \quad k \geq 0 \end{cases}$

b) Solve the difference equation by the application of z-transform:

$x(k+2) + 3x(k+1) + 2x(k) = 0$ with conditions $x(0) = 0$, $x(1) = 1$. [5]

6. a) A tightly stretched string with fixed ends at $x = 0$ and $x = l$ is initially at rest in its equilibrium position. Find the deflection $u(x, t)$ if it is set vibrating by giving to each of its points a velocity $3(lx-x^2)$. [10]

   b) Derive two dimensional heat equation. [10]

7. a) Obtain the Fourier sine integral representation of $e^{-x}\cos x$ and hence show that

$$\int_0^{\infty} \frac{\omega^3 \sin \omega x}{\omega^4 + 4} d\omega = \frac{\pi}{2} e^{-x} \cos x, \quad x > 0.$$ [5]

   b) Find the Fourier Cosine transform of $f(x) = e^{-x}, x > 0$ and hence by Parseval's identity, show that [5]

$$\int_0^{\infty} \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}.$$

14 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2074 Bhadra

Exam.		Regular	
Level	BE	Full Marks	80
Programme	BGE, BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define an analytic function for a function of complex variable. Derive Cauchy Riemann equations in Cartesian form. [1+4]

   b) Define linear fractional mapping. Find bilinear mapping which maps the points $z = 0, 1, -1$ to $w = i, 2, 4$. [1+4]

2. a) State and Prove Cauchy integral theorem. [5]

   b) Point out difference between Taylor's series and Laurent's series. Find Laurent' series of function $f(z) = \frac{\sin z}{z^6}$, $0 < |z| < TR$ [1+4]

3. a) Define pole of order m. Using Cauchy's residue theorem evaluate $\int_c \cot z \; dz$; where C is $|z| = 1$. [1+4]

   b) Using Counter integration evaluate, [5]

   $$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}.$$

4. a) Find the z-transform of: [2+3]

   (i) $\cos at$ (ii) te^{-at}

   b) State final value theorem. If $x(t) = 0$ for $t < 0$ and $Z[x(t)] = X(z)$ for $t \geq 0$ then prove that: [1+4]

   $$Z[x(t+nT)] = z^n\left[X(z) - \sum_{k=0}^{n-1} x(kT)z^k\right].$$

5. a) Obtain inverse Z-transform of $\frac{z(3z^2 - 6z + 4)}{(z-1)^2(z-2)}$. [5]

   b) Solve the difference equation by the application of z-transform:

   $x(k+2) - 4x(k+1) + 4x(k) = 0$; with conditions $x(0) = 1$; $x(1) = 0$. [5]

6. a) Derive one dimensional wave equation and solve it completely. [5+5]

   b) A uniform rod of length ℓ has its end maintained at a temperature $0°C$ and the initial temperature of the rod is:

   $$u(x,0) = 3\sin\frac{\pi x}{\ell} \quad \text{for } 0<x<\ell.$$

   Find the temperature $u(x, t)$. [10]

7. a) Find Fourier integral of the function [5]

   $$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

   b) Verify the convolution theorem for Fourier transform for the functions $f(x) = g(x) = e^{-x^2}$. [5]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2073 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ *Attempt **All** questions.*
✓ *The figures in the margin indicate **Full Marks**.*
✓ *Assume suitable data if necessary.*

1. a) Define harmonic function of complex variable. Determine the analytical function

   $f(z) = u + iv$ if $u = y^3 - 3x^2y$ [1+4]

   b) Derive Cauchy-Reimann equations if function of complex variable $f(z) = u + iv$ is analytic in cartesian form. [5]

2. a) What do you mean by conformal mapping? Find the linear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = 0, w_2 = 1, w_3 = \infty$. [1+4]

   b) State and prove Cauchy's integral formula. [5]

3. a) State Taylor's theorem. Find the Laurent's series representation of the function

   $f(z) = \dfrac{z}{(z+1)(z+2)}$ in the annular region between $|z| = 1$ and $|z| = 2$. [1+4]

   b) Define zero of order m of function of complex variable .Determine the poles and residue at poles of the functions $f(z) = \dfrac{1+z}{(z+2)(1-z)^2}$. [1+4]

