Chapter 4

The Semiconductor in Equilibrium

Outline

4.1 Charge Carriers in Semiconductors
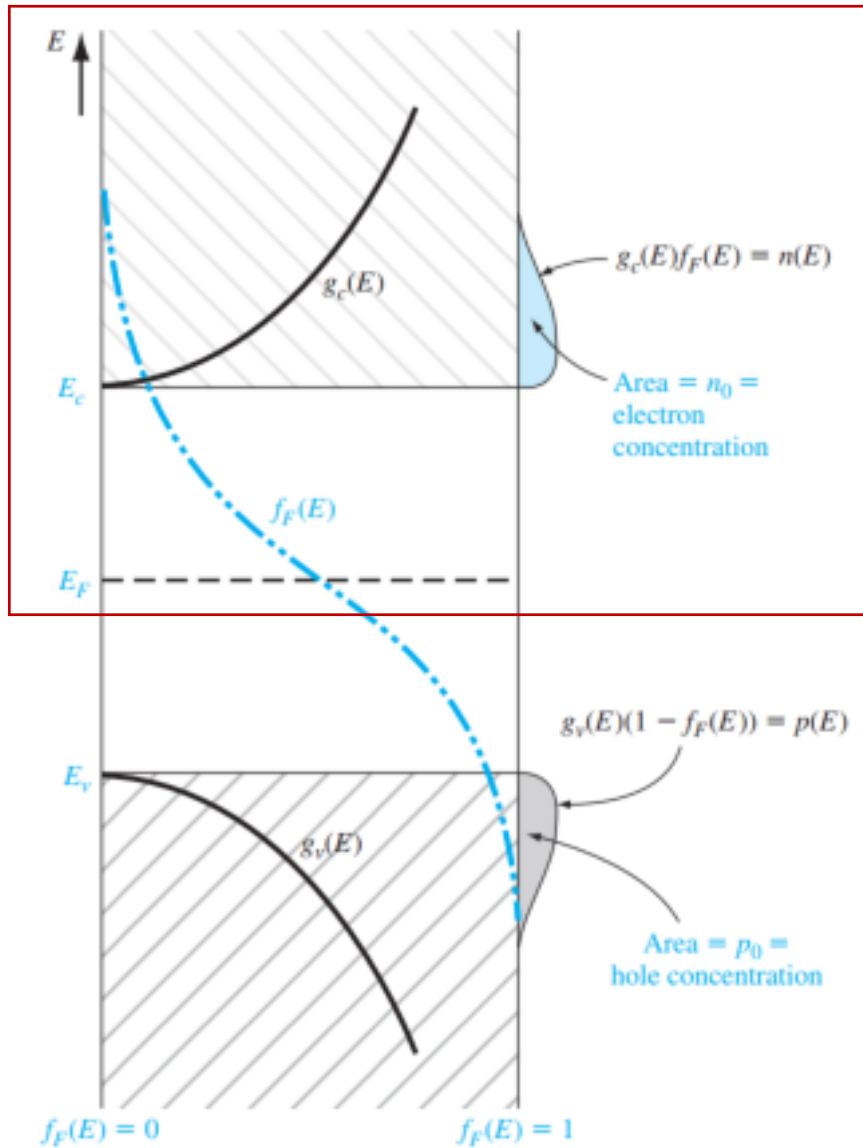
4.2 Dopant Atoms and Energy Levels

4.3 The Extrinsic Semiconductor

4.4 Statistics of Donors and Acceptors

4.5 Charge Neutrality

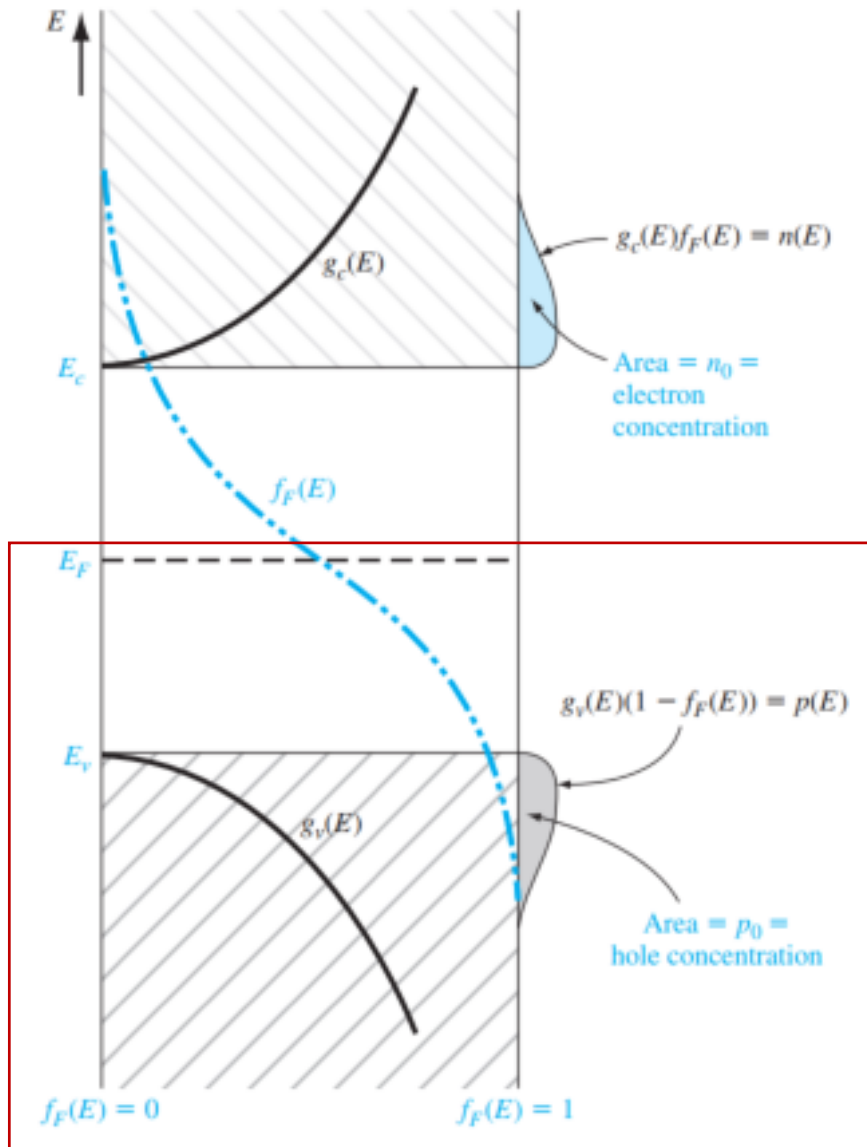4.6 Position of Fermi Energy Level

平衡時電子濃度

$$n(E) = g_c(E) f_F(E)$$

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

└──→ 單位體積中傳導帶電子的數量

$$p(E) = g_v(E)\left[1 - f_F(E)\right]$$

$$p_o = \int_{-\infty}^{E_v} g_v(E)\left[1 - f_F(E)\right]dE$$

⌐→ 單位體積中價帶電洞的數量

N_c : The effective density of states function in the conduction band.
N_v : The effective density of states function in the valence band.

電子濃度 $\quad n = \dfrac{2\left(2\pi m_n^* kT\right)^{3/2}}{h^3} \exp\left(-\dfrac{E_c - E_F}{kT}\right) = N_c \exp\left(-\dfrac{E_c - E_F}{kT}\right)$

電洞濃度 $\quad p = \dfrac{2\left(2\pi m_p^* kT\right)^{3/2}}{h^3} \exp\left(-\dfrac{E_F - E_v}{kT}\right) = N_v \exp\left(-\dfrac{E_F - E_v}{kT}\right)$

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Example 4.1

Objective: Calculate the probability that a quantum state in the conduction band at $E = E_c + kT/2$ is occupied by an electron, and calculate the thermal-equilibrium electron concentration in silicon at $T = 300$ K.

   Assume the Fermi energy is 0.25 eV below the conduction band. The value of N_c for silicon at $T = 300$ K is $N_c = 2.8 \times 10^{19}$ cm^{-3} (see Appendix B).

Example 4.1

Objective: Calculate the probability that a quantum state in the conduction band at $E = E_c + kT/2$ is occupied by an electron, and calculate the thermal-equilibrium electron concentration in silicon at $T = 300$ K.

Assume the Fermi energy is 0.25 eV below the conduction band. The value of N_c for silicon at $T = 300$ K is $N_c = 2.8 \times 10^{19}$ cm^{-3} (see Appendix B).

$$n = \frac{2\left(2\pi m_n^* kT\right)^{3/2}}{h^3} \exp\left(-\frac{E_c - E_F}{kT}\right) = 2.8 \times 10^{19} \exp\left(-\frac{0.25}{0.0259}\right) = 1.8 \times 10^{15} \quad \text{cm}^{-3}$$

Example 4.2

Objective: Calculate the thermal-equilibrium hole concentration in silicon at $T = 400$ K.

