

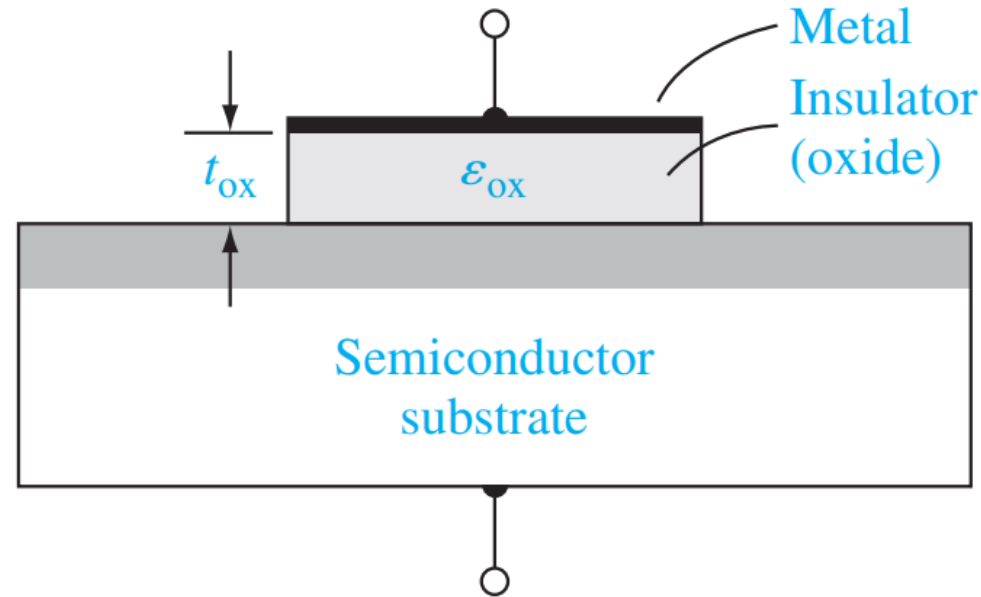
# Chapter 10

## Fundamentals of the Metal-Oxide-Semiconductor Field-Effect Transistor

# The Two-Terminal MOS Structure

- The heart of MOSFET is the MOS capacitor
- The metal may be aluminum (Al) or high-conductivity poly-Si.

$t_{ox}$  Thickness  
 $\epsilon_{ox}$  Permittivity



# The Two-Terminal MOS Structure

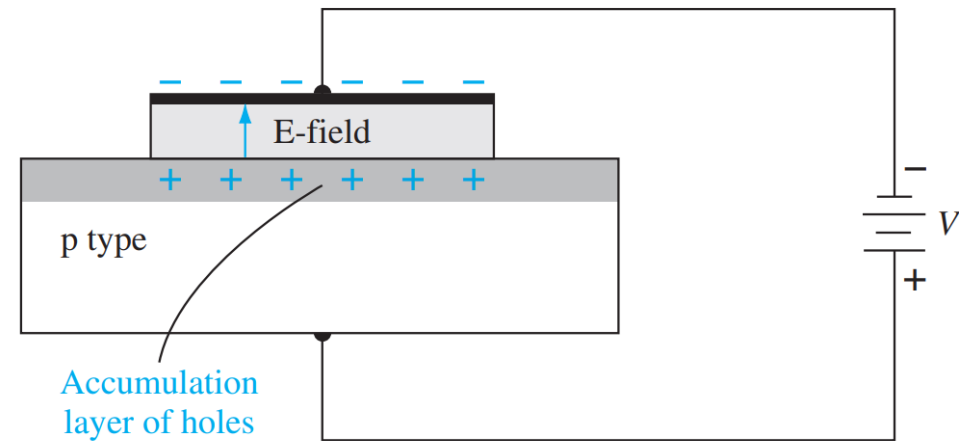
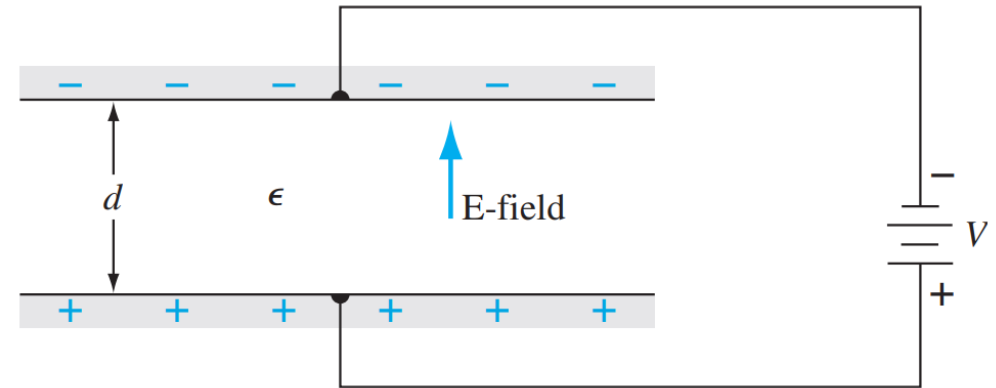
- 電容每單位面積

$$C' = \frac{\epsilon_{ox}}{d}$$

- 電場大小

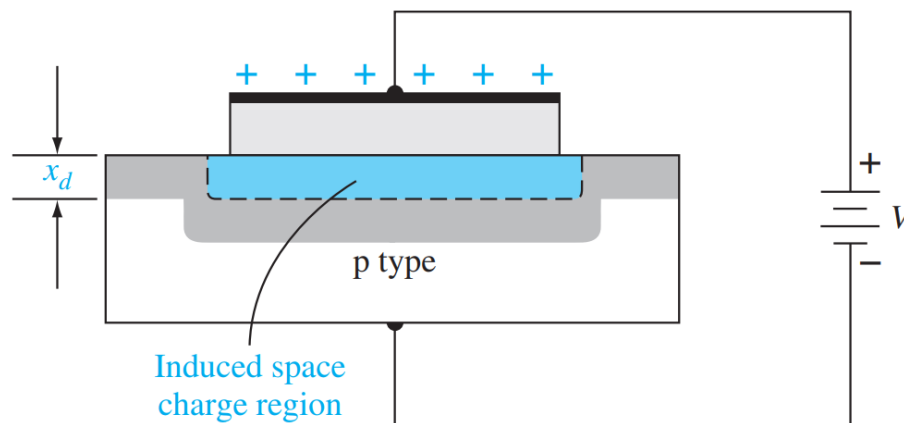
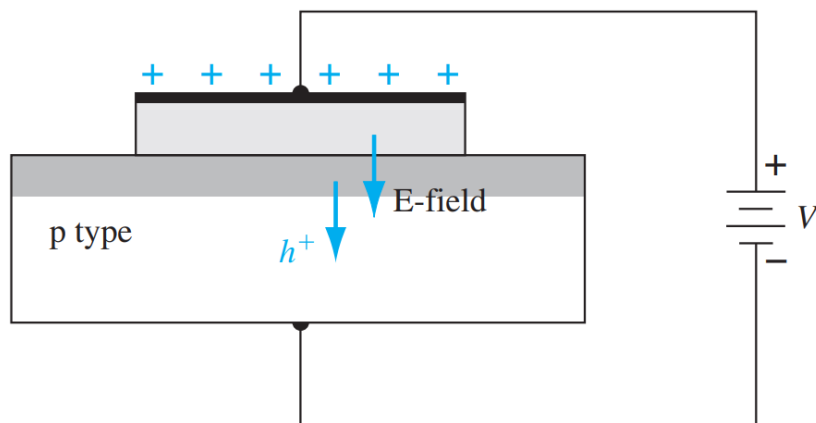
$$E = \frac{V}{d}$$

- $V_{gs} < 0$ ，因施加的外部電場將p型基板內的電洞推向氧化物-半導體介面，而產生一層電洞的累積。(accumulation layer)



