

Computationally Efficient Codes for Adversarial Binary-Erasure Channels

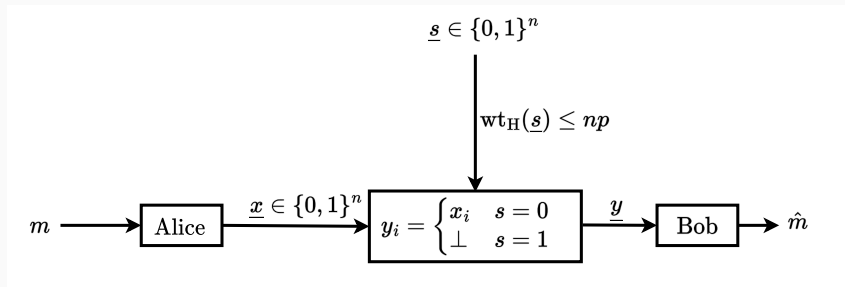
Short informal talk @ Imperial College

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Joint work with Sijie Li (UT Austin), Prasad Krishnan (IIIT Hyderabad), Sidharth Jaggi (U. Bristol), Michael Langberg (U. at Buffalo)

Our good friend the erasure channel

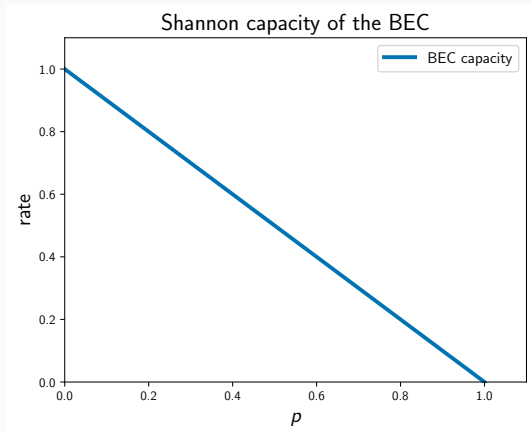


The erasure channel almost needs no introduction...

- Binary input, ternary output.
- Think of erasures as a state sequence \underline{s} where 1 means “erase.”
- Fraction of erased bits upper bounded by p , either exactly (coding theory) or with high probability (Shannon theory).

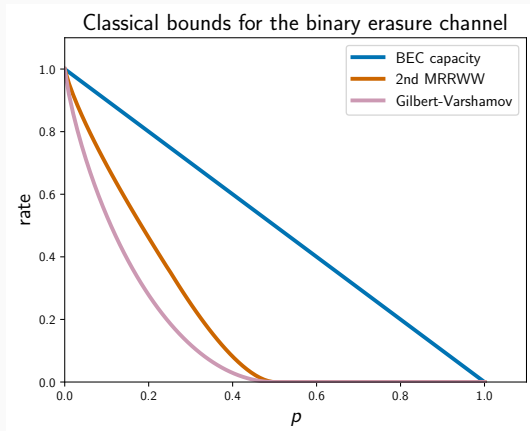
Optimism versus pessimism

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Optimistic: *Random* erasures (BEC):
erasures are i.i.d. Bernoulli. Shannon
capacity is $1 - p$.

Optimism versus pessimism



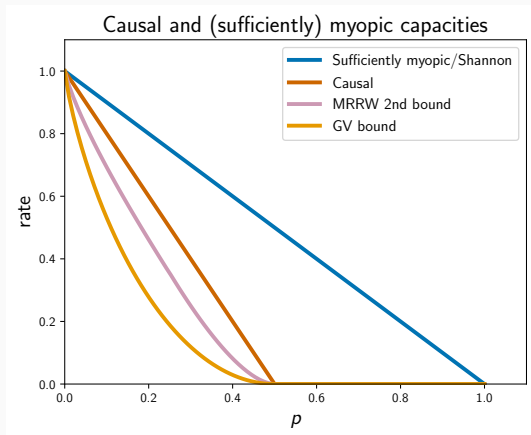
Optimistic: *Random* erasures (BEC):
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Pessimistic: *Adversarial* erasures: erasures
can depend on transmitted codeword.

Capacity unknown!

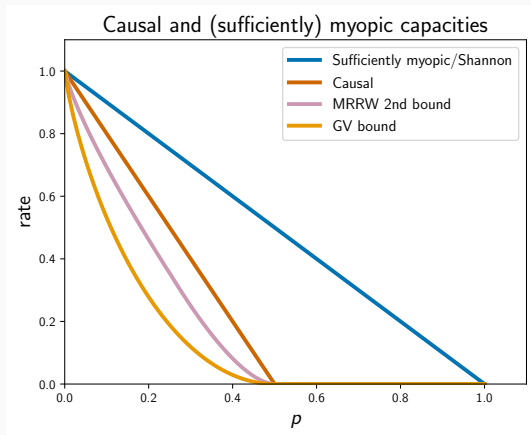
- Lower bound: Gilbert-Varshamov (and **linear codes** work.)
- Upper bound from linear programming (MRRW “LP” bound).

Models in the middle: causal and myopic



(Chen, Jaggi, Langberg 2015) (Sarwate 2010) (Dey et al. 2016) (Dey et al. 2019)

Models in the middle: causal and myopic

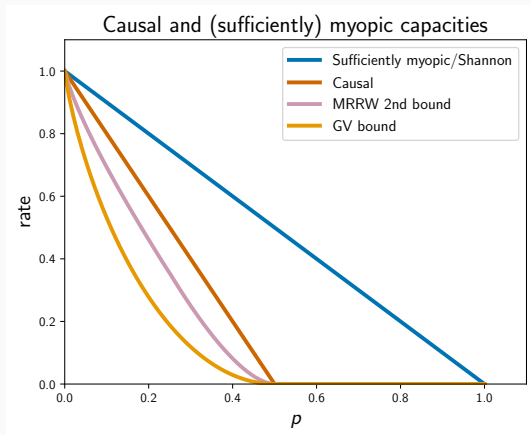


Causal adversaries: erasures can depend on the current and past input only.