   OR

   Evaluate the real integral $\int_{-\infty}^{\infty} \dfrac{x^2}{(1+x^2)^3} dx$ by contour integration in the complex plane. [5]

4. a) Define z-transform. How does it differ from Fourier transform? Obtain z-transform of

   (i) t^2a^t (ii) cosat [1+1+1.5+1.5]

   b) State initial value theorem for z transform. Find the initial value x(0) and x(1) for the function. [1+4]

   $$X(z) = \frac{(1-e^T)z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$$

5. a) Obtain the inverse z-transform of $X(z) = \dfrac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Apply method of z-transform to solve the difference equation [5]

   $x(k+2) - 4x(k+1) + 4x(k) = 0; x(0) = 0, x(1) = 1$

6. Solve completely one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions: [10]

$u(0,t) = 0, u(l,t) = 0, u(x,0) = 0$ and $\left(\frac{\partial u}{\partial t}\right)_{\text{at } t=0} = 3(lx - x^2)$

7. Derive one dimensional heat equation and solve it completely. [10]

8. a) State convolution theorem for Fourier transform. Give its importance with suitable example. [2+3]

b) Find the Fourier cosine integral of the function $f(x) = e^{-kx} (x > 0, k > 0)$ and hence show that $\int_0^\infty \frac{\cos \omega x d\omega}{k^2 + \omega^2} = \frac{\pi}{2k} e^{-kx}; x > 0, k > 0$ [5]

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14 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2073 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- Candidates are required to give their answers in their own words as far as practicable.
- *Attempt **All** questions.*
- *The figures in the margin indicate **Full Marks**.*
- *Assume suitable data if necessary.*

1. a) Define analytical function of complex-variable. Determine the analytic function $f(z) = u + iv$ if $u = \log\sqrt{x^2+y^2}$. [1+4]

   b) Express Cauchy Riemann equations $U_x = V_y$ and $U_y = -V_x$ into polar form. [5]

2. a) Define bilinear transformation. Obtain the linear transformation which maps points $z_1 = -i, z_2 = 0, z_3 = i$ into $w_1 = -1, w_2 = i, w_3 = 1$ [1+4]

   b) Evaluate $\int_c \frac{e^{2z}}{z^2-3z+2}dz$ in the circle $|z| = 3$ by using Cauchy integral formula. [5]

3. a) State Laurent's Theorem. Obtain the Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = i$. [1+4]

   b) Define residue at poles. Evaluate $\oint_c \frac{\sin z}{z^6}dz$, $C: |z| = 1$ by residue method. [1+4]

   OR

   Evaluate real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ by contour integration in the complex plane. [5]

4. a) Define Z-transform and its region of convergence. Find the Z-transform of [1+1+1.5+1.5]

   i) t^2e^{-at} ii) sinat

   b) State and prove final value theorem for Z-transform. [1+4]

5. a) Find the inverse Z-transform of $f(z) = \frac{z-4}{(z-1)(z-2)^2}$ by partial fraction method. [5]

   b) Use the method of Z-transform to solve the difference equation. [5]

   $x(k+2) + 2x(k+1) + 3x(k) = 0 : x(0) = 0, x(1) = 2$

6. Derive one dimensional wave equation and solve it completely. [10]

7. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions: [10]

   $$u(o,y) = u(l,y) = u(x,0) = 0, u(x,\infty) = \sin\left(\frac{n\pi x}{l}\right)$$

8. a) Define Fourier transform of a function. How does it differ from Fourier series? Support your answer with suitable example. [1.5+1.5+2]

   b) Find the Fourier Sine transform of e^{-x}, $x \geq 0$ and hence show that

   $$\int_0^\infty \frac{x^2}{(1+x^2)^2}dx = \frac{\pi}{4}$$ [5]