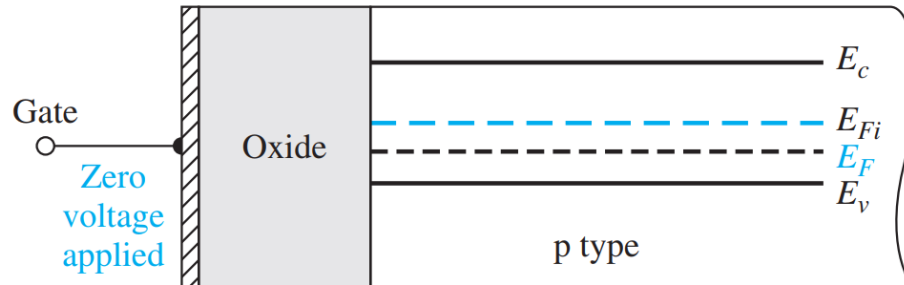
# The Two-Terminal MOS Structure

- $V_{gs} > 0$ ，改變外部電場方向
- 導致氧化物-半導體介面下方產生帶負電的空乏區

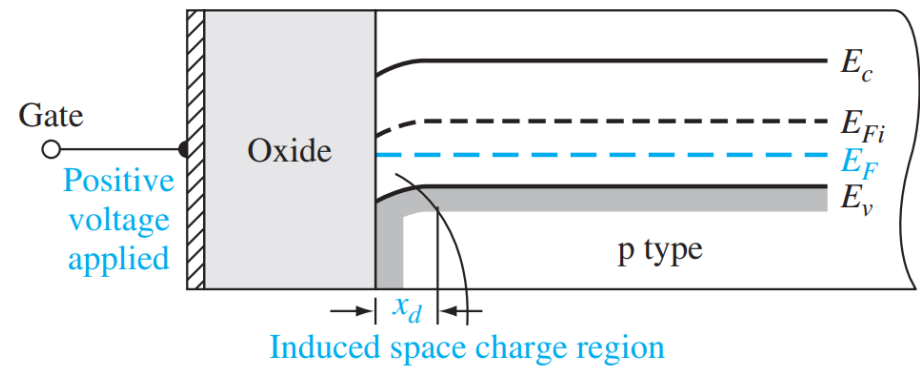


# MOS (p-substrate)

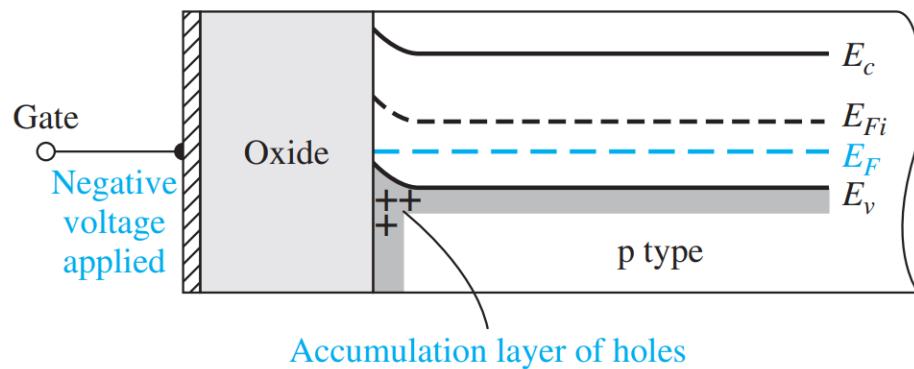
$$V_{gs} = 0$$



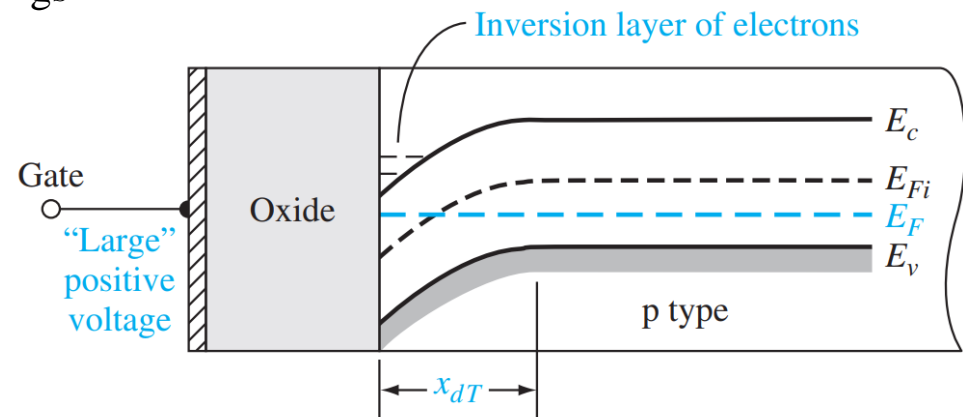
$$V_{gs} > 0$$



$$V_{gs} < 0$$

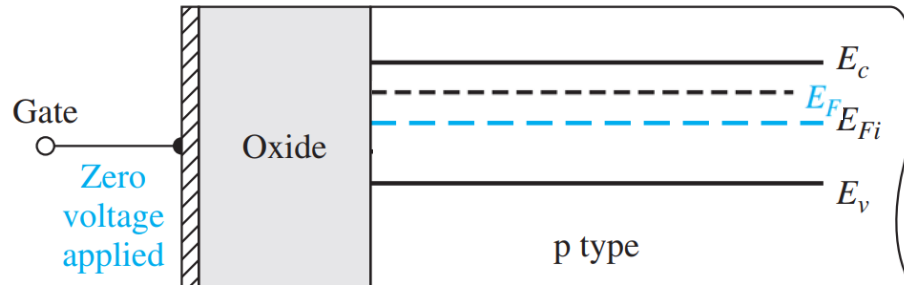


$$V_{gs} > 0$$

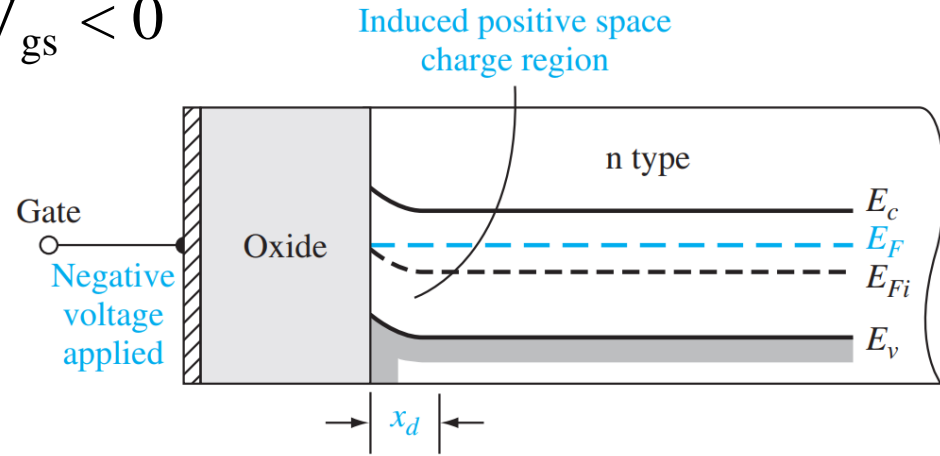


# MOS (n-substrate)

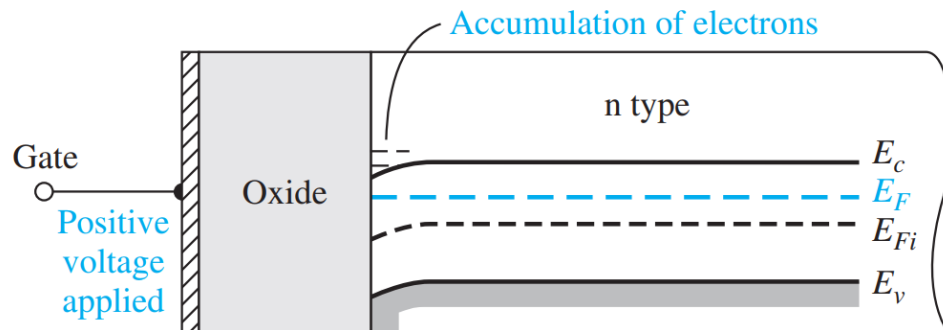
$$V_{gs} = 0$$



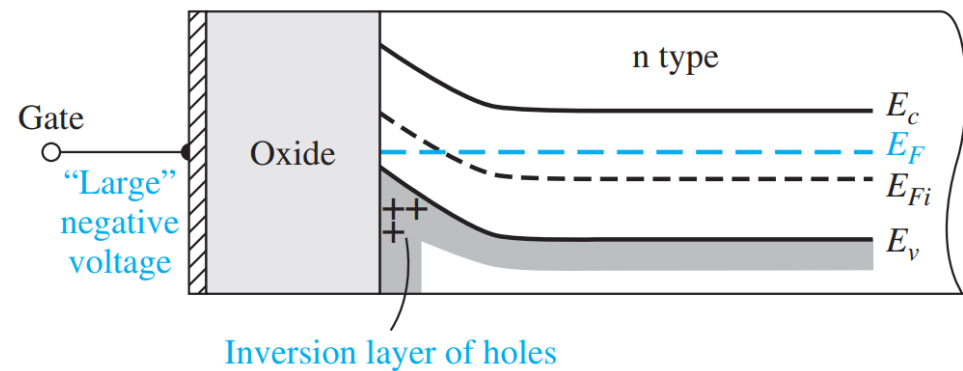
$$V_{gs} < 0$$



$$V_{gs} > 0$$



$$V_{gs} < 0$$



# MOS (p-substrate)

$$V_{gs} > 0$$

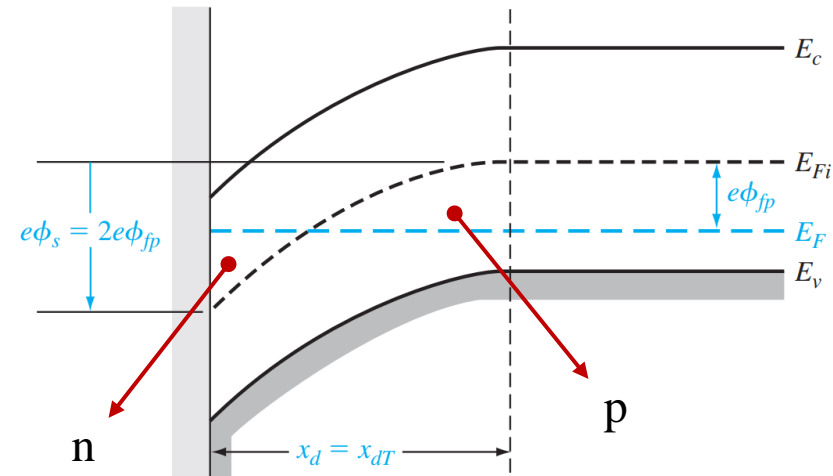
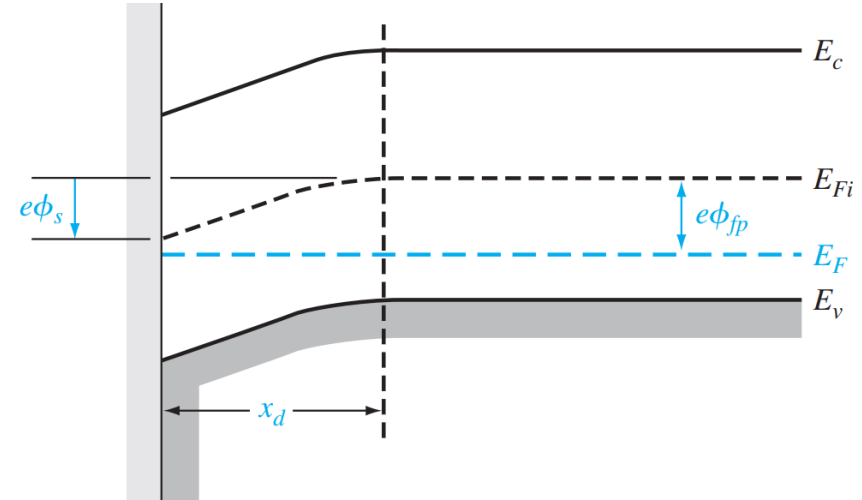
$$\phi_{fp} = kT \ln \left( \frac{N_a}{n_i} \right) \quad \text{Ch. 07}$$

$$x_d = \sqrt{\frac{2\epsilon_s \phi_s}{eN_a}} \quad \text{one-sided junction}$$

Threshold inversion point  $\phi_s = 2\phi_{fp}$

Threshold voltage  $V_{th}$

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fp}}{eN_a}}$$



# MOS (n-substrate)

$$V_{gs} < 0$$

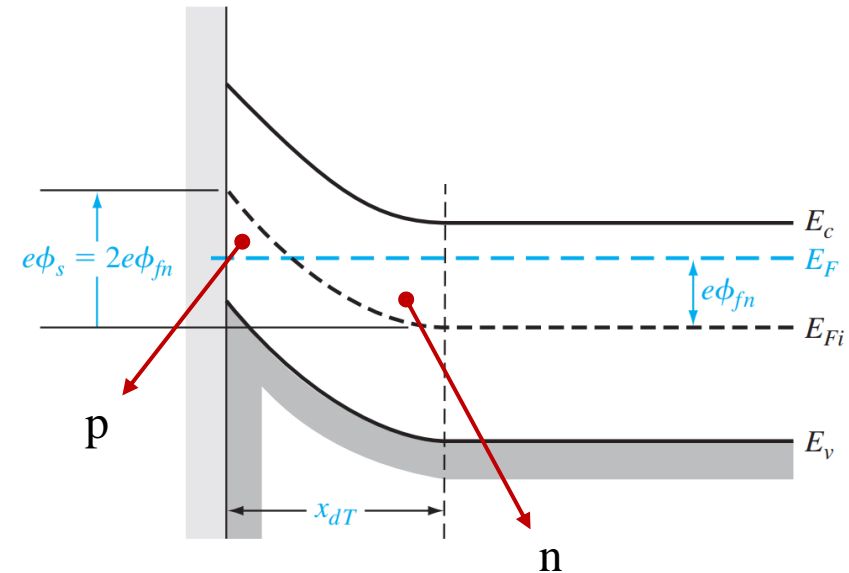
$$\phi_{fn} = kT \ln \left( \frac{N_d}{n_i} \right) \quad \text{From Ch. 7}$$

$$x_d = \sqrt{\frac{2\epsilon_s \phi_s}{eN_d}} \quad \text{one-sided junction}$$

Threshold inversion point  $\phi_s = 2\phi_{fn}$

Threshold voltage  $V_{th}$

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fn}}{eN_d}}$$

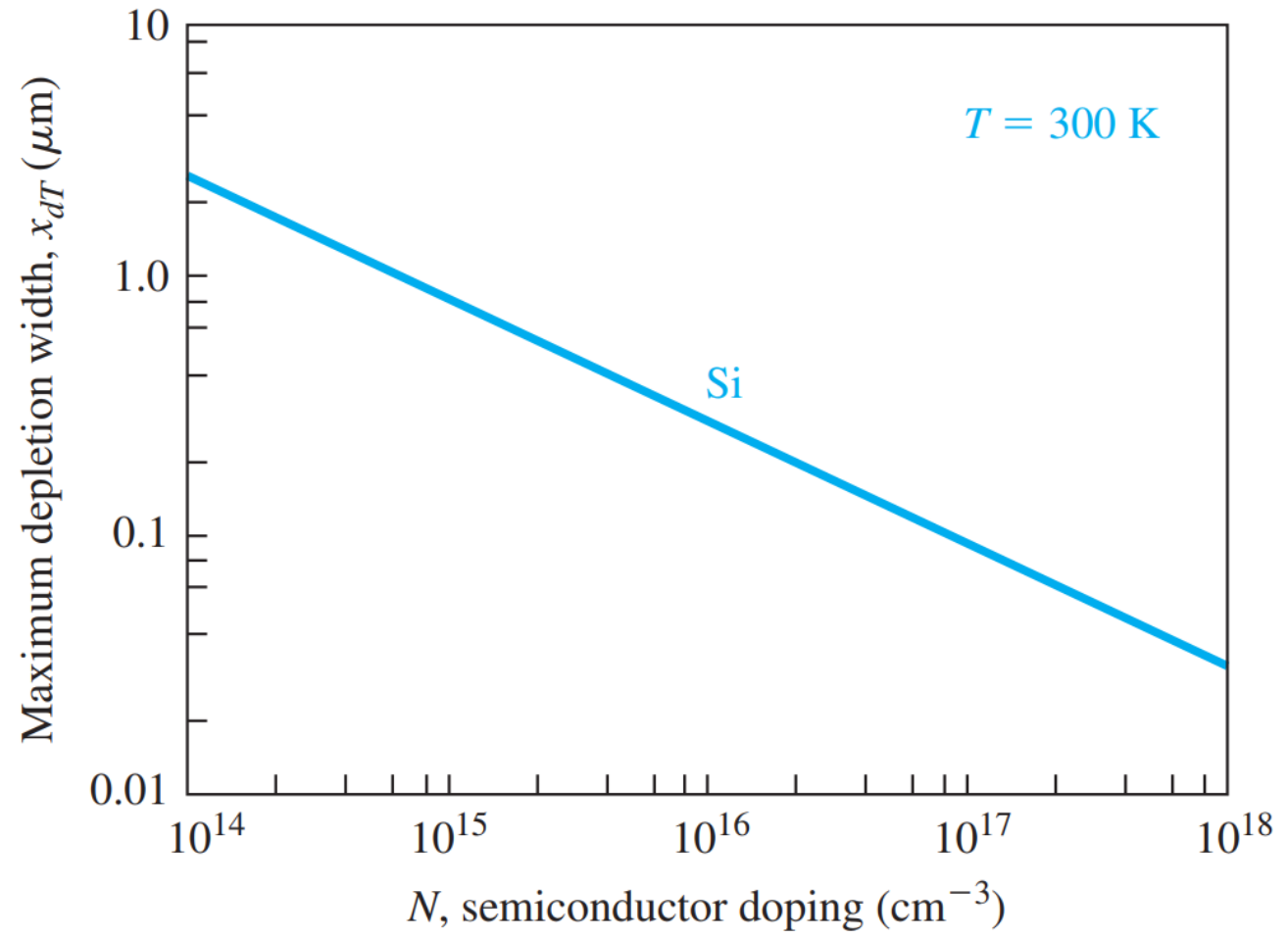




# 空乏區寬度

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fp}}{eN_a}}$$

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fn}}{eN_d}}$$



# Example 10.1

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**Objective:** Calculate the maximum space charge width for a given semiconductor doping concentration.

Consider silicon at  $T = 300$  K doped to  $N_a = 10^{16} \text{ cm}^{-3}$ . The intrinsic carrier concentration is  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ .

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$$\phi_{fp} = V_t \ln \left( \frac{N_a}{n_i} \right) = (0.0259) \ln \left( \frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.3473 \text{ V}$$

$$x_{dT} = \left[ \frac{4\epsilon_s \phi_{fp}}{eN_a} \right]^{1/2} = \left[ \frac{4(11.7)(8.85 \times 10^{-14})(0.3473)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2}$$

$$x_{dT} \cong 0.30 \times 10^{-4} \text{ cm} = 0.30 \text{ } \mu\text{m}$$

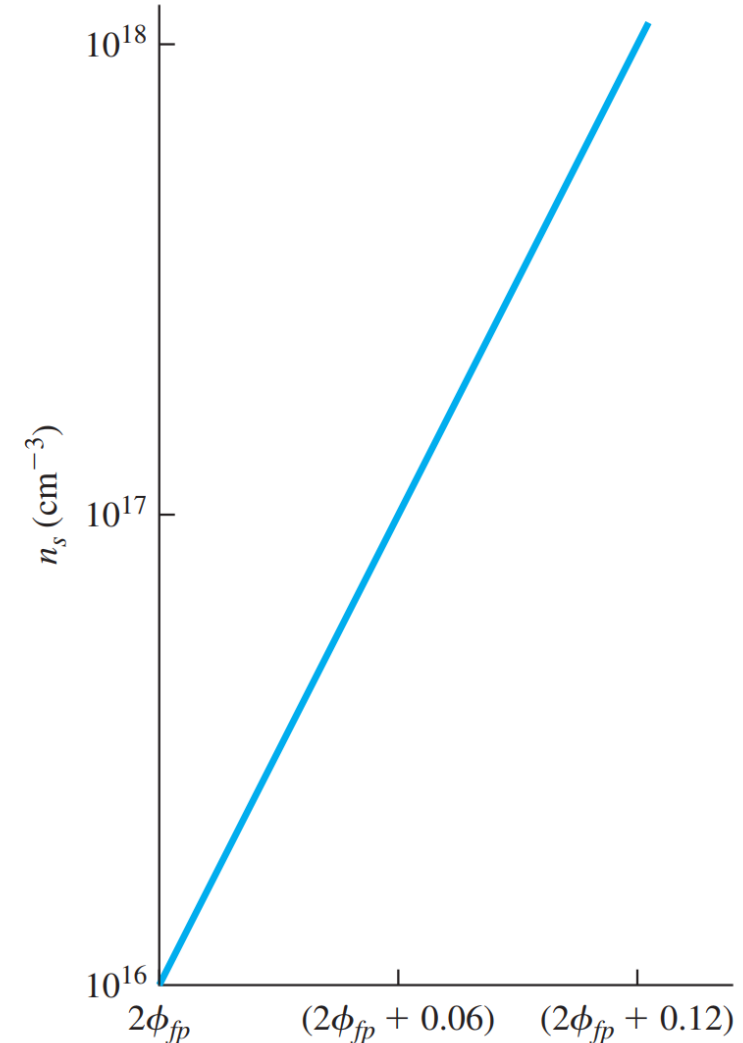
# 空乏區表面電荷密度

From Ch. 4

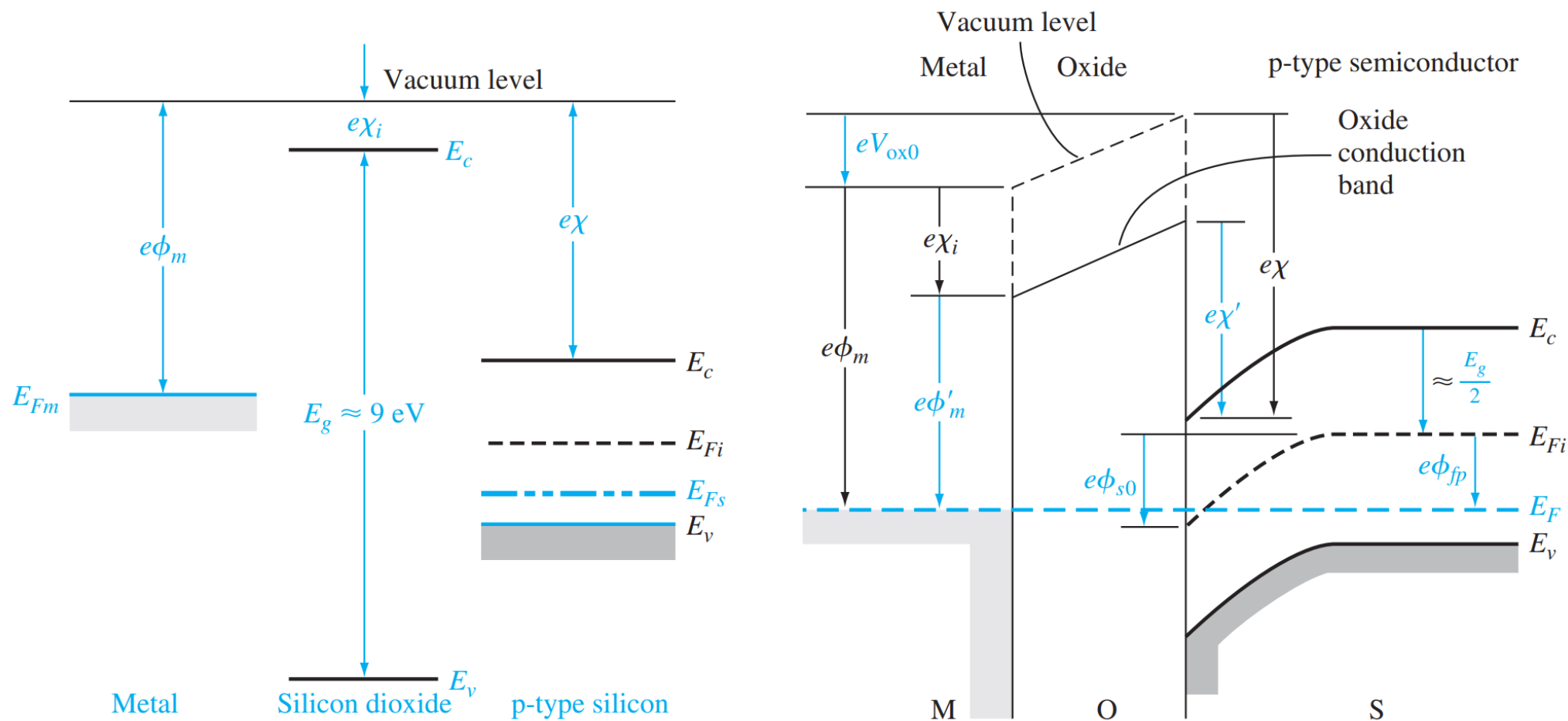
$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) = n_i \exp\left[\frac{e(\phi_{fp} + \Delta\phi_s)}{kT}\right]$$

$$n_s = n_i \exp\left(\frac{e\phi_{fp}}{kT}\right) \exp\left(\frac{e\Delta\phi_s}{kT}\right) = n_{st} \exp\left(\frac{e\Delta\phi_s}{kT}\right)$$

- $n_{st}$ : threshold inversion charge density
- The inversion charge density increases by a factor of 10 with a 60-mV increase in surface potential
- The charge density increases rapidly, means the space charge width essentially reaches a maximum value.