Assume that the Fermi energy is 0.27 eV above the valence-band energy. The value of N_v for silicon at $T = 300$ K is $N_v = 1.04 \times 10^{19}$ cm^{-3}. (See Appendix B)

Example 4.2

Objective: Calculate the thermal-equilibrium hole concentration in silicon at $T = 400$ K.

Assume that the Fermi energy is 0.27 eV above the valence-band energy. The value of N_v for silicon at $T = 300$ K is $N_v = 1.04 \times 10^{19}$ cm^{-3}. (See Appendix B)

$$p = \frac{2\left(2\pi m_p^* kT\right)^{3/2}}{h^3} \exp\left(-\frac{E_F - E_v}{kT}\right) = 1.04 \times 10^{19} \exp\left(-\frac{0.27}{0.0259}\right)$$

$$p = 1.04 \times 10^{19} \times \left(\frac{400}{300}\right)^{3/2} \exp\left[-\frac{0.27}{0.0259\left(400/300\right)}\right] = 6.43 \times 10^{15} \quad \text{cm}^{-3}$$

平衡時載子濃度(未摻雜半導體)

對一本質半導體(intrinsic semiconductor)而言，電子濃度必等於電洞濃度，因為一顆電子跳上去傳導帶，即產生一顆電洞在價帶。

$$\begin{cases} n_i = N_c \exp\left(-\frac{E_c - E_F}{kT} \right) \\ p_i = N_v \exp\left(-\frac{E_F - E_v}{kT} \right) \end{cases}$$

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

$$n_i p_i = N_c N_v \exp\left(-E_g / kT \right)$$

$$n_i^2 = N_c N_v \exp\left(-E_g / kT \right)$$

Example 4.3

Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

Example 4.3

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right) \qquad N_c, N_v \propto T^{3/2}$$

$$250\text{k}: \quad n_i^2 = 2.8 \times 10^{19} \times 1.04 \times 10^{19} \times \left(\frac{250}{300}\right)^3 \times \exp\left(-\frac{1.12}{0.0259(250/300)}\right) = 4.9 \times 10^{15}$$

$$n_i = 7.0 \times 10^7$$

$$400\text{k}: \quad n_i^2 = 2.8 \times 10^{19} \times 1.04 \times 10^{19} \times \left(\frac{400}{300}\right)^3 \times \exp\left(-\frac{1.12}{0.0259(400/300)}\right) = 5.67 \times 10^{24}$$

$$n_i = 2.38 \times 10^{12}$$

載子濃度與溫度關係（本質半導體）
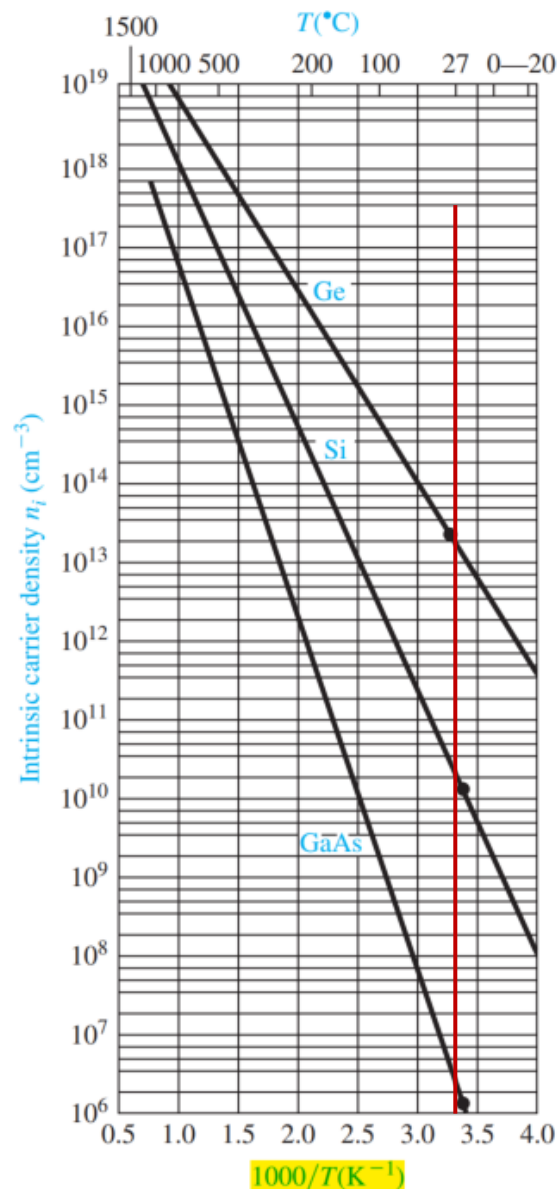
$$\frac{1000}{T} = 3.33 \Rightarrow T = 300$$

$$\frac{1000}{T} = 4 \Rightarrow T = 250$$

費米能階位置（本質半導體）

$$E_c$$

$$E_{Fi}$$

$$E_{mid}$$

$$E_v$$
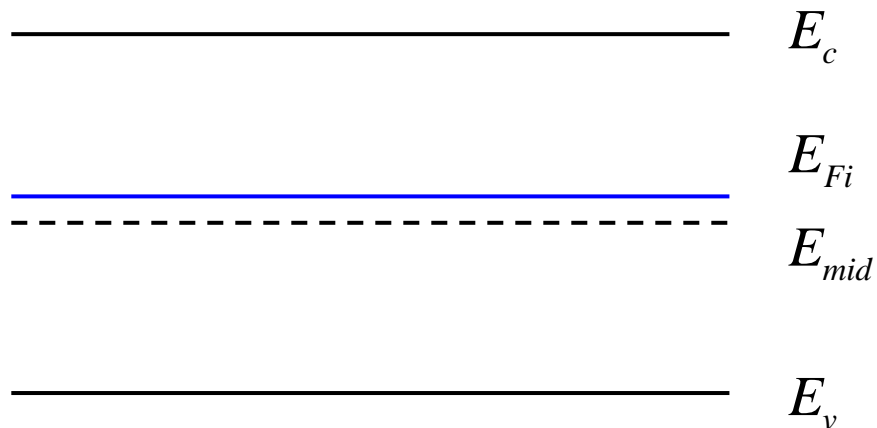
$$n = p \Rightarrow E_{Fi} - E_{mid} = \frac{1}{2}kT\ln\left(\frac{N_v}{N_c}\right) = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

常溫時 E_{Fi} 幾乎位於 E_c 和 E_v 的中央位置

Example 4.4

Example 4.4

Objective: Calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at $T = 300$ K.

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ and $m_p^* = 0.56m_0$.

Example 4.4

Objective: Calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at $T = 300$ K.

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ and $m_p^* = 0.56m_0$.

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4} (0.0259) \ln\left(\frac{0.56}{1.08}\right)$$

$$E_{Fi} - E_{\text{midgap}} = -0.0128 \text{ eV} = -12.8 \text{ meV}$$

An extrinsic semiconductor:

A semiconductor in which controlled amounts of specific dopant or impurity atoms have been added so that the thermal-equilibrium electron and hole concentrations are different from the intrinsic carrier concentration.

Pure Silicon
● Silicon nuclei

N-Type Silicon
● Phosphorous nucleus

The phosphorous atom creates an extra electron.

P-Type Silicon
● Boron nucleus

The boron atom creates a hole. ○
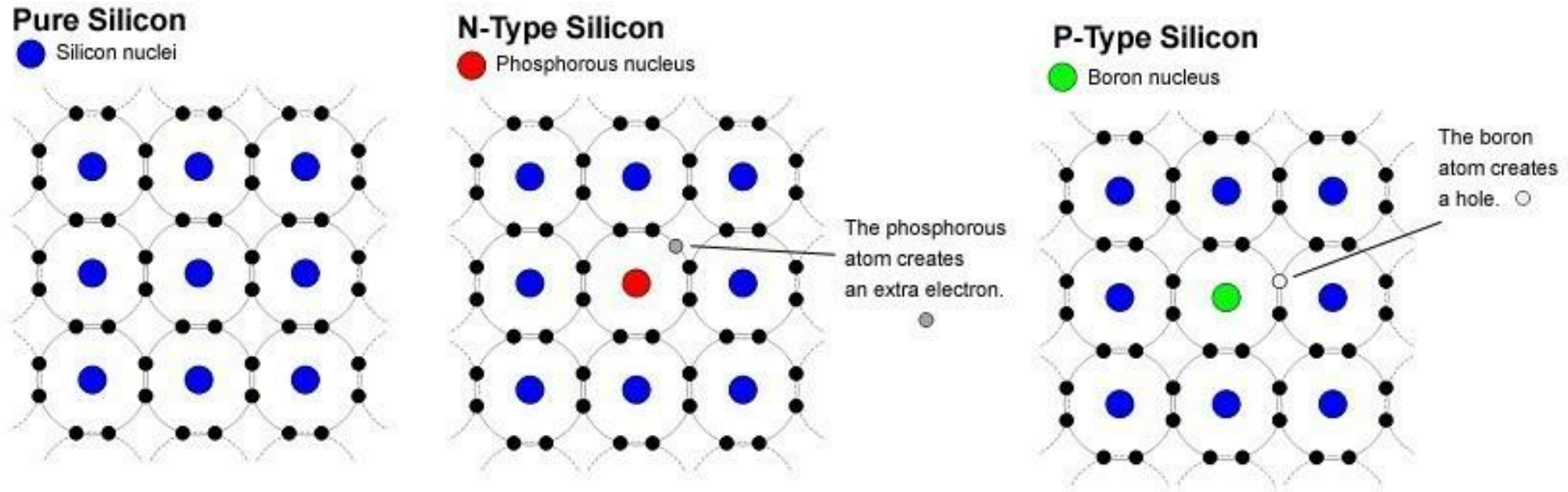
Qualitative Description for Donor

The atom donates an electron to the CB is called a donor impurity atom.

