Chapter 7

The pn Junction

Outline

7.1 Basic Structure of The pn Junction

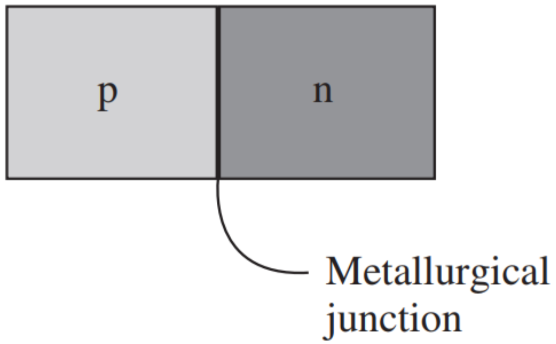
7.2 Zero Applied Bias

7.3 Reverse Applied Bias

7.4 Junction Breakdown

7.5 Nonuniformly Doped Junctions

7.1 Basic Structure of pn Junction



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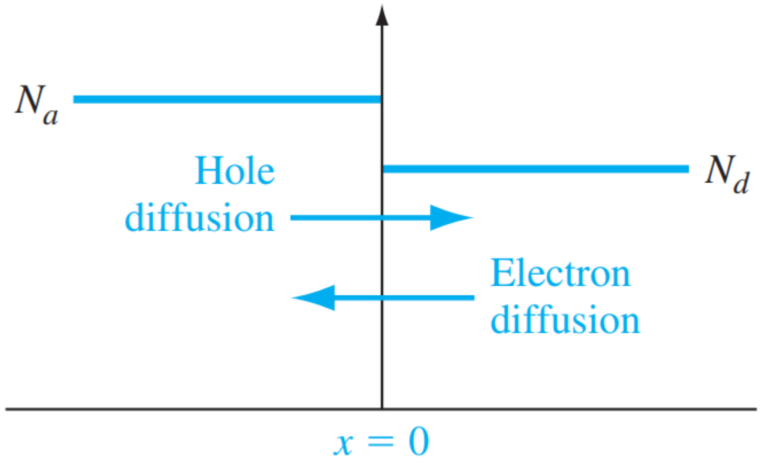
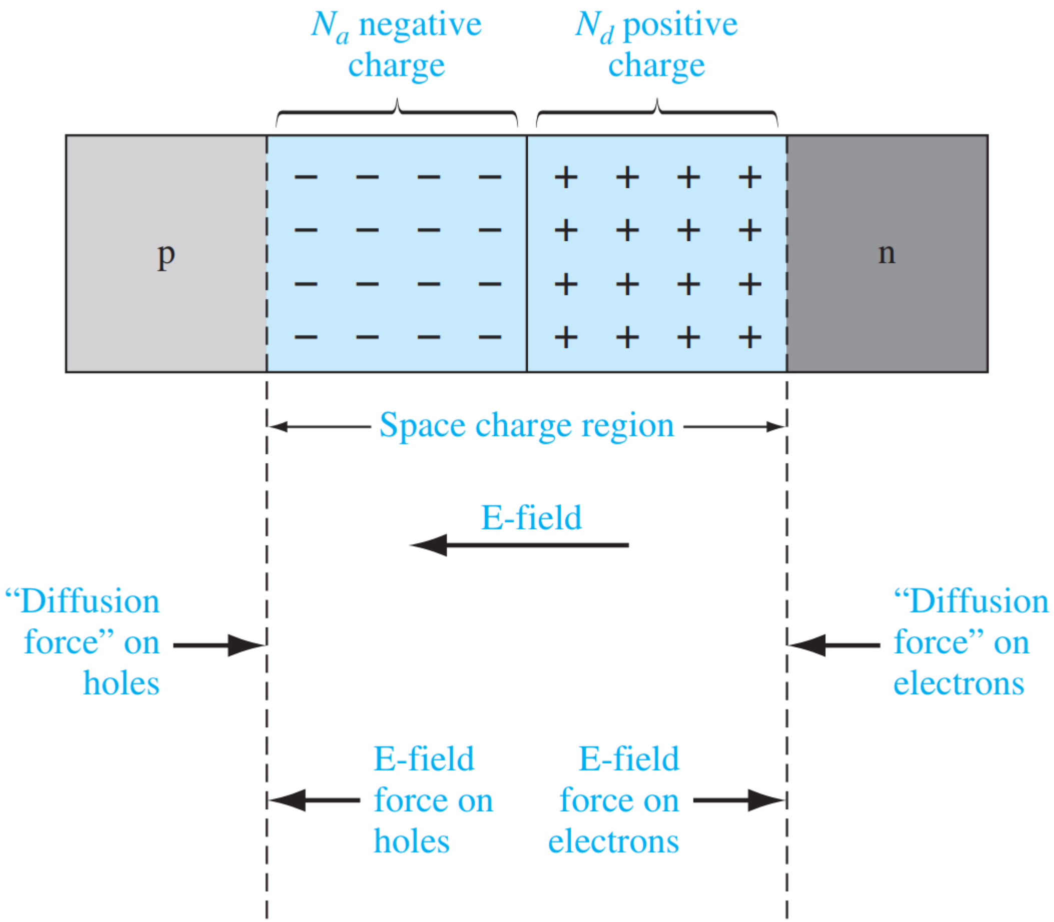
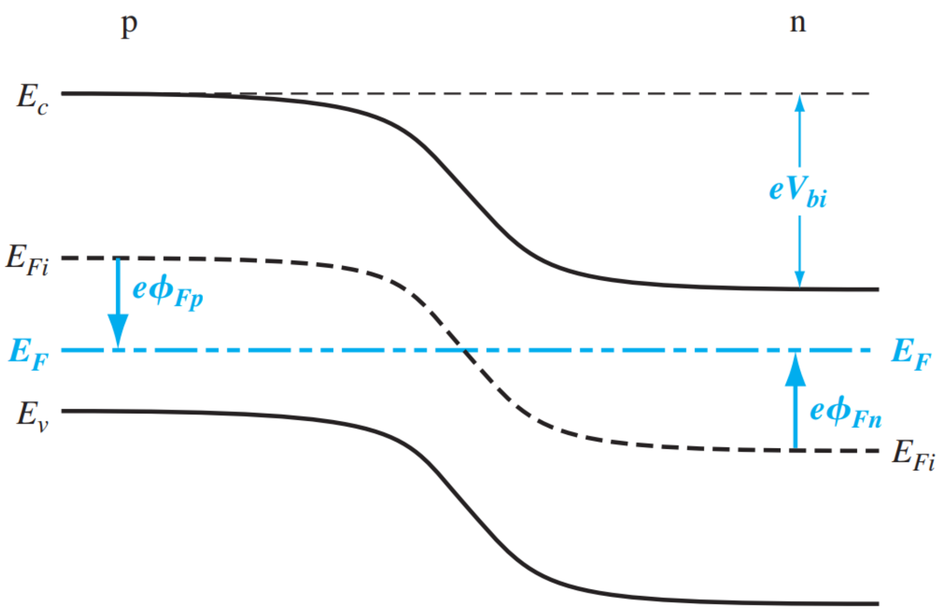