# Work Function Difference



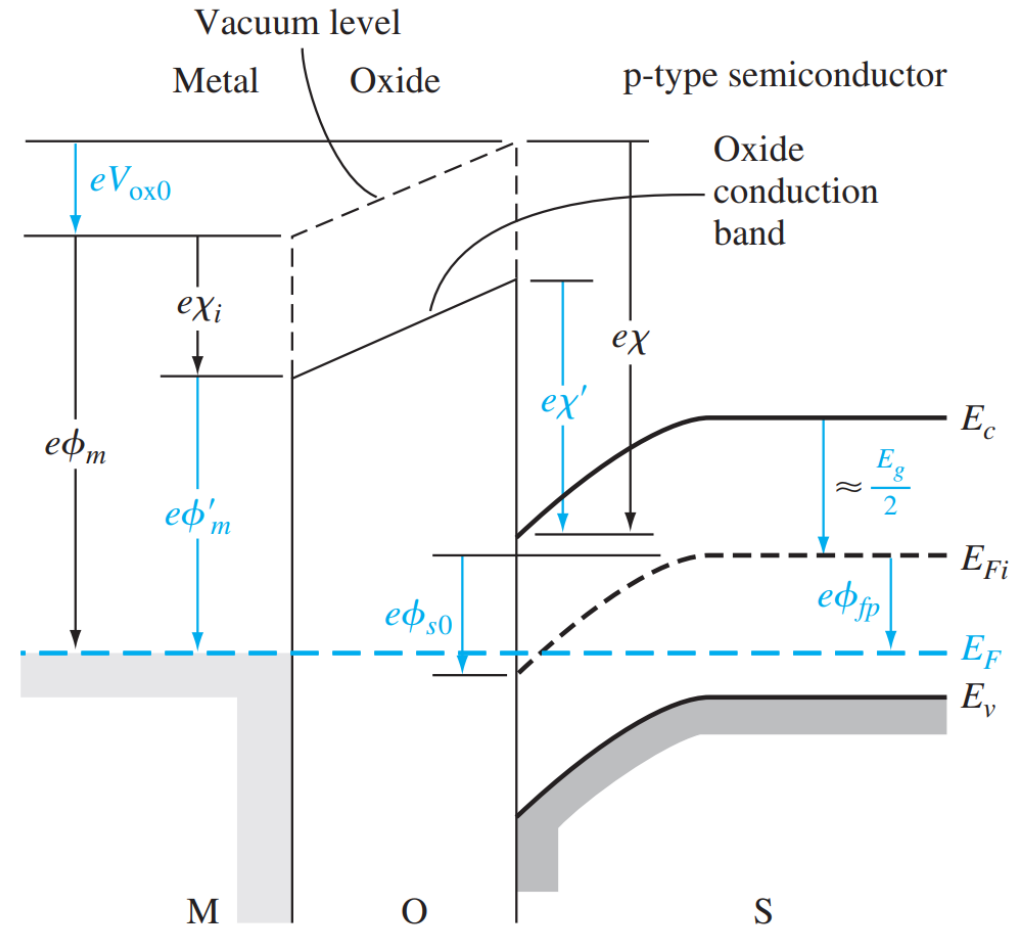
# Work Function Difference

$$e\phi'_m + eV_{ox0} = e\chi' + \frac{E_g}{2} - (e\phi_{s0} - e\phi_{fp})$$

$$V_{ox0} + \phi_{s0} = - \left[ \phi'_m - \left( \chi' + \frac{E_g}{2e} + \phi_{fp} \right) \right]$$

Metal-semiconductor work function difference

$$\phi_{ms} = \phi'_m - \left( \chi' + \frac{E_g}{2e} + \phi_{fp} \right) \quad \text{p-substrate}$$



# Example 10.2

**Objective:** Determine the metal–semiconductor work function difference,  $\phi_{ms}$ , for a given MOS system and semiconductor doping.

For an aluminum–silicon dioxide junction,  $\phi'_m = 3.20$  V and, for a silicon–silicon dioxide junction,  $\chi' = 3.25$  V. We may assume that  $E_g = 1.12$  V. Let the p-type doping be  $N_a = 10^{15} \text{ cm}^{-3}$ .

## Example 10.2

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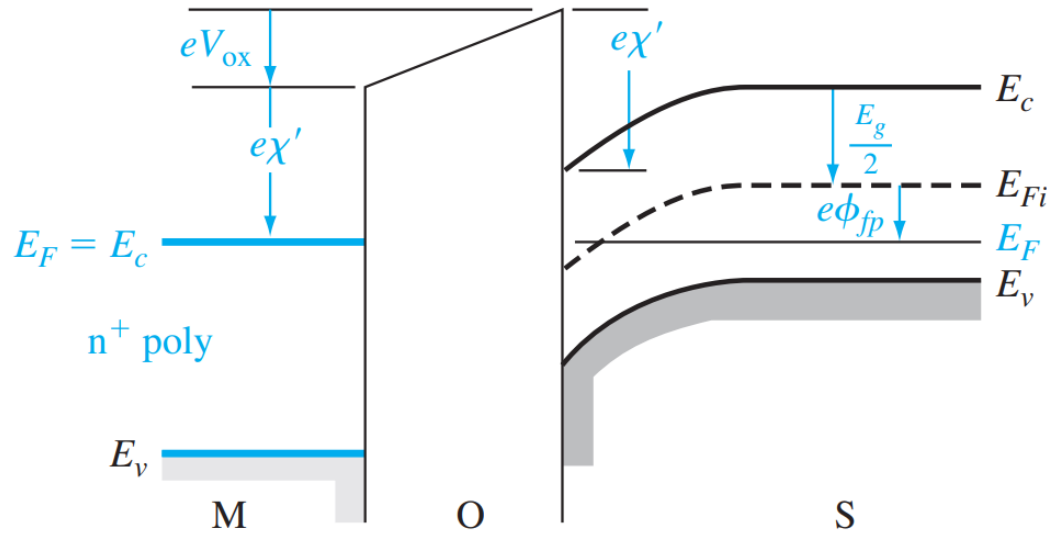
$$\phi_{fp} = V_t \ln \left( \frac{N_a}{n_j} \right) = (0.0259) \ln \left( \frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.288 \text{ V}$$

$$\phi_{ms} = \phi'_m - \left( \chi' + \frac{E_g}{2e} + \phi_{fp} \right) = 3.20 - (3.25 + 0.560 + 0.288)$$

$$\phi_{ms} = -0.898 \text{ V}$$



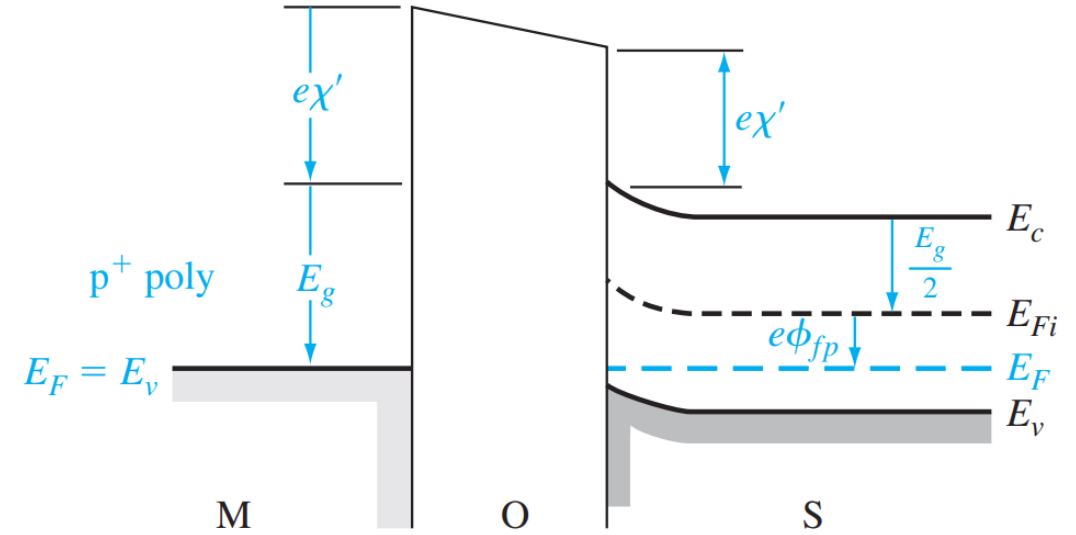
# Polysilicon – Oxide – Semiconductor



$n^+ - O - p$

$$\phi'_m = \chi'$$

$$\phi_{ms} = \phi'_m - \left( \chi' + \frac{E_g}{2e} + \phi_{fp} \right) = - \left( \frac{E_g}{2e} + \phi_{fp} \right)$$

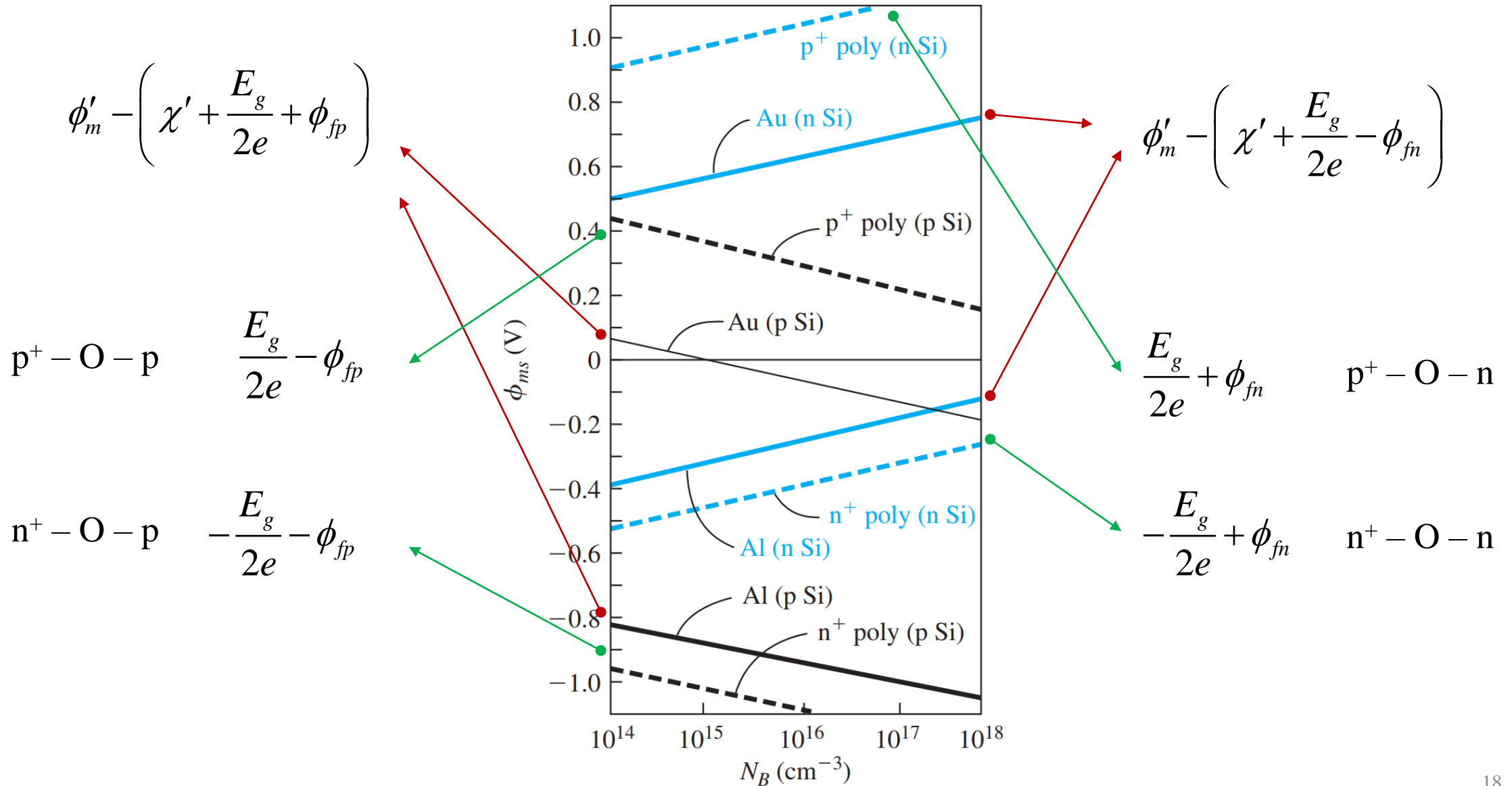


$p^+ - O - p$

$$\phi'_m = \chi' + E_g / e$$

$$\phi_{ms} = \phi'_m - \left( \chi' + \frac{E_g}{2e} + \phi_{fp} \right) = \frac{E_g}{2e} - \phi_{fp}$$

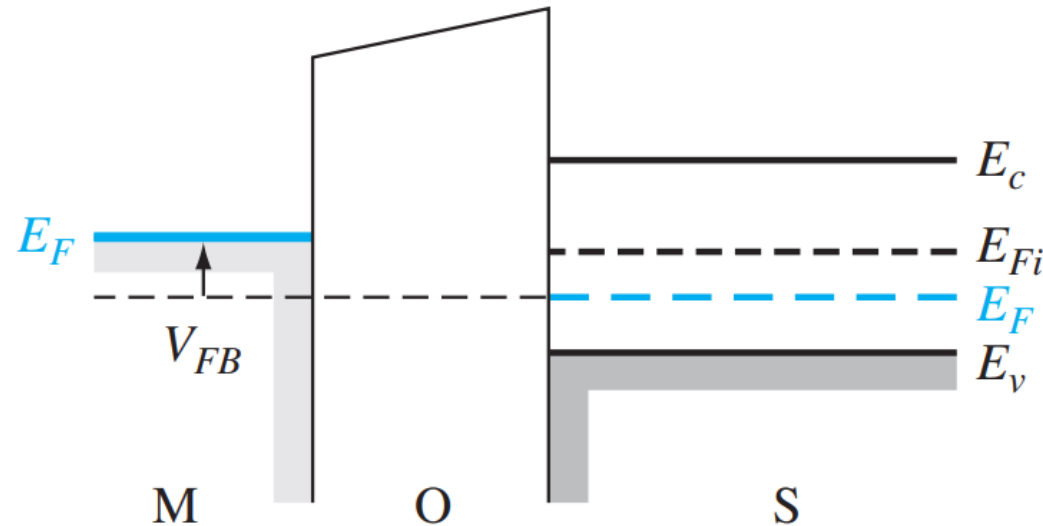
# Work Function Difference vs Doping



# Flat-Band Voltage

平帶電壓：施加的電壓剛好可以讓半導體區域能帶不彎曲，或者說無空乏區。

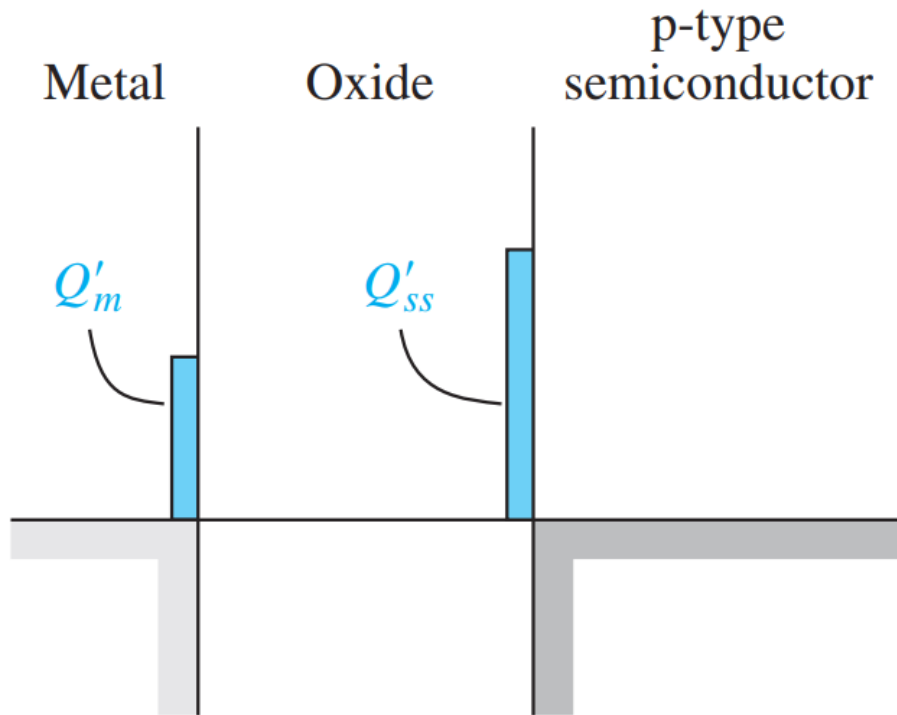
The *flat-band voltage* is defined as the applied gate voltage such that there is no band bending in the semiconductor and, as a result, zero net space charge in this region.



# Fixed Charges

在氧化物和半導體的邊界處，因為懸鍵導致氧化物表面帶正電。

Some positive charge,  $Q'_{ss}$ , has been identified with broken or dangling covalent bonds near oxide-semiconductor interface due to excessive silicon.



$$V_{ox0} + \phi_{s0} = -\phi_{ms}$$

$$V_G = \Delta V_{ox} + \Delta \phi_s = (V_{ox} - V_{ox0}) + (\phi_s - \phi_{s0}) = V_{ox} + \phi_s + \phi_{ms}$$

$$Q'_m + Q'_{ss} = 0 \Rightarrow V_{ox} = -\frac{Q'_{ss}}{C_{ox}}$$

$$V_{FB} = -\frac{Q'_{ss}}{C_{ox}} + \phi_{ms} \quad \text{flat-band for this MOS device.}$$

## Example 10.3

**Objective:** Calculate the flat-band voltage for a MOS capacitor with a p-type semiconductor substrate.

Consider a MOS capacitor with a p-type silicon substrate doped to  $N_a = 10^{16} \text{ cm}^{-3}$ , a silicon dioxide insulator with a thickness of  $t_{ox} = 20 \text{ nm} = 200 \text{ \AA}$ , and an  $n^+$  polysilicon gate. Assume that  $Q'_{ss} = 5 \times 10^{10}$  electronic charges per  $\text{cm}^2$ .