15 TRIBHUVAN UNIVERSITY	Exam.	Regular		
INSTITUTE OF ENGINEERING	Level	BE	Full Marks	80
Examination Control Division	Programme	BEL, BEX, BCT, BGE	Pass Marks	32
2072 Ashwin	Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) If $u=(x-1)^3-3xy^2+3y^2$, determine V so that u + iv is an analytic function of x+iy. [5]

   b) Define an analytic function. Express Cauchy Riemann equations $u_x=v_y$ and $u_y=-v_x$ in polar from. [5]

2. a) Find the bilinear transformation which maps points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=-1, w_3=-i$ respectively. [5]

   b) Evaluate $\int_0^{1+i}(x^2+iy)dz$ along the path $y=x^2$ [5]

3. a) Express $f(z)=\dfrac{1}{(z^2-3z+2)}$ as Laurent's series in the region $1<|z|<2$. [5]

   b) Evalute $\int_0^{2\pi}\dfrac{1}{5-4\sin\theta}d\theta$ by contour integration method in complex plane. [5]

4. a) Find z-transform of: [5]
   i) te^{-at}
   ii) sinat

   b) State and prove final value theorem for z- transform. [5]

5. a) Find the inverse z-transform of $\dfrac{2z^2-5z}{(z-2)(z-3)}$ by using partial fraction method. [5]

   b) Solve difference equation $x(k+2)-3x(k+1)+2x(k)=4^k$ for $x(0)=0$ and $x(1)=1$. [5]

6. Derive one dimensional wave equation and obtain its solution. [10]

7. Solve one dimensional heat equation: [10]

   $\dfrac{\partial u}{\partial t}=c^2\dfrac{\partial^2 u}{\partial x^2}$ under the conditions:

   i) u is not infinite as $t\to\infty$

   ii) $\dfrac{\partial u}{\partial x}=0$ for $x=0$ and $x=l$

   iii) $u(x,0)=lx-x^2$ for $t=0$; between $x=0$ and $x=l$

8. a) Find Fourier integral representation of $f(x)=e^{-x}, x>0$ and hence evaluate $\int_0^{\infty}\dfrac{\cos(sx)}{s^2+1}ds$ [5]

   b) Find the Fourier cosine transform of $f(x)=e^{-|x|}$ and hence, by Parseval's identity, shown that $\int_0^{\infty}\dfrac{1}{(1+x^2)^2}dx=\dfrac{\pi}{4}$ [5]

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05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH451)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ ***All*** *questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$

2. Find the extreme value of $x^2+y^2+z^2$ connected by the relation $x+y+z=3a$

3. Evaluate: $\iint r\sin\theta\, dr.d\theta$ over the area of the cardioid $r = a(l+\cos\theta)$ above the initial line.

4. Evaluate: $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} y^2.\sqrt{(x^2+y^2)}\ dy.dx$ by changing polar coordinates.

   OR

   Evaluate: $\iiint x^{l-1}.y^{m-1}.z^{n-1}dx.dy.dz$

   Evaluate: x, y, z are all positive but $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r \le 1$

5. Find the length of perpendicular from the point (3, -1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also obtain the equation of the perpendicular.

6. Find the magnitude of the line of the shortest distance between the lines $\frac{X}{4}=\frac{Y+1}{3}=\frac{Z-2}{2}$, $5x-2y-3z+6=0,\ x-3y+2z-3=0$

7. Find the centre and radius of the circle in which the sphere $x^2+y^2+z^2-8x+4y+8z-45=0$ is cut by the phane $x-2y+2z=3$

8. The plane through OX and OY include an angle α, show that their line of intersection lies on the cone $z^2(x^2+y^2+z^2)=x^2y^2\tan^2\alpha$

9. Solve by power series method of the differential equation y"-y=0

10. Express $f(x)=x^3-5x^2+x+2$ in terms of Legendre's polynomials.

11. Prove the Bessel's function: $J_{3/2}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x}-\cos x\right)$

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors then prove that

$$\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \frac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0$$

13. If $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j} + at\tan\alpha\,\vec{k}$, find $\left[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\right]$

14. Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of victor $\vec{i} + 2\vec{j} + 2\vec{k}$