7.2 Zero Applied Bias

- 平衡系統內，費米能階必處處相等
- 內建位能屏障 Built-in Potential Barrier V_{bi}



$$\begin{cases} n_0 = n_i \exp\left(-\dfrac{e\phi_{Fn}}{kT}\right) \approx N_d \\ \phi_{Fn} = -\dfrac{kT}{e}\ln\left(\dfrac{N_d}{n_i}\right) \end{cases}$$

$$\begin{cases} p_0 = n_i \exp\left(\dfrac{e\phi_{Fp}}{kT}\right) \approx N_a \\ \phi_{Fp} = \dfrac{kT}{e}\ln\left(\dfrac{N_a}{n_i}\right) \end{cases}$$

$$V_{bi} = \left|\phi_{Fp}\right| + \left|\phi_{Fn}\right| = \frac{kT}{e}\ln\left(\frac{N_d}{n_i}\right) + \frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right) \Rightarrow V_{bi} = \frac{kT}{e}\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

Example 7.1

Objective: Calculate the built-in potential barrier in a pn junction.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

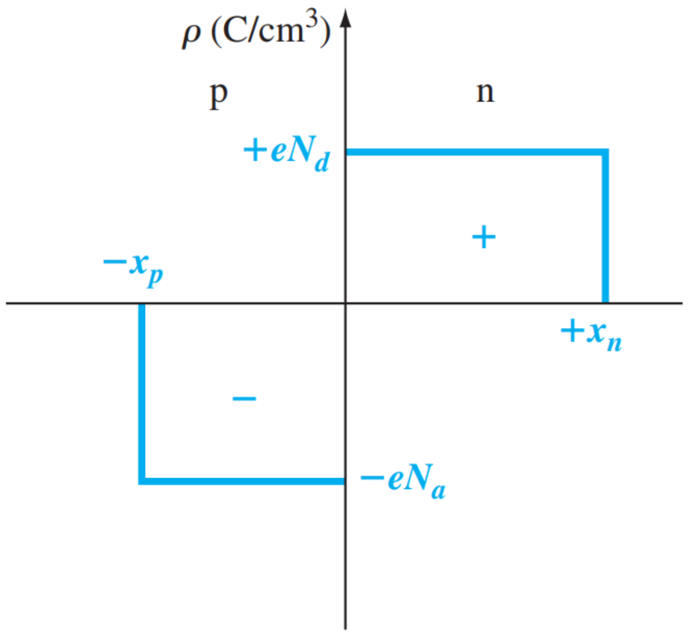
Example 7.1

Objective: Calculate the built-in potential barrier in a pn junction.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left[\frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.713 \text{ V}$$

Electric Field



一維高斯定律

電場

$$\frac{dE}{dx}=\frac{\rho}{\varepsilon}\Rightarrow E=\int\frac{\rho}{\varepsilon}dx$$

P區

$$E=\int\frac{\rho}{\varepsilon}dx=\int\frac{(-e)N_a}{\varepsilon_s}dx=-\frac{eN_a}{\varepsilon_s}x+C_1$$

$$E(-x_p)=0\Rightarrow C_1=-\frac{eN_a x_p}{\varepsilon_s}$$

$$E=-\frac{eN_a}{\varepsilon_s}(x+x_p)$$

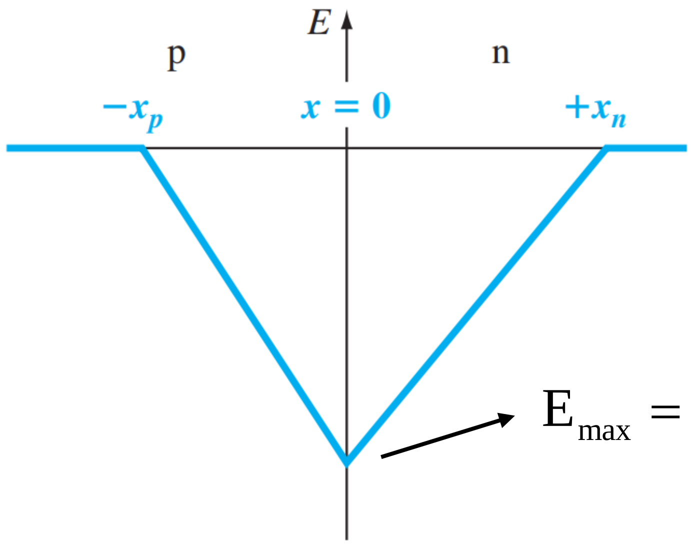
N區

$$E=\int\frac{\rho}{\varepsilon}dx=\int\frac{eN_d}{\varepsilon_s}dx=\frac{eN_d}{\varepsilon_s}x+C_2$$

$$E(x_n)=0\Rightarrow C_2=-\frac{eN_d x_n}{\varepsilon_s}$$

$$E=\frac{eN_d}{\varepsilon_s}(x-x_n)$$

Electric Field



$$E_{max} = -\frac{eN_d x_n}{\varepsilon_s}$$

在x=0位置，左右兩邊電場必相等

$$E_{max} = -\frac{eN_a x_p}{\varepsilon_s} = -\frac{eN_d x_n}{\varepsilon_s} \Rightarrow N_a x_p = N_d x_n$$

電場與電位關係

$$-\frac{d\phi}{dx} = E \Rightarrow \phi = -\int E dx$$

P區 $$\phi = \frac{eN_a}{\varepsilon_s}\left(\frac{x^2}{2} + x_p \cdot x\right) + C_1'$$

N區 $$\phi = \frac{eN_d}{\varepsilon_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2'$$

Electric Potential Barrier

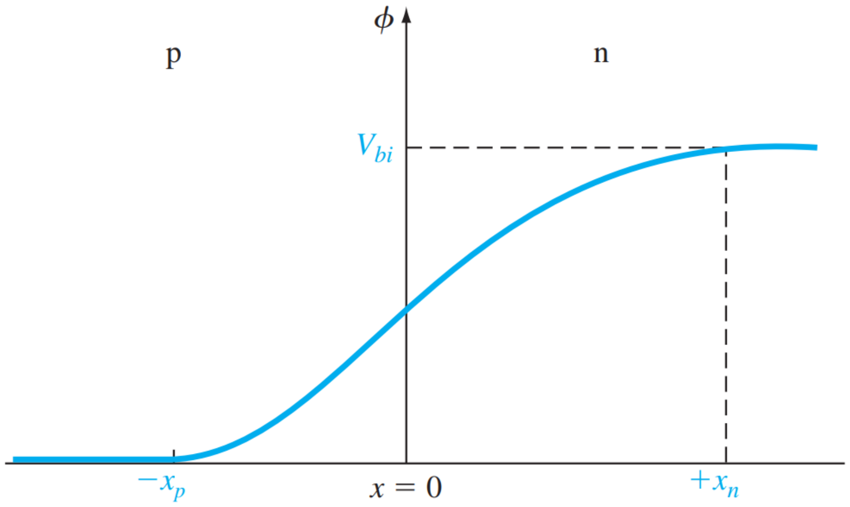


Figure 7.6 | Electric potential through the space charge region of a uniformly doped pn junction.