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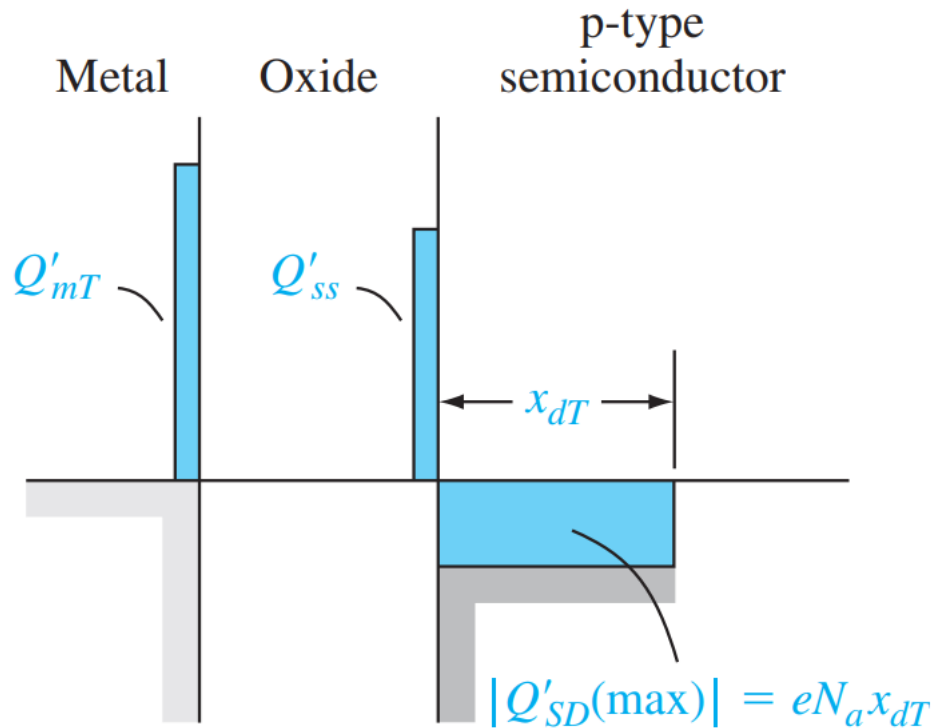
$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}} = 1.726 \times 10^{-7} \text{ F/cm}^2$$

$$Q'_{ss} = (5 \times 10^{10})(1.6 \times 10^{-19}) = 8 \times 10^{-9} \text{ C/cm}^2$$

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} = -1.1 \frac{-8 \times 10^{-9}}{1.726 \times 10^{-7}} = -1.15 \text{ V}$$

# Threshold Voltage

The threshold voltage is defined as the applied gate voltage required to achieve the threshold inversion point.  $\phi_s = 2\phi_{fp}$



$$Q'_{mT} + Q'_{ss} = |Q'_{SD}(\max)| = eN_a x_{dT}$$

$$V_G = V_{ox} + \phi_s + \phi_{ms}$$

$$V_{TN} = V_{oxT} + 2\phi_{fp} + \phi_{ms} = \frac{Q'_{mT}}{C_{ox}} + 2\phi_{fp} + \phi_{ms}$$

$$= \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + 2\phi_{fp} + \phi_{ms}$$

$$= \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

# Example 10.4

**Objective:** Calculate the threshold voltage of a MOS system using an aluminum gate.

Consider a p-type silicon substrate at  $T = 300$  K doped to  $N_a = 10^{15} \text{ cm}^{-3}$ . Let  $Q'_{ss} = 10^{10} \text{ cm}^{-2}$ ,  $t_{ox} = 12 \text{ nm} = 120 \text{ \AA}$ , and assume the oxide is silicon dioxide.



# Example 10.4

**Objective:** Calculate the threshold voltage of a MOS system using an aluminum gate.

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$$\phi_{fp} = V_t \ln \left( \frac{N_a}{n_i} \right) = (0.0259) \ln \left( \frac{10^{15}}{1.5 \times 10^{10}} \right) = 0.2877 \text{ V}$$

$$x_{dT} = \left\{ \frac{4\epsilon_s \phi_{fp}}{eN_a} \right\}^{1/2} = \left\{ \frac{4(11.7)(8.85 \times 10^{-14})(0.2877)}{(1.6 \times 10^{-19})(10^{15})} \right\}^{1/2} = 8.63 \times 10^{-5} \text{ cm}$$

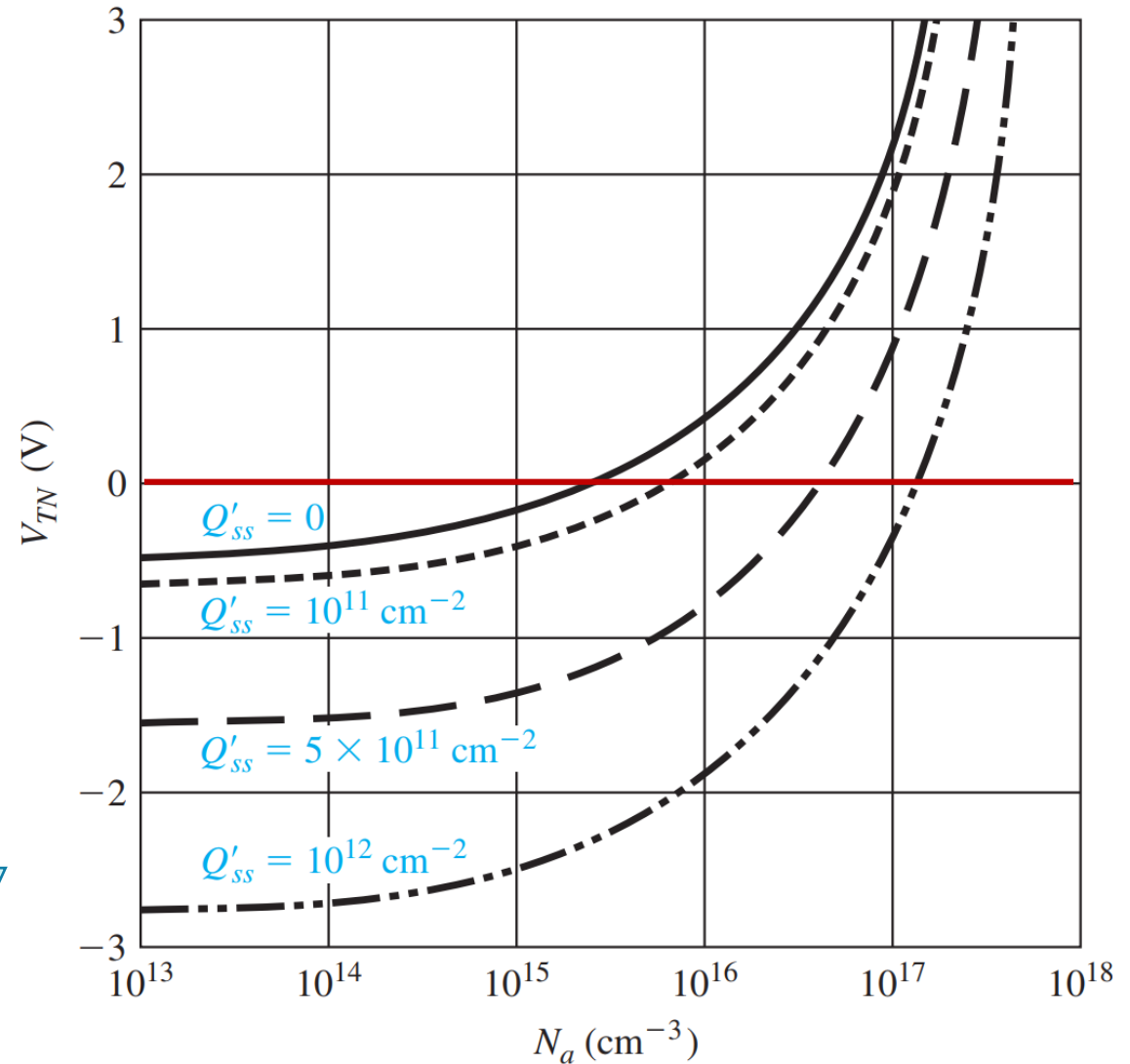
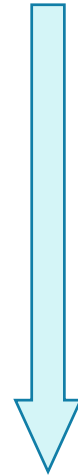
$$|Q'_{SD}(\text{max})| = eN_a x_{dT} = (1.6 \times 10^{-19})(10^{15})(8.63 \times 10^{-5}) = 1.381 \times 10^{-8} \text{ C/cm}^2$$

$$\begin{aligned} V_{TN} &= (|Q'_{SD}(\text{max})| - Q'_{ss}) \left( \frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} + 2\phi_{fp} \\ &= [(1.381 \times 10^{-8}) - (10^{10})(1.6 \times 10^{-19})] \cdot \left[ \frac{120 \times 10^{-8}}{(3.9)(8.85 \times 10^{-14})} \right] \\ &\quad + (-0.88) + 2(0.2877) = -0.262 \text{ V} \end{aligned}$$

# Threshold Voltage

$$V_{TN} = \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + 2\phi_{fp} + \phi_{ms}$$
$$= \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

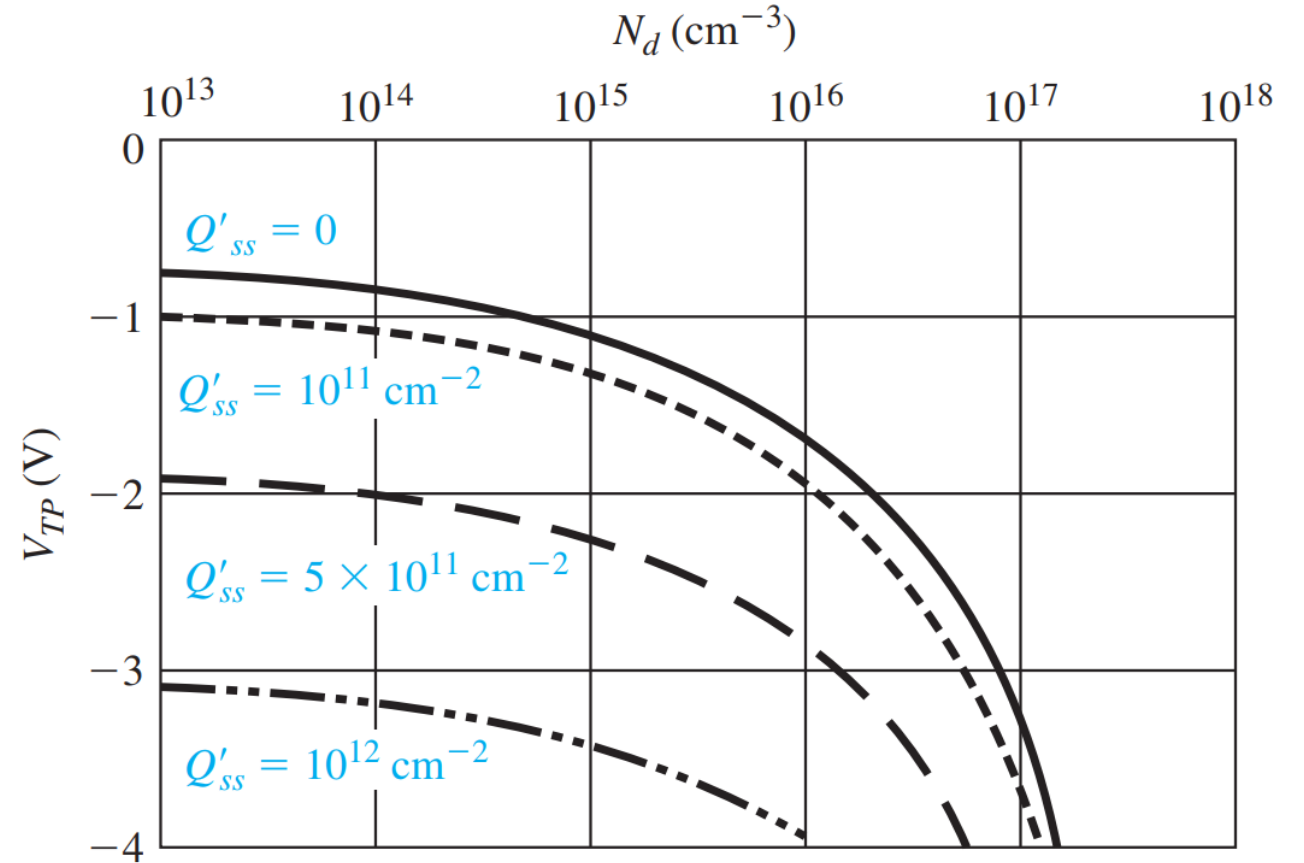
還沒給電壓就已經達到反轉，所以必須做高濃度摻雜才能維持在 enhancement mode。



# Threshold Voltage

$$V_{TP} = \frac{-|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} - 2\phi_{fn} + \phi_n$$

$$= \frac{-|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} - 2\phi_{fn}$$



# Example 10.5

**Objective:** Determine the gate material and design the semiconductor doping concentration to yield a specified threshold voltage.

Consider a MOS device with silicon dioxide and an n-type silicon substrate. The oxide thickness is  $t_{ox} = 12 \text{ nm} = 120 \text{ \AA}$  and the oxide charge is  $Q'_{ss} = 2 \times 10^{10} \text{ cm}^{-2}$ . The threshold voltage is to be approximately  $V_{TP} = -0.3 \text{ V}$ .

# Example 10.5

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$$\phi_{fn} = V_t \ln \left( \frac{N_d}{n_i} \right) = (0.0259) \ln \left( \frac{10^{17}}{1.5 \times 10^{10}} \right) = 0.407 \text{ V}$$

$$x_{dT} = \left( \frac{4\epsilon_s \phi_{fn}}{eN_d} \right)^{1/2} = \left\{ \frac{4(11.7)(8.85 \times 10^{-14})(0.407)}{(1.6 \times 10^{-19})(10^{17})} \right\}^{1/2}$$
$$= 1.026 \times 10^{-5} \text{ cm}$$

$$|Q'_{SD}(\text{max})| = eN_d x_{dT} = (1.6 \times 10^{-19})(10^{17})(1.026 \times 10^{-5})$$
$$= 1.642 \times 10^{-7} \text{ C/cm}^2$$

$$V_{TP} = [-|Q'_{SD}(\text{max})| - Q'_{ss}] \cdot \left( \frac{t_{ox}}{\epsilon_{ox}} \right) + \phi_{ms} - 2\phi_{fn}$$

$$V_{TP} = \frac{[-(1.642 \times 10^{-7}) - (2 \times 10^{10})(1.6 \times 10^{-19})] \cdot (120 \times 10^{-8})}{(3.9)(8.85 \times 10^{-14})} + 1.1 - 2(0.407)$$

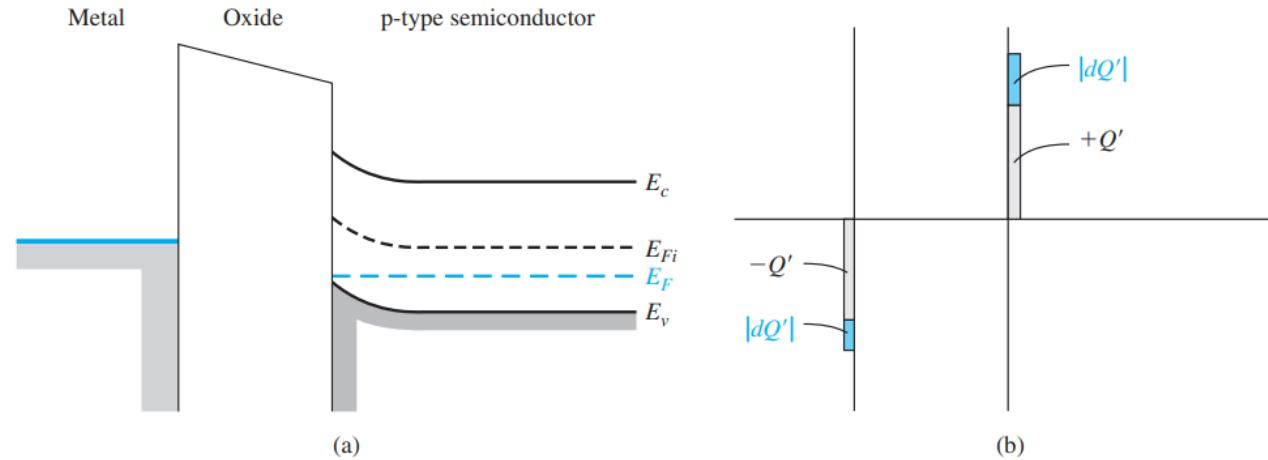
$$V_{TP} = -0.296 \text{ V} \cong -0.3 \text{ V}$$