Donor energy level

The atom accepts an electron from the VB is called an <mark>acceptor impurity atom</mark>.
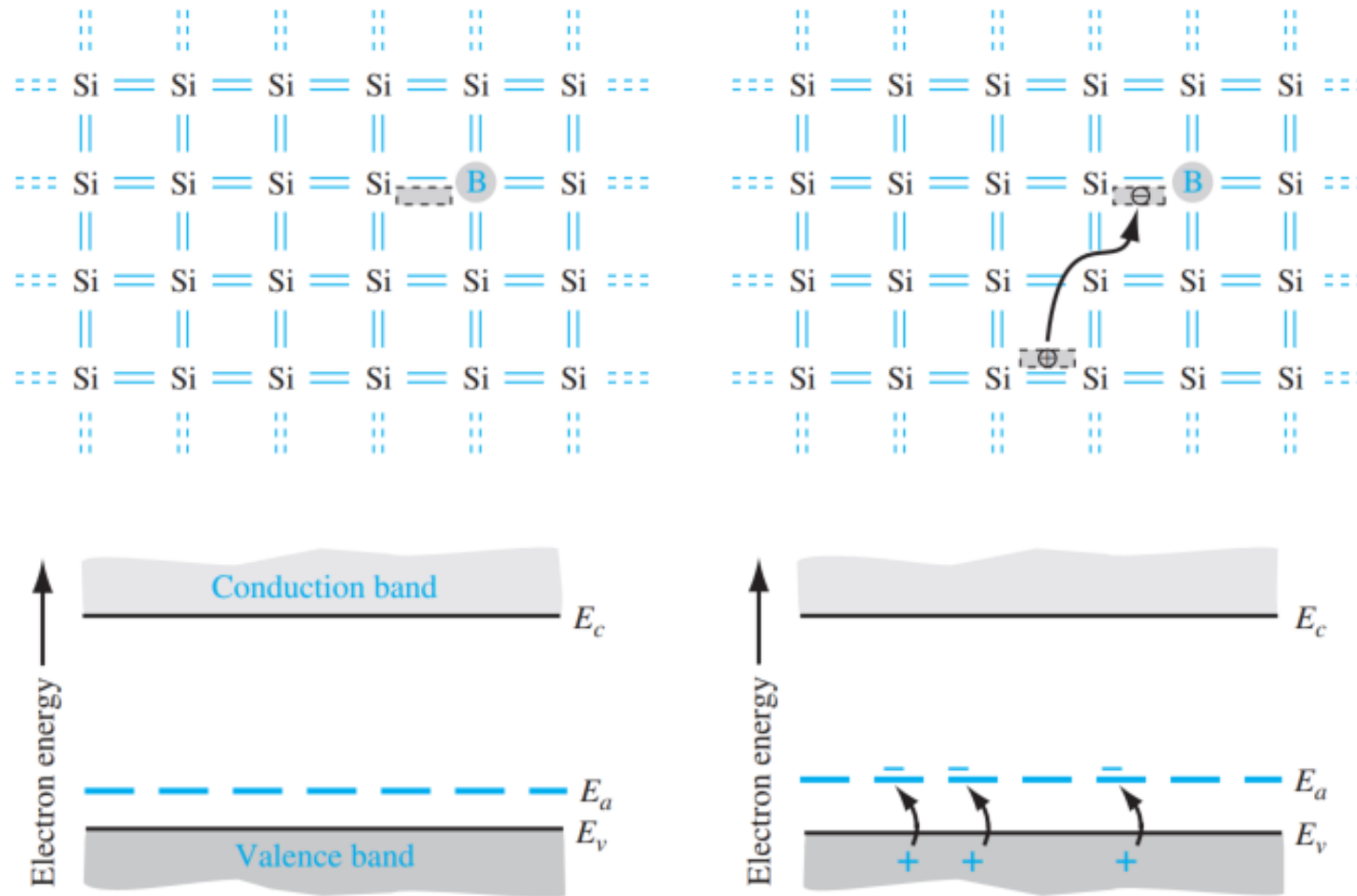
Acceptor energy level

摻雜濃度

Intrinsic semiconductor
本質半導體、無摻雜半導體

Extrinsic semiconductor
異質半導體、摻雜半導體
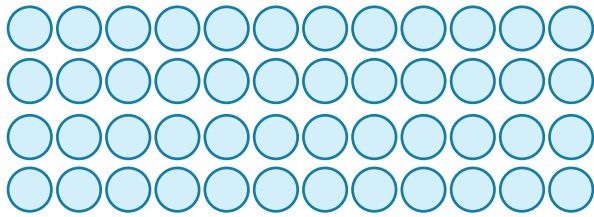
矽濃度: $\sim 10^{22}$ (cm^{-3})

電子濃度: $\sim 10^{12}$ (cm^{-3})
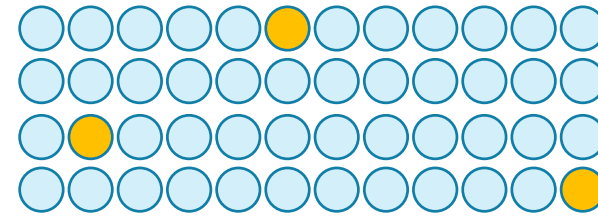
輕度摻雜: 10^{15}-10^{18} (cm^{-3})

重度摻雜: $> 10^{19}$ (cm^{-3})

摻雜會改變費米能階的位置
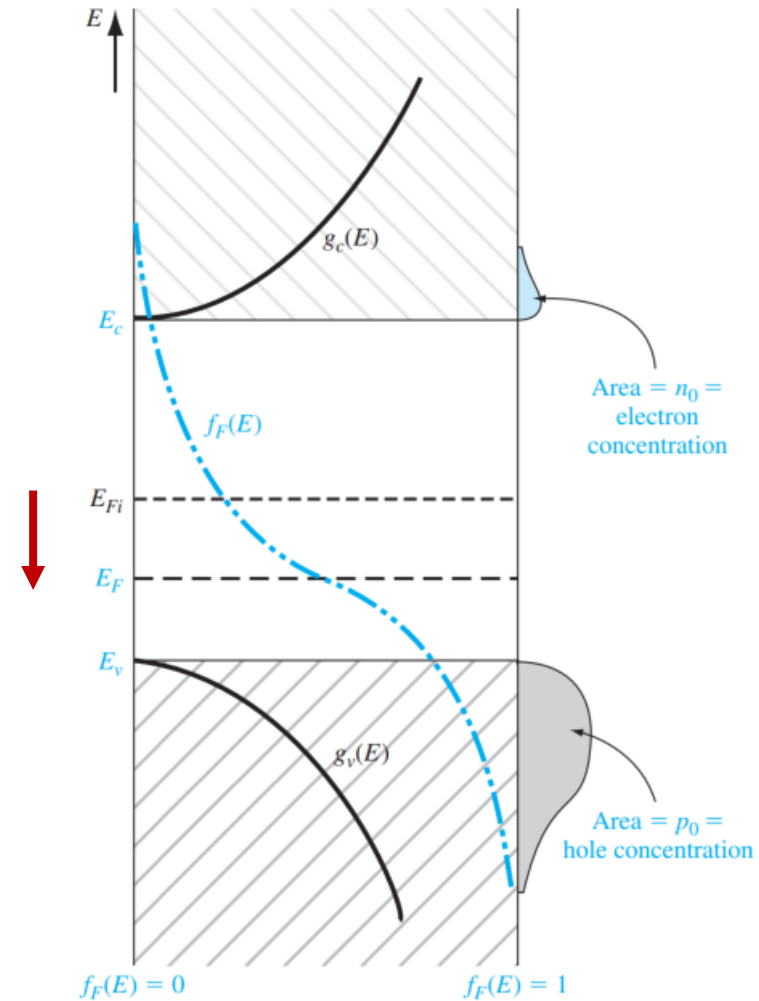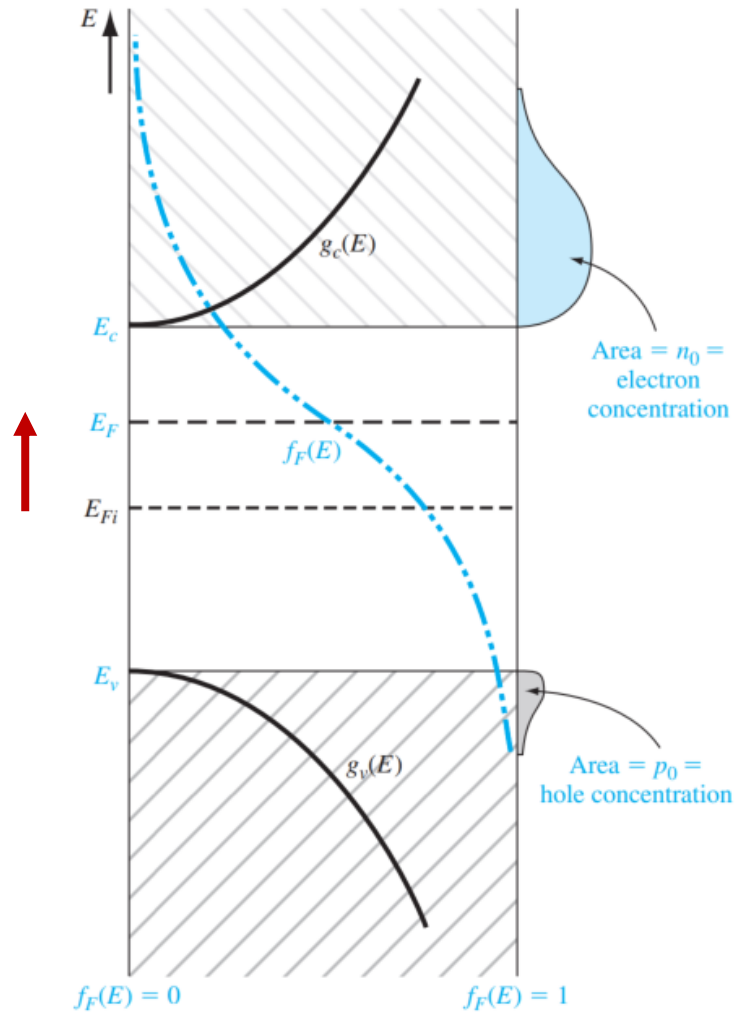
平衡時載子濃度(輕度摻雜半導體)

