

# Chapter 9

## Metal-Semiconductor and Semiconductor Heterojunctions

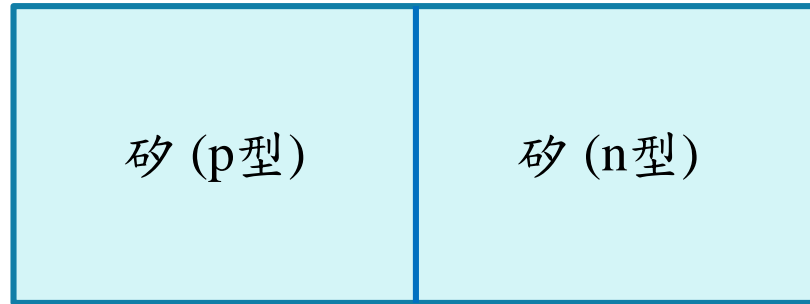
# Outline

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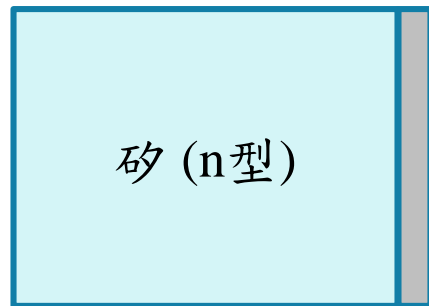
- 9.1 The Schottky Barrier Diode
- 9.2 Metal–semiconductor Ohmic Contacts
- 9.3 Heterojunctions

# 同質介面與異質介面

兩邊材料基質相同時稱之為同質介面 Homojunction



兩邊材料基質不同時稱之為異質介面 Heterojunction

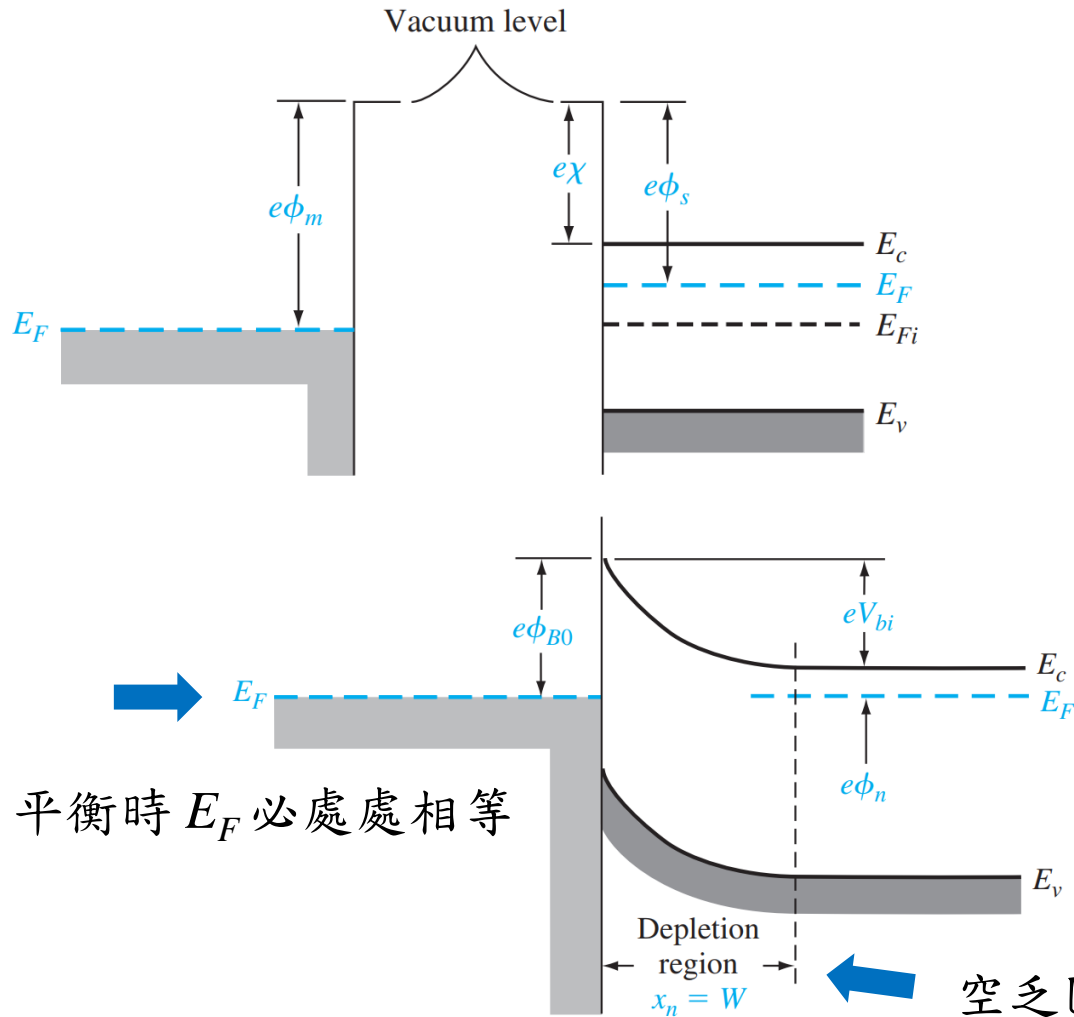


金屬

- Schottky Diode
- Ohmic contact
- Tunneling Barrier

# Schottky Diode

Metal-semiconductor contact is made on n-type semiconductor.



$\phi_m$  金屬功函數 work function :  
使電子脫離金屬所需能量

$\phi_s$  半導體功函數 semiconductor work function :  
使電子脫離費米能階所需能量

$\chi$  電子親和能 electron affinity :  
使電子脫離傳導帶所需能量

假設：  $\phi_m > \phi_s$

空乏區(電子擴散至金屬)

# 金屬功函數與電子親和能

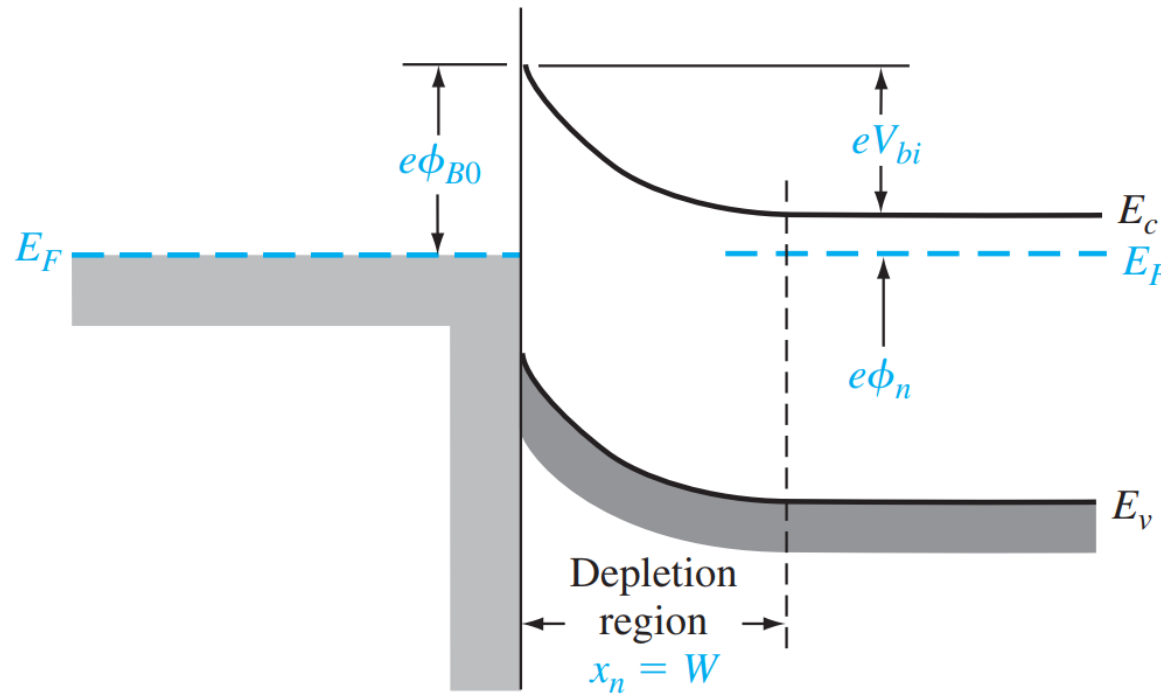
**Table 9.1** | Work functions of some elements

Element	Work function, $\phi_m$
Ag, silver	4.26
Al, aluminum	4.28
Au, gold	5.1
Cr, chromium	4.5
Mo, molybdenum	4.6
Ni, nickel	5.15
Pd, palladium	5.12
Pt, platinum	5.65
Ti, titanium	4.33
W, tungsten	4.55

**Table 9.2** | Electron affinity of some semiconductors

Element	Electron affinity, $\chi$
Ge, germanium	4.13
Si, silicon	4.01
GaAs, gallium arsenide	4.07
AlAs, aluminum arsenide	3.5