P區 $$\phi = \frac{eN_a}{\varepsilon_s}\left(\frac{x^2}{2} + x_p \cdot x + \frac{x_p^2}{2}\right)$$

N區 $$\phi = \frac{eN_d}{\varepsilon_s}\left(-\frac{x^2}{2} + x_n \cdot x\right) + \frac{eN_a x_p^2}{2\varepsilon_s}$$

$$V_{bi} = \phi(x_n) = \frac{e}{2\varepsilon_s}\left(N_d x_n^2 + N_a x_p^2\right)$$

$$N_a x_p = N_d x_n \Rightarrow x_p = \frac{N_d}{N_a} x_n$$

上一頁結果

代入

Electric Potential Barrier

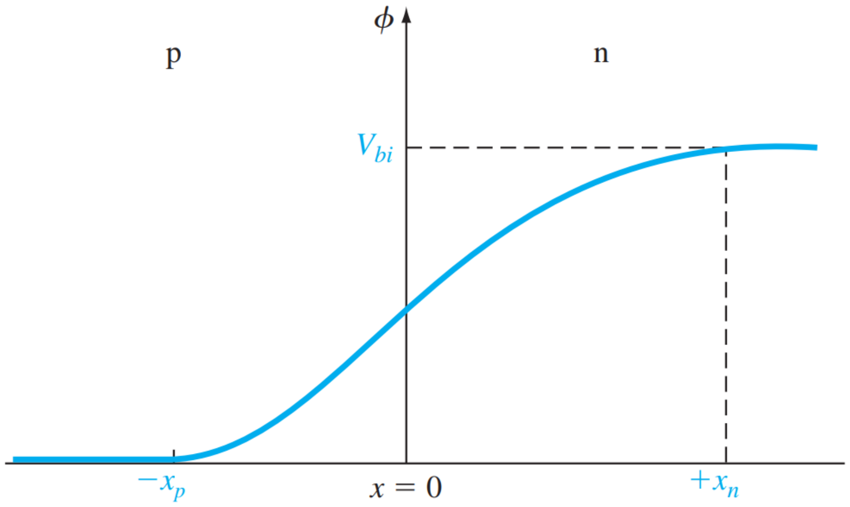


Figure 7.6 | Electric potential through the space charge region of a uniformly doped pn junction.

$$V_{bi} = \frac{e}{2\varepsilon_s}\left(N_d x_n^2 + N_a \frac{N_d^2 x_n^2}{N_a^2}\right) = \frac{e}{2\varepsilon_s}\left(N_d + \frac{N_d^2}{N_a}\right) x_n^2$$

$$\Rightarrow x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}$$

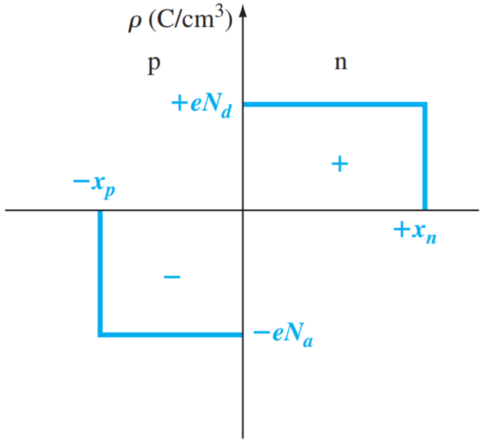
$$\Rightarrow x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

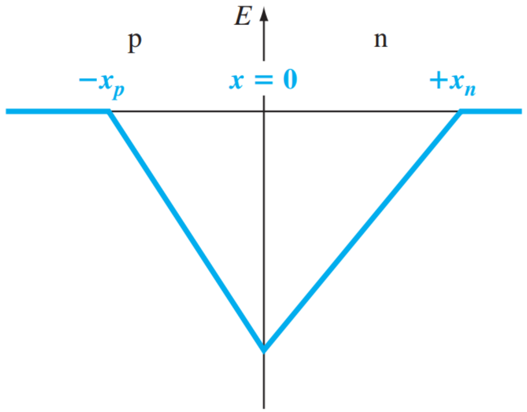
$$\Rightarrow W = x_n + x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]}$$ 空乏區寬度

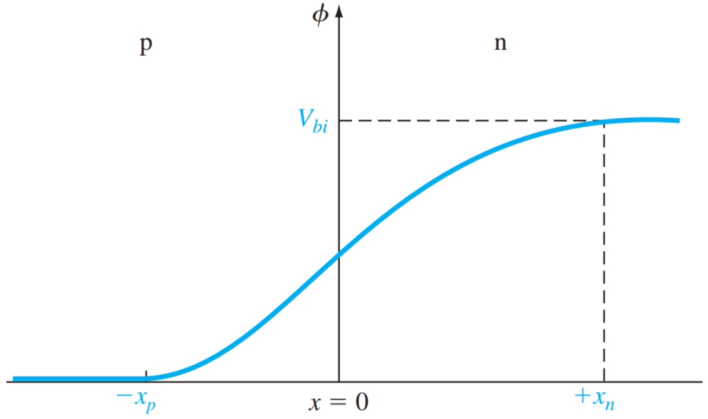
$$\Rightarrow \mathrm{E}_{\max} = -\frac{2V_{bi}}{W}$$

Summary

電荷密度 → 電場強度 → 電位







$$\mathrm{E}_{\max} = -\frac{eN_a x_p}{\varepsilon_s} = -\frac{eN_d x_n}{\varepsilon_s}$$

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}$$

$$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]}$$

$$\mathrm{E}_{\max} = -\frac{2V_{bi}}{W}$$

Example 7.2

Objective: Calculate the space charge width and electric field in a pn junction for zero bias.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}\ \text{cm}^{-3}$ and $N_d = 10^{15}\ \text{cm}^{-3}$.