對輕度摻雜之半導體，公式依然適用

形式1

$$n_o = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$

$$p_o = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$$

形式2

$$n_o = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$p_o = p_i \exp\left(-\frac{E_F - E_{Fi}}{kT}\right)$$

$$n_o p_o = n_i p_i = n_i^2 \quad \text{但} \quad {\color{red}n_o \neq p_o}$$

Example 4.5

Objective: Calculate the thermal equilibrium concentrations of electrons and holes for a given Fermi energy.

Consider silicon at $T = 300$ K so that $N_c = 2.8 \times 10^{19}$ cm^{-3} and $N_v = 1.04 \times 10^{19}$ cm^{-3}. Assume that the Fermi energy is 0.25 eV below the conduction band. If we assume that the bandgap energy of silicon is 1.12 eV, then the Fermi energy will be 0.87 eV above the valence band.

$$n_o = N_c \exp\left(-\frac{E_c - E_F}{kT}\right) = 2.8 \times 10^{19} \exp\left(-\frac{0.25}{0.0259}\right) = 1.8 \times 10^{15}$$

$$p_o = N_v \exp\left(-\frac{E_F - E_v}{kT}\right) = 1.04 \times 10^{19} \exp\left(-\frac{0.87}{0.0259}\right) = 2.7 \times 10^{4}$$

公式適用範圍

以能帶的角度來看：

$$E_c - E_F > 3kT$$
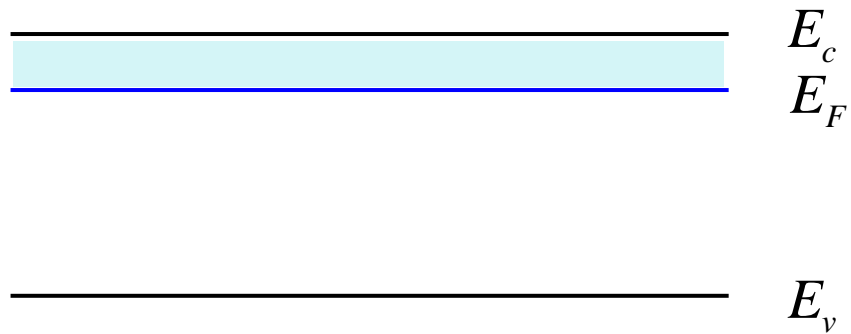
$$E_F - E_v > 3kT$$

費米能階不可以太靠近傳導帶，否則公式失效。
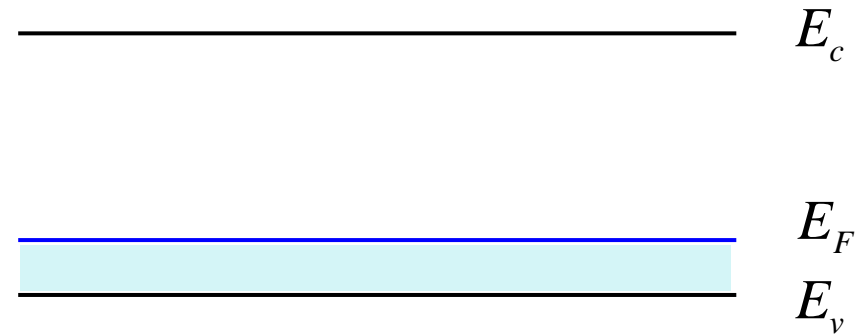
費米能階不可以太靠近價帶，否則公式失效。

Degenerate Semiconductors

If the doping level is **low,** the dopant atoms <u>are spread far enough apart</u> so that there is no interaction between donor electrons or acceptor holes.
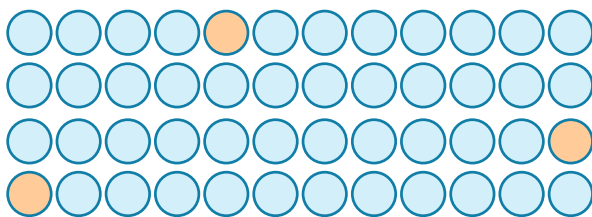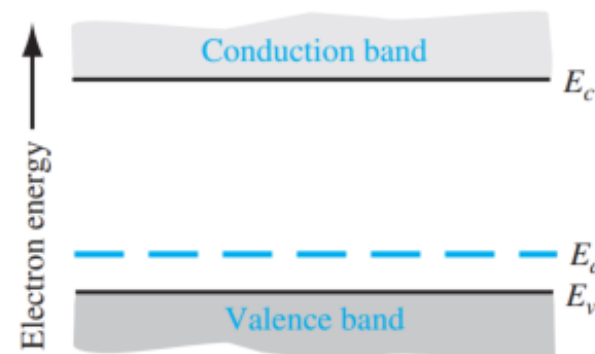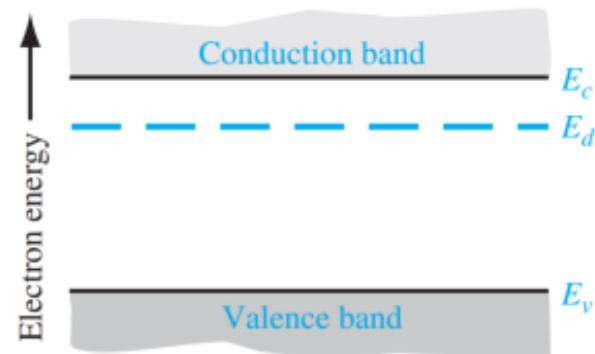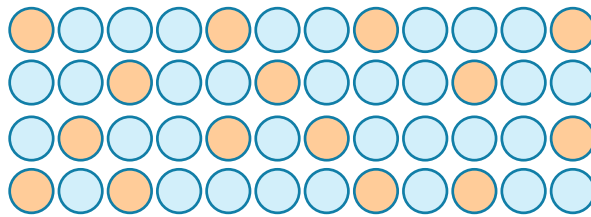