# Schottky Diode



Difference of conduction

$$\phi_{B0} = \phi_m - \chi \quad \text{Schottky barrier}$$

Difference of Fermi level

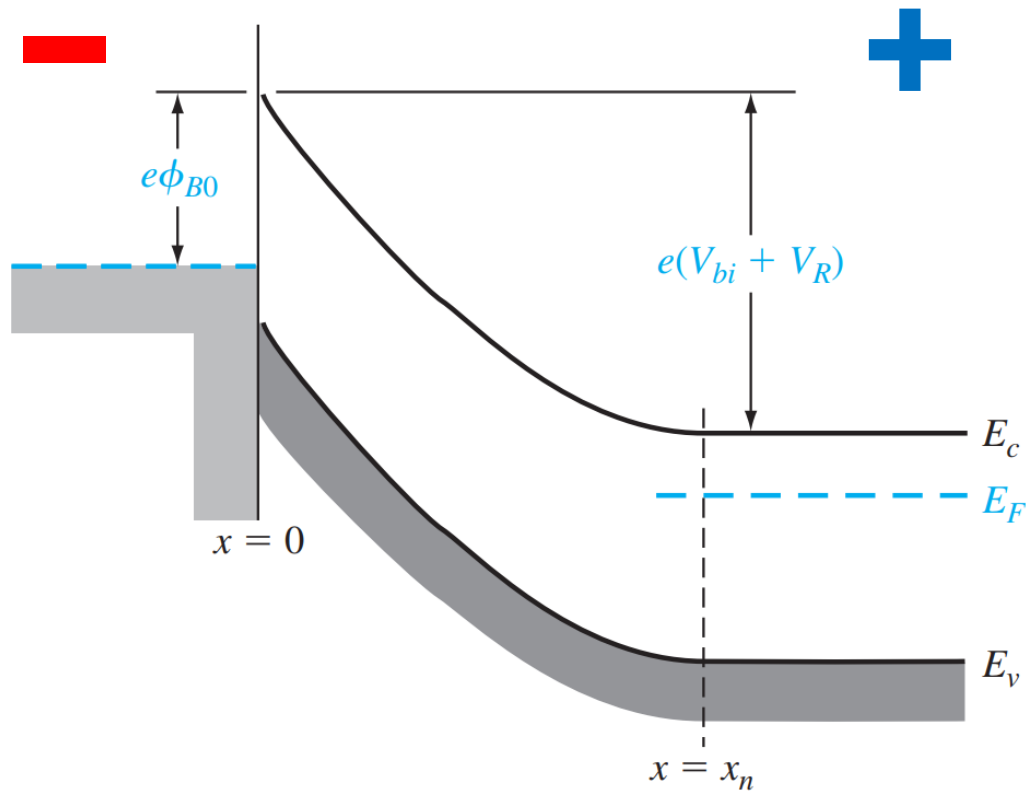
$$V_{bi} = \phi_{B0} - \phi_n$$

$$\phi_n = E_c - E_F$$

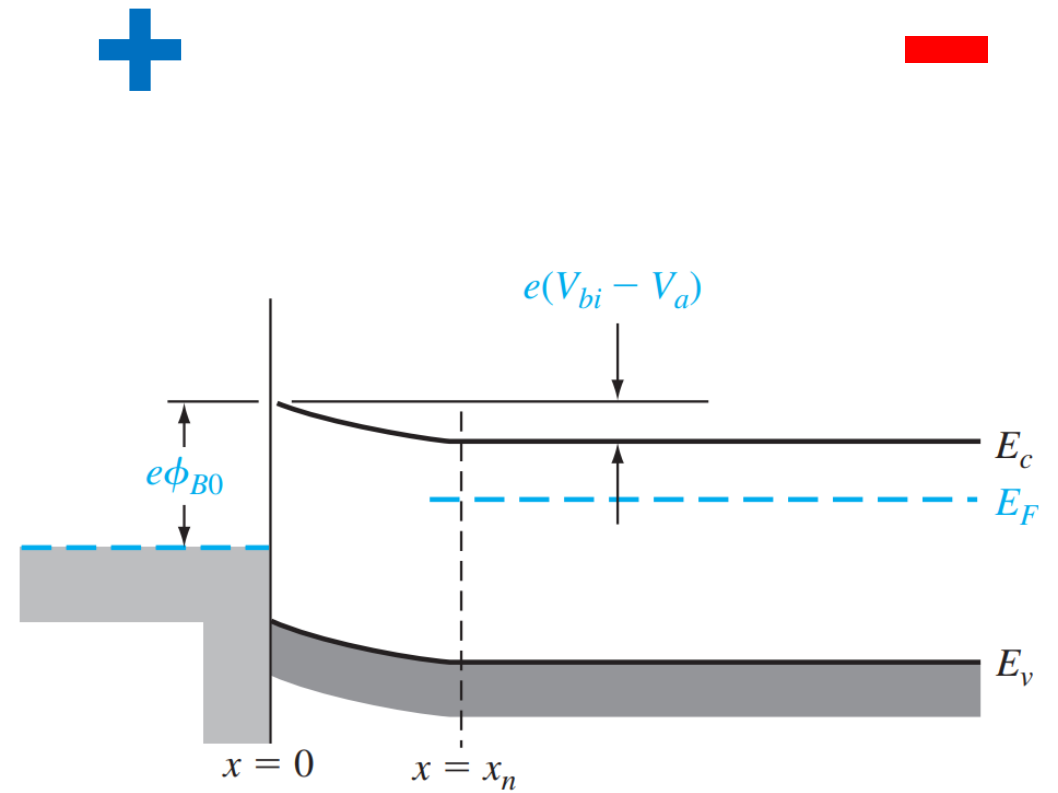
與摻雜濃度有關，濃度愈高，差值愈小

# Schottky Diode

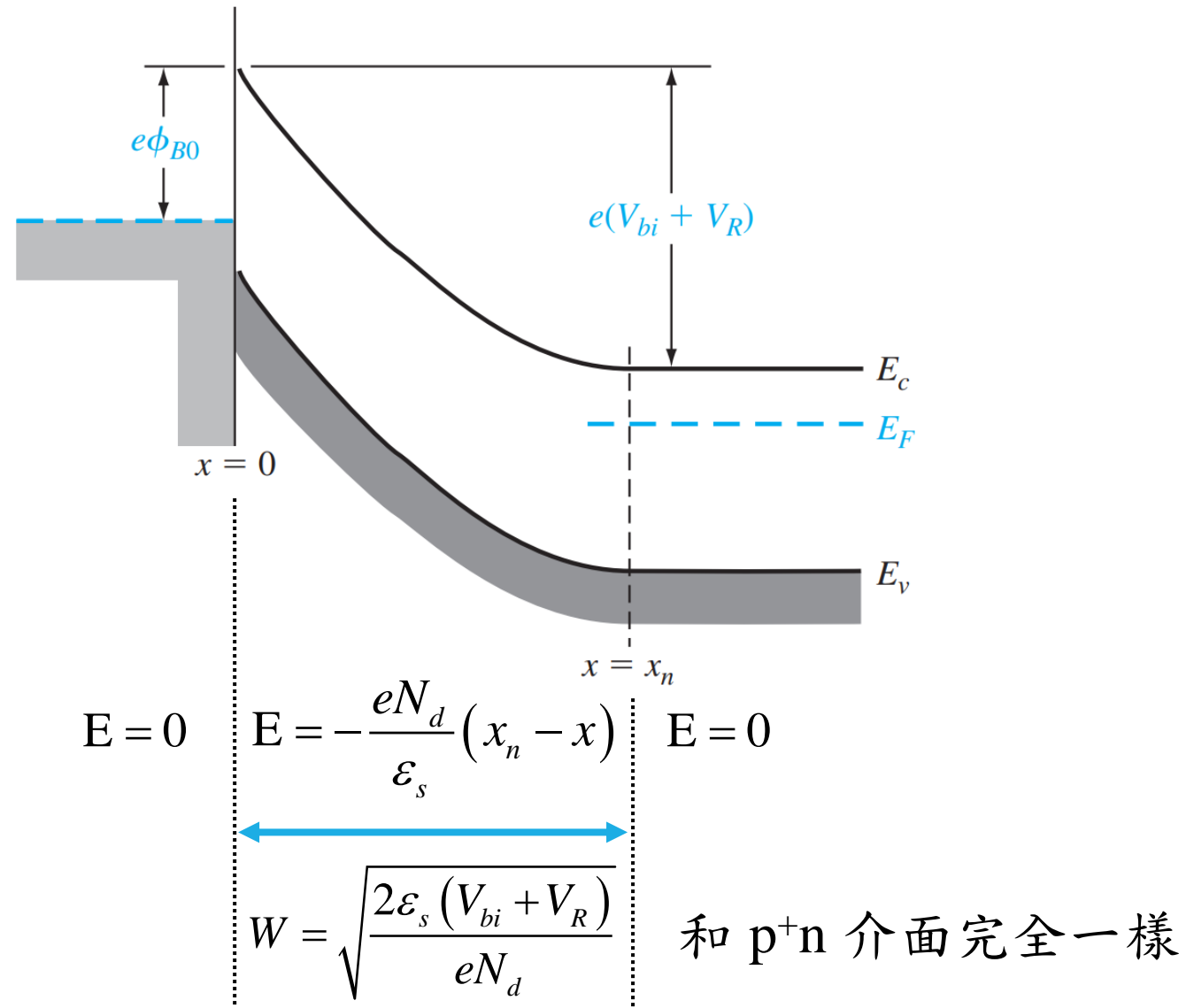
逆向偏壓  
(無電流)



順向偏壓  
(有電流)



# 電場與空乏區寬度





# Example 9.1

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**Objective:** Determine the theoretical barrier height, built-in potential barrier, and maximum electric field in a metal–semiconductor diode for zero applied bias.

Consider a contact between tungsten and n-type silicon doped to  $N_d = 10^{16} \text{ cm}^{-3}$  at  $T = 300 \text{ K}$ .

# Example 9.1

**Objective:** Determine the theoretical barrier height, built-in potential barrier, and maximum electric field in a metal–semiconductor diode for zero applied bias.

Consider a contact between tungsten and n-type silicon doped to  $N_d = 10^{16} \text{ cm}^{-3}$  at  $T = 300 \text{ K}$ .

$$\phi_{B0} = \phi_m - \chi = 4.55 - 4.01 = 0.54 \text{ V}$$

$$|E_{\max}| = \frac{eN_dx_n}{\epsilon_s} = \frac{(1.6 \times 10^{-19})(10^{16})(0.208 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})}$$

$$\phi_n = \frac{kT}{e} \ln \left( \frac{N_c}{N_d} \right) = 0.0259 \ln \left( \frac{2.8 \times 10^{19}}{10^{16}} \right) = 0.206 \text{ V}$$

$$|E_{\max}| = 3.21 \times 10^4 \text{ V/cm}$$

$$V_{bi} = \phi_{B0} - \phi_n = 0.54 - 0.206 = 0.334 \text{ V}$$

$$x_n = \left[ \frac{2\epsilon_s V_{bi}}{eN_d} \right]^{1/2} = \left[ \frac{2(11.7)(8.85 \times 10^{-14})(0.334)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

$$x_n = 0.208 \times 10^{-4} \text{ cm}$$

評論：

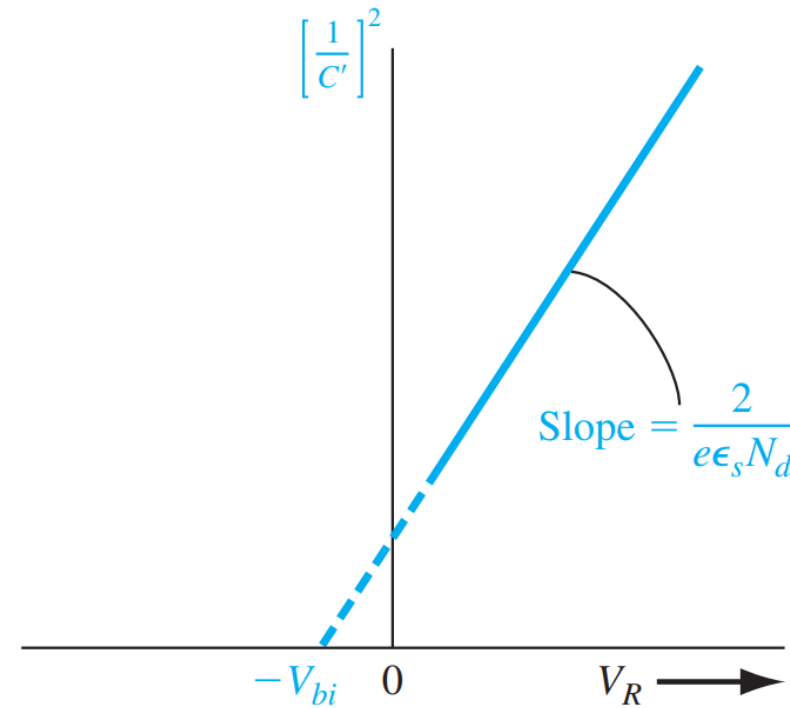
空間電荷寬度和電場的值與 pn 接面所獲得的值非常相似

# Junction Capacitance

- 和 p<sup>+</sup>n 介面完全一樣
- 實驗上可得知材料的內建電位差  $V_{bi}$  以及摻雜濃度

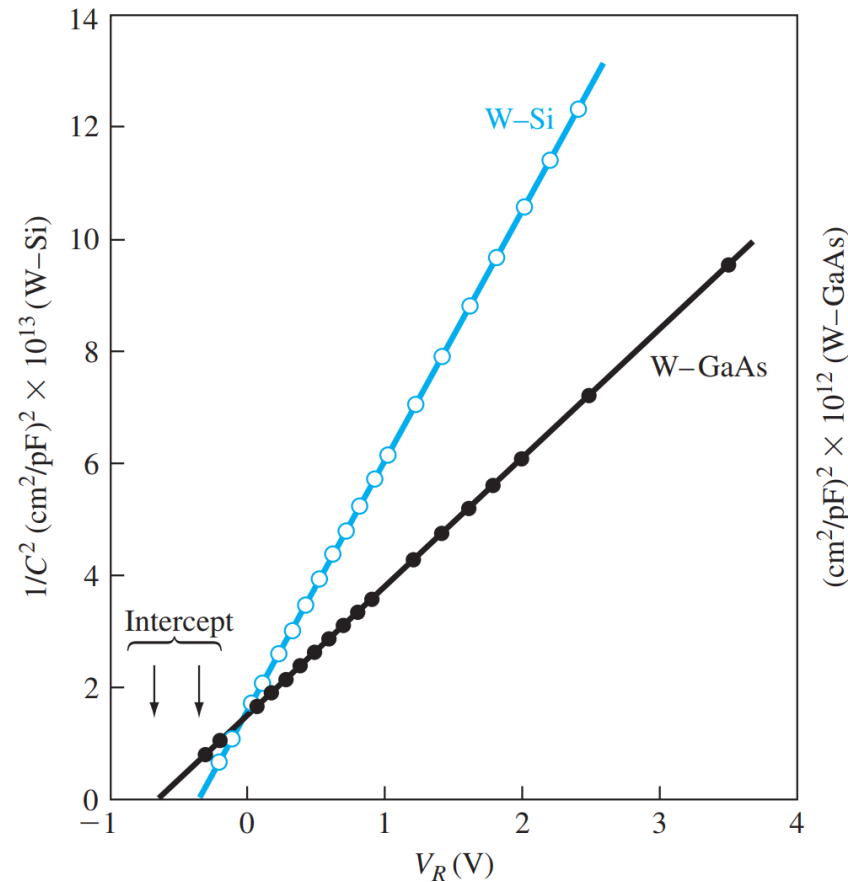
$$C' = \frac{dQ'}{dV_R} \approx \sqrt{\frac{\epsilon_s e N_d}{2(V_{bi} + V_R)}}$$

$$\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{\epsilon_s e N_d}$$



# Example 9.2

**Objective:** To calculate the semiconductor doping concentration and Schottky barrier height from the silicon diode experimental capacitance data shown in Figure 9.3. Assume  $T = 300$  K.



## Example 9.2

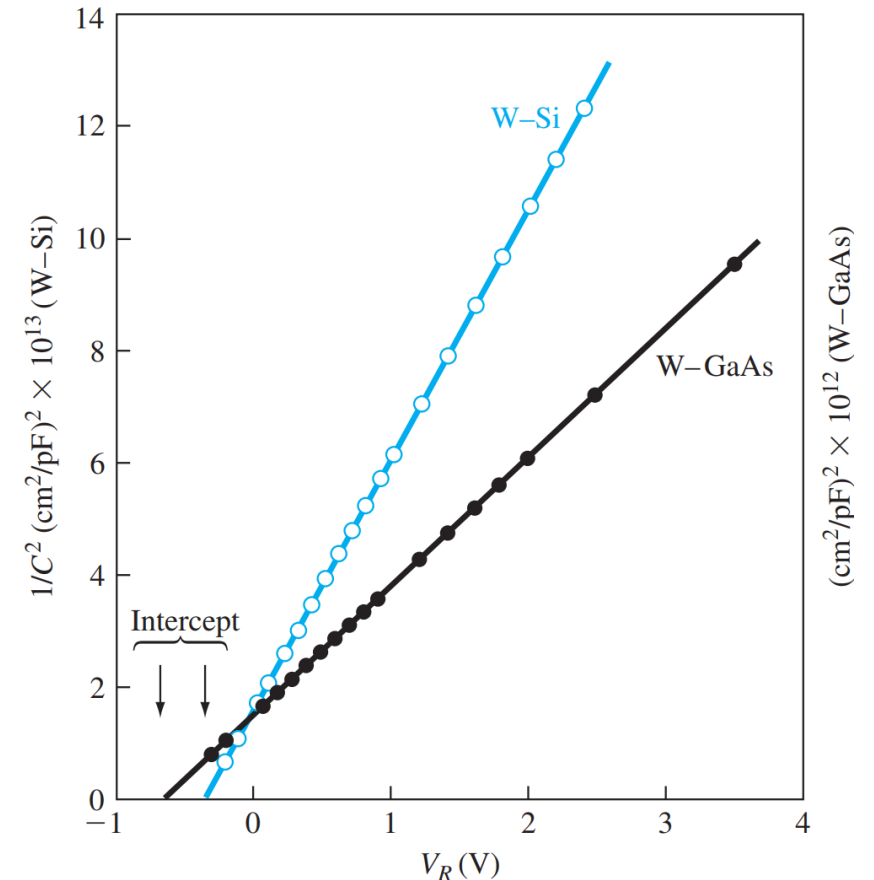
**Objective:** To calculate the semiconductor doping concentration and Schottky barrier height from the silicon diode experimental capacitance data shown in Figure 9.3. Assume  $T = 300$  K.