OR

If $\vec{r}$ be the position vector and $\vec{a}$ is constant vector then prove that $\nabla.\left(\frac{\vec{a} \times \vec{r}}{r}\right) = 0$

15. Determine whether the series $\sum \frac{n}{1 + n\sqrt{n+1}}$ is convergent or divergent.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n.2^n}$

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Bhādra

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Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BOE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ ***All*** *questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function $f(z) = u + iv$ if $u = \log\sqrt{x^2 + y^2}$.
2. State and prove Cauchy's integral formula.
3. Find the Taylor's series of $f(z) = \frac{1}{1-z}$ about $z = 3i$.
4. Evaluate the integral: $\oint_C \frac{e^{2z}}{(z+1)(z+3)}$ where C: $|z| = 4$, using residue theorem.
5. Define conformal mapping, show that $w = \frac{az+b}{cz+d}$ is invariant to
$$\left(\frac{w-w_1}{w-w_3}\right)\times\left(\frac{w_2-w_3}{w_2-w_1}\right) = \left(\frac{z-z_1}{z-z_3}\right)\times\left(\frac{z_2-z_3}{z_2-z_1}\right)$$
6. Using contour integration, evaluate real integral: $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$
7. Find the z-transform of $x(z) = \cosh t \sinh t$.
8. State and prove "final value theorem" for the z-transform.
9. Find the inverse z-transform of $x(z) = \frac{z}{z^2 + 7z + 10}$.
10. Using z-transform solve the difference equation:
$x(K+2) + 6x(K+1) + 9x(K) = 2^K;\ x_0 = x_1 = 0.$
11. Derive one-dimensional heat equation.
12. Solve the wave equation for a tightly stretched string of length 'l' fixed at both ends if the initial deflection in $y(x, 0) = lx - x^2$ and the initial velocity is zero.
13. ~~Solve~~ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions $u(0, y) = u(l, y) = u(x, 0) = 0, u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$
14. Derive the wave equation (vibrating of a string).
15. Find the Fourier cosine transform of $f(x) = e^{-m|x|}$ and hence show that $\int_0^{\infty} \frac{\cos py}{y^2 + \beta^2} dy = \frac{\pi}{2\beta} e^{-p\beta}$.
16. Find the Fourier integral representation of the function $f(x) = e^{-x}, x \geq 0$ with $f(-x) = f(x)$.
Hence evaluate $\int_0^{\infty} \frac{\cos(sx)}{s^2+1} ds$.

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12 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2a 2071 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT, BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks.***
- ✓ *Assume suitable data if necessary.*

1. a) Determine the analytic function $f(z) = u + iv$ if $u = 3x^2y - y^3$. [5]

   b) Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed points of the transformation. [5]

2. a) State and prove Cauchy's integral formula. [5]

   b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. [5]

3. a) Find the first four terms of the Taylor's series expansion of the complex function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the centre $z = 2$. [5]

   b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [5]

   OR

   Evaluate $\int_0^{2\pi} \frac{1}{\cos\theta + 2} d\theta$ by contour integration in the complex plane.

4. Derive one dimensional heat equation $u_t = c^2 u_{xx}$ and solve it completely. [10]

5. Find all possible solution of Laplace equation $u_{xx} + u_{yy} = 0$. Using this, hence solve $u_{xx} + u_{yy} = 0$, under the conditions $u(0, y) = 0$, $u(x, y) = 0$ when $y \to \infty$ and $u(x, 0) = \sin x$. [10]

6. a) Find the z-transform of $\sin K\theta$. Use it to find the $z[a^K \sin K\theta]$. [5]

   b) If $z[x(K)] = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the value of $x(2)$ and $x(3)$. [5]

7. a) Find the inverse z-transform of $x(z) = \frac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

   b) Using z-transform solve the difference equation $x(K+2) - 4x(K+1) + 4x(K) = 2^K$ given that $x(0) = 0$, $x(1) = 1$. [5]