參雜濃度低

雜質原子距離很遠

可忽略交互作用

波函數不重疊

不形成能帶

E_c-E_F>3kT 法則仍適用
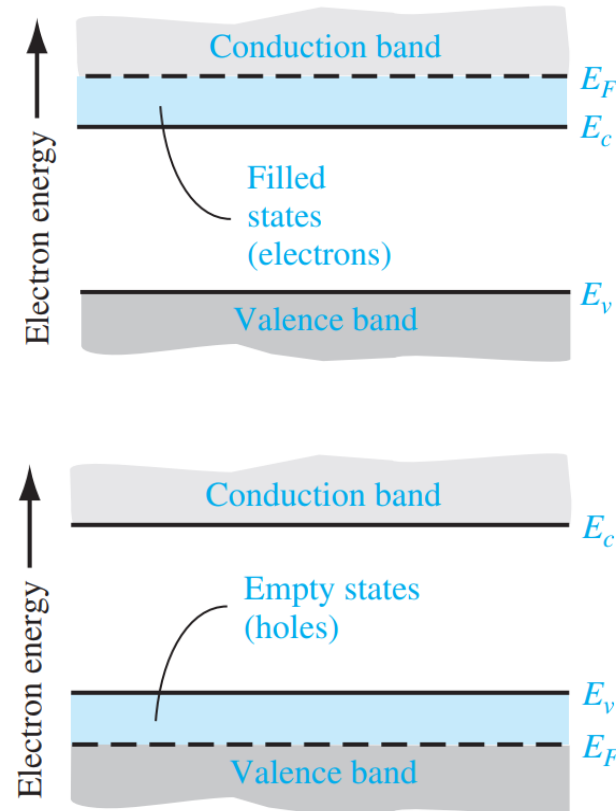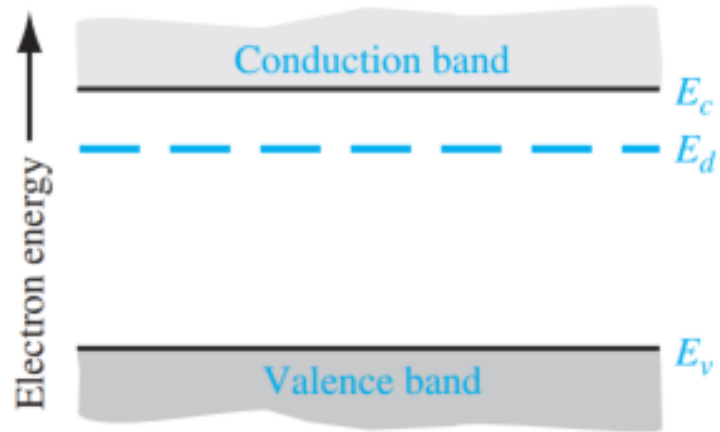
Nondegenerate Semiconductors

If the doping level is **high**, the dopant atoms <u>are close enough</u> so that the donor electron or acceptor holes will begin to interact, and an impurity energy band will be formed.

參雜濃度高

雜質原子距離很近

不可忽略交互作用
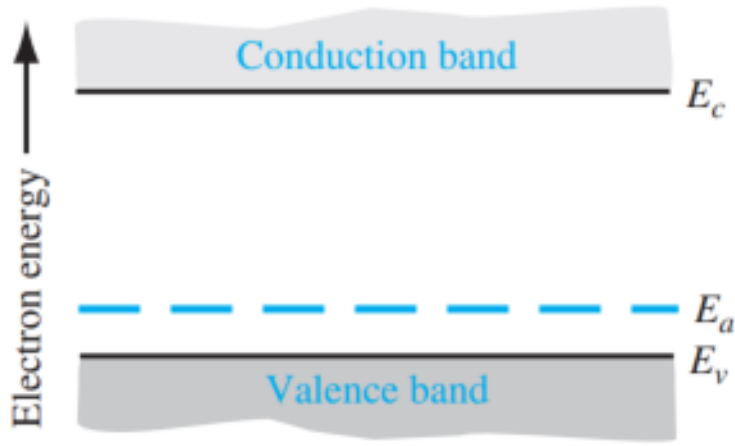
波函數重疊

形成能帶

E_c-E_F<3kT 法則不適用

離子化 Ionization

未離子化濃度
donor 摻雜濃度

$$\frac{n_d}{N_d} = \frac{1}{1 + \dfrac{1}{2}\exp\left(\dfrac{E_d - E_F}{kT}\right)}$$

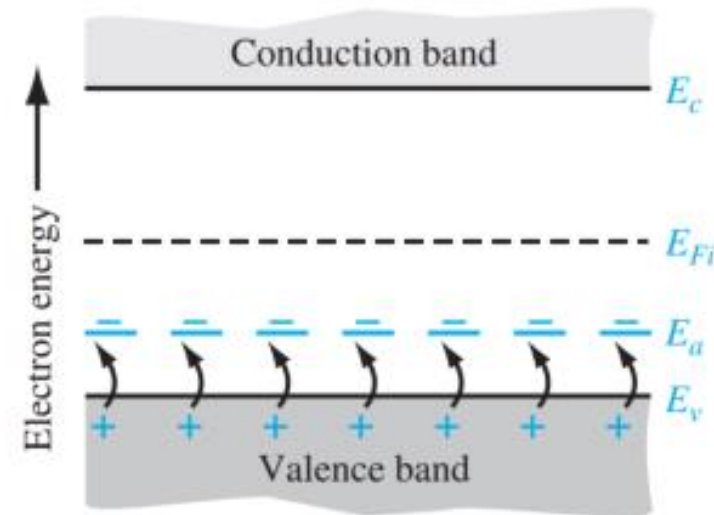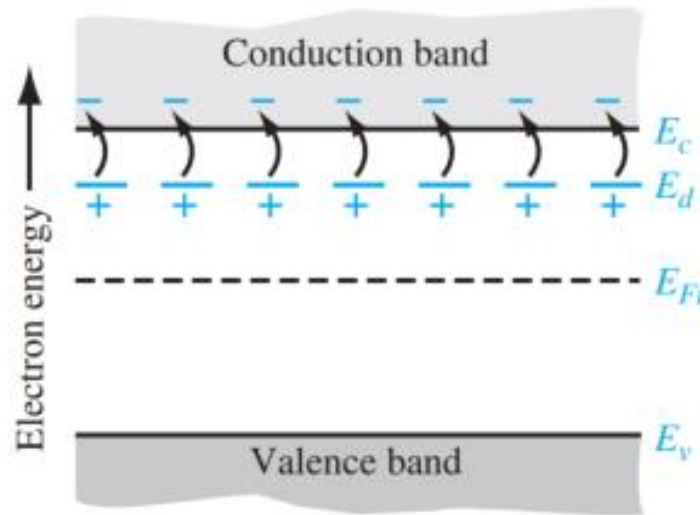
未離子化濃度
Acceptor摻雜濃度

$$\frac{p_a}{N_a} = \frac{1}{1 + \dfrac{1}{4}\exp\left(\dfrac{E_F - E_a}{kT}\right)}$$

離子化與溫度變化

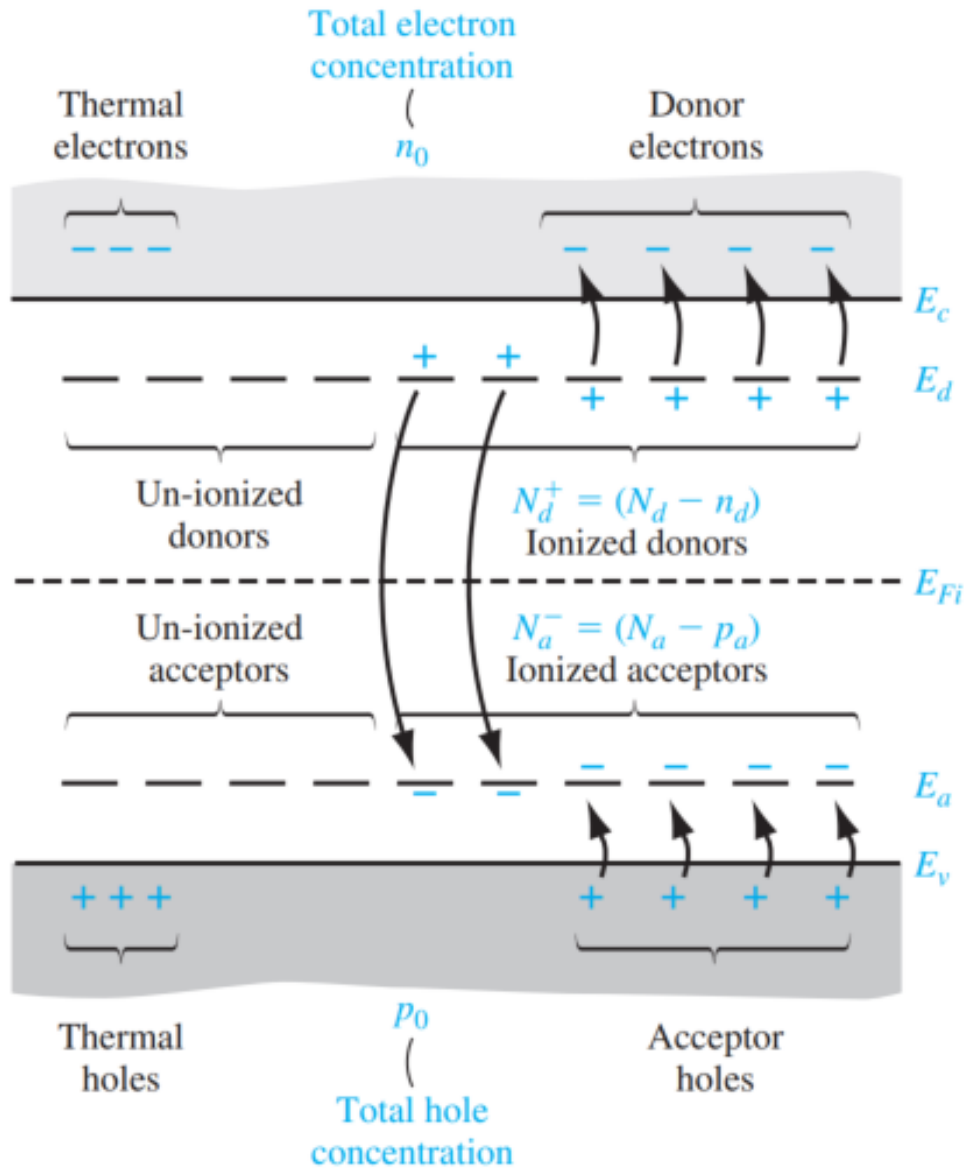| 絕對溫度 K	攝氏溫度 °C	游離率
50	-223	4%
100	-173	88.4%
200	-73	99%
300	23	99.6%

幾乎全部游離

互補半導體 Compensated Semiconductor

在同一區域內同時摻雜了 donor 和 acceptor.