$$\frac{d(1/C')^2}{dV_R} \approx \frac{\Delta(1/C')^2}{\Delta V_R} = \frac{2}{e\epsilon_s N_d} \approx 4.4 \times 10^{13}$$

$$N_d = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(4.4 \times 10^{13})} = 2.7 \times 10^{17} \text{ cm}^{-3}$$

$$\phi_n = \frac{kT}{e} \ln \left( \frac{N_c}{N_d} \right) = (0.0259) \ln \left( \frac{2.8 \times 10^{19}}{2.7 \times 10^{17}} \right) = 0.12 \text{ V}$$

$$\phi_{Bn} = V_{bi} + \phi_n = 0.40 + 0.12 = 0.52 \text{ V}$$

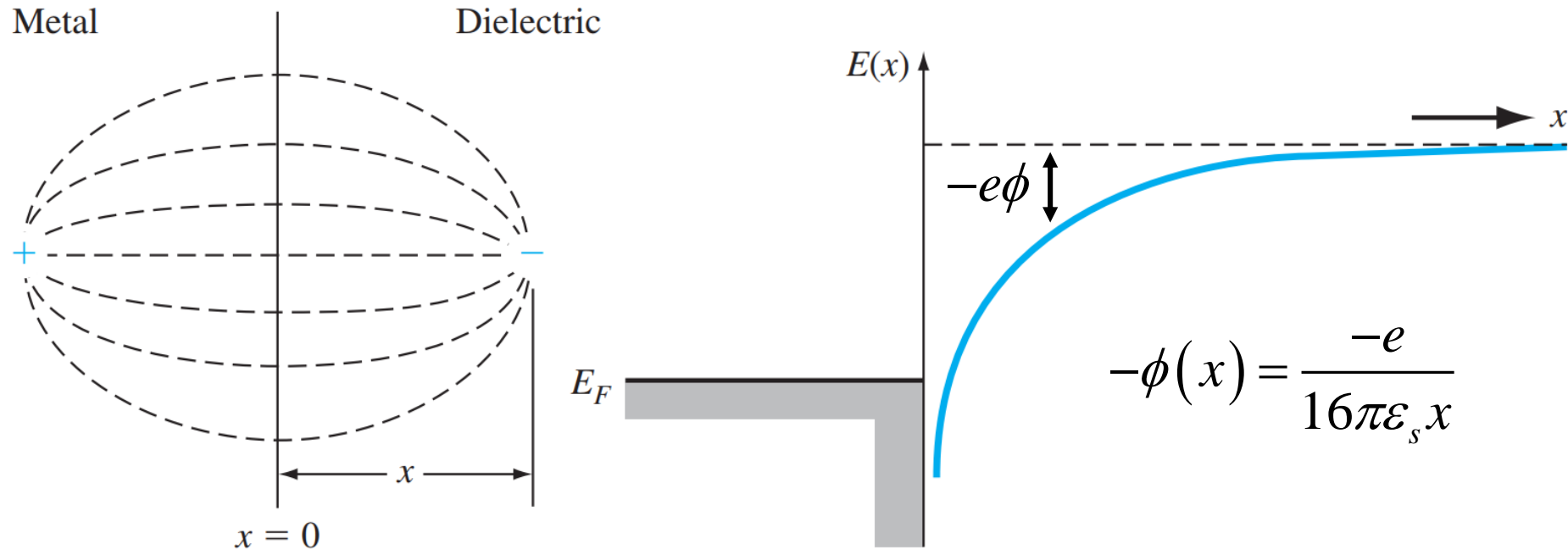


評論：

實驗值為0.52伏特，可以與理想的屏障高度0.54伏特進行比較。

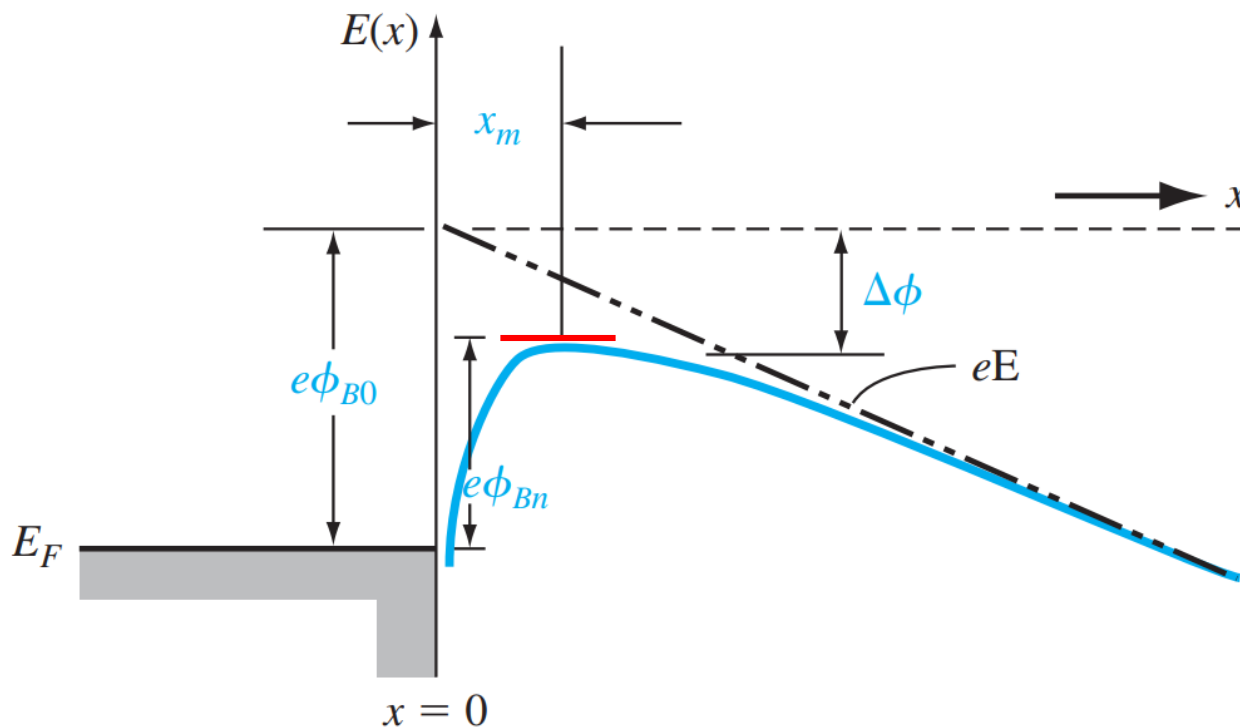
# 位能障壁上的非理想效應

效應1：Schottky Effect / image-force-induced lowering



Coulomb force:  $F = \frac{-e^2}{4\pi\epsilon_s (2x)^2} = -eE$

# 位能障壁上的非理想效應



$$\frac{d[e\phi(x)]}{dx} = 0$$

$$-\phi(x) = \frac{-e}{16\pi\epsilon_s x} - Ex$$

$$x_m = \sqrt{\frac{e}{16\pi\epsilon_s E}}$$

$$\Delta\phi = \sqrt{\frac{eE}{4\pi\epsilon_s}}$$

$$\phi_{Bn} = \phi_{B0} - \Delta\phi$$

## Example 9.3

**Objective:** Calculate the Schottky barrier lowering and the position of the maximum barrier height.

Consider a gallium arsenide metal–semiconductor contact in which the electric field in the semiconductor is assumed to be  $E = 6.8 \times 10^4$  V/cm.

$$\Delta\phi = \sqrt{\frac{eE}{4\pi\epsilon_s}} = \sqrt{\frac{(1.6 \times 10^{-19})(6.8 \times 10^4)}{4\pi(13.1)(8.85 \times 10^{-14})}} = 0.0273 \text{ V}$$

$$x_m = \sqrt{\frac{e}{16\pi\epsilon_s E}} = \sqrt{\frac{(1.6 \times 10^{-19})}{16\pi(13.1)(8.85 \times 10^{-14})(6.8 \times 10^4)}}$$

$$x_m = 2 \times 10^{-7} \text{ cm} = 20 \text{ \AA}$$

評論:

儘管肖特基勢壘降低可能看似一個小值，但在電流-電壓關係中，勢壘高度和勢壘降低將以指數項的形式呈現。因此，勢壘高度的微小變化可能會對肖特基勢壘二極管中的電流產生顯著影響。



# 位能障壁上的非理想效應

## 效應2：Interface States

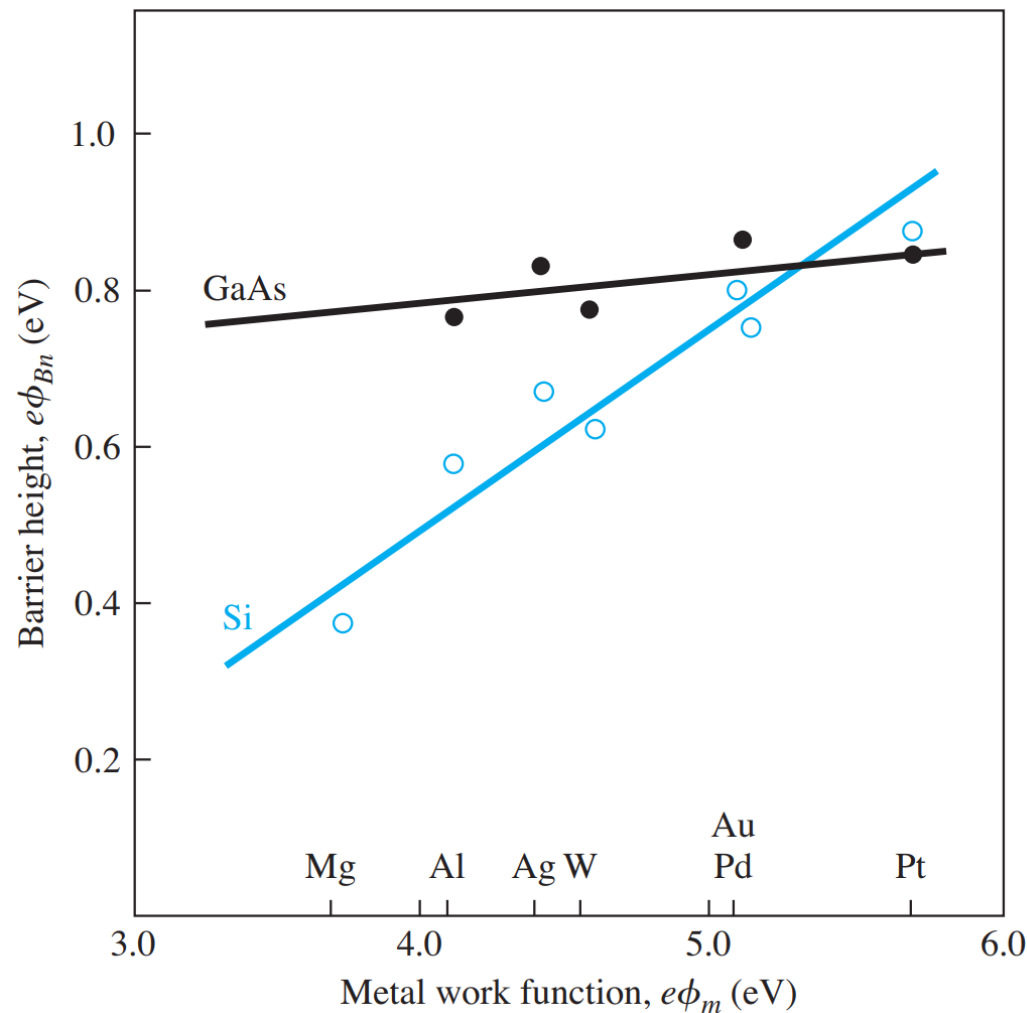
理論上障壁的高度應該要滿足下式：

$$\phi_{B0} = \phi_m - \chi$$

基本上可以看到GaAs的障壁高度幾乎不變

實驗結果顯示 GaAs 卻是符合下式：

$$\phi_{Bn} = \frac{1}{e} (E_g - e\phi_0)$$



# 位能障壁上的非理想效應

	Donor-like	Acceptor-like
Neutrality	Contain e-	Not contain e-
Negatively		Contain e-
Positively	Not contain e-	

$$Q_{ss} = \frac{eN_d x_n}{eD_{it}} \left( E_g - e\phi_0 - e\phi_{Bn} \right) - \frac{C \cdot \Delta V}{eD_{it} \delta} \left[ \phi_m - (\chi + \phi_{Bn}) \right]$$

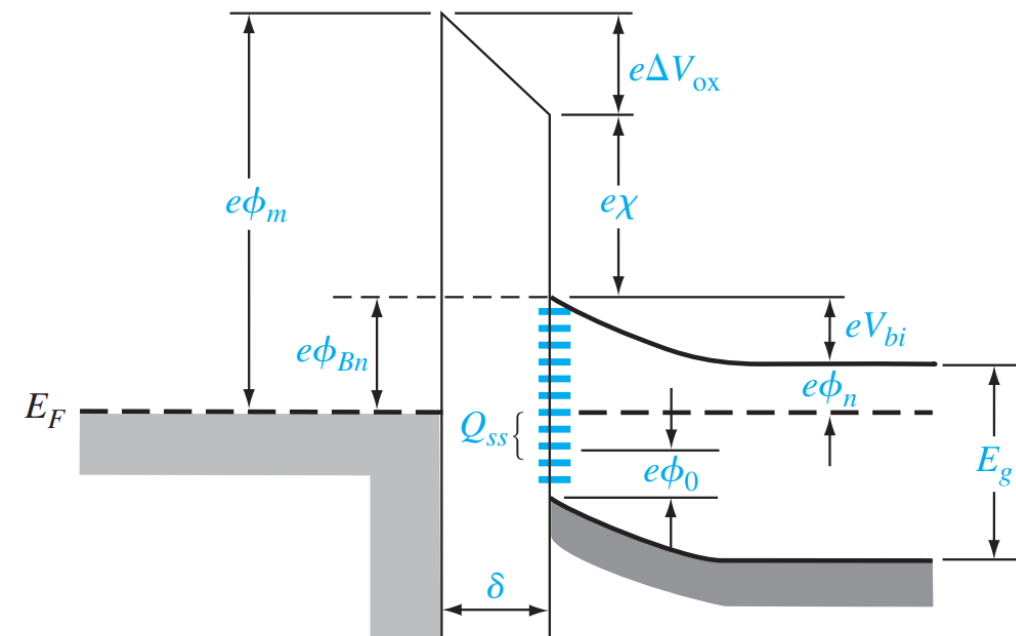
$D_{it} \left( \# / cm^2 - eV \right)$

**Case 1** Let  $D_{it} \rightarrow \infty$ . In this case, the right side of Equation (9.16) goes to zero. We then have (給任何材料都不動，pinned住。)