Example 7.2

Objective: Calculate the space charge width and electric field in a pn junction for zero bias.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

$$V_{bi} = 0.0259 \ln\left(\frac{N_d N_a}{n_i^2}\right) = 0.635 \quad \text{V}$$

$$W = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2}$$

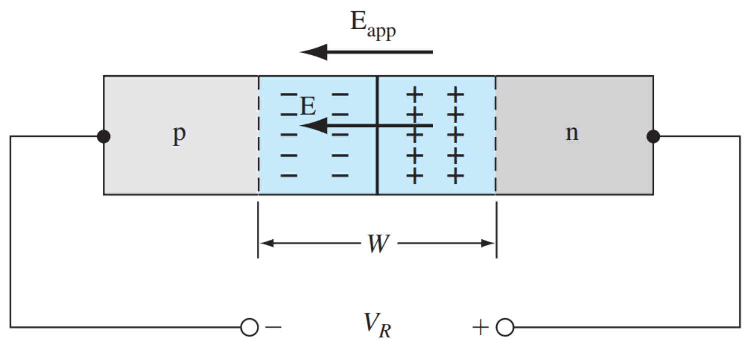
$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

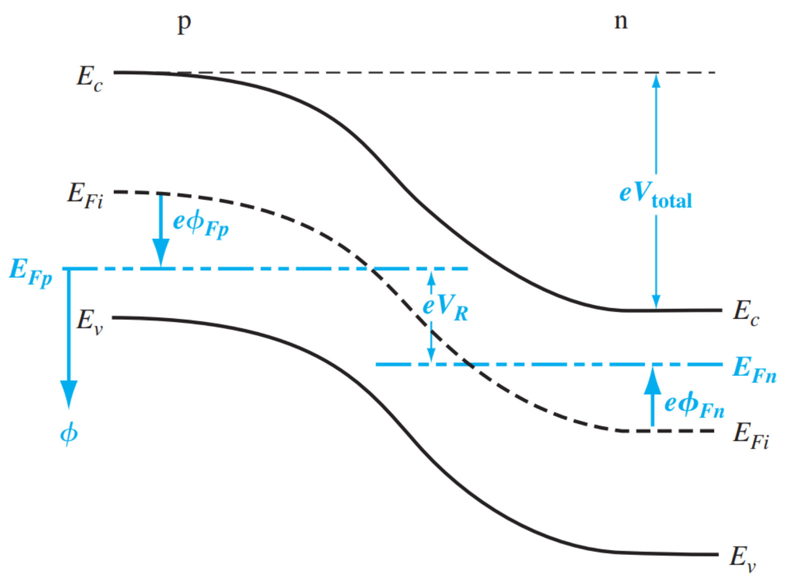
$$= 0.951 \times 10^{-4} \text{ cm} = 0.951 \ \mu\text{m}$$

$$\mathrm{E}_{\max} = -\frac{2V_{bi}}{W} = -\frac{2 \times 0.635}{0.951 \times 10^{-4}} = -1.34 \times 10^{4} \quad \text{V/cm}$$

7.3 Reverse Applied Bias

- 施加負向偏壓





$$V_{total} = |\phi_{Fp}| + |\phi_{Fn}| + V_R = V_{bi} + V_R$$

$$\Rightarrow W = x_n + x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]}$$

$$\Rightarrow \mathrm{E}_{\max} = -\frac{2(V_{bi} + V_R)}{W}$$

Example 7.3

Objective: Calculate the width of the space charge region in a pn junction when a reverse-biased voltage is applied.

Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Assume that $n_i = 1.5 \times 10^{10}$ cm^{-3} and $V_R = 5$ V.

Example 7.3

Objective: Calculate the width of the space charge region in a pn junction when a reverse-biased voltage is applied.

Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Assume that $n_i = 1.5 \times 10^{10}$ cm^{-3} and $V_R = 5$ V.

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83\ \mu\text{m}$$

Example 7.4

Objective: Design a pn junction to meet maximum electric field and voltage specifications.

Consider a silicon pn junction at $T = 300$ K with a p-type doping concentration of $N_a = 2 \times 10^{17}$ cm^{-3}. Determine the n-type doping concentration such that the maximum electric field is $|\mathrm{E}_{max}| = 2.5 \times 10^5$ V/cm at a reverse-biased voltage of $V_R = 25$ V.

Example 7.4

Objective: Design a pn junction to meet maximum electric field and voltage specifications.

Consider a silicon pn junction at $T = 300$ K with a p-type doping concentration of $N_a = 2 \times 10^{17}$ cm^{-3}. Determine the n-type doping concentration such that the maximum electric field is $|\mathrm{E}_{\max}| = 2.5 \times 10^5$ V/cm at a reverse-biased voltage of $V_R = 25$ V.

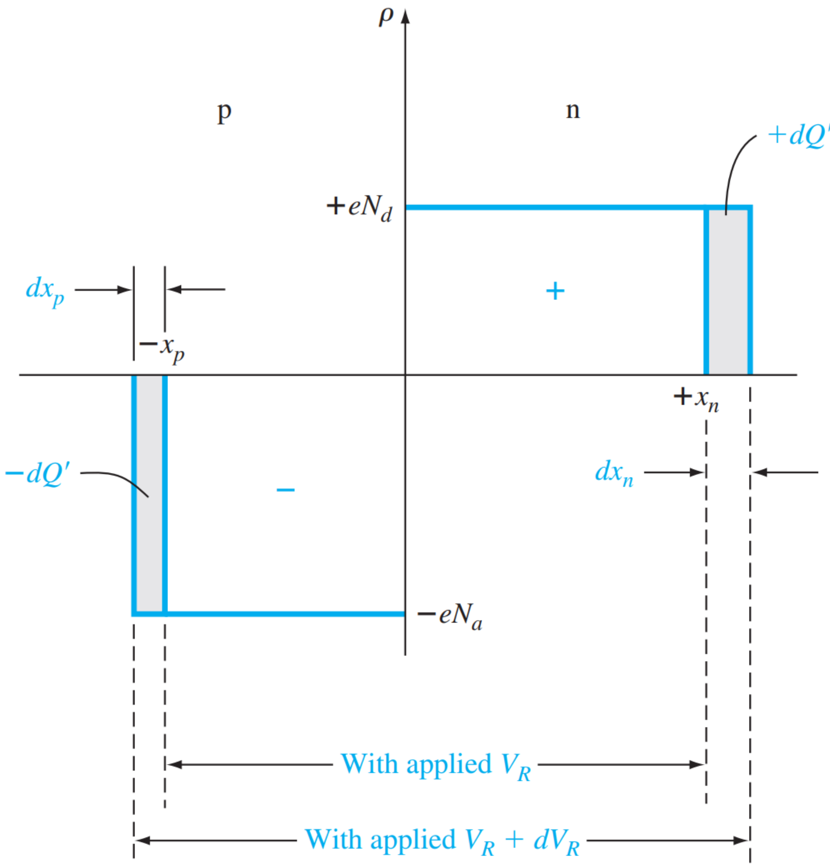
$$|\mathrm{E}_{\max}| \cong \left\{\frac{2e\mathrm{V}_R}{\epsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)\right\}^{1/2}$$

$$2.5 \times 10^5 = \left\{\frac{2(1.6 \times 10^{-19})(25)}{(11.7)(8.85 \times 10^{-14})}\left[\frac{(2 \times 10^{17})N_d}{2 \times 10^{17} + N_d}\right]\right\}^{1/2}$$

$$N_d = 8.43 \times 10^{15} \text{ cm}^{-3}$$

7.3.2 Junction Capacitance

- 當施加逆向偏壓時，會改變空乏區的寬度(電荷量改變)
- 定義 Junction capacitance $C' = \dfrac{dQ'}{dV_R}$ $\quad Q' = Q/A$ 單位面積的電荷量



$$Q' = eN_d x_n = eN_d \sqrt{\frac{2\varepsilon_s\left(V_{bi}+V_R\right)}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

$$\frac{dQ'}{dV_R} = \sqrt{\frac{\varepsilon_s e N_d N_a}{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}} = \frac{\varepsilon_s}{W}$$

Example 7.5

Objective: Calculate the junction capacitance of a pn junction.