- n-type compensated semiconductor $N_d > N_a$

- p-type compensated semiconductor $N_d < N_a$

- Completely compensated semiconductor $N_d = N_a$

電中性原理

整塊材料的淨電荷量必須是零

$$n_0 + N_a^- = p_0 + N_d^+$$

$$n_0 + \left(N_a - p_a\right) = p_0 + \left(N_d - n_d\right)$$

假設完全游離 $\Rightarrow n_d \approx 0, p_a \approx 0$

$$n_0 + N_a = p_0 + N_d$$

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$n_0^2 - \left(N_d - N_a\right)n_0 - n_i^2 = 0$$

電中性原理

傳導帶電子 $\quad n_0 = \dfrac{N_d - N_a}{2} + \sqrt{\left(\dfrac{N_d - N_a}{2}\right)^2 + n_i^2}$

價帶電洞 $\quad p_0 = \dfrac{N_a - N_d}{2} + \sqrt{\left(\dfrac{N_a - N_d}{2}\right)^2 + n_i^2}$

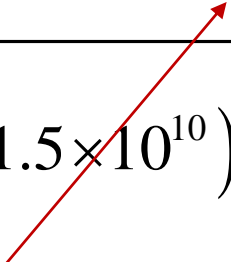|	
Silicon	$n_i = 1.5 \times 10^{10} \ \text{cm}^{-3}$
Gallium arsenide	$n_i = 1.8 \times 10^{6} \ \text{cm}^{-3}$
Germanium	$n_i = 2.4 \times 10^{13} \ \text{cm}^{-3}$

Example 4.9

Objective: Determine the thermal-equilibrium electron and hole concentrations in silicon at $T = 300$ K for given doping concentrations. (a) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (b) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}.

   Recall that $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at $T = 300$ K.

Example 4.9

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Objective: Determine the thermal-equilibrium electron and hole concentrations in silicon at $T = 300$ K for given doping concentrations. (a) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (b) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}.

Recall that $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at $T = 300$ K.

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + \left(1.5 \times 10^{10}\right)^2} \approx 10^{16}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{16}} = 2.25 \times 10^4$$

Example 4.10

Objective: Calculate the thermal-equilibrium electron and hole concentrations in germanium for a given doping concentration.

Consider a germanium sample at $T = 300$ K in which $N_d = 2 \times 10^{14}$ cm^{-3} and $N_a = 0$. Assume that $n_i = 2.4 \times 10^{13}$ cm^{-3}.

Example 4.10

Objective: Calculate the thermal-equilibrium electron and hole concentrations in germanium for a given doping concentration.

Consider a germanium sample at $T = 300$ K in which $N_d = 2 \times 10^{14}$ cm^{-3} and $N_a = 0$. Assume that $n_i = 2.4 \times 10^{13}$ cm^{-3}.

$$n_0 = \frac{2 \times 10^{14}}{2} + \sqrt{\left(\frac{2 \times 10^{14}}{2}\right)^2 + \left(2.4 \times 10^{13}\right)^2} \approx 2.028 \times 10^{14}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(2.4 \times 10^{13}\right)^2}{2.028 \times 10^{14}} = 2.84 \times 10^{12}$$

Example 4.11
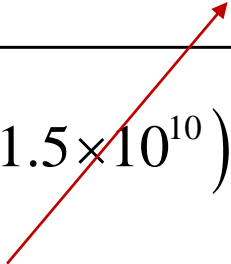
Objective: Calculate the thermal-equilibrium electron and hole concentrations in a compensated p-type semiconductor.

Consider a silicon semiconductor at $T = 300$ K in which $N_a = 10^{16}$ cm^{-3} and $N_d = 3 \times 10^{15}$ cm^{-3}. Assume $n_i = 1.5 \times 10^{10}$ cm^{-3}.

Example 4.11

Example 4.11

Objective: Calculate the thermal-equilibrium electron and hole concentrations in a compensated p-type semiconductor.

Consider a silicon semiconductor at $T = 300$ K in which $N_a = 10^{16}$ cm^{-3} and $N_d = 3 \times 10^{15}$ cm^{-3}. Assume $n_i = 1.5 \times 10^{10}$ cm^{-3}.

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} = \frac{7 \times 10^{15}}{2} + \sqrt{\left(\frac{7 \times 10^{15}}{2}\right)^2 + \left(1.5 \times 10^{10}\right)^2} \approx 7 \times 10^{15}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{\left(1.5 \times 10^{10}\right)^2}{7 \times 10^{15}} = 3.21 \times 10^4$$
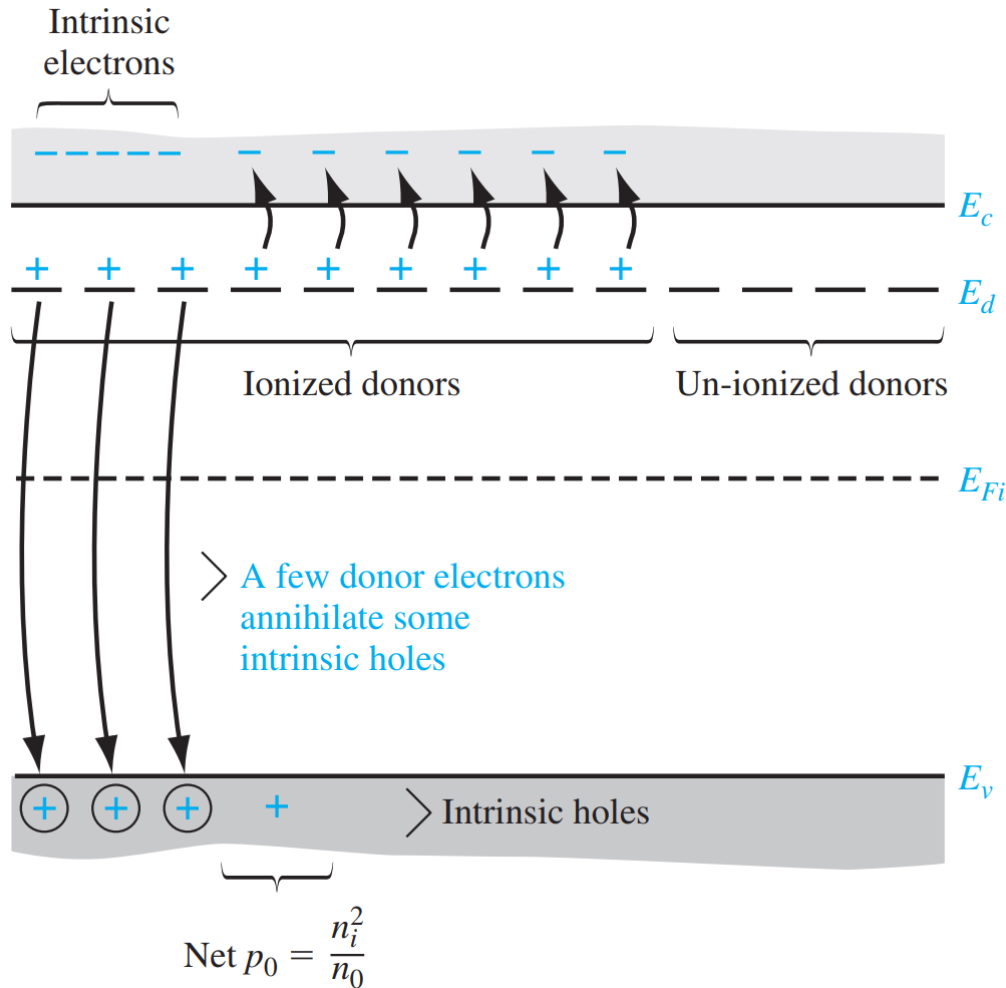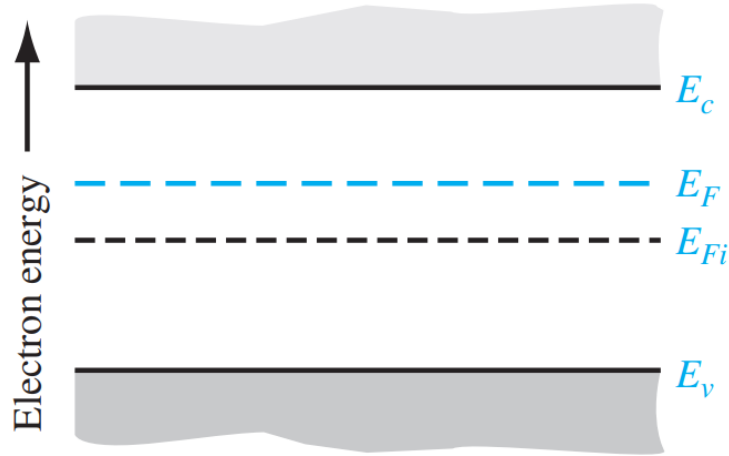
- 傳導帶電子因N型摻雜而增加.