# Ideal C–V Characteristics

- Initially considered there is zero charge trapped in the oxide and also that there is no charge trapped at the oxide–semiconductor interface

negative voltage is applied to the gate

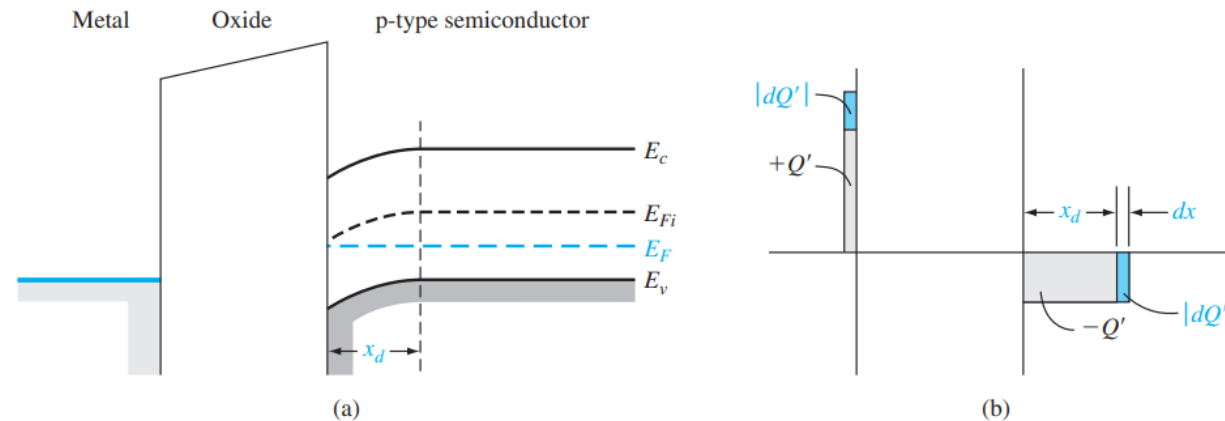
$$C'(acc) = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$



positive voltage is applied to the gate

$$\frac{1}{C'(depl)} = \frac{1}{C_{ox}} + \frac{1}{C'_{SD}} = \frac{1}{\frac{\epsilon_{ox}}{t_{ox}}} + \frac{1}{\frac{\epsilon_s}{x_d}}$$

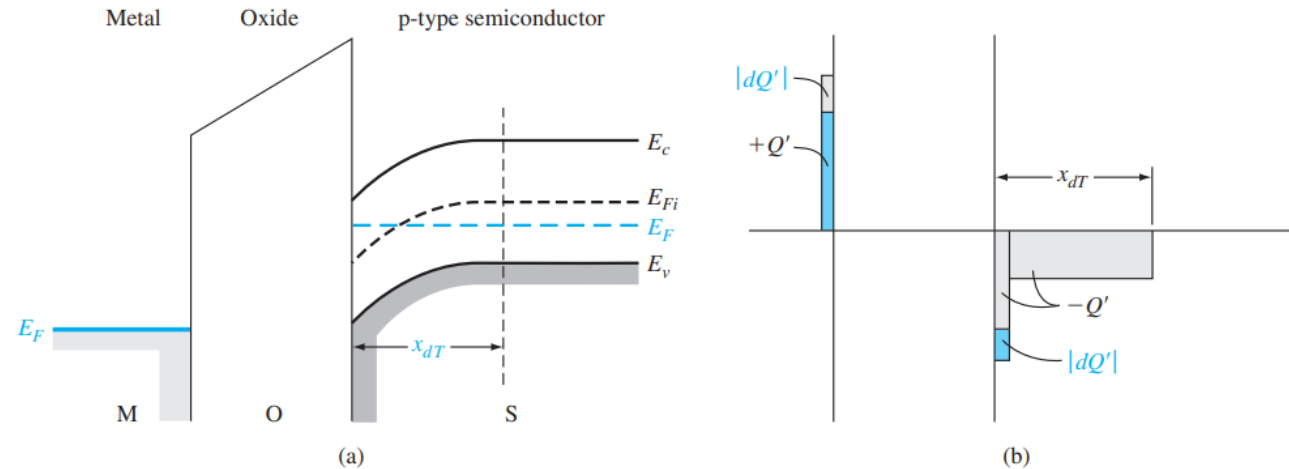
$$C'_{min} = \frac{C_{ox}}{1 + \frac{C_{ox}}{C'_{SD,min}}} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)x_{dT}}$$



# Ideal C–V Characteristics

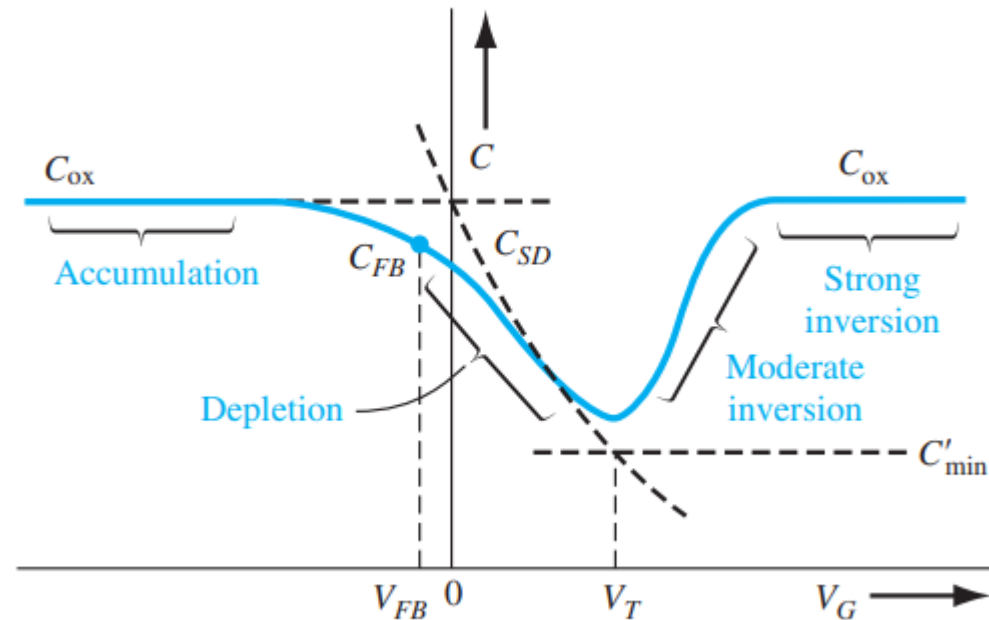
large positive voltage is applied to the gate

$$C'(inv) = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$



- Ideal low-frequency capacitance versus gate voltage of a MOS capacitor with a p-type substrate.

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left( \frac{\epsilon_{ox}}{\epsilon_s} \right) \sqrt{\frac{kT}{e} \left( \frac{\epsilon_s}{eN_a} \right)}}$$



# EXAMPLE 10.6

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**Objective:** Calculate  $C_{ox}$ ,  $C'_{min}$ , and  $C'_{FB}$  for a MOS capacitor.

Consider a p-type silicon substrate at  $T = 300$  K doped to  $N_a = 10^{16} \text{ cm}^{-3}$ .

The oxide is silicon dioxide with a thickness of  $t_{ox} = 18 \text{ nm} = 180 \text{ \AA}$ , and the gate is aluminum.



# EXAMPLE 10.6

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The oxide is silicon dioxide with a thickness of  $t_{ox} = 18 \text{ nm} = 180 \text{ \AA}$ , and the gate is aluminum.

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8}} = 1.9175 \times 10^{-7} \text{ F/cm}^2$$

$$\phi_{fp} = V_t \ln \left( \frac{N_a}{n_i} \right) = (0.0259) \ln \left( \frac{10^{16}}{1.5 \times 10^{10}} \right) = 0.3473 \text{ V}$$

$$x_{dT} = \left\{ \frac{4\epsilon_s \phi_{fp}}{eN_a} \right\}^{1/2} = \left\{ \frac{4(11.7)(8.85 \times 10^{-14})(0.3473)}{(1.6 \times 10^{-19})(10^{16})} \right\}^{1/2}$$

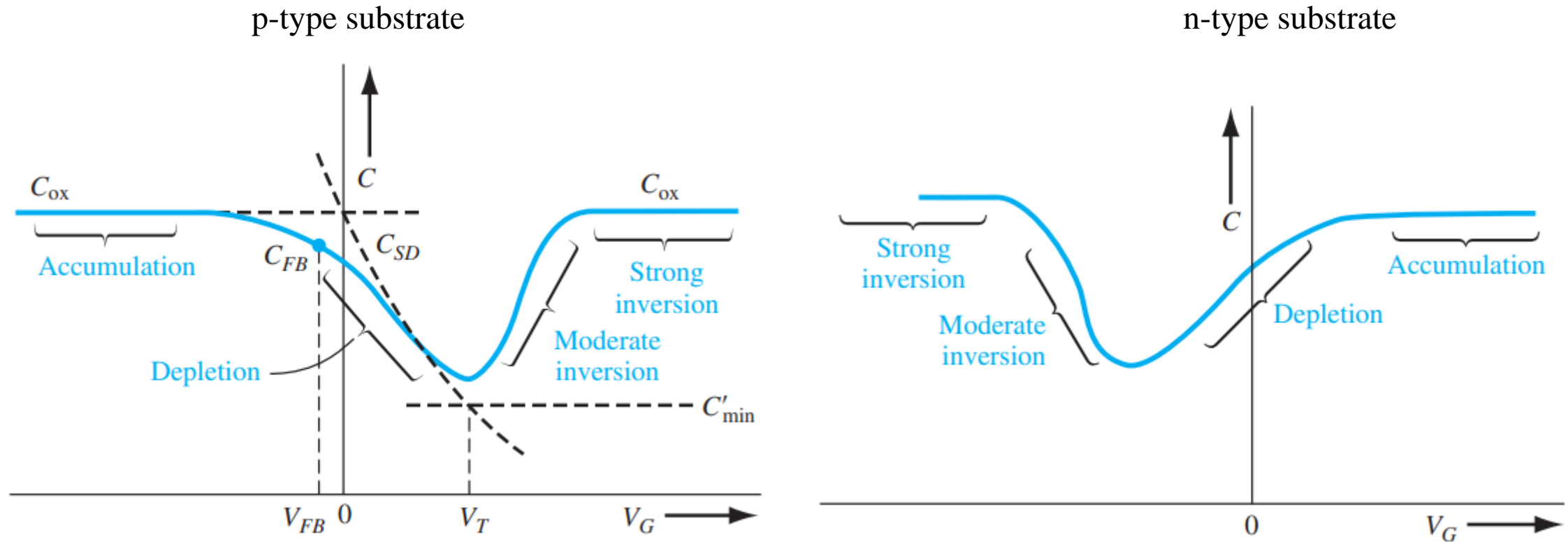
$$\cong 0.30 \times 10^{-4} \text{ cm}$$

$$C'_{min} = \frac{\epsilon_{ox}}{t_{ox} + \left( \frac{\epsilon_{ox}}{\epsilon_s} \right) x_{dT}} = \frac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8} + \left( \frac{3.9}{11.7} \right) (0.30 \times 10^{-4})}$$

$$= 2.925 \times 10^{-8} \text{ F/cm}^2$$

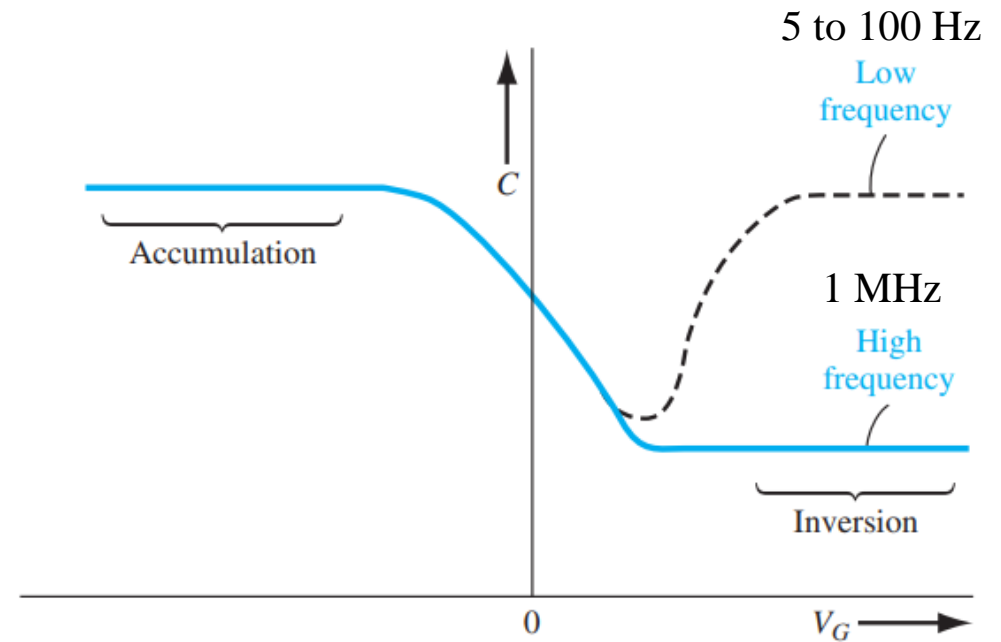
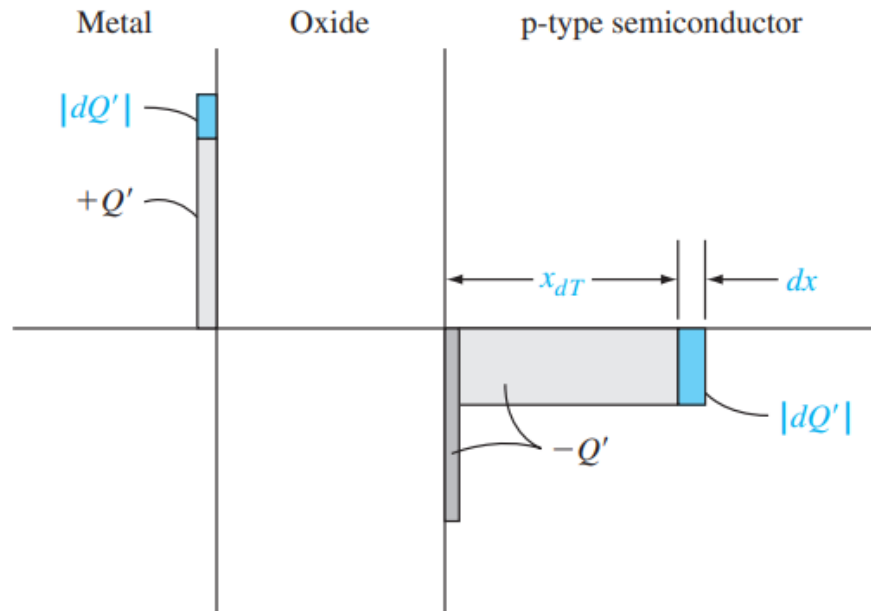
$$\begin{aligned} C'_{FB} &= \frac{\epsilon_{ox}}{t_{ox} + \left( \frac{\epsilon_{ox}}{\epsilon_s} \right) \sqrt{\frac{V_t \epsilon_s}{eN_a}}} \\ &= \frac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8} + \left( \frac{3.9}{11.7} \right) \sqrt{\frac{(0.0259)(11.7)(8.85 \times 10^{-14})}{(1.6 \times 10^{-19})(10^{16})}}} \\ &= 1.091 \times 10^{-7} \text{ F/cm}^2 \end{aligned}$$

