Small-Signal Admittance(Mathematical Analysis)

$V_a = V_0 + v_1(t)$ (V_0 is the dc quiescent bias voltage)

$$p_n(0,t) = p_{n0}\exp\left\{\frac{e\left[V_0 + v_1(t)\right]}{kT}\right\} = p_{dc}\exp\frac{ev_1(t)}{kT}$$

assume $\left|v_1(t)\right| \ll (kT/e) = V_t$

then using taylor's expansion

$$p_n(0,t) \approx p_{dc}\left[1 + \frac{v_1(t)}{V_t}\right] = p_{dc}\left[1 + \frac{\hat{V}_1}{V_t}e^{j\omega t}\right]$$

In neutral n region (x>0),E=0

$$D_p\frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$$

we can assume steady state solution for δp_n to be of the form :

$\delta p_n(x,t) = \delta p_0(x) + p_1(x)e^{j\omega t}$;

δp_0 is the dc excess carrier concentration same as previous;

p_1 is magnitude of ac component of excess carrier

$$D_p\left\{\frac{\partial^2\left[\delta p_0(x)\right]}{\partial x^2} + \frac{\partial^2 p_1(x)}{\partial x^2}e^{j\omega t}\right\} - \frac{\delta p_0(x) + p_1(x)e^{j\omega t}}{\tau_{p0}}$$

$$= j\omega p_1(x)e^{j\omega t}$$

Re *write*

$$\left\{\frac{D_p\partial^2\left[\delta p_0(x)\right]}{\partial x^2} - \frac{\delta p_0(x)}{\tau_{p0}}\right\} + \left[D_p\frac{\partial^2 p_1(x)}{\partial^2 x^2} - \frac{p_1(x)}{\tau_{p0}} - j\omega p_1(x)\right]e^{j\omega t}$$

$$= 0$$

Same as DC

Small-Signal Admittance(Mathematical Analysis)

$$\left\{\frac{D_p\partial^2\left[\delta p_0(x)\right]}{\partial x^2}-\frac{\delta p_0(x)}{\tau_{p0}}\right\}+\left[D_p\frac{\partial^2 p_1(x)}{\partial^2 x^2}-\frac{p_1(x)}{\tau_{p0}}-j\omega p_1(x)\right]e^{j\omega t}$$

$$=0$$

$$D_p\frac{\partial^2 p_1(x)}{\partial^2 x^2}-\frac{p_1(x)}{\tau_{p0}}-j\omega p_1(x)=0$$

$$\frac{d^2 p_1(x)}{d^2x^2}-\frac{(1+j\omega\tau_{p0})}{L_p^2}p_1(x)=\frac{d^2 p_1(x)}{d^2x^2}-C_p^2 p_1(x)=0$$

$$p_1(x)=K_1e^{-C_px}+K_2e^{C_px}$$

$$B.C.\begin{cases} p_1(x)= & K_1e^{-C_px}\left(p_1(x\to+\infty)=0\right)\\ p_1(0)=K_1= & p_{dc}\left(\dfrac{\hat{V}_1}{V_t}\right)\end{cases}$$

$$J_p=-eD_p\frac{\partial p_n}{\partial x}\Big|_{x=0}=-eD_p\frac{\partial\delta p_n}{\partial x}\Big|_{x=0}$$

$$=-eD_p\frac{\partial\delta p_0(x)}{\partial x}\Big|_{x=0}-eD_p\frac{\partial p_1(x)}{\partial x}\Big|_{x=0}e^{j\omega t}$$

$$J_p=J_{p0}+j_p(t)$$

where $J_{p0}=-eD_p\dfrac{\partial\delta p_0(x)}{\partial x}\Big|_{x=0}=\dfrac{eD_pp_{n0}}{L_p}\left[\exp\left(\dfrac{eV_a}{kT}\right)-1\right]$

$$j_p(t)=\hat{J}_pe^{j\omega t}=-eD_p\frac{\partial p_1(x)}{\partial x}\Big|_{x=0}e^{j\omega t}$$

$$\hat{J}_p=-eD_p(-C_p)\left[p_{dc}\left(\frac{\hat{V}_1}{V_t}\right)\right]e^{-C_px}\Big|_{x=0}$$

Small-Signal Admittance(Mathematical Analysis)

$$\hat{J}_p = -eD_p\left(-C_p\right)\left[p_{dc}\left(\frac{\hat{V}_1}{V_t}\right)\right]e^{-C_p x}\Big|_{x=0}$$

$$\hat{I}_p = A\hat{J}_p = eAD_p C_p p_{dc}\left(\frac{\hat{V}_1}{V_t}\right) = \frac{eAD_p p_{dc}}{L_p}\sqrt{1+j\omega\tau_{p0}}\left(\frac{\hat{V}_1}{V_t}\right)$$

$$= I_{p0}\sqrt{1+j\omega\tau_{p0}}\left(\frac{\hat{V}_1}{V_t}\right)$$

Going through the same type of analysis

for the minority carrier electrons in the p region

$$\hat{I}_n = I_{n0}\sqrt{1+j\omega\tau_{n0}}\left(\frac{\hat{V}_1}{V_t}\right) \text{ where } I_{n0} = \frac{eAD_n n_{p0}}{L_n}\exp\left(\frac{eV_0}{kT}\right)$$

$$Y = \frac{\hat{I}}{\hat{V}_1} = \frac{\hat{I}_p + \hat{I}_n}{\hat{V}_1}$$

$$= \left(\frac{1}{V_t}\right)\left[I_{p0}\sqrt{1+j\omega\tau_{p0}} + I_{n0}\sqrt{1+j\omega\tau_{n0}}\right]$$