- 價帶電洞因N型摻雜而減少

- 因大部分電子跳上傳導帶，少部分跳到價帶並填入電洞
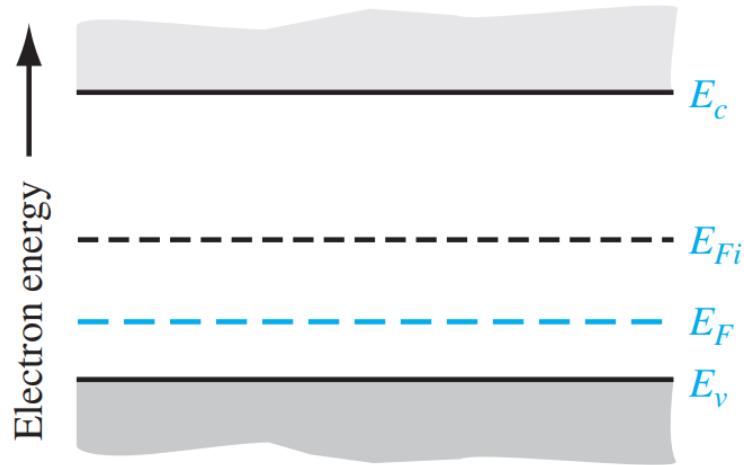
$$n_o p_o = n_i^2 \Rightarrow p_o = \frac{n_i^2}{n_o}$$
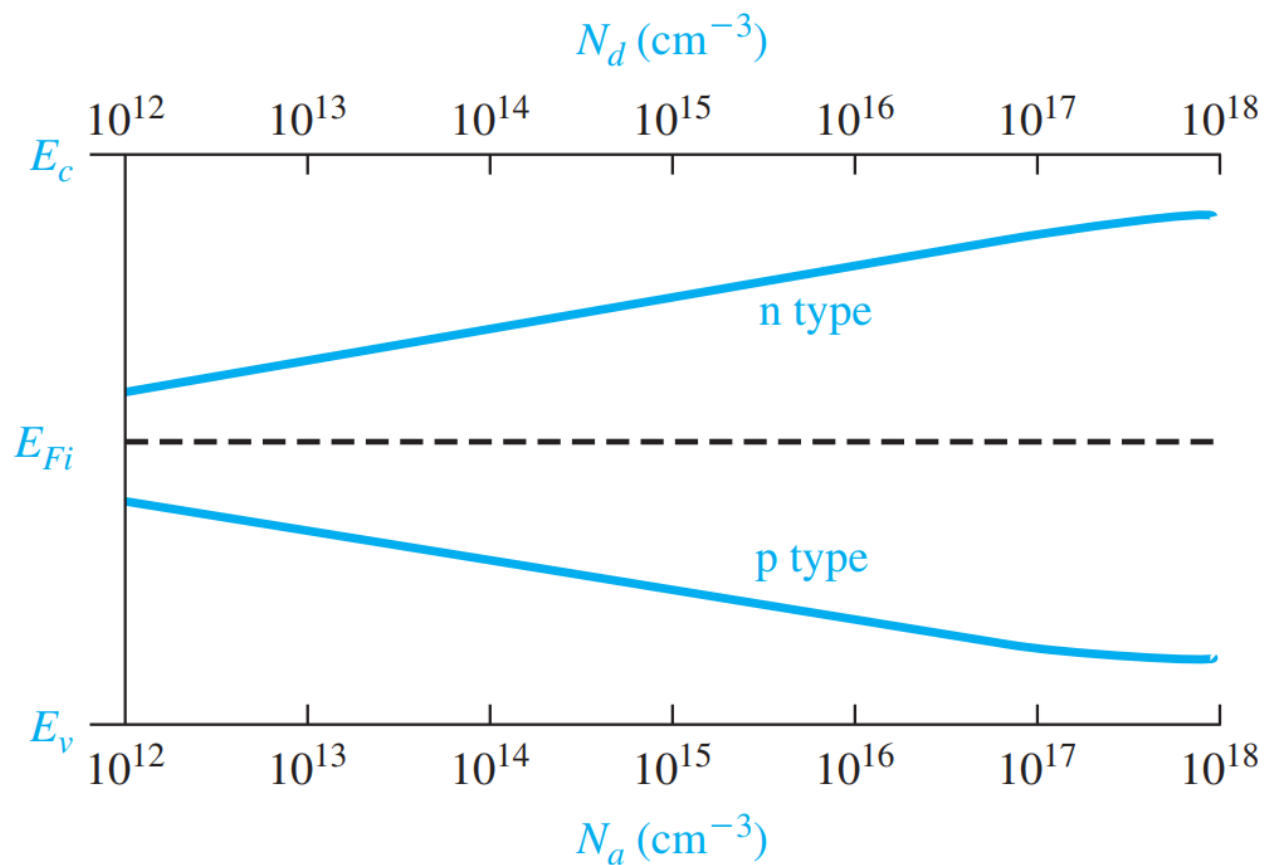
載子濃度與溫度關係 (n-type uncompensated)

$$n_0 = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Intrinsic

Extrinsic

$N_d = 5 \times 10^{14}$

Partial ionization

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

n_i

$n_0 \, (\text{cm}^{-3})$

10^{16}

10^{15}

5

10^{14}

10^{13}

10^{12}

100 200 300 400 500 600 700

T (K)
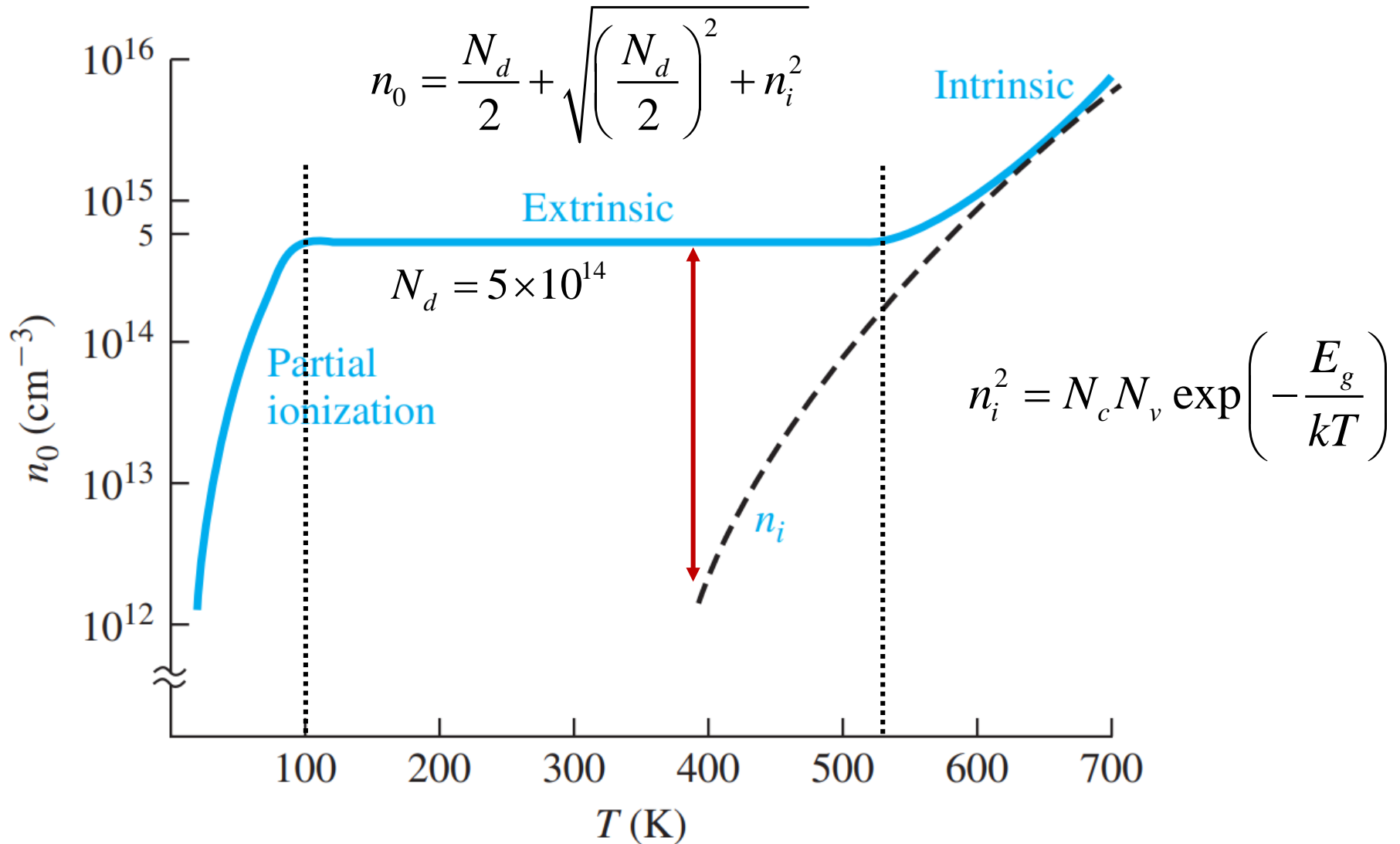
$$\begin{cases} E_c - E_F \approx kT \ln\left(\dfrac{N_c}{N_d}\right) \\[2em] E_F - E_{Fi} \approx kT \ln\left(\dfrac{N_d}{n_i}\right) \end{cases}$$