Consider the same pn junction as that in Example 7.3. Again assume that $V_R = 5$ V.

Example 7.5

Objective: Calculate the junction capacitance of a pn junction.

Consider the same pn junction as that in Example 7.3. Again assume that $V_R = 5$ V.

$$C' = \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})(10^{15})}{2(0.635 + 5)(10^{16} + 10^{15})} \right\}^{1/2}$$

$$C' = 3.66 \times 10^{-9} \text{ F/cm}^2$$

If the cross-sectional area of the pn junction is, for example, $A = 10^{-4}$ cm^2, then the total junction capacitance is

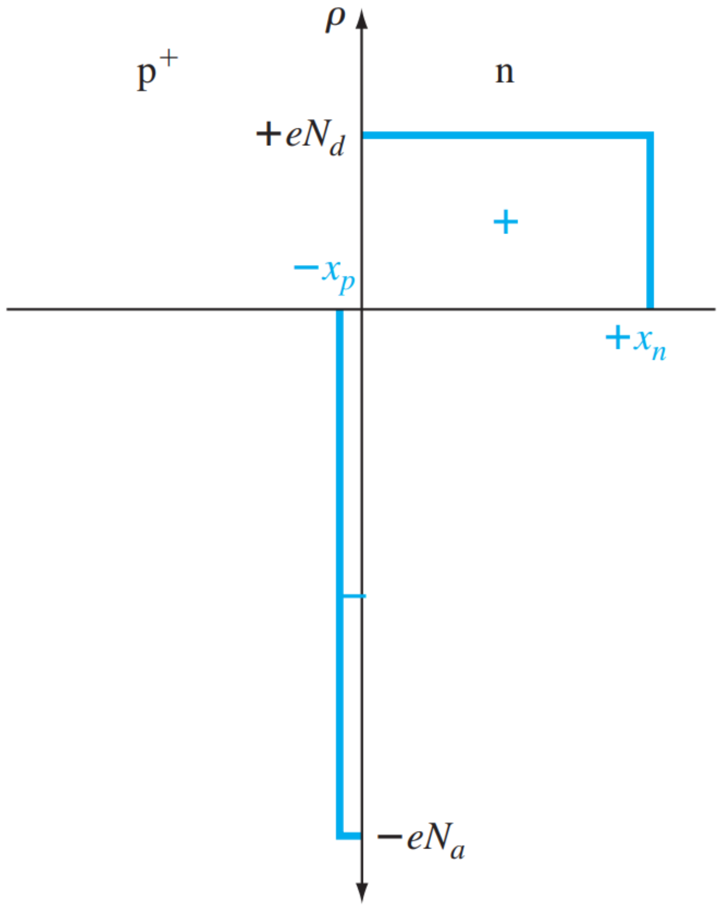
$$C = C' \cdot A = 0.366 \times 10^{-12} \text{ F} = 0.366 \text{ pF}$$

7.3.3 One-Sided Junctions (p^+n junction, $N_a >> N_d$)

$$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{1}{N_d}}$$

$$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a^2}}$$ 可忽略

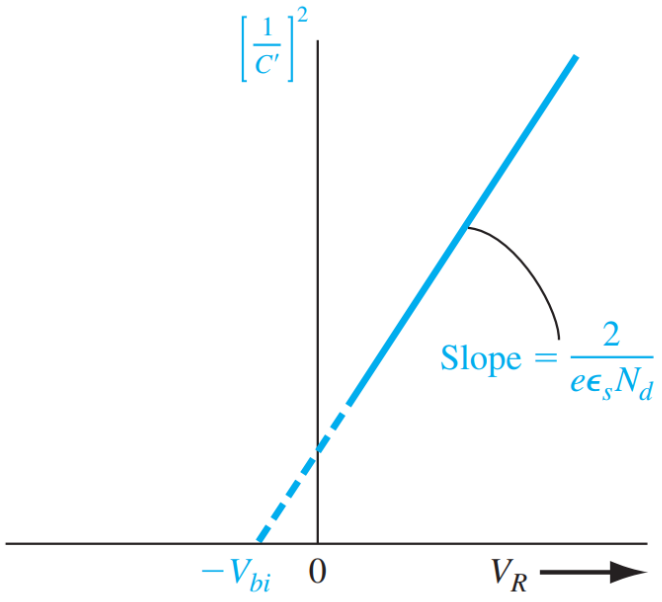
$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a+N_d}{N_aN_d}\right]} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{1}{N_d}}$$



7.3.3 One-Sided Junctions (p+n junction, $N_a >> N_d$)

$$C' = \frac{dQ'}{dV_R} = \sqrt{\frac{\varepsilon_s e N_d N_a}{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}} \approx \sqrt{\frac{\varepsilon_s e N_d}{2\left(V_{bi}+V_R\right)}}$$

$$\underbrace{\left(\frac{1}{C'}\right)^2}_{\text{實驗上可量測到}} = \frac{2\overbrace{\left(V_{bi}+V_R\right)}^{\text{實驗變量}}}{\varepsilon_s e N_d}$$



Example 7.6

Objective: Determine the impurity doping concentrations in a p^+n junction given the parameters from Figure 7.11.

Assume that the intercept and the slope of the curve in Figure 7.11 are $V_{bi} = 0.725$ V and 6.15×10^{15} $(\text{F/cm}^2)^{-2}$ $(\text{V})^{-1}$, respectively, for a silicon p^+n junction at $T = 300$ K.