$$\phi_{Bn} = \frac{1}{e} (E_g - e\phi_0) \text{ 材料決定 } E_g \text{ 和 } \phi_0$$

**Case 2** Let  $D_{it} \delta \rightarrow 0$ . Equation (9.16) reduces to

$$\phi_{Bn} = \phi_m - \chi$$



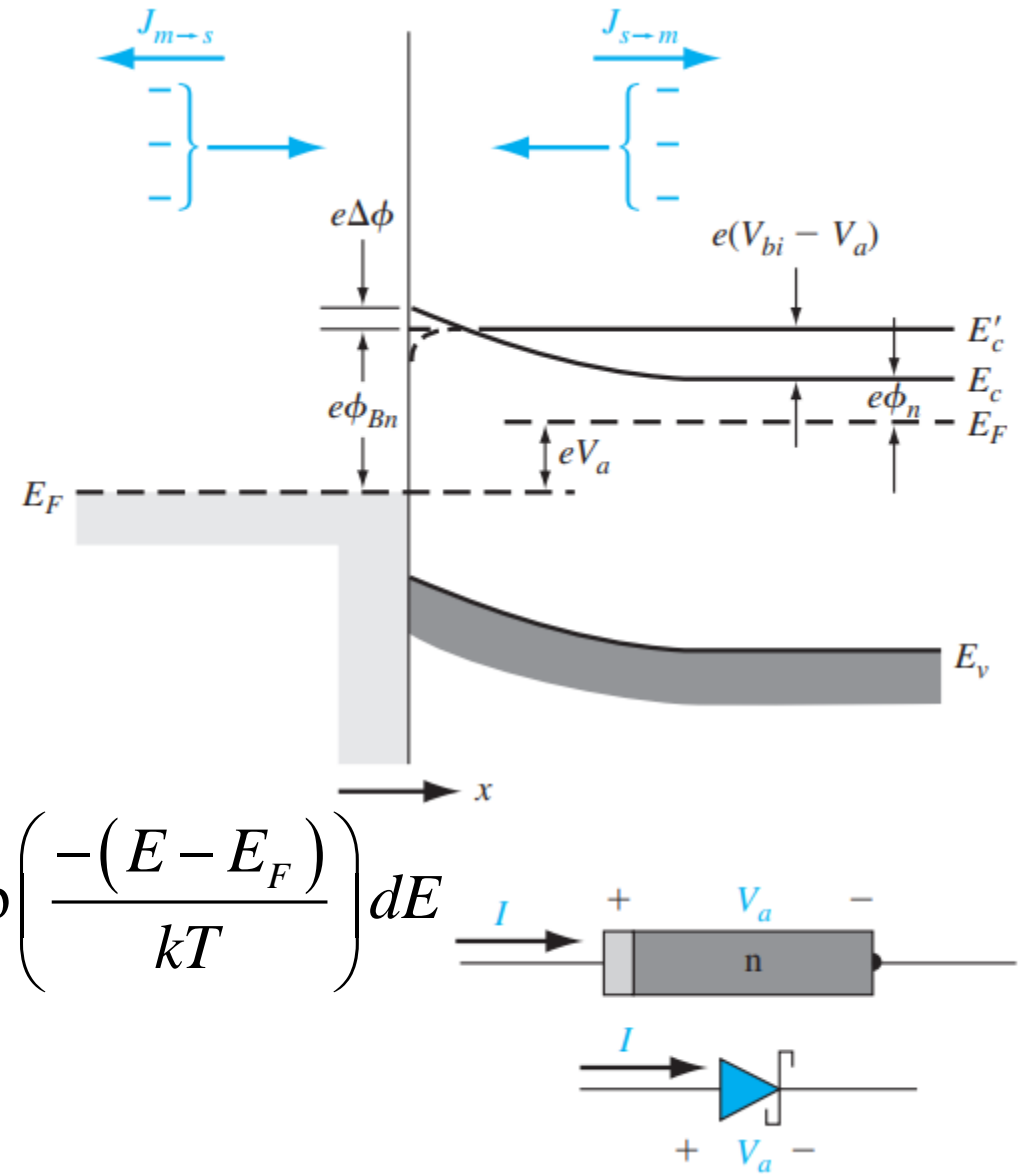
# 電流－電壓關係

- The current transport in a metal–semiconductor junction is due mainly to majority carriers as opposed to minority carriers in a pn junction
- The thermionic emission characteristics are derived by assuming that the barrier height is much larger than  $kT$ , so that the Maxwell–Boltzmann approximation applies

$$J_{s \rightarrow m} = e \int_{E_c(\text{min energy})}^{\infty} v_x dn$$

$$dn = g_c(E) f_F(E) dE = \frac{4\pi (2m_n^*)^3}{h^3} \sqrt{E - E_c} \exp\left(\frac{-(E - E_F)}{kT}\right) dE$$

$$\frac{1}{2} m_n^* v^2 = E - E_c$$



# 電流－電壓關係

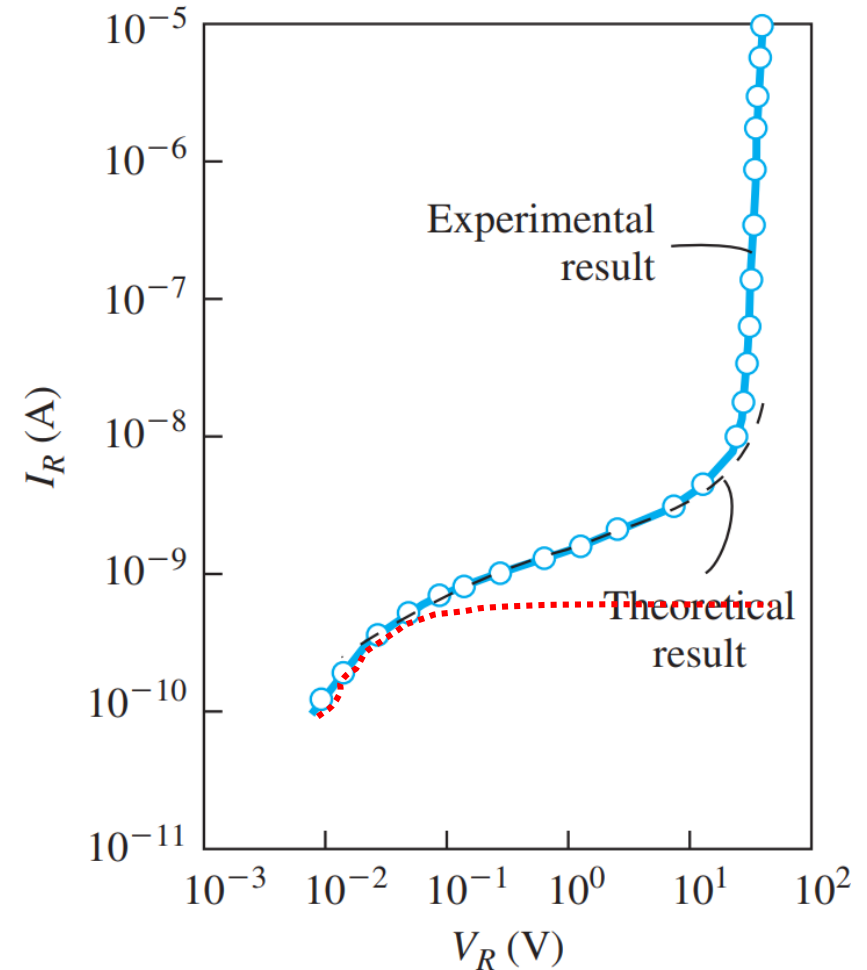
- Schottky diode 的電流主要是來自多數載子的移動，與 pn junction 不同。
- Thermionic Emission Theory 解出

$$J = J_{s \rightarrow m} - J_{m \rightarrow s}$$

$$J = J_{sT} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

$$J_{sT} = A^* T^2 \exp\left(-\frac{e\phi_{Bn}}{kT}\right) = A^* T^2 \exp\left(\frac{-e\phi_{B0}}{kT}\right) \exp\left(\frac{e\Delta\phi}{kT}\right)$$

$$A^* = \frac{4\pi e m_n^* k^2}{h^3}$$



# Example 9.4

**Objective:** Determine the effective Richardson constant from the current–voltage characteristics.

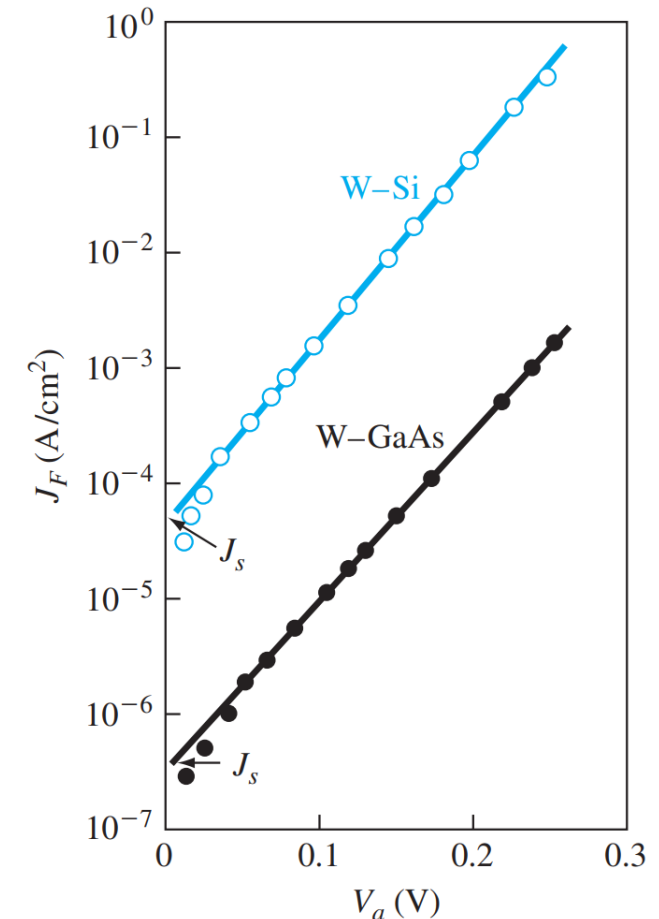
Consider the tungsten–silicon diode curve in Figure 9.9 and assume a barrier height of  $\phi_{Bn} = 0.67$  V. From the figure,  $J_{sT} \approx 6 \times 10^{-5}$  A/cm<sup>2</sup>.

$$J_{sT} = A^* T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right)$$

$$A^* = \frac{J_{sT}}{T^2} \exp\left(\frac{+e\phi_{Bn}}{kT}\right)$$

$$A^* = \frac{6 \times 10^{-5}}{(300)^2} \exp\left(\frac{0.67}{0.0259}\right) = 114 \text{ A/K}^2\text{-cm}^2$$

評論:兩個材料有效質量不一樣，障壁高度也不一樣



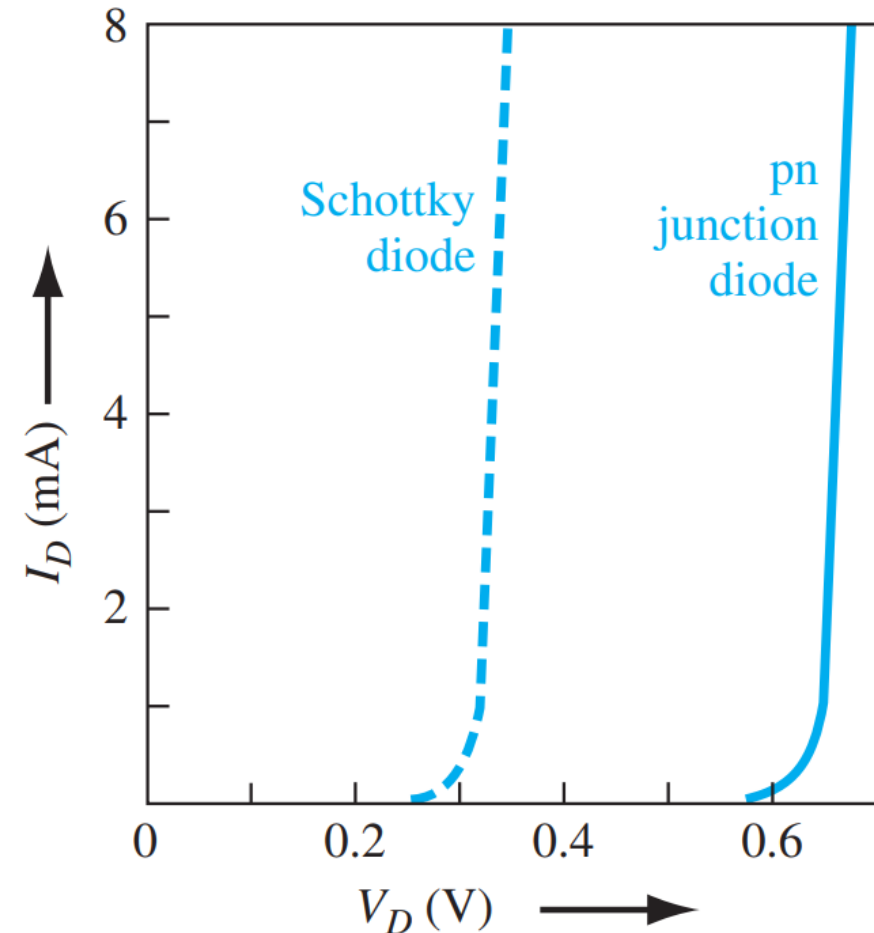
# Comparison of Schottky and pn Junction

因為  $J_{sT} \gg J_s$ ，Schottky diode 開啟的電壓比較小

$$J = J_{sT} \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

Schottky diode  $J_{sT} = A^* T^2 \exp\left(-\frac{e\phi_{Bn}}{kT}\right)$

pn junction  $J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$



# Example 9.5

**Objective:** Calculate the ideal reverse-saturation current densities of a Schottky barrier diode and a pn junction diode.