# p-type substrate v.s. n-type substrate

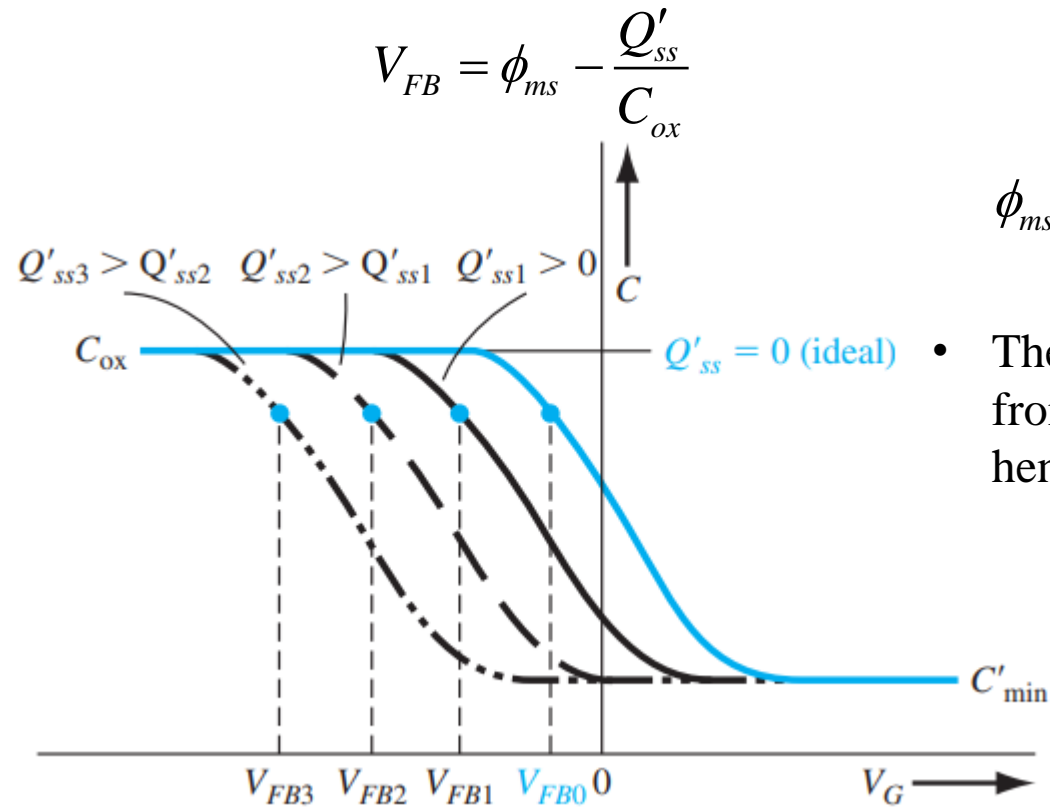


# Frequency Effects

- We must consider the source of electrons that produces a change in the inversion charge density
- First source: y diffusion of minority carrier electrons from the p-type substrate across the space charge region
- Second source: thermal generation of electron–hole pairs within the space charge region
- The electron concentration in the inversion layer, then, cannot change instantaneously



# Fixed Oxide



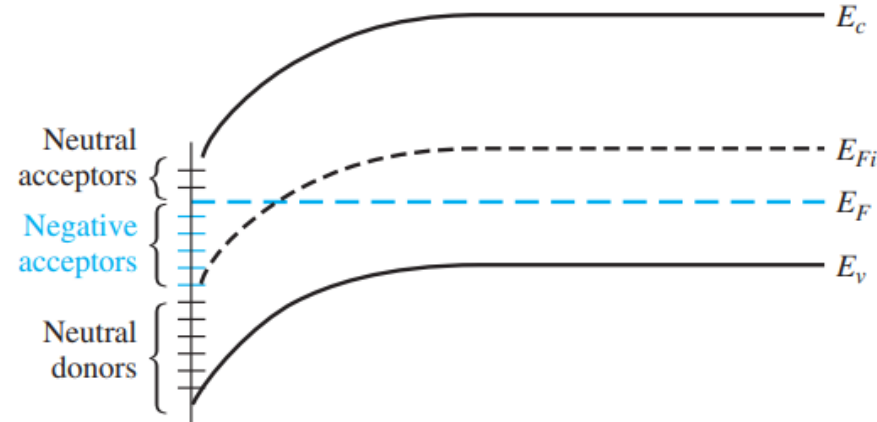
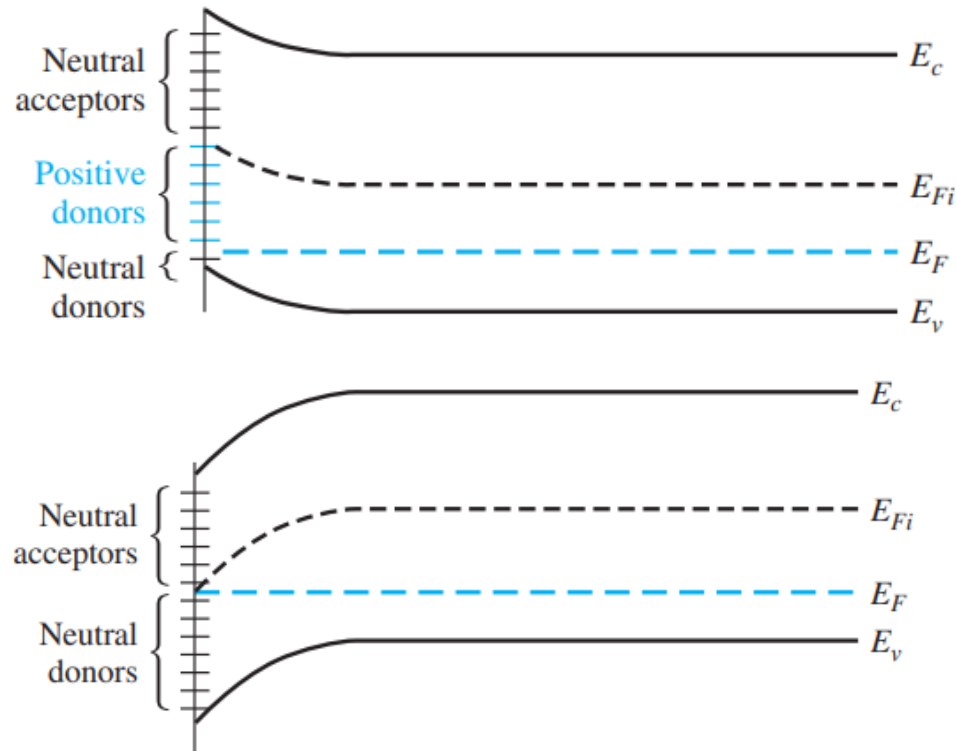
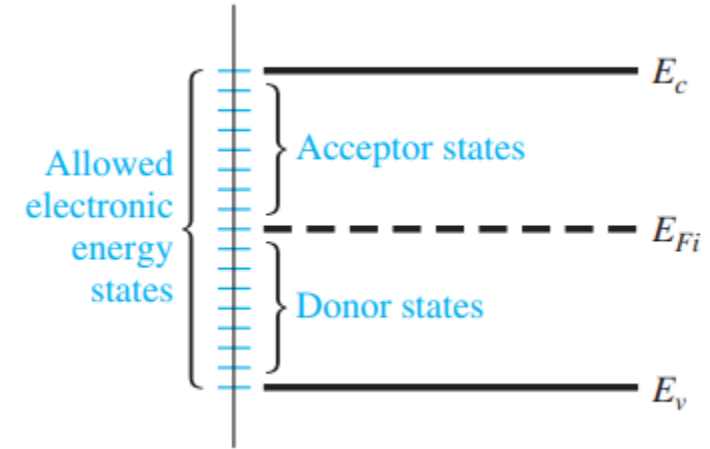
$Q'_{ss}$  : equivalent fixed oxide charge

$\phi_{ms}$  : metal ? semiconductor work function difference

- The experimental value of flat-band voltage can be measured from the C-V curve, and the value of fixed oxide charge can then be determined (早期用在太空辐射)

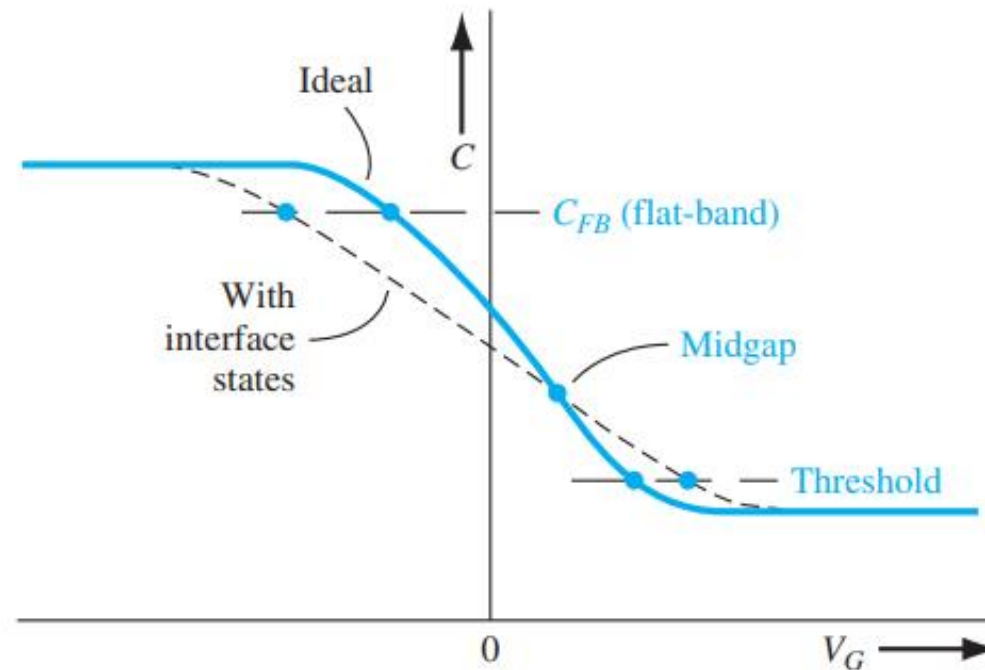
# Interface Charge Effects

- The periodic nature of the semiconductor is abruptly terminated at the interface so that allowed electronic energy levels will exist within the forbidden bandgap (interface states)
- Charge can flow between the semiconductor and interface states, in contrast to the fixed oxide charge

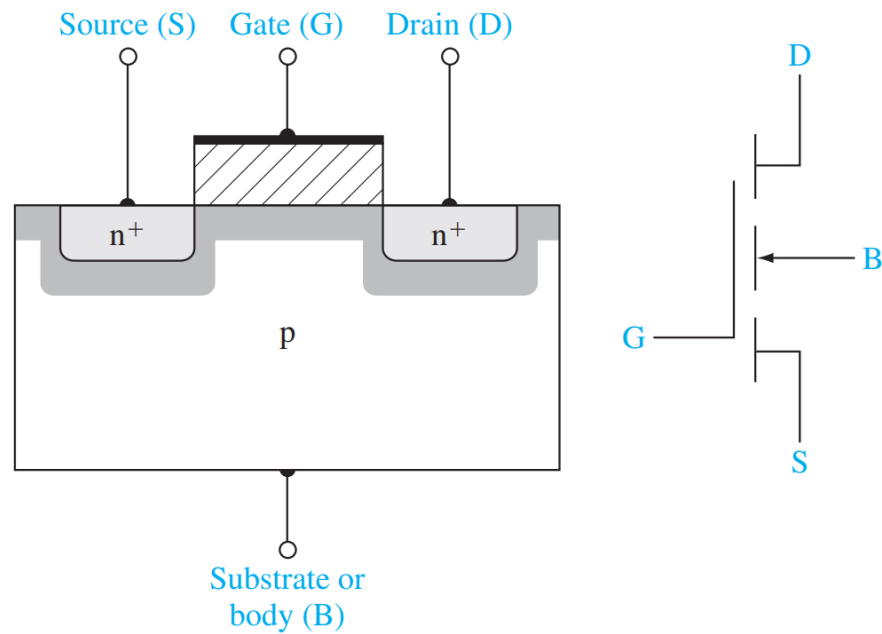


# Smearing out of the curve

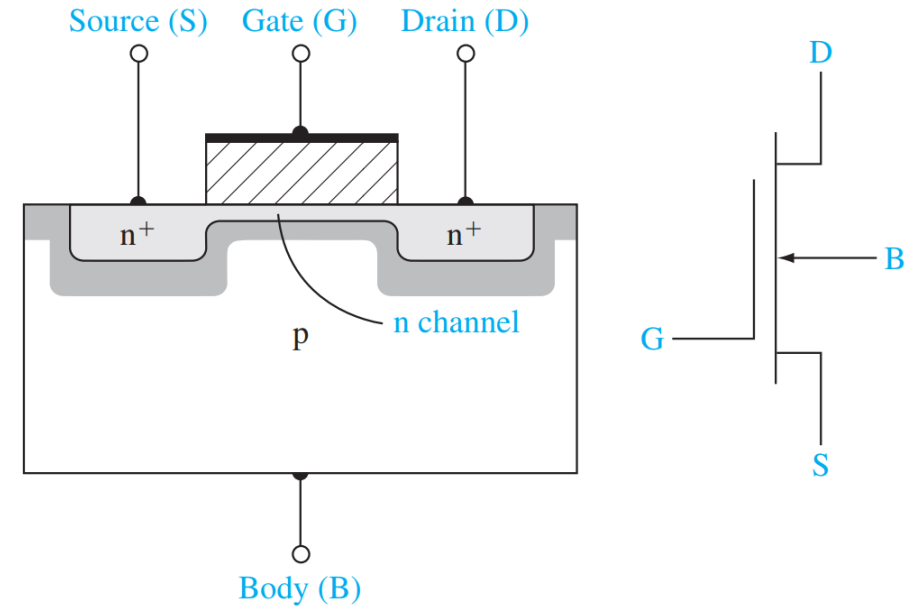
- Smearing out in the experimental curve indicates the presence of interface states and any parallel shift indicates the presence of fixed oxide charge



# The Basic MOSFET Operation

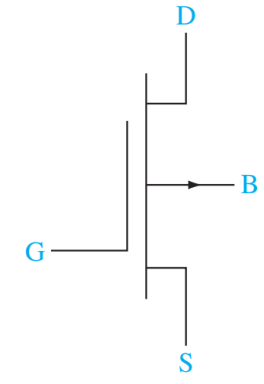
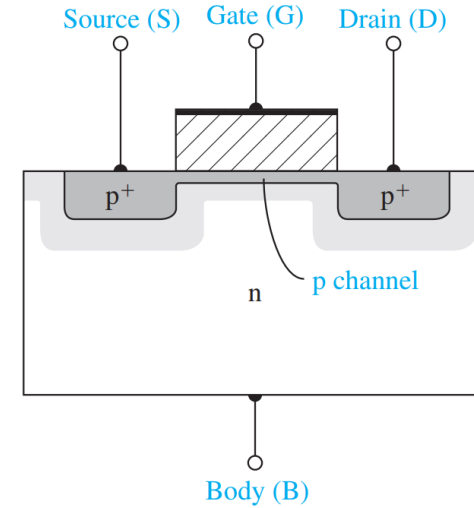
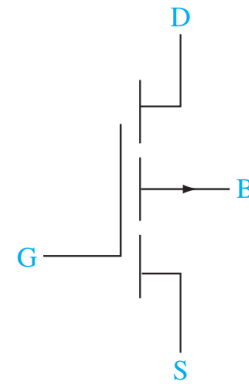
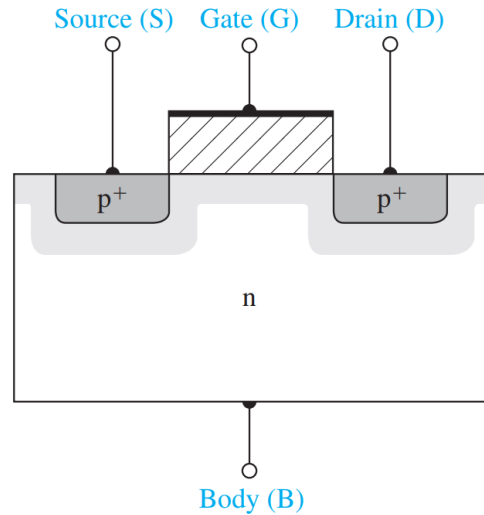


**n-channel enhancement mode:** the substrate is not inverted directly under  $V_G=0$ .



**n-channel depletion mode:** an n-channel region exists under the oxide at  $V_G=0$ .

# The Basic MOSFET Operation

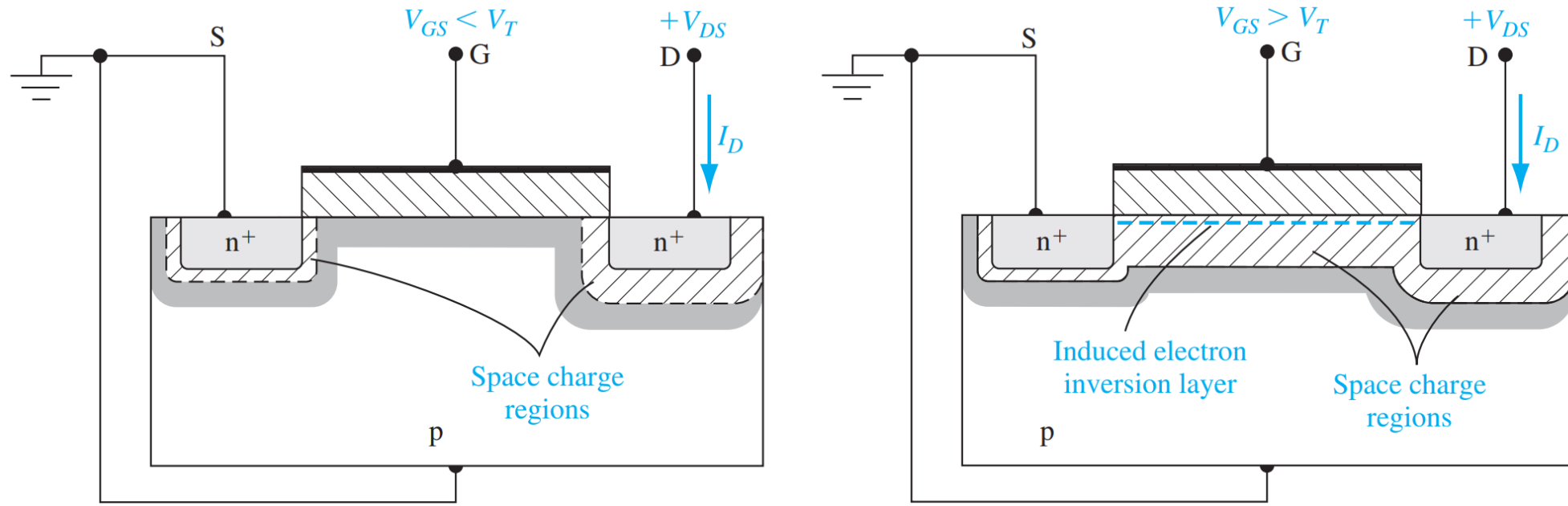


**p-channel enhancement mode:** the substrate is not inverted directly under  $V_G=0$ .

**p-channel depletion mode:** an n-channel region exists under the oxide at  $V_G=0$ .



# Current-Voltage Relationship Concepts



- $V_G < V_T$  , no inversion layer , 無法導通 , 無電流
- $V_G > V_T$  , an inversion layer has been created , 可導通