$$N_d = \frac{2}{e\,\epsilon_s} \cdot \frac{1}{slope} = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(6.15 \times 10^{15})}$$

$$N_d = 1.96 \times 10^{15} \text{ cm}^{-3}$$

From the expression for V_{bi}, which is

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

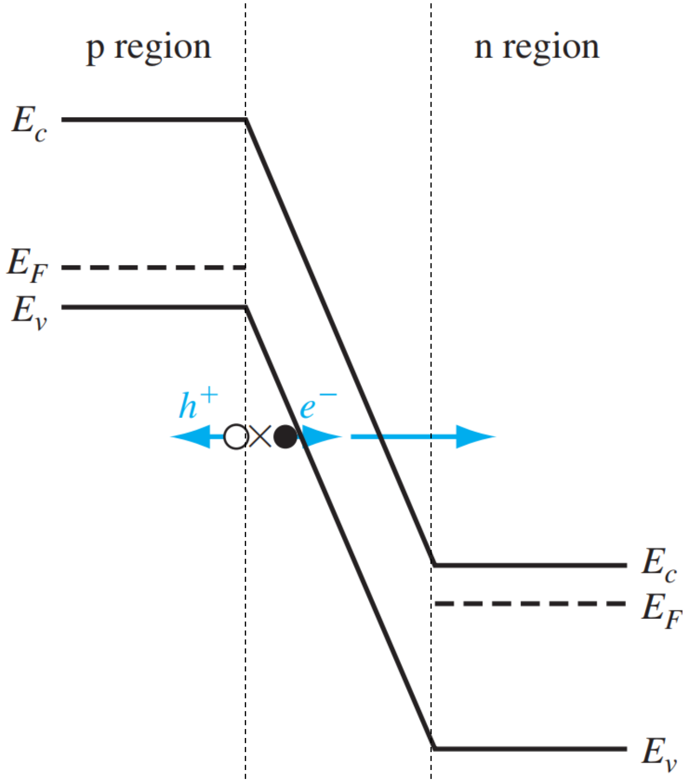
we can solve for N_a as

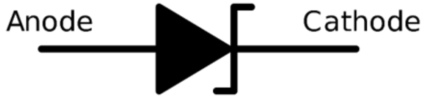
$$N_a = \frac{n_i^2}{N_d} \exp\left(\frac{V_{bi}}{V_t}\right) = \frac{(1.5 \times 10^{10})^2}{1.963 \times 10^{15}} \exp\left(\frac{0.725}{0.0259}\right) = 1.64 \times 10^{17} \text{ cm}^{-3}$$

7.4 Junction Breakdown

Zener effect (Highly-doped pn junctions)

- 高濃度摻雜，空乏區很窄
- p價帶電子能量比n傳導帶電子還高，加上窄空乏區，穿隧效應提升。





齊納二極體

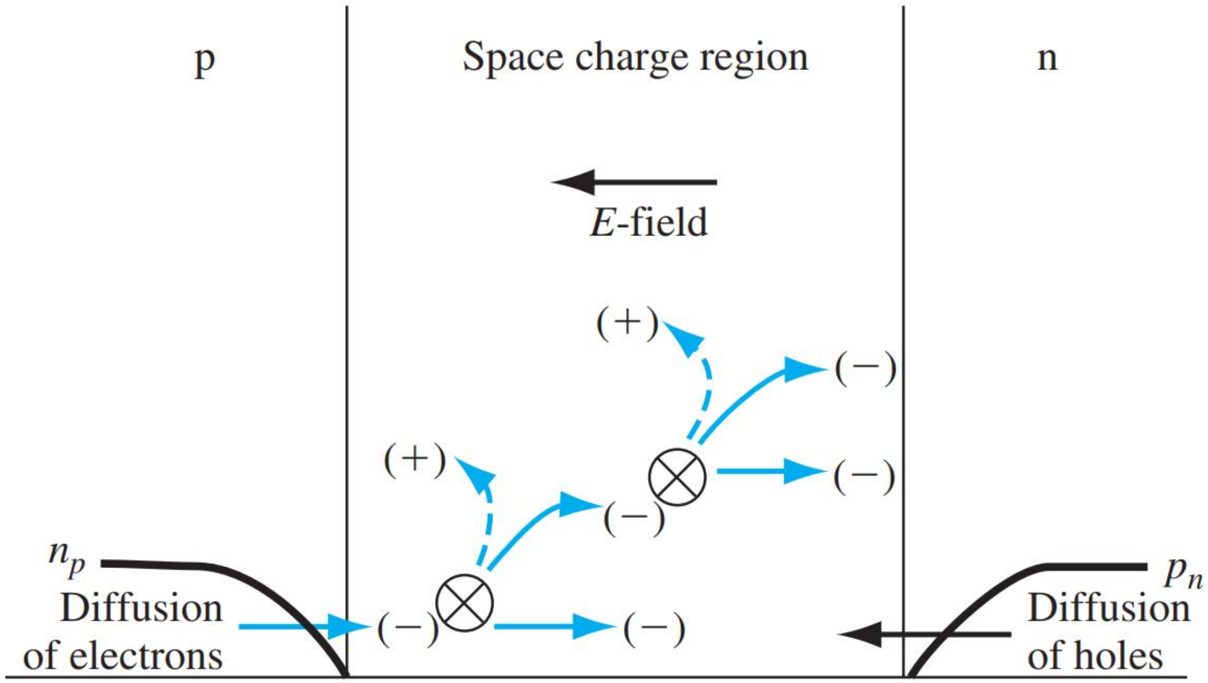
$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s\left(V_{bi}+V_R\right)}{e}\left[\frac{N_a+N_d}{N_a N_d}\right]}$$

7.4 Junction Breakdown

Avalanche effect

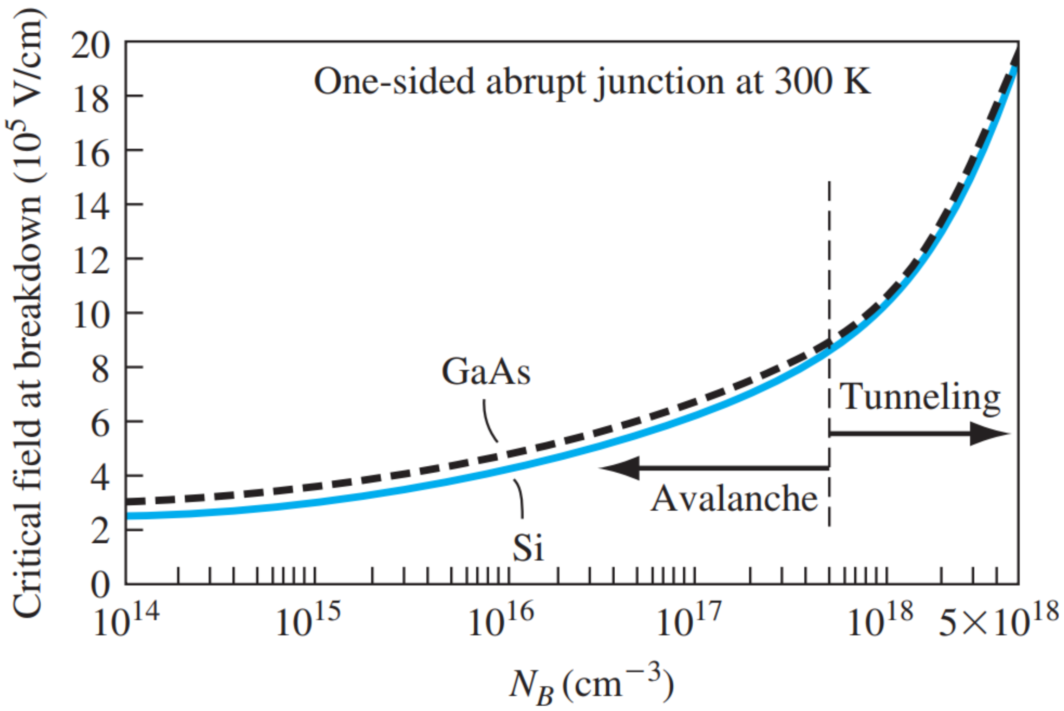
- p區的電子為少數載子，因擴散效應而有微量的流動
- 當電子進入空乏區後，在高電場的加速下撞出更多電子電洞對，造成雪崩式效應。