Consider a tungsten barrier on silicon with a measured barrier height of  $e\phi_{Bn} = 0.67$  eV. The effective Richardson constant is  $A^* = 114$  A/K<sup>2</sup>-cm<sup>2</sup>. Let  $T = 300$  K.

$$J_{sT} = A^* T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right) = (114)(300)^2 \exp\left(\frac{-0.67}{0.0259}\right) = 5.98 \times 10^{-5} \text{ A/cm}^2$$

$$\begin{array}{llll} N_a = 10^{18} \text{ cm}^{-3} & N_d = 10^{16} \text{ cm}^{-3} & L_p = 1.0 \times 10^{-3} \text{ cm} & L_n = 1.58 \times 10^{-3} \text{ cm} \\ D_p = 10 \text{ cm}^2/\text{s} & D_n = 25 \text{ cm}^2/\text{s} & p_{no} = 2.25 \times 10^4 \text{ cm}^{-3} & n_{po} = 2.25 \times 10^2 \text{ cm}^{-3} \\ \tau_{po} = 10^{-7} \text{ s} & \tau_{no} = 10^{-7} \text{ s} & & \end{array}$$

$$\begin{aligned} J_s &= \frac{(1.6 \times 10^{-19})(25)(2.25 \times 10^2)}{(1.58 \times 10^{-3})} + \frac{(1.6 \times 10^{-19})(10)(2.25 \times 10^4)}{(1.0 \times 10^{-3})} \\ &= 5.7 \times 10^{-13} + 3.6 \times 10^{-11} = 3.66 \times 10^{-11} \text{ A/cm}^2 \end{aligned}$$

## Example 9.6

**Objective:** Calculate the forward-bias voltage required to induce a forward-bias current density of  $10 \text{ A/cm}^2$  in a Schottky barrier diode and a pn junction diode.

Consider diodes with the parameters given in Example 9.5. We can assume that the pn junction diode will be sufficiently forward biased so that the ideal diffusion current will dominate. Let  $T = 300 \text{ K}$ .

$$J = J_{sT} \left[ \exp \left( \frac{eV_a}{kT} \right) - 1 \right]$$

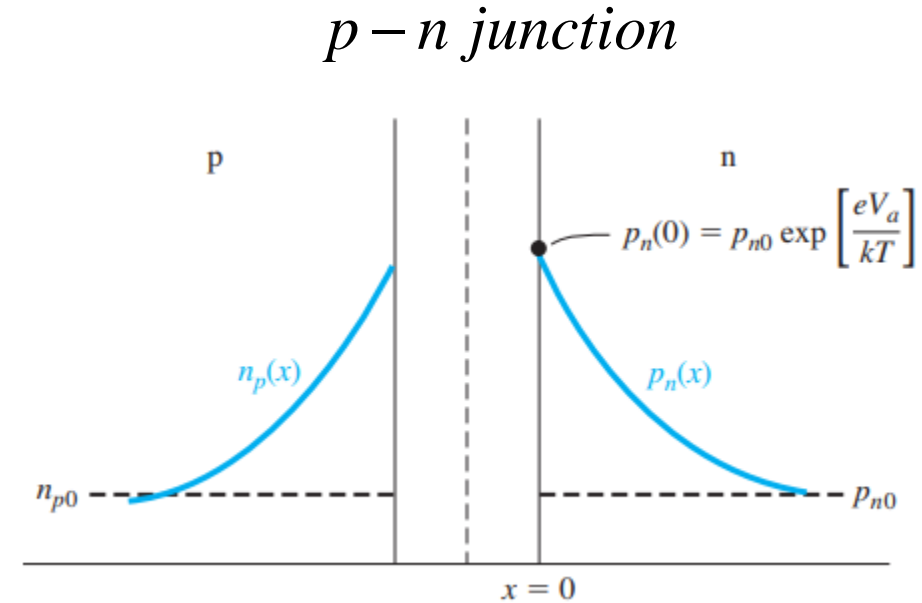
$$V_a = \left( \frac{kT}{e} \right) \ln \left( \frac{J}{J_{sT}} \right) = V_t \ln \left( \frac{J}{J_{sT}} \right) = (0.0259) \ln \left( \frac{10}{5.98 \times 10^{-5}} \right) = 0.312 \text{ V}$$

$$V_a = V_t \ln \left( \frac{J}{J_s} \right) = (0.0259) \ln \left( \frac{10}{3.66 \times 10^{-11}} \right) = 0.682 \text{ V}$$



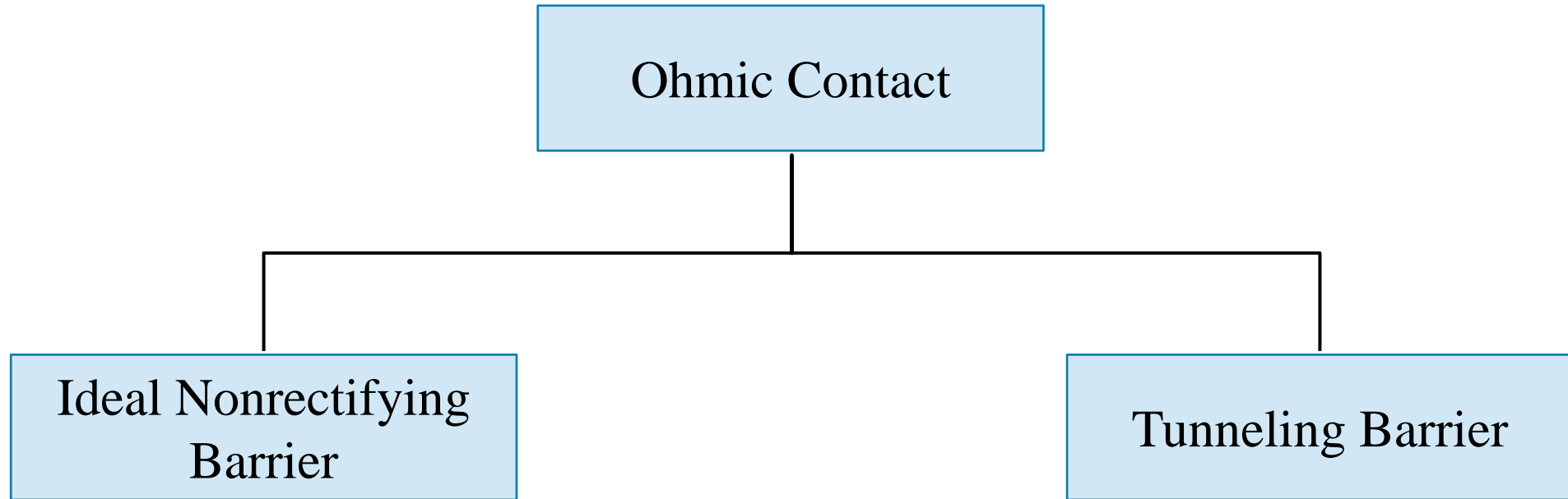
# Frequency response

- The second difference: **frequency response**.
- The current in a Schottky diode is due to the injection of **majority**. This fact means that there is **no diffusion capacitance**.
- When switching a Schottky diode from forward to reverse bias, there is **no minority carrier stored charge** to remove.
- A typical switching time for a Schottky diode is in the **picosecond** range, while for a pn junction it is in **nanosecond** range.



# Ohmic Contact

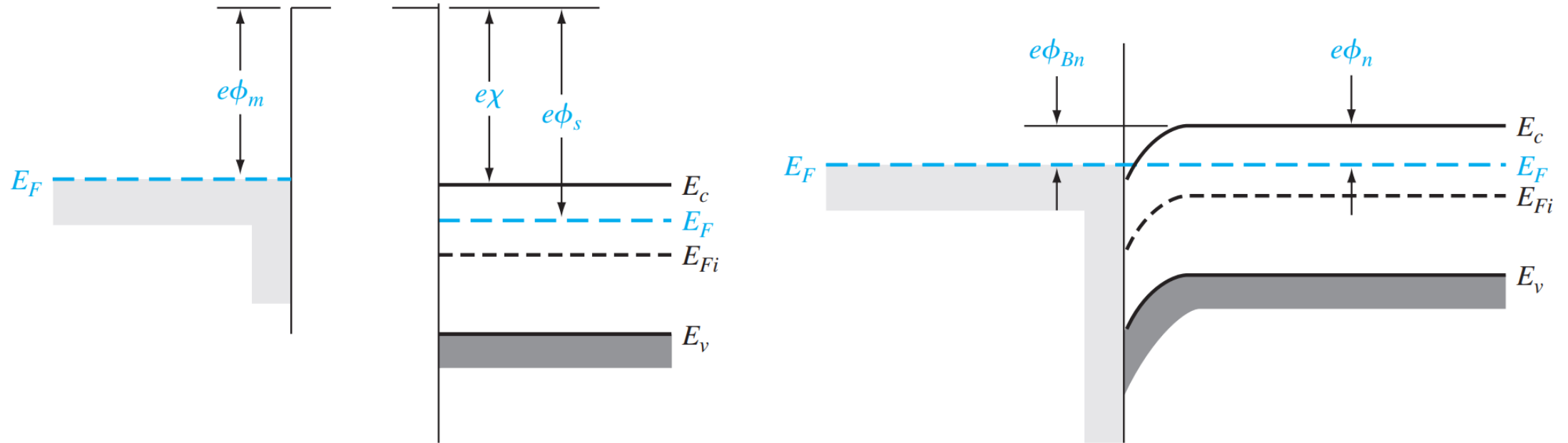
無論電位差的方向，電子都非常容易流動，電阻值低，適合拿來連接金屬導線，這類接觸就稱為歐姆接觸 (ohmic contact)



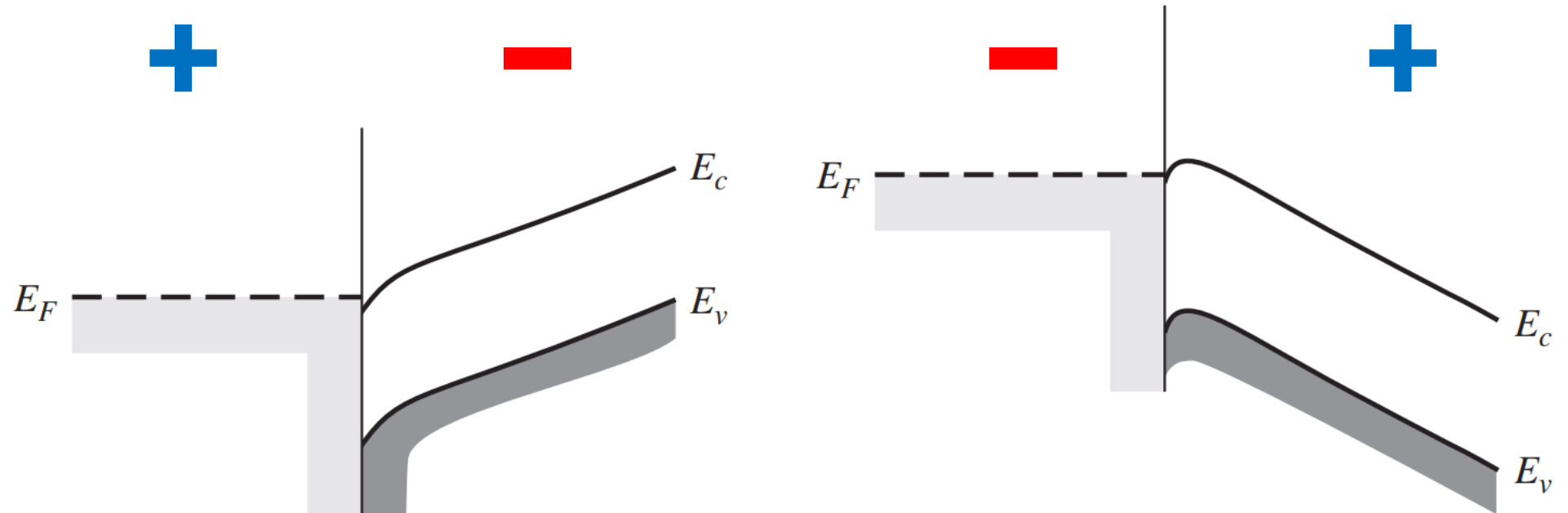
主要

# Ideal Nonrectifying Barrier (n type)

$\phi_m < \phi_s$  , 多數載子為電子

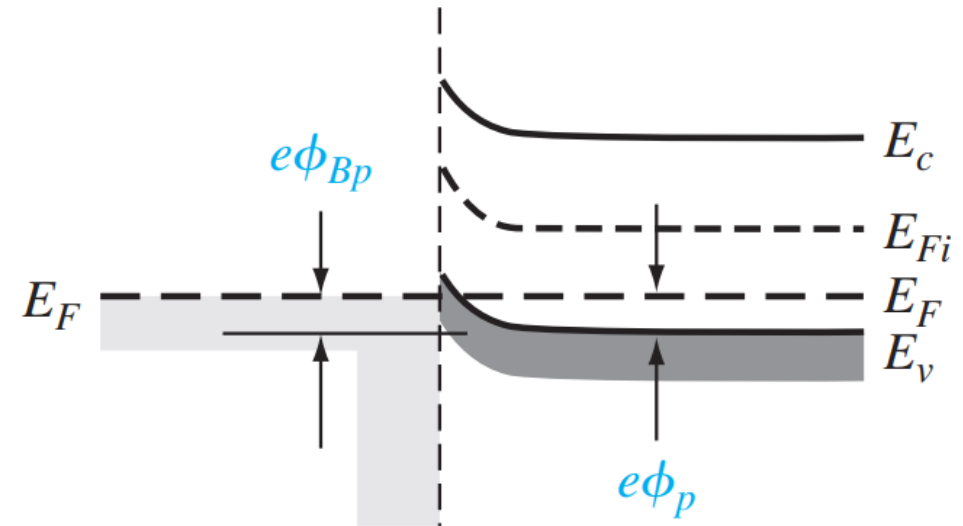
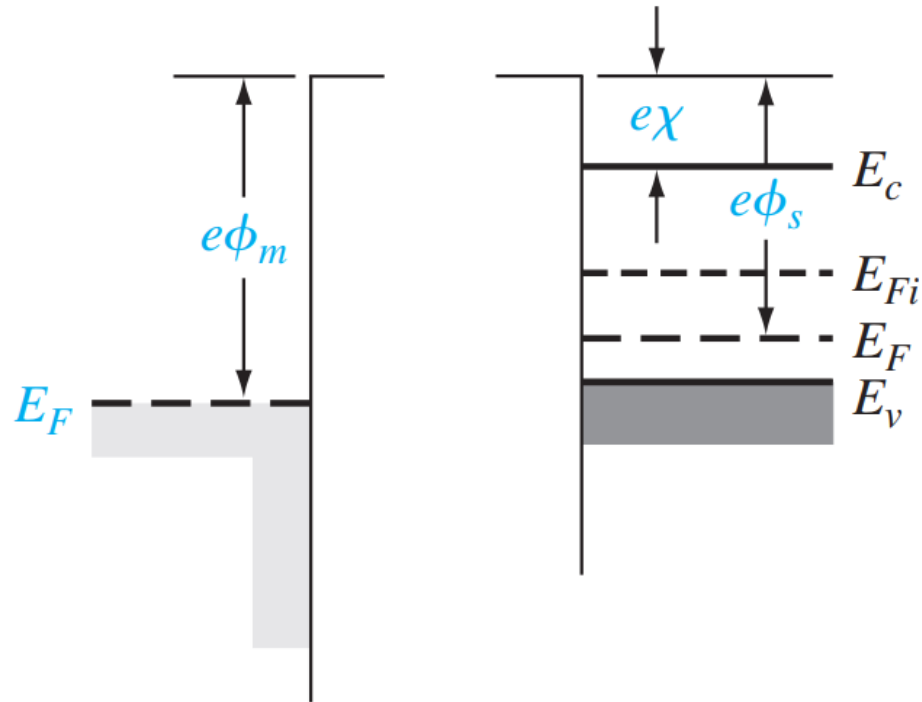