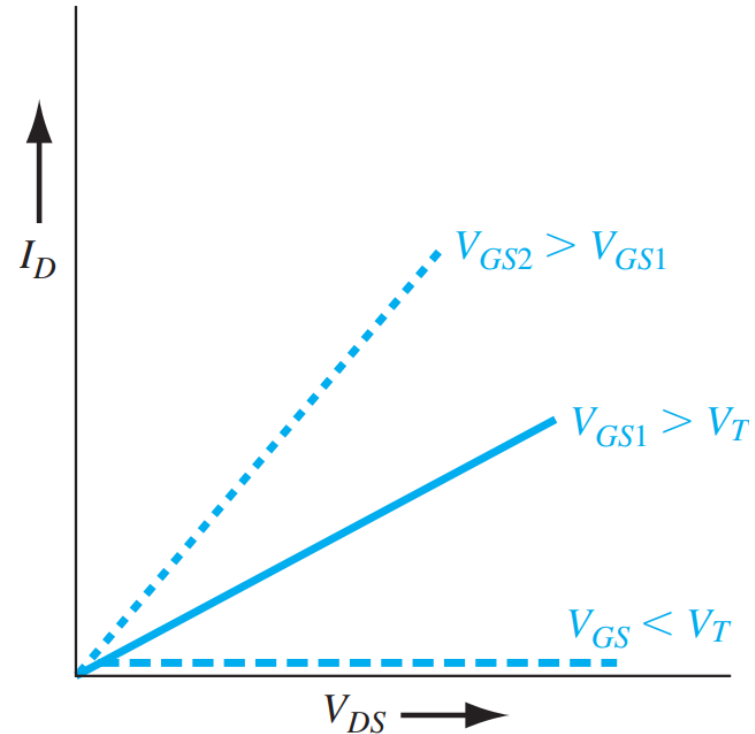
# Current-Voltage Relationship Concepts

For **small**  $V_{DS}$ , the channel region has the characteristic of a resistor

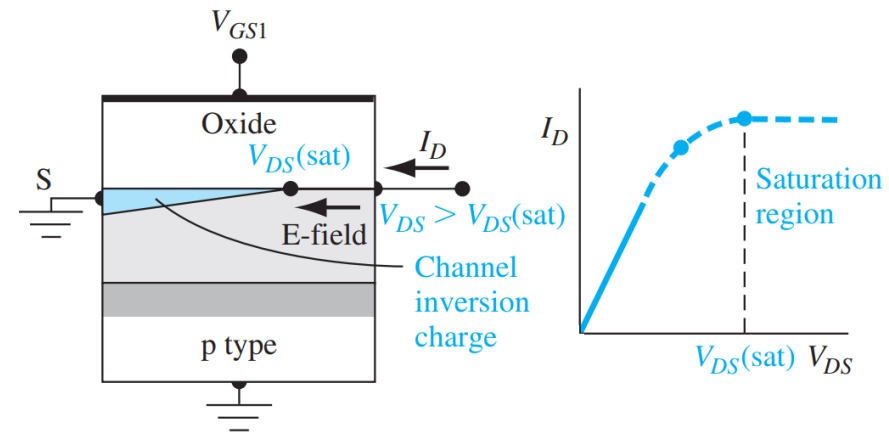
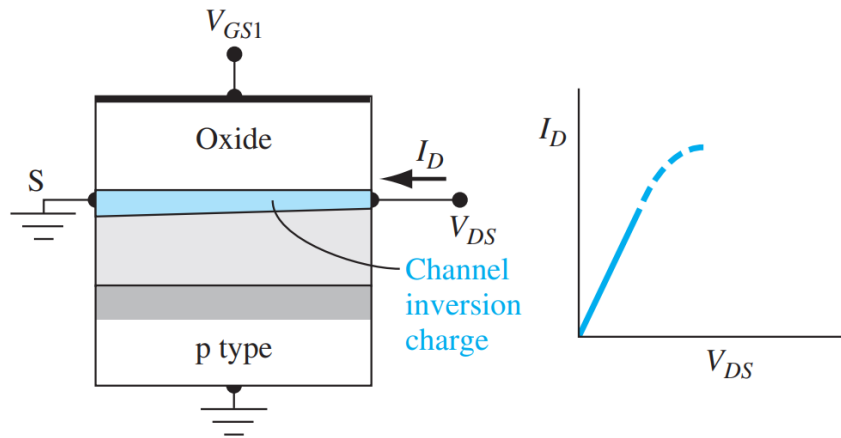
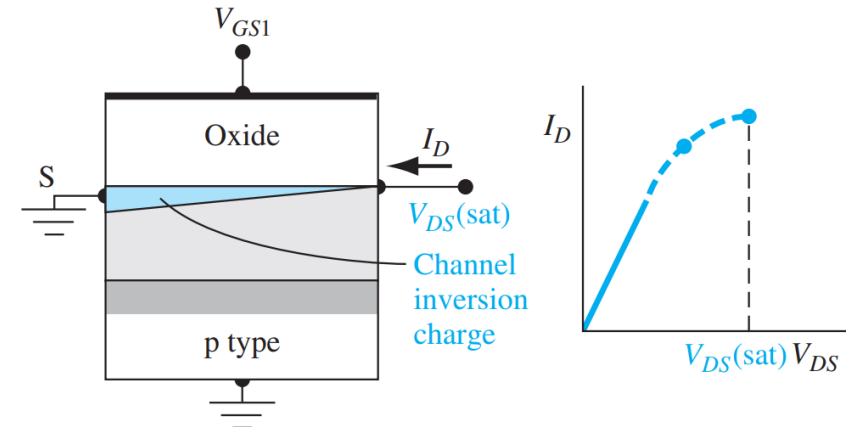
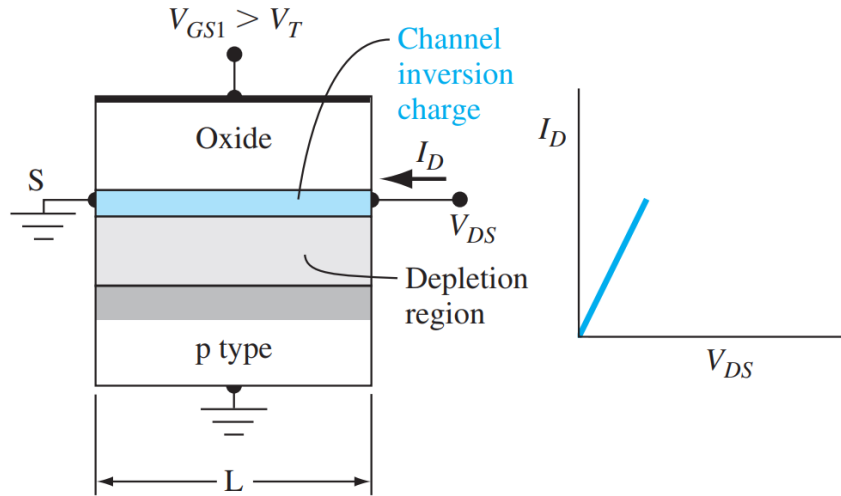
$$I_D = g_v V_{DS}$$



channel conductance

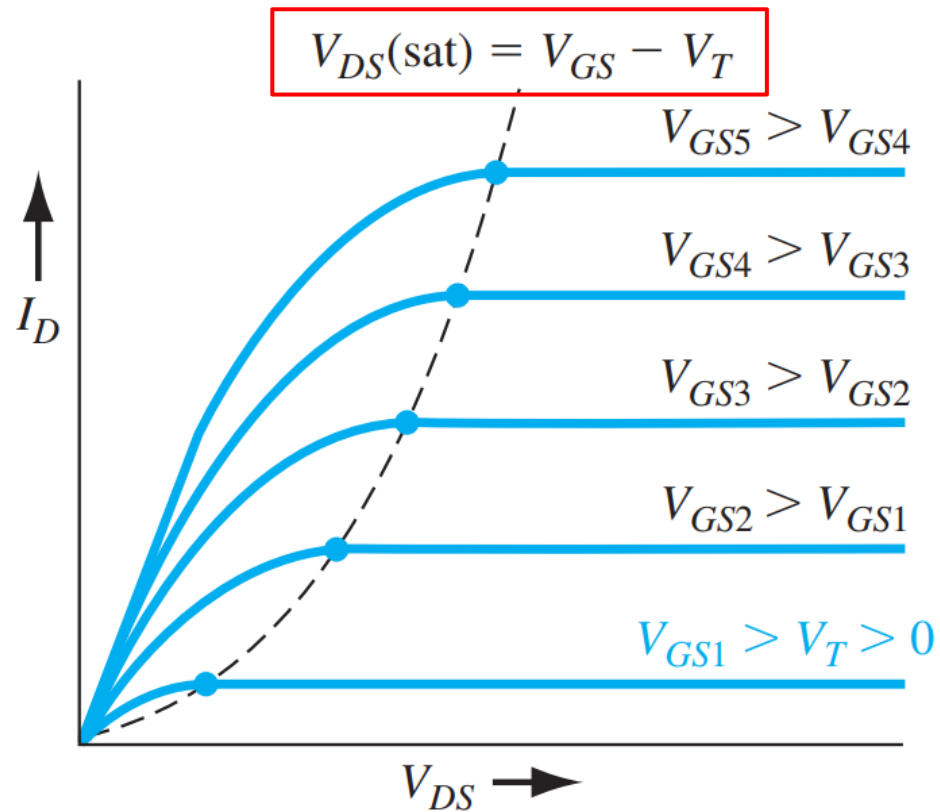


# Current-Voltage Relationship Concepts

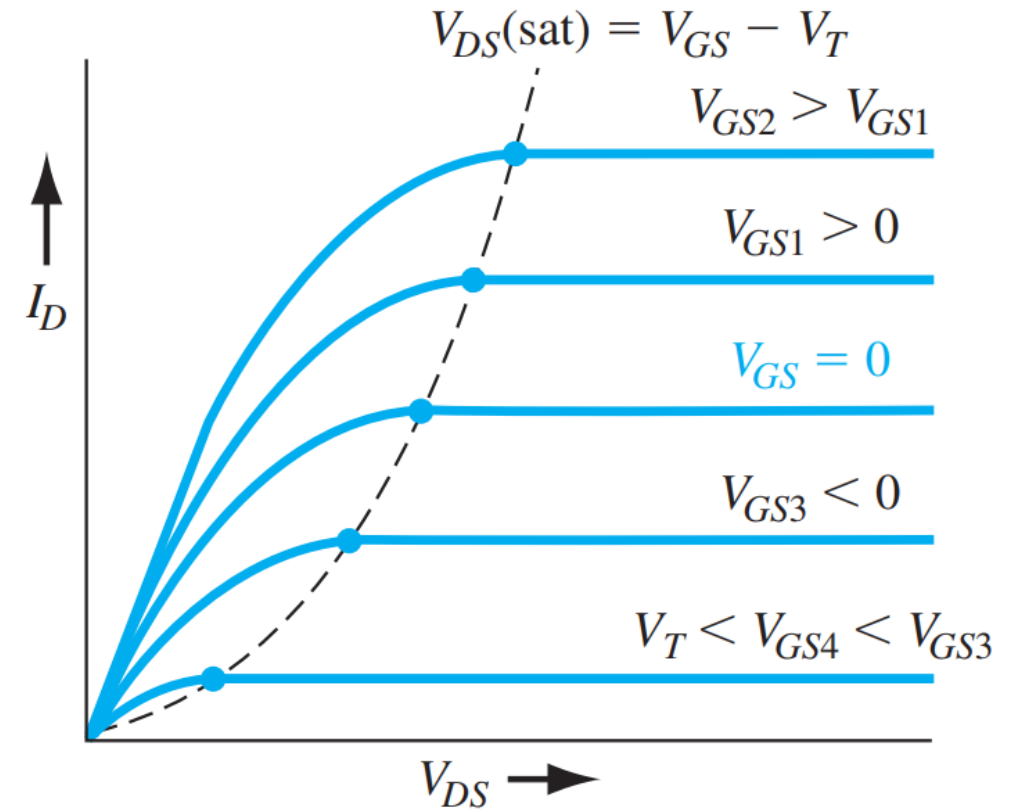


# Current-Voltage Relationship Concepts

**n-channel enhancement mode**



**n-channel depletion mode**



# The Ideal I-V Relation for n-channel MOSFET:

$$I_D = \frac{W}{2L} \mu_n C_{ox} \left[ 2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$$

$$I_D = \frac{W_n}{2L} k' \left[ 2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$$

conductance parameter (A/V<sup>2</sup>).

$$I_D = K_n \left[ 2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$$

process conductance parameter (A/V<sup>2</sup>).

$$V_{DS} = V_{GS} - V_T \Rightarrow I_D = K_n (V_{GS} - V_T)^2$$

# Example 10.7

**Objective:** Design the width of a MOSFET such that a specified current is induced for a given applied bias.

Consider an ideal n-channel MOSFET with parameters  $L = 1.25 \mu\text{m}$ ,  $\mu_n = 650 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_{\text{ox}} = 6.9 \times 10^{-8} \text{ F/cm}^2$ , and  $V_T = 0.65 \text{ V}$ . Design the channel width  $W$  such that  $I_D(\text{sat}) = 4 \text{ mA}$  for  $V_{GS} = 5 \text{ V}$ .

$$I_D(\text{sat}) = \frac{W\mu_n C_{\text{ox}}}{2L} (V_{GS} - V_T)^2$$

$$4 \times 10^{-3} = \frac{W(650)(6.9 \times 10^{-8})}{2(1.25 \times 10^{-4})} \cdot (5 - 0.65)^2 = 3.39 W$$

$$W = 11.8 \mu\text{m}$$

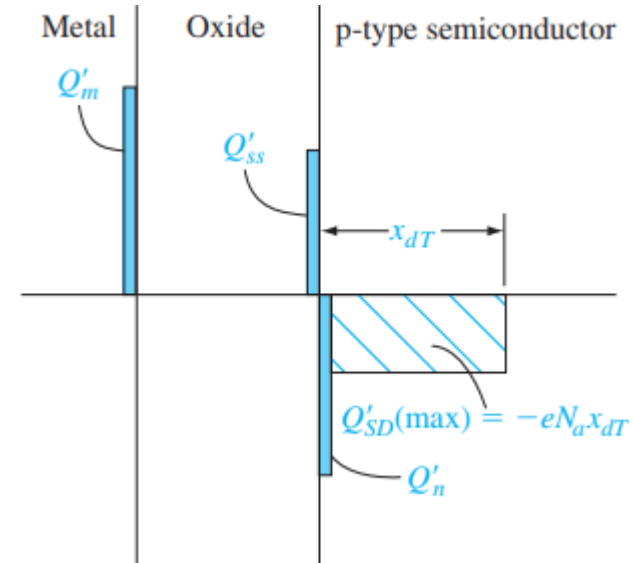
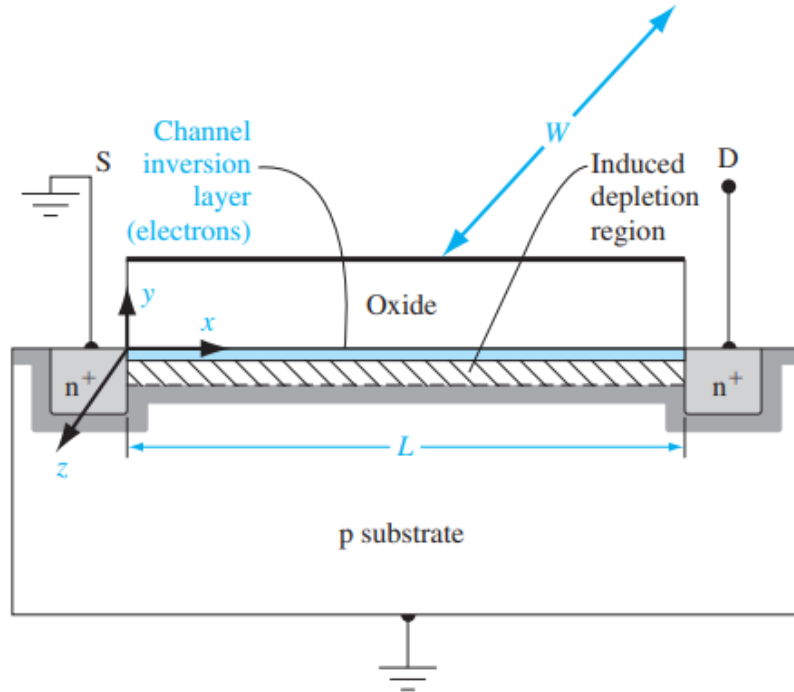
# Current–Voltage Relationship—Mathematical\*