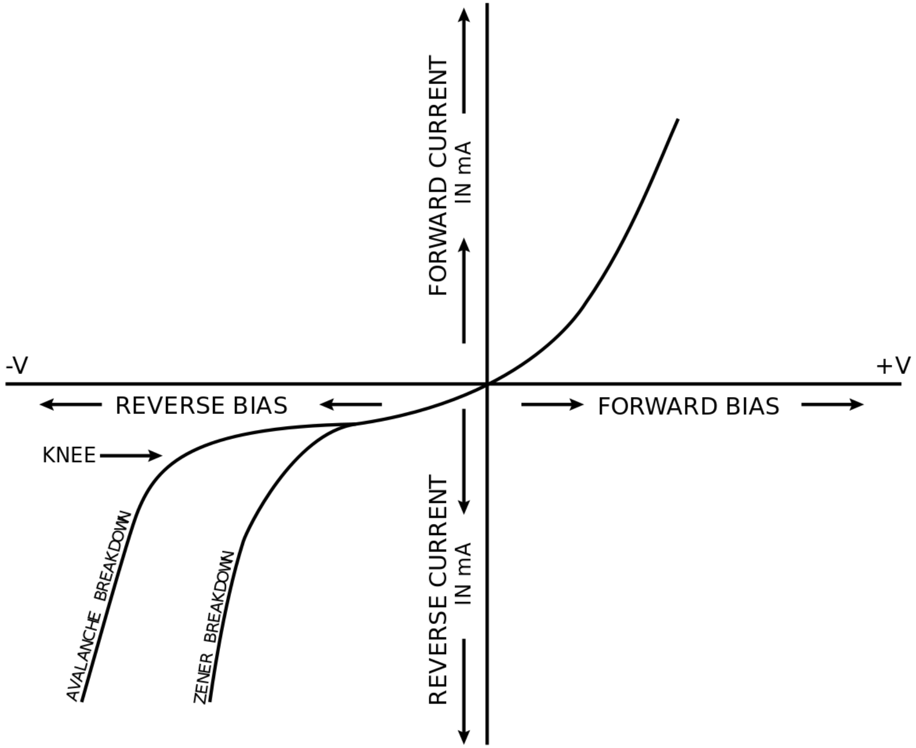


Breakdown Voltage and Critical Electric Field

崩潰電壓

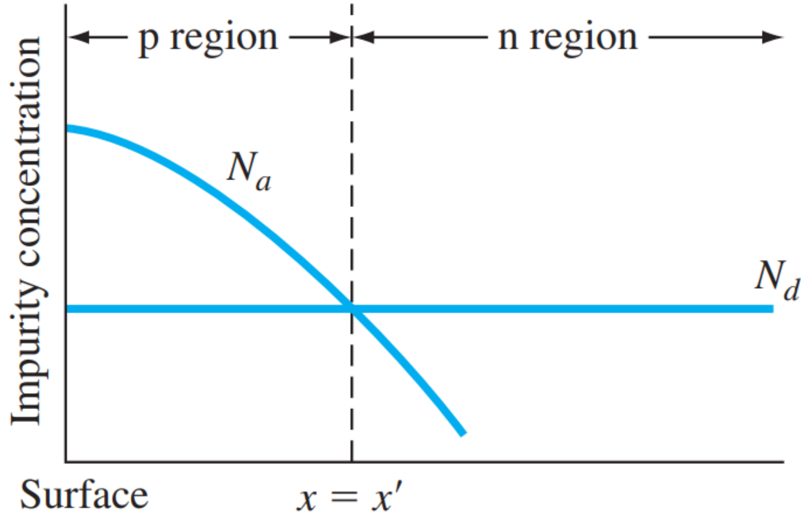
$$V_B = \frac{\epsilon_s E_{crit}^2}{2eN_B}$$