# Ideal Nonrectifying Barrier (n type)



# Ideal Nonrectifying Barrier (p type)

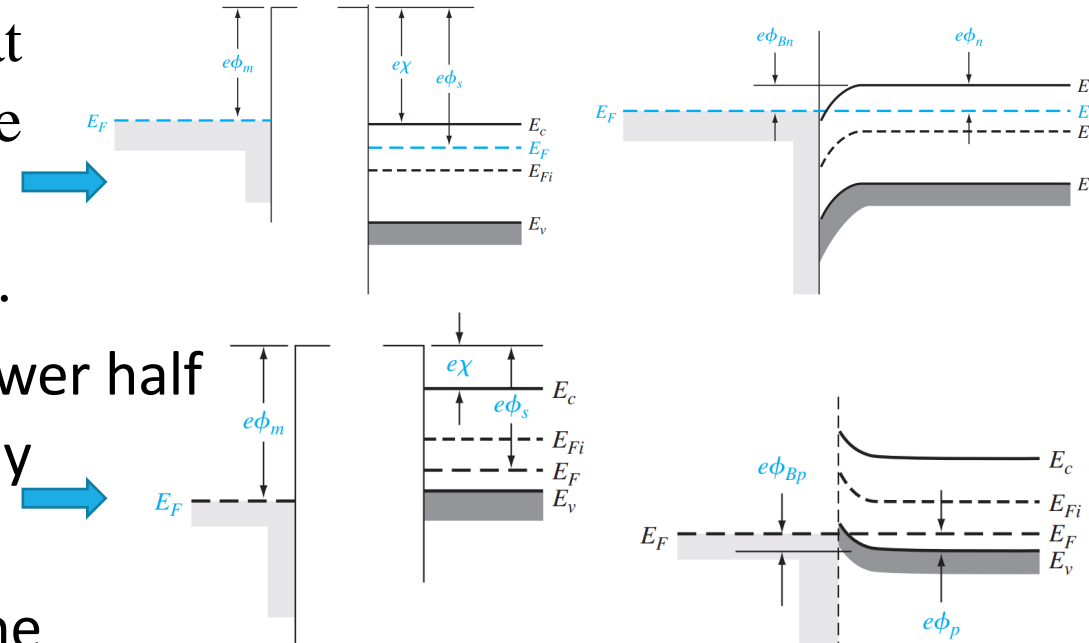
$\phi_m > \phi_s$  , 多數載子為電洞



P-type外加偏壓一樣可以畫：

# Surface states

- Consider the effect of surface states. If we assume that **acceptor surface states exist in the upper half** of the semiconductor bandgap, these surface states will be negatively charged and alter the energy band diagram.
- If we assume that **donor surface states exist in the lower half** of the bandgap, then all donor states will be positively charged
- Therefore, with large surface states, if  $\phi_m < \phi_s$ , for the metal-n-type semiconductor contact, and if  $\phi_m > \phi_s$ , for the metal-p-type semiconductor contact, we may not necessarily form a good ohmic contact.



# Tunneling Barrier

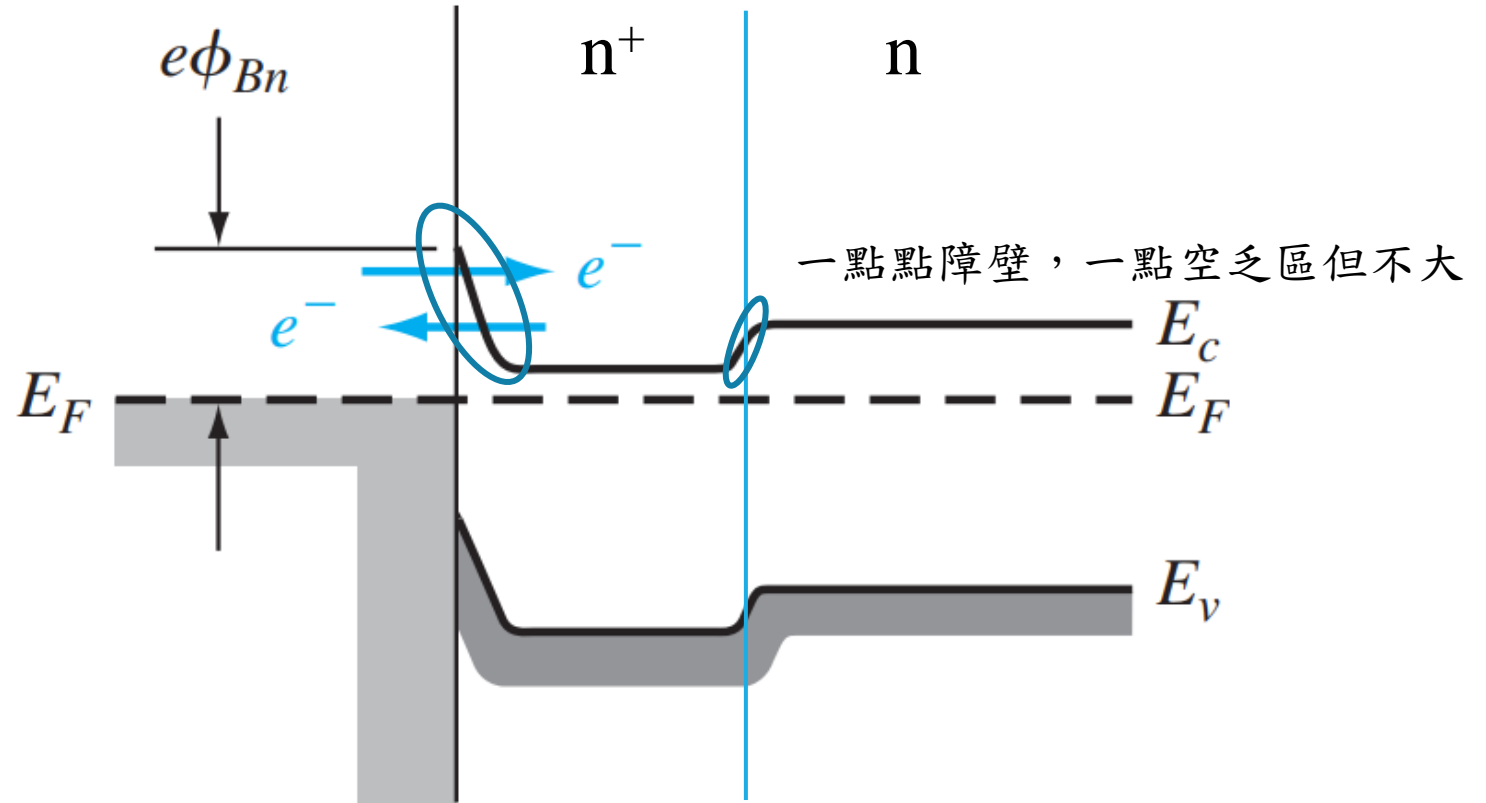
Tunneling Current :

$$J_t \propto \exp\left(\frac{-e\phi_{Bn}}{E_{oo}}\right)$$

$$E_{oo} = \frac{e\hbar}{2} \sqrt{\frac{N_d}{\epsilon_s m_n^*}}$$

摻雜愈濃愈容易穿隧

$$W = \sqrt{\frac{2\epsilon_s (V_{bi} + V_R)}{eN_d}}$$



# Example 9.7

**Objective:** Calculate the space charge width for a Schottky barrier on a heavily doped semiconductor.

Consider silicon at  $T = 300$  K doped at  $N_d = 7 \times 10^{18} \text{ cm}^{-3}$ . Assume a Schottky barrier with  $\phi_{Bn} = 0.67$  V. For this case, we can assume that  $V_{bi} \approx \phi_{B0}$ . Neglect the barrier lowering effect.

$$x_n = \left[ \frac{2\epsilon_s V_{bi}}{eN_d} \right]^{1/2} = \left[ \frac{2(11.7)(8.85 \times 10^{-14})(0.67)}{(1.6 \times 10^{-19})(7 \times 10^{18})} \right]^{1/2}$$

$$x_n = 1.1 \times 10^{-6} \text{ cm} = 110 \text{ \AA}$$



# Tunneling Barrier

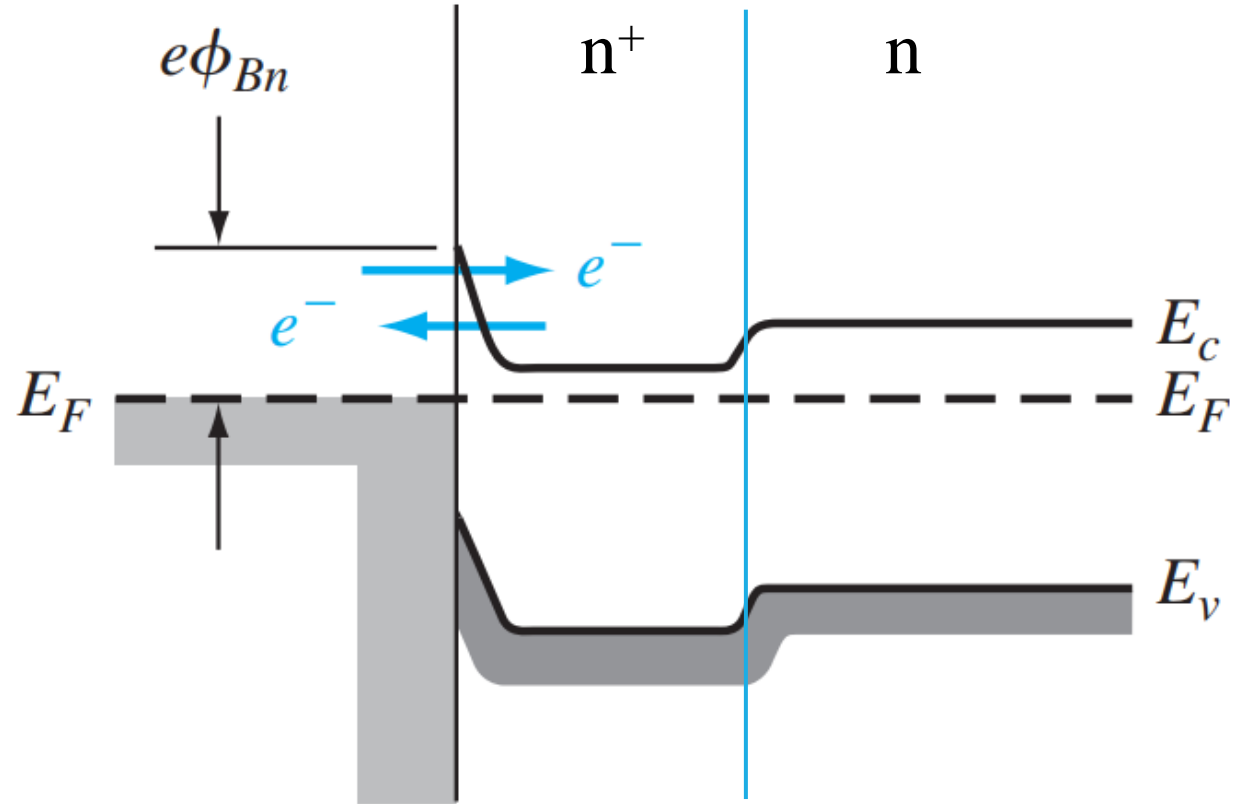
Tunneling Current :

$$J_t \propto \exp\left(\frac{-e\phi_{Bn}}{E_{oo}}\right)$$

$$E_{oo} = \frac{e\hbar}{2} \sqrt{\frac{N_d}{\epsilon_s m_n^*}}$$

摻雜愈濃愈容易穿隧

$$W = \sqrt{\frac{2\epsilon_s (V_{bi} + V_R)}{eN_d}}$$



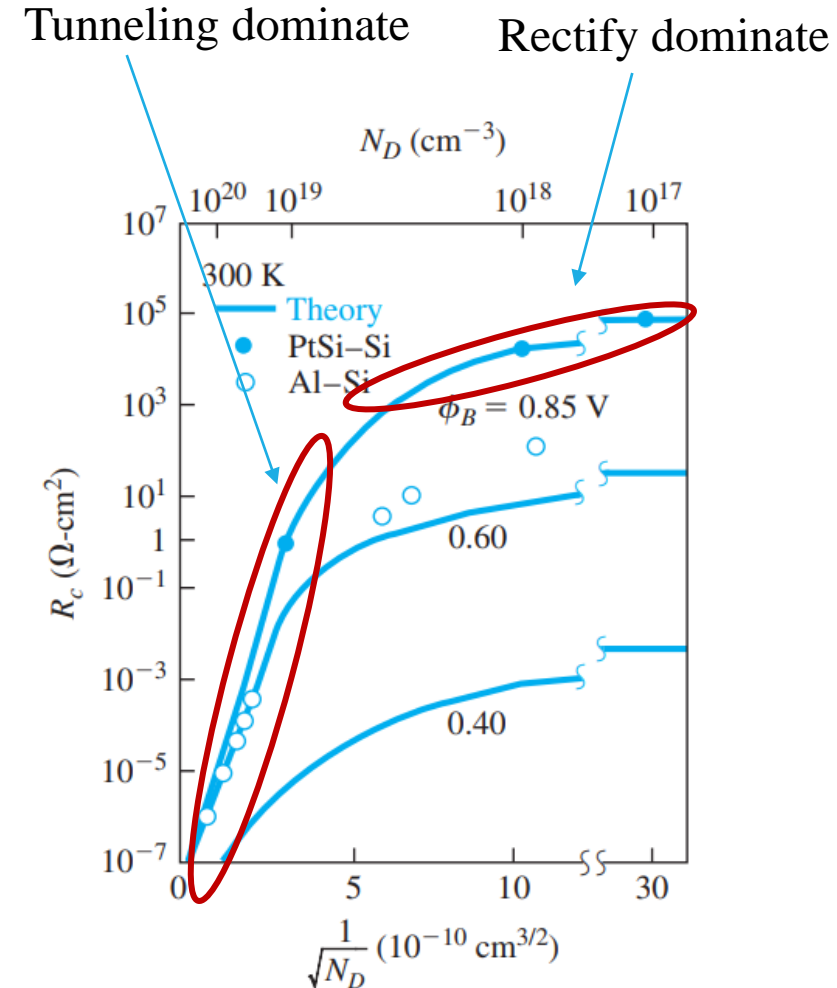
# Specific Contact Resistance 1

$$R_c = \left( \frac{\partial J}{\partial V} \right)^{-1} \Big|_{V=0} \quad (\Omega - \text{cm}^2)$$

We want  $R_c$  to be as small as possible for an ohmic contact.