8. a) Find the Fourier sine integral of the function $f(x) = e^{-Kx}$ and hence show that [5]

$$\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + \beta^2} d\lambda = \frac{\pi}{2} e^{-Kx}, \quad x > 0, K > 0$$

   b) Find the Fourier sine transform of e^{-x}, $x \geq 0$ and hence show that [5]

$$\int_0^\infty \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, \quad m > 0$$

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25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2070 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics ***(SH551)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Show that $u(x, y) = x^2 + 2xy - y^2$ is a harmonic function and determine $v(x, y)$ in such a way that $f(z) = u(x, y) + iv(x, y)$ is analytic. [5]
2. Define complex integral. State and prove Cauchy integral formula. [5]

OR

Obtain bilinear transformation which maps $-i, o, i$ to $-1, i, 1$. [5]

3. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is $|z| = 3$ using Cauchy's integral formula. [5]
4. Obtain the Laurent series which represents the function $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ $2 < |z| < 3$. [5]
5. Find the Laurent series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]
6. State and prove Taylor series of a function f(z). [5]
7. Derive one dimensional wave equation $u_{tt} = c^2 u_{xx}$ and solve it completely. [10]
8. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary condition $\frac{\partial u}{\partial x} = 0$ when $x = 0$ and $x = L$ and initial condition $u(x, 0) = x$ for $0 < x < L$. [10]
9. Find Z transform of (a) te^{-at} and (b) sin at. [5]
10. Find the inverse z-transform (a) $\frac{z-4}{(z-1)(z-2)^2}$ (b) $\frac{z}{z^2 - 3z + 2}$. [5]
11. Obtain the Z transform of $x(t) = (1 - e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [5]
12. Solve using z-transform the difference equation $x(K + 2) + 2x(K + 1) + 3x(K) = 0$. [5]
13. Find the Fourier sine transform of $f(x) = e^{-x}$, $x \geq 0$ and hence evaluate $\int_0^\infty \frac{x \sin x}{(1+x^2)} dx$. [5]
14. State and prove convolution theorem of Fourier transform. [5]

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25 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	**Full Marks**	80
Programme	BEL, BEX, BCT	**Pass Marks**	32
Year / Part	II / II	**Time**	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define analytic function. Show that the function $f(z) = \frac{1}{z^4}$ is analytic except $z = 0$ [5]
2. Define complex integral. Evaluate $\int_c \log z\, dz; c: |z| = 1$ [5]

OR

Obtain a bilinear transformation which maps -i, 0, i to -1, i, 1.

3. Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path $y = x$. [5]
4. Find the Taylor series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]
5. Evaluate the integrals by residue theorem $\int_c \frac{1-\cos z}{z^3} dz$ [5]
6. State Cauchy's Residue theorem and use it to evaluate $\int_c \frac{z^2}{3+4z+z^2} dz$ where C is $|z| = 2$ [5]

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2}$ by contour integration in complex plane.

7. Derive the one dimensional wave equation. [10]
8. A rod of length L has its ends A and B maintained at 0° and 100°respectively until steady state prevails. If the changes are made by reducing the temperature of end B to 85° and increasing that of end A to 15°, then find the temperature distribution in the rod at a time t. [10]
9. Find the z-transform of (i) $e^{-at} \sin wt$ (ii) cos at [5]
10. Obtain inverse Z-transform of (i) $\frac{z+2}{(z-2)(z-3)}$, (ii) $\frac{z}{(z-2)(z-1)}$ [5]
11. If $x(k) = 0$ for $k < 0$ and $Z\{x(k)\} = X(z)$ for $k > 0$ then prove that $Z\{x(k+n)\} = z^n X(z) - z^n \sum_{k=0}^{n-1} \chi(k) z^{-k}$ where n = 0, 1, 2.... [5]
12. Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions, $x(0) = 0, x(1) = 1$ [5]
13. Find the cosine transform of $f(x) = e^{-mx}$ $m > 0$ show that $\int_0^\infty \frac{\cos pr}{r^2 + B^2} = \frac{\Pi}{2B} e^{-PB}$ [5]
14. Find the Fourier transform of $g(x) = \begin{cases} 1 - x^2 & \text{if } -1 < x < 1; \\ 0, & \text{if otherwise.} \end{cases}$ [5]

and hence use it to evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) \cos(x/2) dx$

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TRIBHUVAN UNIVERSITY INSTITUTE OF ENGINEERING **Examination Control Division** **2069 Bhadra**	Exam.	Regular (2066 & Later Batch)		
	Level	BE	Full Marks	80
	Programme	BEL, BEX, BCT	Pass Marks	32
	Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Determine the analytic function $f(z) = u(x,y) + iv(x,y)$ if $u(x,y) = x^2-y^2$.