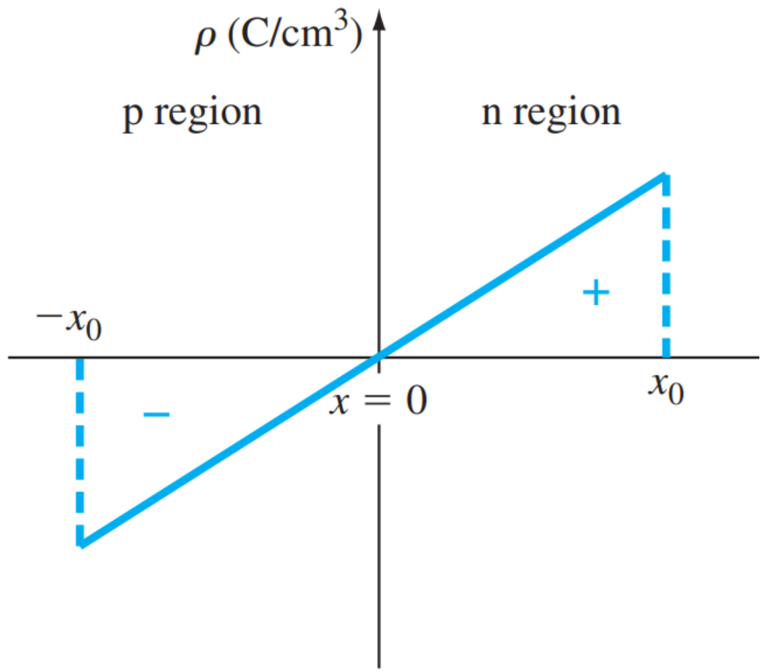




7.5 Nonuniformly Doped Junctions

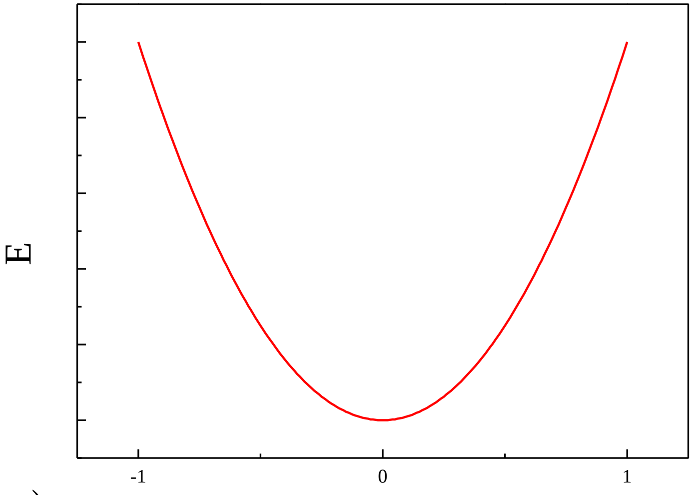
Linearly Graded Junctions

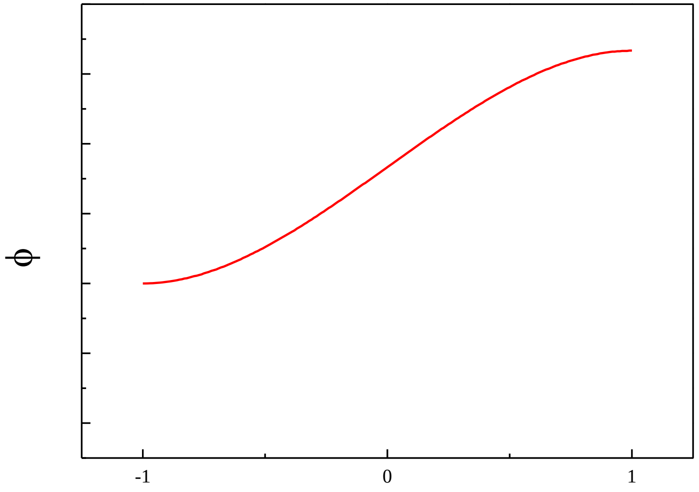




$$\rho(x) = eax$$

Electric field and potential





$$\frac{dE}{dx}=\frac{\rho(x)}{\varepsilon_s}=\frac{eax}{\varepsilon_s}$$ a(is the gradient of the net impurity concentration)

$$E=\int\frac{eax}{\varepsilon_s}dx=\frac{ea}{2\varepsilon_s}\left(x^2-x_0{}^2\right)$$

$$\phi(-x_0)=0$$

$$\phi(x)=-\int Edx=\frac{-ea}{2\varepsilon_s}\left(\frac{x^3}{3}-x_0{}^2x\right)+\frac{ea}{3\varepsilon_s}x_0{}^3$$

$$\phi(x_0)=\frac{2}{3}\frac{eax_0{}^3}{\varepsilon_s}=V_{bi}$$

Linearly Graded Junctions

$$V_{bi} = V_t \ln\left[\frac{N_d(x_0)N_a(-x_0)}{n_i^2}\right]$$

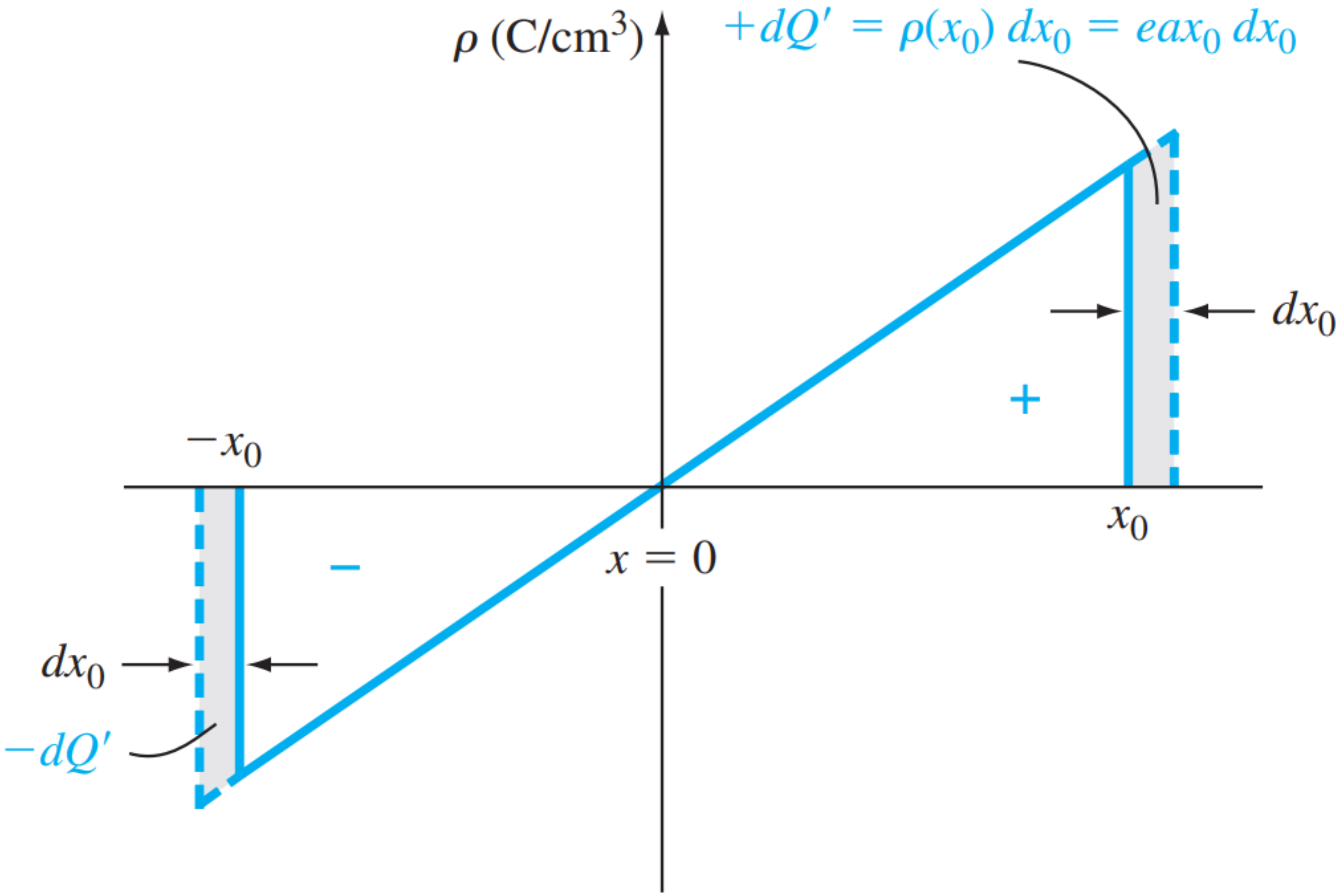
$$N_d(x_0) = ax_0 ; N_a(-x_0) = ax_0$$

$$V_{bi} = V_t \ln\left(\frac{ax_0}{n_i}\right)^2$$

From last page: $x_0 = \left\{\frac{3}{2}\frac{\varepsilon_s}{ea}(V_{bi} + V_R)\right\}^{1/3}$

$$\rho = eax$$

$$C' = \frac{dQ'}{dV_R} = eax_0\frac{dx_0}{dV_R} = \left[\frac{ea\varepsilon_s^2}{12(V_{bi} + V_R)}\right]^{1/3}$$



Hyperabrupt Junctions

$$N = Bx^m$$

$$C' = \left[\frac{eB\varepsilon_s^{m+1}}{(m+2)(V_{bi}+V_R)} \right]^{1/(m+2)}$$