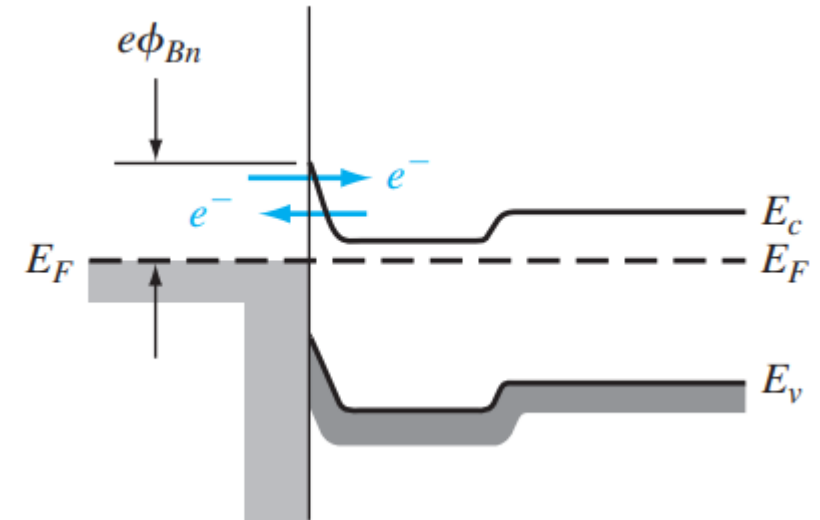
$$J = A^* T^2 \exp\left(-\frac{e\phi_{Bn}}{kT}\right) \left[ \exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

$$R_c = \frac{\frac{kT}{e} \exp\left(\frac{e\phi_{Bn}}{kT}\right)}{A^* T^2} \propto \exp\left( \frac{2\sqrt{\epsilon_s m_n^*}}{\hbar} \frac{\phi_{Bn}}{\sqrt{N_d}} \right)$$



# Specific Contact Resistance 2

- the  $n^+$ -n junction also has a **specific contact** resistance, since there is a **barrier** associated with this junction
- For a fairly **low doped n** region, this contact resistance may actually dominate the total resistance of the junction.
- To form a good ohmic contact, we need to create a **low barrier** and use a **highly doped** semiconductor
- It is **difficult** to make a good contact on **wide bandgap** materials, low barriers **are not possible** on these materials. So, heavily doped is needed.
- **Heavily doped** is limited  $5 \times 10^{19} \text{ cm}^{-3}$  for n-type GaAs.



# Energy-Band Diagrams of Heterojunctions

Straddling  
(most cases)



(a)

Staggered



(b)

Broken gap



(c)

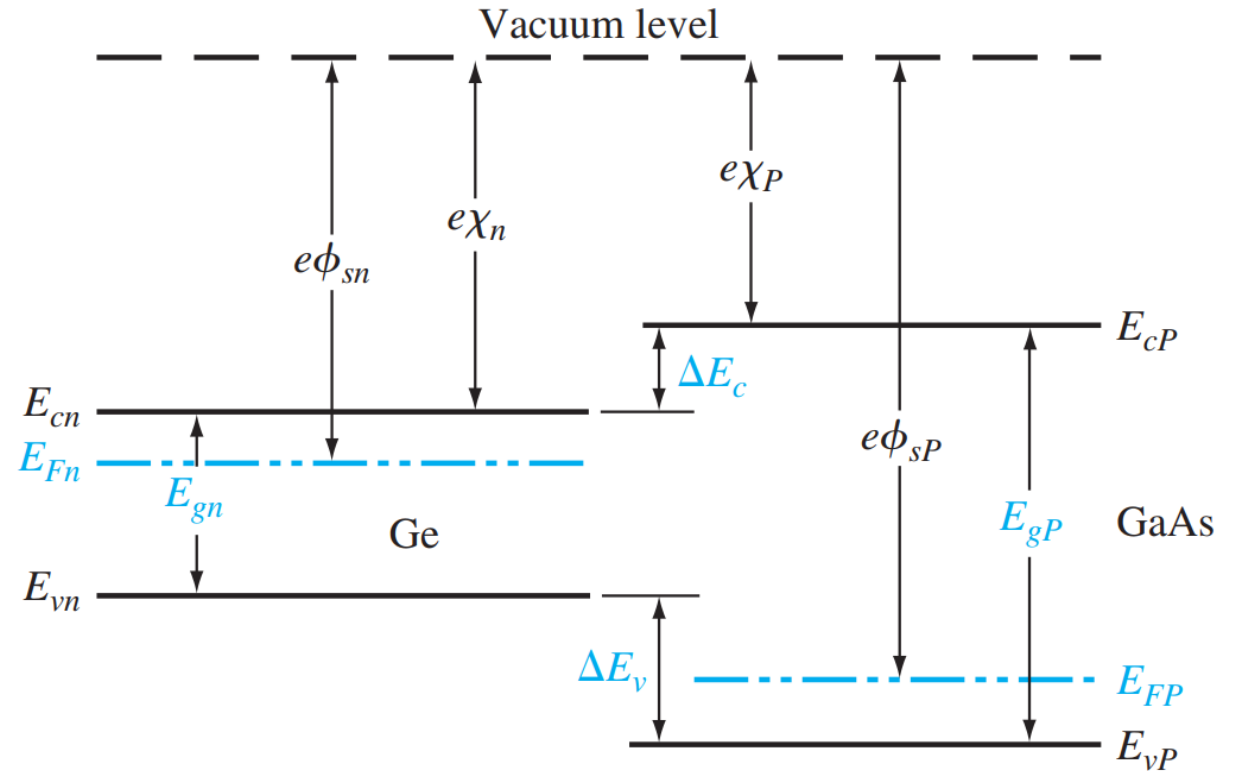
# nP – Straddling (未接觸)

nP：大寫P表示能帶間隙比較大

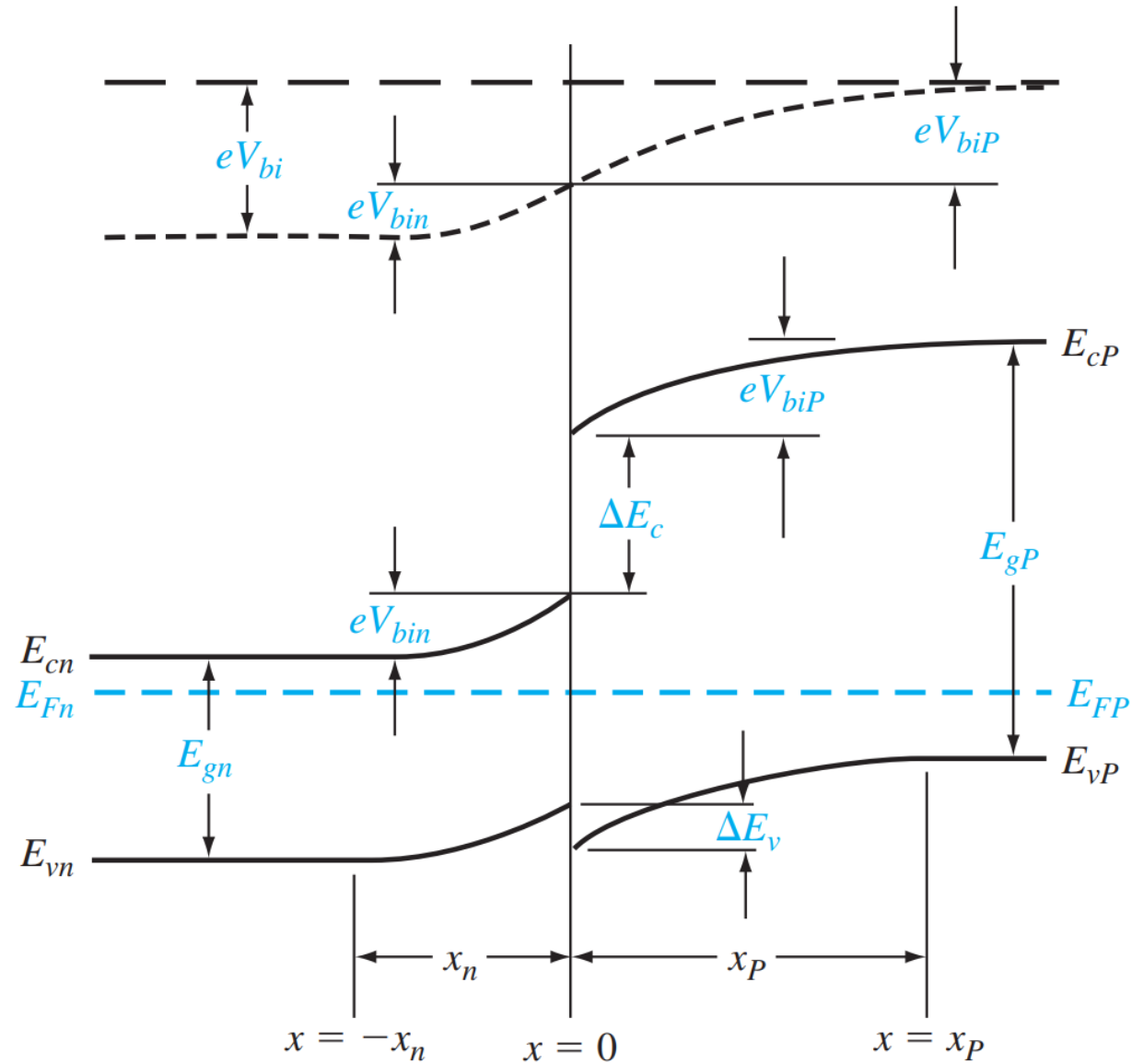
$$\Delta E_c = e(\chi_n - \chi_p)$$

$$\Delta E_c + \Delta E_v = E_{gP} - E_{gn} = \Delta E_g$$

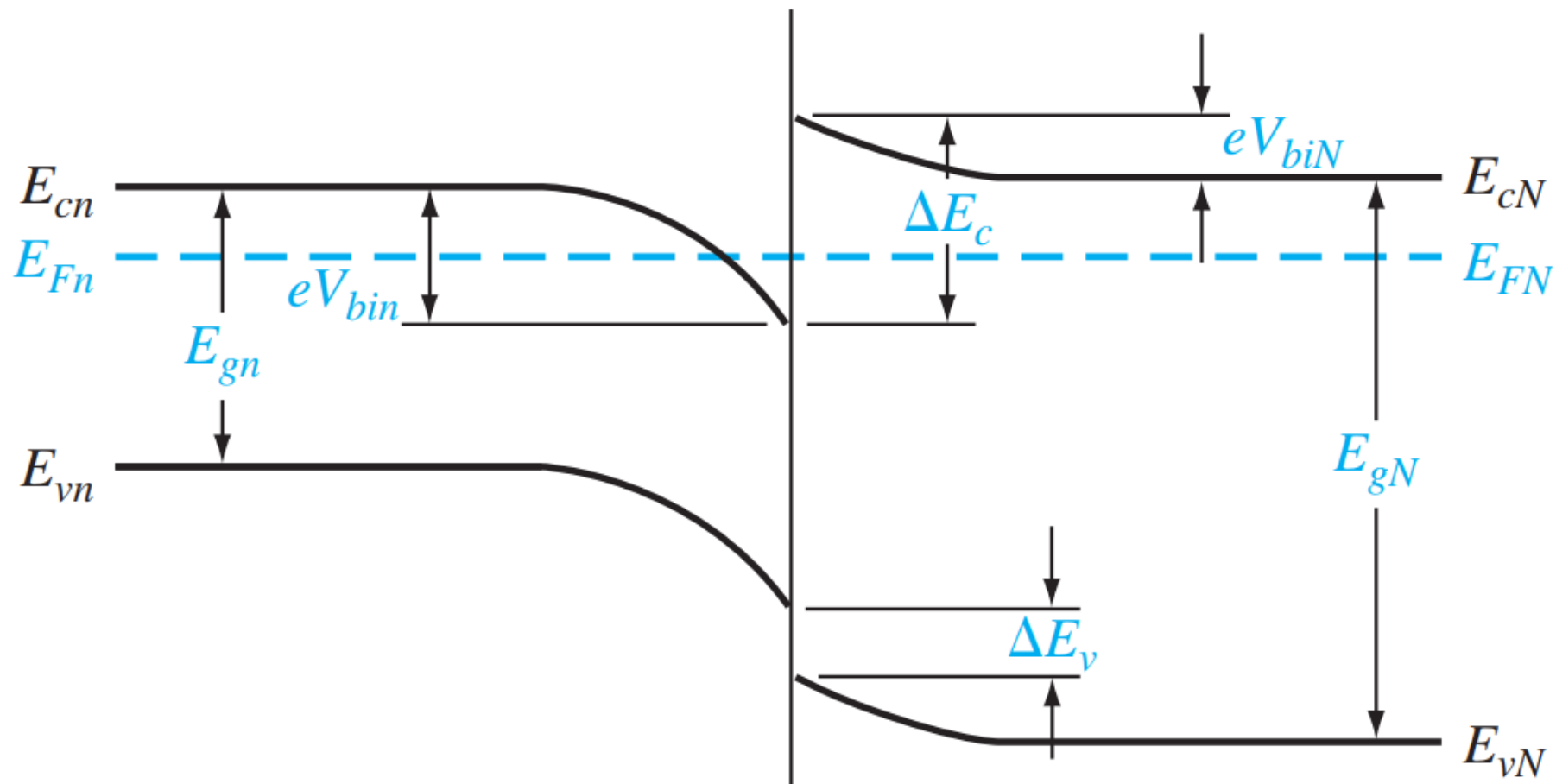
- In the ideal abrupt heterojunction using nondegenerately doped semiconductors, the vacuum level is parallel to both conduction bands and valence bands.
- If the vacuum level is continuous, then the same  $E_c$  and  $E_v$  discontinuities.