2. Define complex integral. Evaluate: $\oint_C (z+1)dz$ where C is the square with vertices at $z = 0$, $z = 1$, $z = 1+i$ and $z = i$.

OR

Find linear fractional transformation mapping of: $-2 \mapsto \infty, 0 \mapsto \frac{1}{2}, 2 \mapsto \frac{3}{4}$.

3. a) State Cauchy's integral formula and evaluate the integral $\oint \frac{4-3z}{z(z-1)(z-2)}dz$, where C is circle $|z| = \frac{3}{2}$.

   b) Obtain the Laurent series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1<|z|<2$.

4. a) Find the Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = i$.

   b) Evaluate $\int_C \tan z\, dz$ where C is a circle $|z| = 2$ by Cauchy's residue theorem.

OR

Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ by contour integration in the complex plane.

5. Find the z-transforms of: (i) cos h (at) sin (bt) (ii) n.(n-1); n = k

6. Find the inverse z-transforms of: (i) $\frac{Z}{Z^2-3Z+2}$ (ii) $\frac{Z}{(Z+1)^2(Z-1)}$.

7. a) State and prove convolution theorem for z-transform.

   b) Solve by using z-transform the difference equation $x(k+2) + 2x(k+1) + 3x(k) = 0$ given that $x(0) = 0$ and $x(1) = 2$

8. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given that $u = 0$ as $t \to \infty$ as well as $u = 0$ at $x = 0$ and $x = l$

9. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the condition $u(0,y) = u(L,y) = u(x,0) = 0$ and $u(x,a) = \sin\left(\frac{n\pi x}{L}\right)$.

OR

The diameter of a semi-circular plate of radius a is kept at 0°C and the temperature at the semi-circular boundary is u_0. Find the steady state temperature in the plate.

10. Find the Fourier integral representation of the function $f(x) = e^{-x}, x \geq o$ with $f(-x) = f(x)$.

Hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2+1} ds$.

11. Find the Fourier transform of:

$$f(x) = 1-x^2, |x|<1$$
$$= 0, |x|>1$$ and hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx.$$

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25 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Poush

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics *(SH551)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. If $f(z) = u + iv$ is any analytic function of complex variable z and $u-v = e^x(\cos y - \sin y)$, find f(z) in terms of z.
2. State and prove Cauchy's integral theorem.
3. Using Cauchy integral formal to evaluate: $\oint_c \frac{zdz}{(z-1)(z-3)}$ where $C:|z| = 3/2$.
4. Find the Laurent's series expansion of $f(z) = \frac{1-\cos z}{z^3}, 0 < |z| < R$.
5. Define singular points and poles. compute the residue of $f(z) = \frac{z^2}{(z-2)(z^2+1)}$ at its pole(s).

OR

Using contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$.

6. Find the z-transform of the following: (any two)
   i) K^2a^k for $K \geq 0$ (ii) $a^k \cos k\pi$ for $k \geq 0$ (iii) $e^{at} \cos wt$ for $t \geq 0$.
7. State and prove final value theorem for the z-transform.
8. Solve the difference equation: $y_{n+2} + 2y_{n+1} + y_n = n$ where $y_0 = y_1 = 0$, and $n = k$
9. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y (x, t).
10. Change the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in to polar form.
11. Define Fourier series in complex form. Verify the convolution theorem for $f(x) = g(x) = e^{-x^2}$.
12. Find the Fourier cosine transform of $f(x) = e^{-x}$, x>0 and hence by Parseral's identity, show that $\int_0^\infty \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}$.