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## Assumptions

- The current in the channel is due to drift rather than diffusion
- There is no current through the gate oxide
- A gradual channel approximation is used in which  $\partial E_x / \partial y \gg \partial E_x / \partial x$  ( $E_x$  is constant)
- Any fixed oxide charge is an equivalent charge density at the oxide
- The carrier mobility in the channel is constant

# Current–Voltage Relationship—Mathematical\*



$$J_x = \sigma E_x \text{ (ohm's law)}$$

$$I_x = \int \int_{y,z} J_x dy dz$$

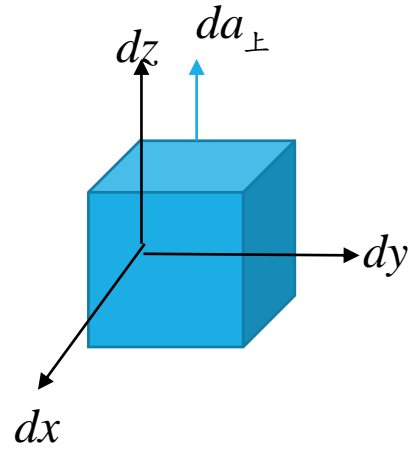
$$Q'_n = - \int e n(y) dy$$

$$I_x = -W \mu_n Q'_n E_x$$

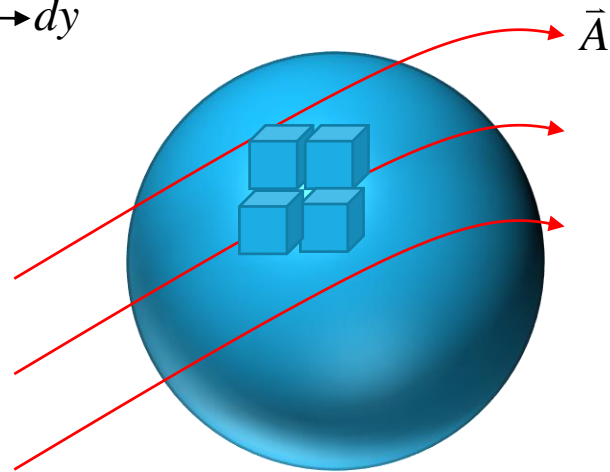
$$Q'_m + Q'_{ss} + Q'_n + Q'_{SD}(\text{max}) = 0 \quad \text{電中性}$$



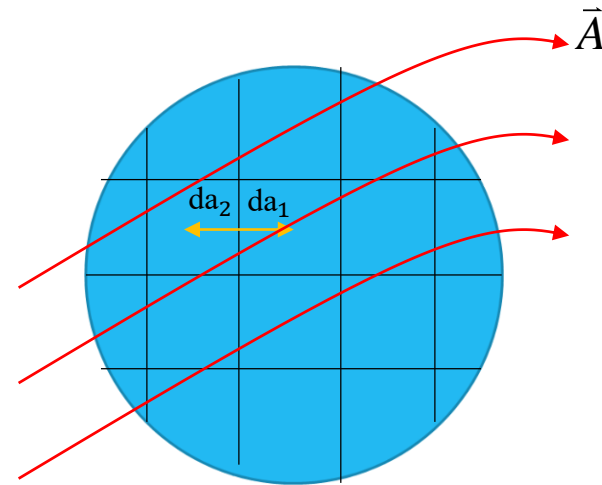
# Divergence theorem



Volume



Corss section

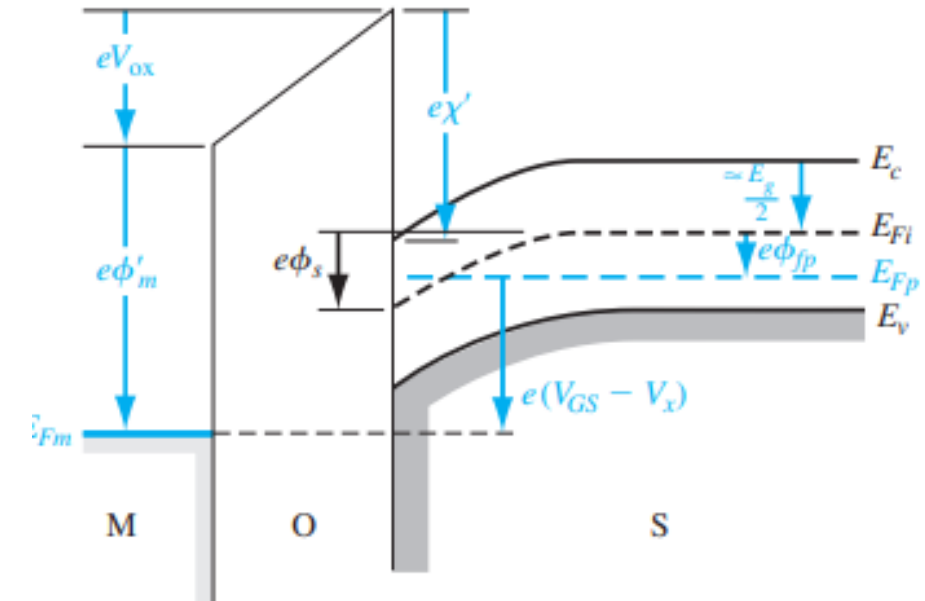
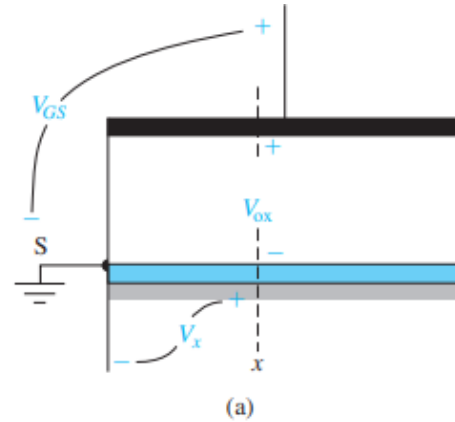
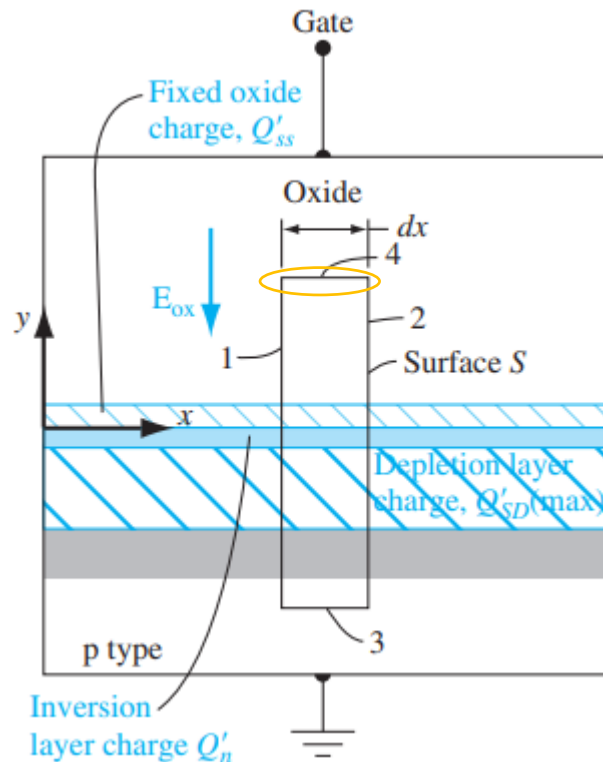


# Current–Voltage Relationship—Mathematical\*

$$\oint_s \epsilon E_n dS = -\epsilon_{ox} E_{ox} W dx = Q_T$$

$$Q_T = [Q'_{ss} + Q'_n + Q'_{SD}(\max)] W dx$$

$$-\epsilon_{ox} E_{ox} = Q'_{ss} + Q'_n + Q'_{SD}(\max)$$



$$E_{Fp} - E_{Fm} = e(V_{GS} - V_x)e(\phi'_m + V_{ox}) - \left( \chi' + \frac{E_g}{2e} - \phi_s + \phi_{fp} \right) = V_{ox} + 2\phi_{fp} + \phi_{ms}$$

Inversion condition

$$-\epsilon_{ox} E_{ox} = -\epsilon_{ox} \frac{V_{ox}}{t_{ox}} = -\epsilon_{ox} \frac{[(V_{GS} - V_x) - (\phi_{ms} + 2\phi_{fp})]}{t_{ox}} = Q'_{ss} + Q'_n + Q'_{SD}(\max)$$

$$I_x = -W \mu_n Q'_n E = -W \mu_n C_{ox} \frac{dV_x}{dx} [(V_{GS} - V_x) - V_T]$$

# Current–Voltage Relationship—Mathematical\*

Integrate over the length of channel

$$\int_0^L I_x dx = -W \mu_n C_{ox} \int_{V_x(0)}^{V_x(L)} [(V_{GS} - V_T) - V_x] dV_x$$

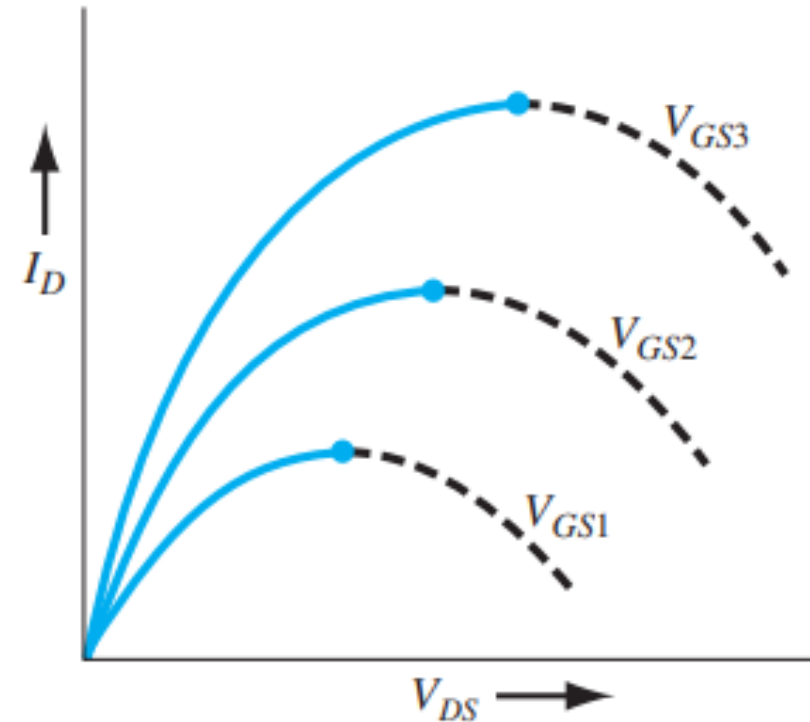
$$I_D = \frac{W \mu_n C_{ox}}{2L} [2(V_{GS} - V_T)V_{DS} - V_{DS}^2] \text{ (適用非飽和區)}$$

$$k'_n = \mu_n C_{ox} \text{ (process\_parameter)}$$

$$K_n = \frac{W \mu_n C_{ox}}{2L} \text{ (conduction\_parameter)}$$

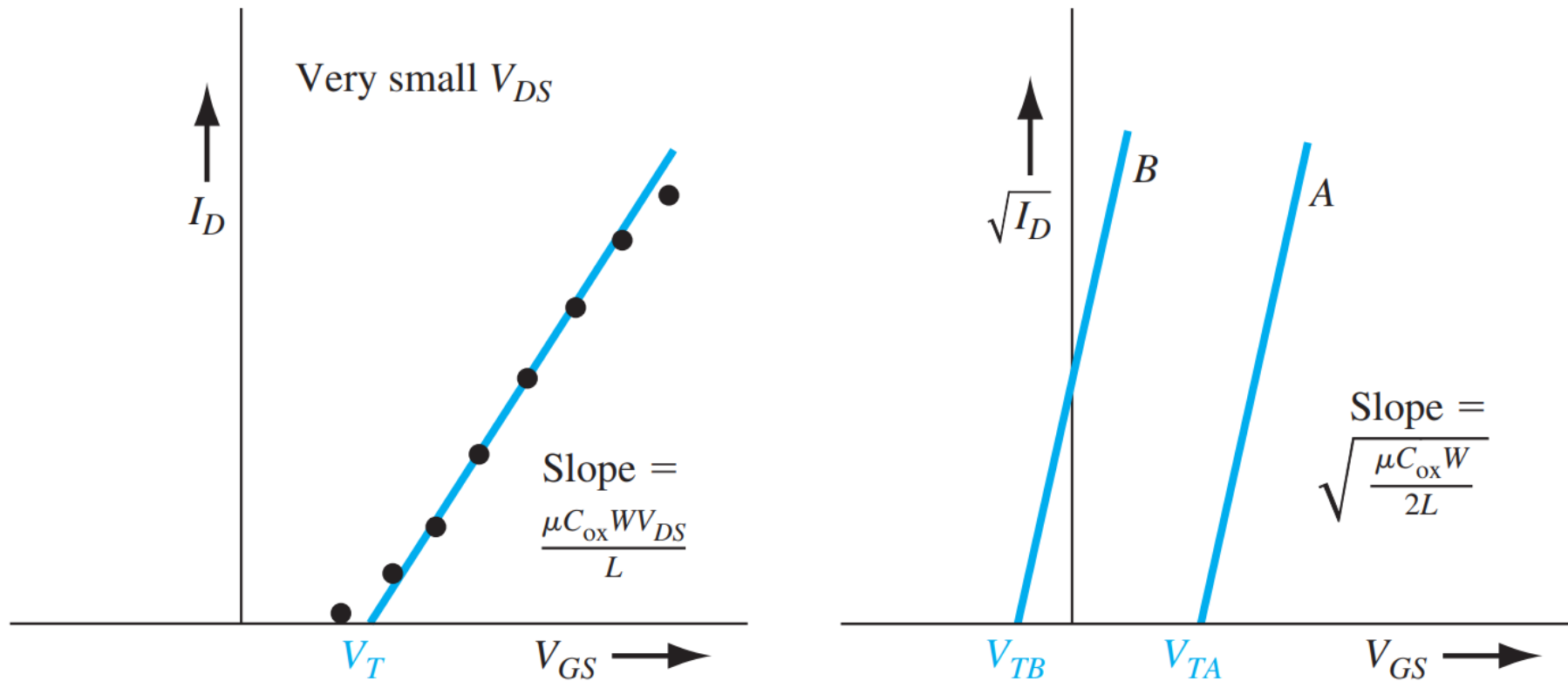
$$\frac{\partial I_D}{\partial V_{DS}} = 0 \Rightarrow V_{DS} = V_{GS} - V_T \text{ 帶回 } I_D$$

$$I_D(sat) = \frac{W \mu_n C_{ox}}{2L} (V_{GS} - V_T)^2$$



# The Ideal I-V Relation for n-channel MOSFET:

$$I_D = K_n \left[ 2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right] \Rightarrow 2K_n (V_{GS} - V_T)V_{DS}$$



# Example 10.8

**Objective:** Determine the inversion carrier mobility from experimental results.

Consider an n-channel MOSFET with  $W = 15 \mu\text{m}$ ,  $L = 2 \mu\text{m}$ , and  $C_{\text{ox}} = 6.9 \times 10^{-8} \text{ F/cm}^2$ . Assume that the drain current in the nonsaturation region for  $V_{DS} = 0.10 \text{ V}$  is  $I_D = 35 \mu\text{A}$  at  $V_{GS} = 1.5 \text{ V}$  and  $I_D = 75 \mu\text{A}$  at  $V_{GS} = 2.5 \text{ V}$ .

$$I_{D2} - I_{D1} = \frac{W\mu_n C_{\text{ox}}}{L} (V_{GS2} - V_{GS1})V_{DS}$$

$$75 \times 10^{-6} - 35 \times 10^{-6} = \left(\frac{15}{2}\right) \mu_n (6.9 \times 10^{-8}) (2.5 - 1.5) (0.10)$$

$$\mu_n = 773 \text{ cm}^2/\text{V-s}$$

$$V_T = 0.625 \text{ V}$$

# The Ideal I-V Relation for p-channel MOSFET:

$$I_D = \frac{W}{2L} \mu_p C_{ox} \left[ 2(V_{SG} + V_T) V_{SD} - V_{SD}^2 \right]$$

$$I_D = \frac{W}{2L} k'_n \left[ 2(V_{SG} + V_T) V_{SD} - V_{SD}^2 \right]$$

$$I_D = K_n \left[ 2(V_{SG} + V_T) V_{SD} - V_{SD}^2 \right]$$

$$V_{SD} = V_{SG} + V_T \Rightarrow I_D = K_n (V_{SG} + V_T)^2$$