$$\begin{cases} E_F - E_v \approx kT \ln\left(\dfrac{N_v}{N_a}\right) \\[2em] E_{Fi} - E_F \approx kT \ln\left(\dfrac{N_a}{p_i}\right) \end{cases}$$
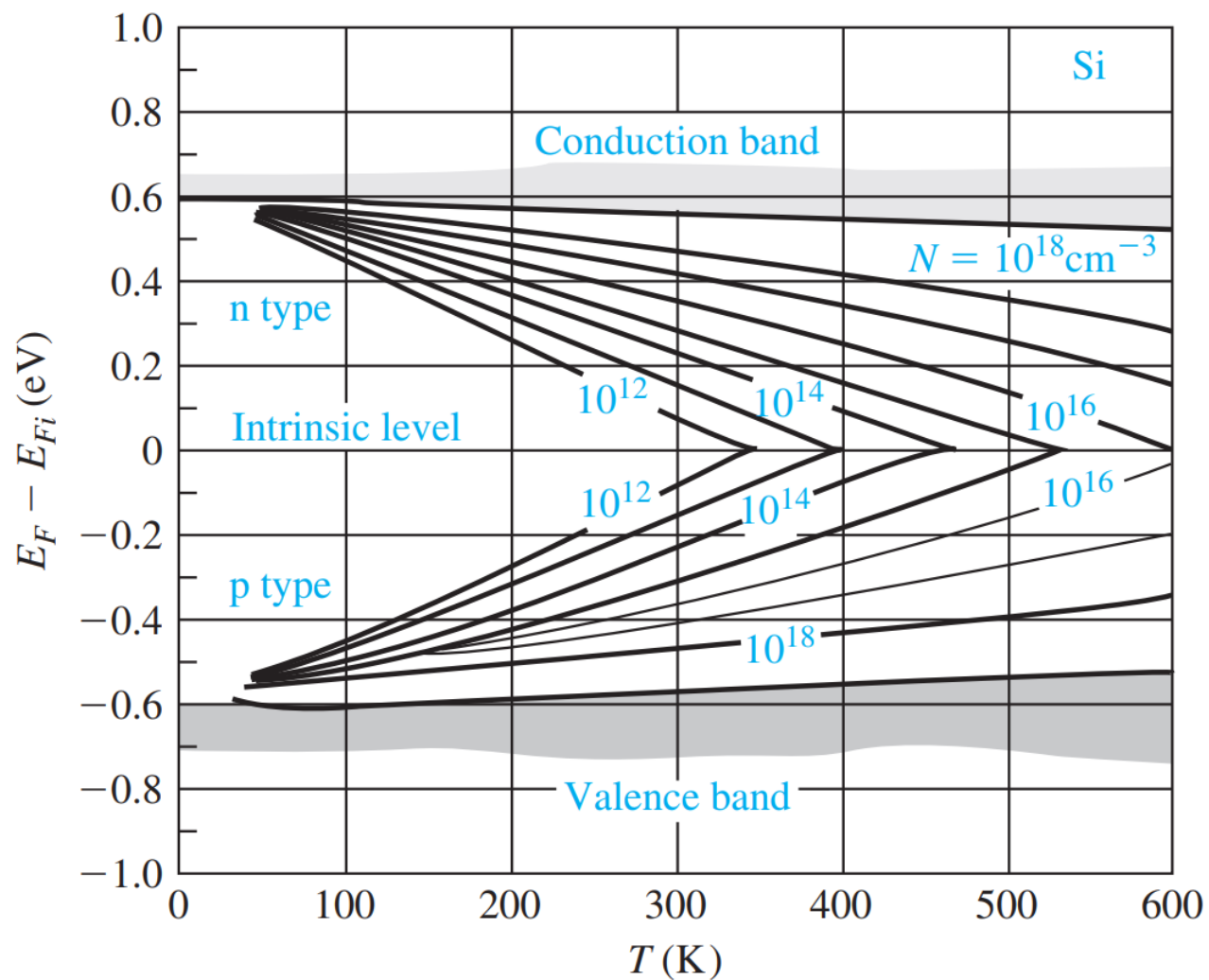
$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$
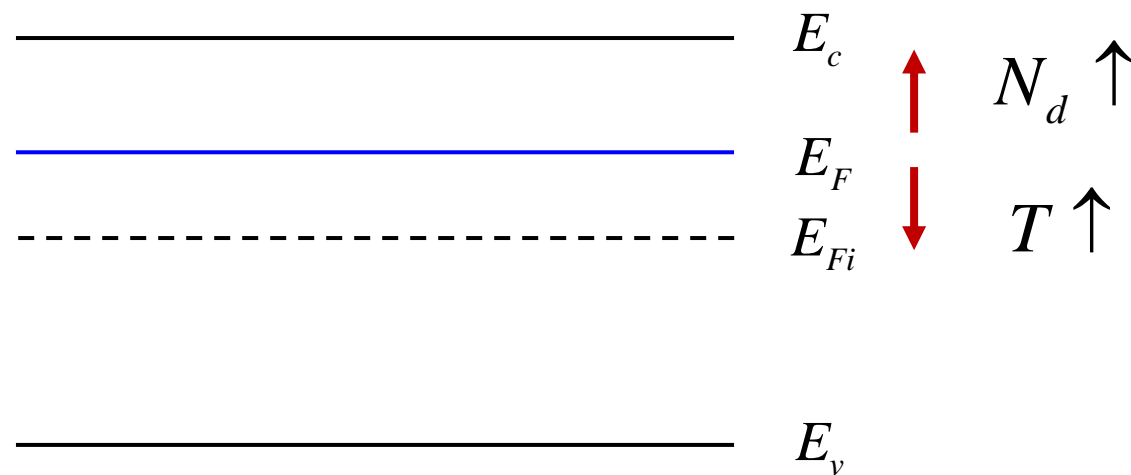
$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{p_i}\right)$$

費米能階位置與溫度之關係
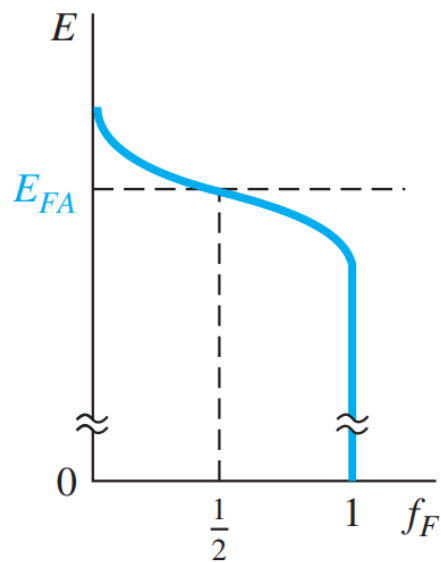

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

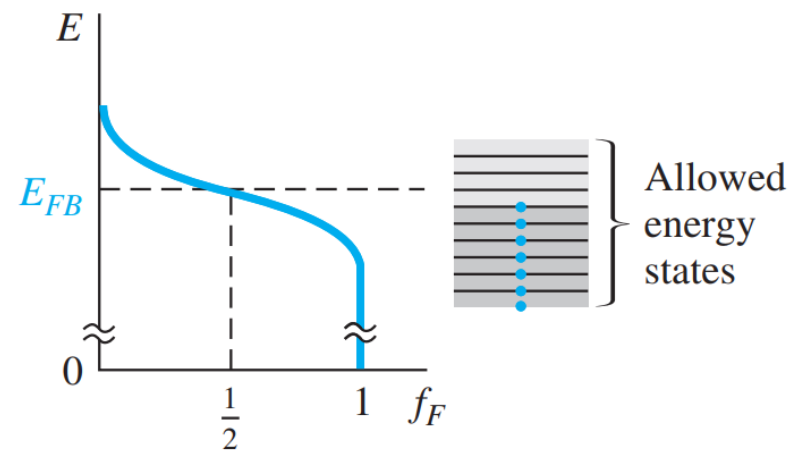$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{p_i}\right)$$

費米能階位置總結

E_c

E_F

E_{Fi}

E_v

$N_d \uparrow$

$T \uparrow$

- 摻雜濃度愈高，費米能階往傳導帶靠
- 溫度愈高，費米能階往中線靠

(a)

(b)

(c)

(d)