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2068 Bhadra

Exam.		Regular	
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ ***All*** *questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. a) State necessary conditions for a function f(z) to be analytic. Show that the function $f(z) = \log z$ is analytic everywhere except at the origin.

   b) Find the linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$ and $z_3 = i$ into points $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively.

2. a) State and prove Cauchy's integral formula.

   b) Write the statement of Cauchy's integral formula. Use it to evaluate the integral $\oint_c \frac{e^z}{(z-1)(z-3)} dz$ where C is the circle $|z| = 2$.

3. a) Write the statement of Taylor's theorem. Find the Laurent series for the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$.

   b) State Cauchy-residue theorem. Using it evaluate $\int_c \frac{\sin z}{z^6} dz$ where C: $|z| = 1$.

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} d\theta$ by contour integration in the complex plane.

4. a) Show that the Z-transform of $\cos k\theta$ is $\frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$. Use this result to find Z-transform of $a^k \cos k\theta$.

   b) Obtain the inverse Z-transform of $\frac{2z^3 + z}{(z-2)^2(z-1)}$, using partial fraction method.

5. a) Solve the difference equation $x(k+2) - x(k+1) + 0.25x(k) = u(k)$ where $x(0) = 1$ and $x(1) = 2$ and u(k) is unit step function.

   b) State and prove shifting theorem of z-transform.

6. Derive one-dimensional wave equation governing transverse vibration of string and solve it completely.

7. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions:

a) u is not infinite as $t \to \infty$

b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$ and

c) $u(x,0) = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$

OR

The diameter of a semi circular plate of radius a is kept at 0°C and temperature at the semi circular boundary is T°C. Show that the steady temperature in the plate is given by $u(r, \theta) \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$

8. a) Find the Fourier cosine integral representation of the function $f(x) = e^{-kx}$ $(x > 0, k > 0)$ and hence show that

$$\int_{0}^{\infty} \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi}{2k} e^{-kx} \quad (x > 0, K > 0)$$

b) Obtain Fourier sine transform of e^{-x}, $(x > 0)$ and hence evaluate $\int_{0}^{\infty} \frac{x^2}{(1+x^2)^2} dx$.

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING

Examination Control Division

2068 Magh

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Define harmonic function and show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and hence determine its harmonic conjugate function. [5]

   b) Define conformal mapping. Find the linear fractional transformation that maps three points $2i, -2, -2i$ onto three given points $-2, -2i, 2$. [5]

2. a) State and prove Cauchy integral theorem. [5]

   b) Evaluate the following integral using Cauchy integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}$ where c is the circle $|z| = 3/2$. [5]

3. a) Find the Taylor's series expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = 1$. [5]

   b) Determine the poles and the residue at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$. [5]

   OR

   Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$ by contour integration in complex plane.

4. a) Find the z-transform of: (any two) [5]

   i) $a^{|k|}$ ii) $\sin\alpha k$ iii) $\cosh at \sin bt$

   b) Find inverse z- transform of $\frac{z-4}{(z-1)(z-2)^2}$ by using inversion integral method. [5]

5. a) State and prove final value theorem for z transform. [5]

   b) Solve by using z transform $y_{k+2} - 4y_{k+1} + 4y_k = 0$ where $y_0 = 1$ and $y_1 = 0$. [5]

6. A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in the position given by $y = y_0 \sin^3 \frac{\pi x}{L}$. If $y(o, t) = y(L, t) = 0$, then find the displacement $y(x, t)$ where initial velocity is zero. [10]

7. Deduce the two dimensional Laplace equation into polar form. [10]

   OR

   Derive one dimensional heat equation for the flow of heat along a metallic rod by conduction and solve it completely.

8. a) State and prove convolution theorem for Fourier transform. [5]

   b) Find the Fourier cosine transform of $f(x) = e^{-mx}$ $(m > 0)$ and hence show that $\int_0^{\infty} \frac{\cos py}{y^2+\beta^2}dy = \frac{\pi}{2\beta}e^{-p\beta}$ $(\beta > 0, p > 0)$. [5]