

chap: 2

the z-transform

Defⁿ

Let $x(t)$ be a discrete time signal function
define for discrete values of 't' where
 $t = KT$

T ← sampling period

$K = 0, 1, 2, \dots$

Then, the z-transform of $x(t)$ is denoted by
 $Z[x(t)]$ & is defined by

$$\begin{aligned} Z[x(t)] &= Z[x(KT)] = \sum_{K=0}^{\infty} x(KT) z^{-K} \\ &= X(z) \end{aligned}$$

Here,

'z' is complex variable

'Z' is operator of z-transform

yo Z ho!!

Above expression is also called one sided or unilateral z-transform.

↳ The one-sided or unilateral z-transform of sequence of $x(k)$ or $x(n)$ is given by:

$$Z[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n} = X(z)$$

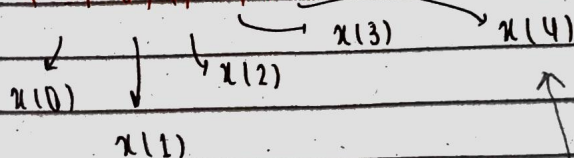
$$Z[x(k)] = \sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

where,

'z' is complex variable

↳ 'Z' is operator of z-transform

1. $x(k) = \{ 1, 8, 4, 3, 7 \}$



Its z-transform is

$$\sum_{k=0}^{\infty} x(k) z^{-k}$$

k=0

For discrete ^{time} signal function $x(t)$ where $-\infty < t < \infty$

Then,

$$\mathcal{Z}[x(t)] = \mathcal{Z}[x(KT)] = \sum_{K=-\infty}^{\infty} x(KT) z^{-K} = X(z)$$

where,

' z ' is complex variable.

T is sampling period

' \mathcal{Z} ' is operator of z -transform

Above exp is called two-sided or bilateral z -transform.

Region of convergence 'ROC'

It is defined as the region where the z -transform of a e^n exists.

The z -transform of sequence $x(n)$ is

$$\mathcal{Z}[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n}$$

ROC is the range of ' z ' for which z -transform converges.

since z -transform is a power series, it converges when $x(k)z^{-k}$ is absolutely summable i.e. finite sum.

stated differently $\sum_{k=0}^{\infty} x(k)z^{-k} < \infty$ must be satisfied for convergence.

• Properties of ROC

- (i) ROC of z -transform is indicated with the circle in the z -plane.
- (ii) ROC can not contain any pole.
- (iii) If $x(k)$ is right sided sequence then ROC extend outward from outermost pole of $x(z)$.

z transform of some std f^n / sequence / signals

(i) unit-step f^n

L f^n defined by $u(k) = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$

is unit step f^n .

By defⁿ of z-transform

$$\mathcal{Z}[u_k] = \sum_{k=0}^{\infty} u_k z^{-k} = \sum_{k=0}^{\infty} 1 \cdot z^{-k}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \dots$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$$

$$\Rightarrow \frac{1}{1 - \frac{1}{z}} \quad \text{for } \left| \frac{1}{z} \right| < 1$$

$$= \frac{z}{z - 1} \quad \text{for } |z| > 1$$

then,

$$\therefore \mathcal{Z}[u_k] = \frac{z}{z - 1}, \quad |z| > 1$$

sum of geometric series.

↳ ROC lies outside of circle $|z| = 1$