Frequency of the ac signal is not too large:

$$\omega\tau_{p0} \ll 1; \omega\tau_{n0} \ll 1$$

$$\sqrt{1+j\omega\tau_{p0}} \approx 1+\frac{j\omega\tau_{p0}}{2}; \sqrt{1+j\omega\tau_{n0}} \approx 1+\frac{j\omega\tau_{n0}}{2}$$

$$Y = \left(\frac{1}{V_t}\right)\left[I_{p0}\left(1+\frac{j\omega\tau_{p0}}{2}\right) + I_{n0}\left(1+\frac{j\omega\tau_{n0}}{2}\right)\right]$$

$$= \left(\frac{1}{V_t}\right)\left(I_{p0}+I_{n0}\right) + j\omega\left[\left(\frac{1}{2V_t}\right)\left(I_{p0}\tau_{p0}+I_{n0}\tau_{n0}\right)\right]$$

Small-Signal Admittance(Mathematical Analysis)

$$Y = \left(\frac{1}{V_t}\right)\left(I_{p0} + I_{n0}\right) + j\omega\left[\left(\frac{1}{2V_t}\right)\left(I_{p0}\tau_{p0} + I_{n0}\tau_{n0}\right)\right]$$

$$= g_d + j\omega C_d$$

g_d is called the diffusion conductance and given by

$$g_d = \left(\frac{1}{V_t}\right)\left(I_{p0} + I_{n0}\right) = \frac{I_{DQ}}{V_t}$$

$$\boxed{C_d = \left(\frac{1}{2V_t}\right)\left(I_{p0}\tau_{p0} + I_{n0}\tau_{n0}\right)}$$

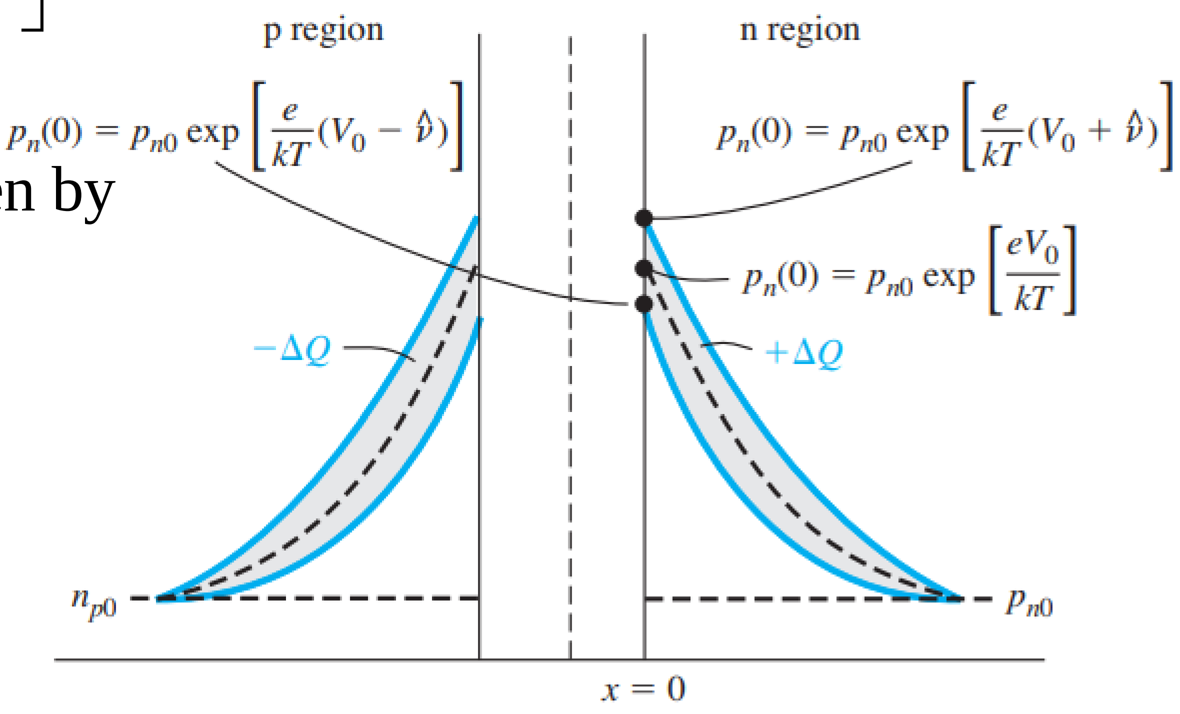

Figure 8.21 | Minority carrier concentration changes with changing forward-bias voltage.

EXAMPLE 8.7

Objective: Calculate the small-signal admittance parameters of a pn junction diode.

This example is intended to give an indication of the magnitude of the diffusion capacitance as compared with the junction capacitance considered in the last chapter. The diffusion resistance will also be calculated. Assume that $N_a \gg N_d$ so that $p_{n0} \gg n_{p0}$. This assumption implies that $I_{p0} \gg I_{n0}$. Let $T = 300$ K, $\tau_{p0} = 10^{-7}$ s, and $I_{p0} = I_{DQ} = 1$ mA.

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The diffusion capacitance, with these assumptions, is given by

$$C_d \approx \left(\frac{1}{2V_t}\right)(I_{p0}\tau_{p0}) = \frac{1}{(2)(0.0259)}(10^{-3})(10^{-7}) = 1.93 \times 10^{-9}\text{ F}$$

The diffusion resistance is

$$r_d = \frac{V_t}{I_{DQ}} = \frac{0.0259\text{ V}}{1\text{ mA}} = 25.9\ \Omega$$