# nP – Straddling (接觸)

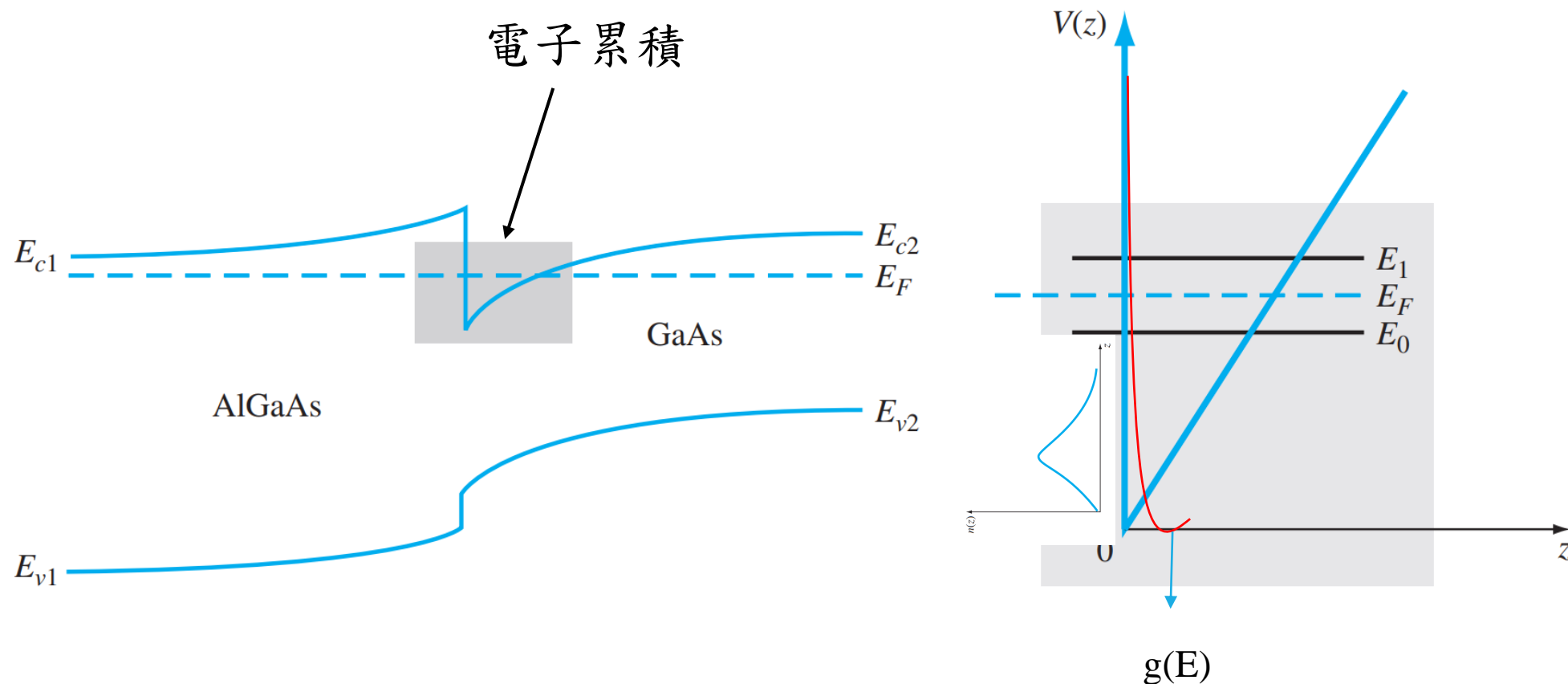


# nN – Straddling (接觸)



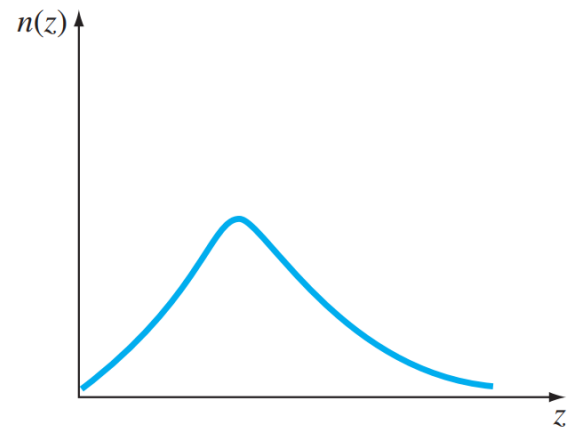
# Two-Dimensional Electron Gas

2DEG：指電子因受到位能限制導致在空間中能量被量子化

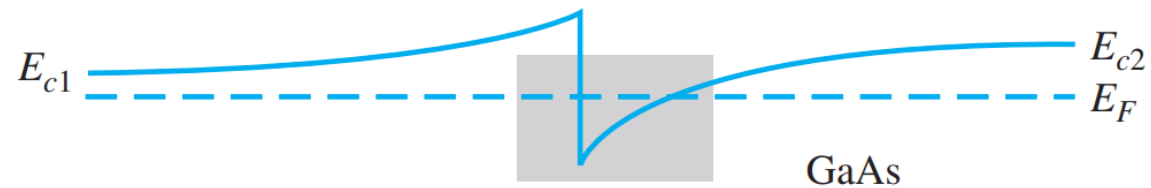
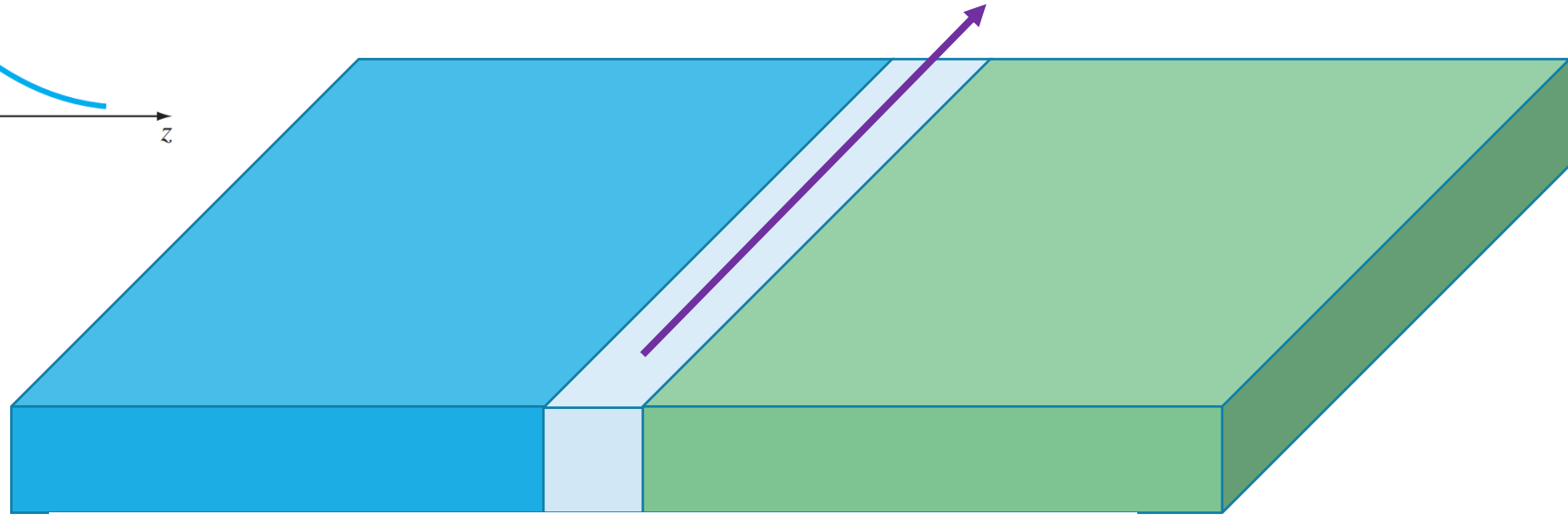




# Two-Dimensional Electron Gas

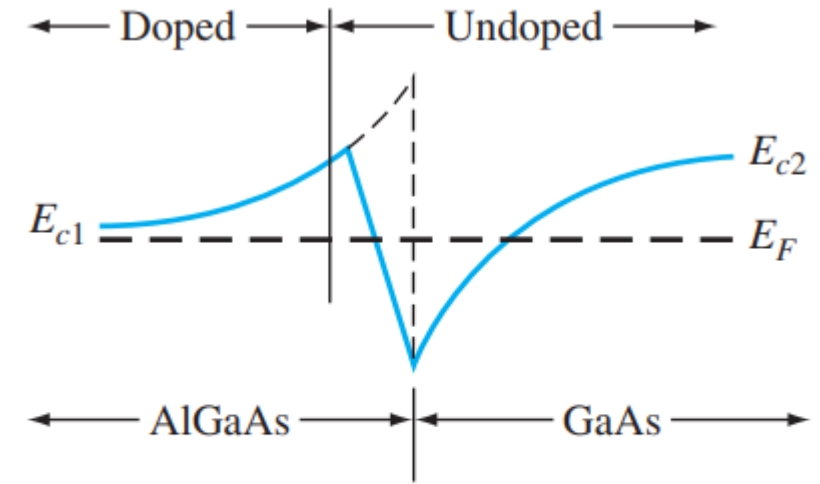


Extremely high-mobility electrons



# Solution

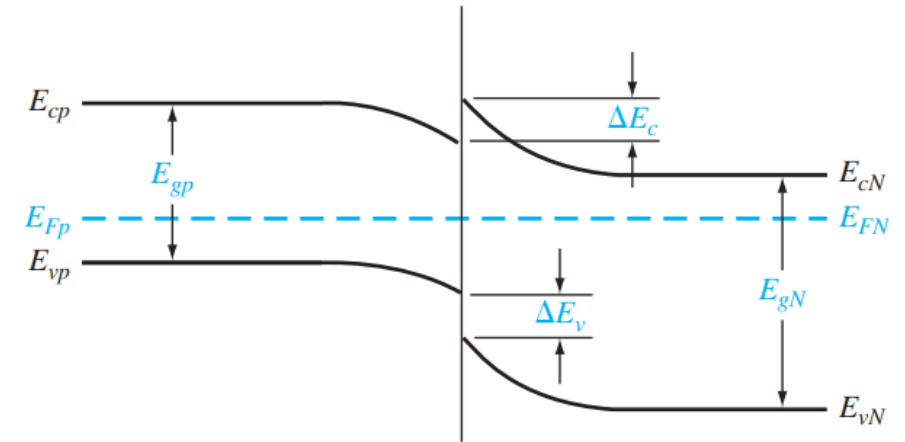
- The movement of the electrons parallel to the interface will still be influenced by the **coulomb attraction** of the ionized impurities in the AlGaAs.
- The effect of these forces can be further reduced by using a **graded** AlGaAs–GaAs heterojunction.
- An intrinsic layer of graded AlGaAs can be sandwiched between the N-type AlGaAs and the intrinsic GaAs.
- The electrons in the potential well are further separated from the ionized impurities so that the electron **mobility is increased**



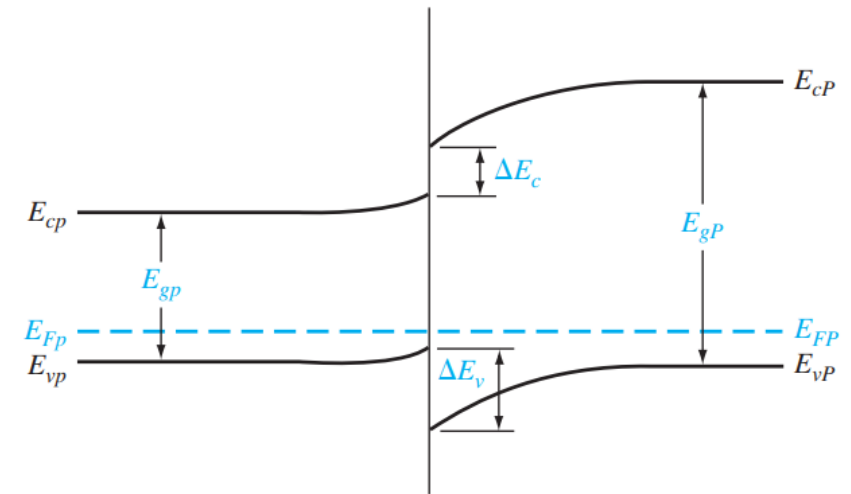
**Figure 9.22** | Conduction-band edge at a graded heterojunction.

# pP and nN

- In pP, **holes** from the wide- bandgap material will flow into the narrow-bandgap material, creating an **accumulation layer of holes** in the narrow-bandgap material at the interface.
- So, accumulation of electron in nN and accumulation of hole in pP (isotype heterojunctions) are obviously **not possible** in a homojunction(同材料不同參雜).



**Figure 9.23** | Ideal energy-band diagram of an Np heterojunction in thermal equilibrium.



**Figure 9.24** | Ideal energy-band diagram of a pP heterojunction in thermal equilibrium.