Capacity is $1 - 2p$.

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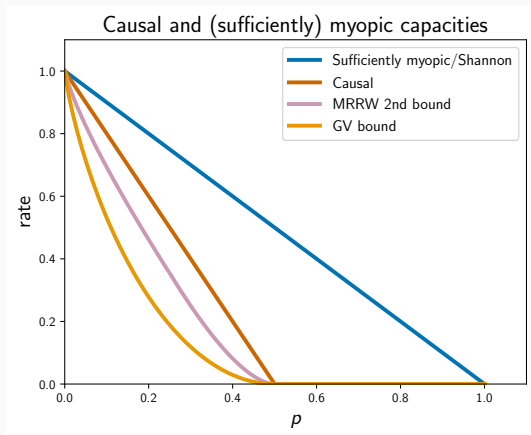
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Myopic adversaries: erasures can depend on a noisy (erased by $\text{BEC}(q)$) version of the codeword.

- If $p < q$, capacity is $1 - p$.
- If $p > q$... see the paper.

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Achievability arguments use **stochastic encoding** and **list decoding** with **nonlinear codes**.

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By **efficient** we mean that they take **polynomial time** to encode, decode, and store.

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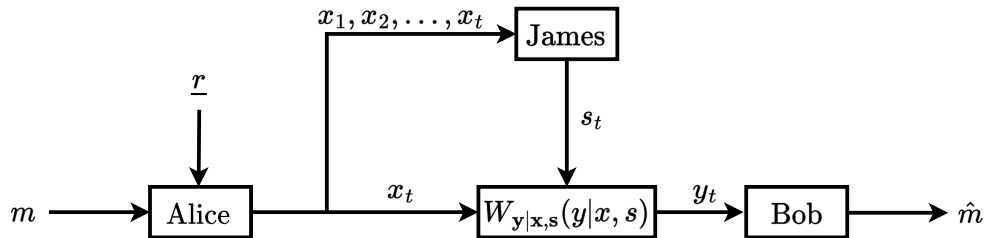
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- **common randomness** is unrealistic.
→ use **limited encoder randomization** to **confuse the adversary**.
- **minimum distance coding** is not efficient in general.
→ use **list decoding** to permit **efficient decoding**.

Causal and Myopic Adversaries

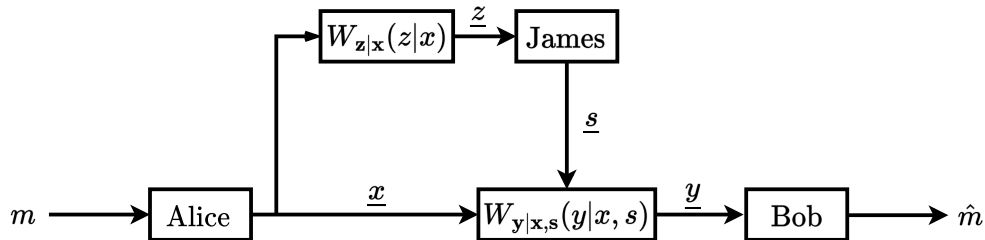
Causal/Online Adversaries



A **causal adversary** can eavesdrop **noiselessly and in real time** on the channel inputs:

- Decision on whether to erase at time t can depend on (x_1, x_2, \dots, x_t) .
- **Adversary's budget:** at most np erasures in codeword of length n .

Myopic adversaries



A **myopic adversary** can eavesdrop **noisily and noncausally** on the channel inputs:

- Decision on whether to erase at time t can depend on \underline{z} formed by passing \underline{x} through a BEC with erasure probability q
- **Adversary's budget:** at most np erasures in codeword of length n .

“Efficient” coding schemes

To get **polynomial complexity**, use

- **a small amount of randomization** to select from a
- **library of random linear codes** and
- uses **list decoding** to reduce the search space

There are different types of complexity we would like to control:

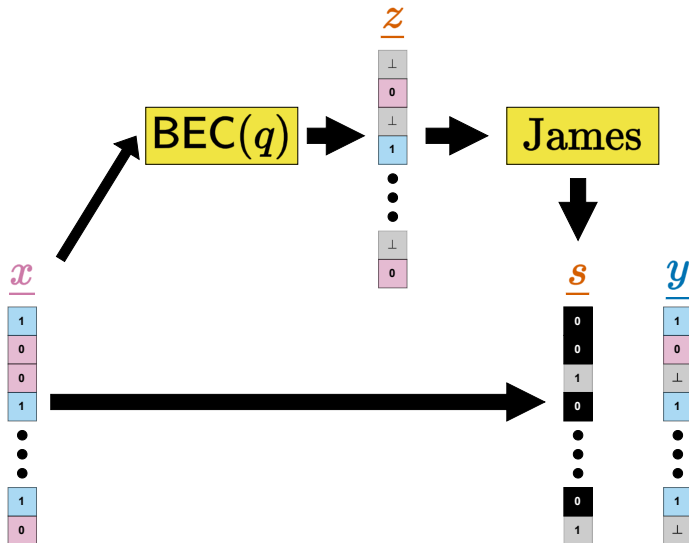
- **Design**: how many bits do we need to generate the code?
- **Storage**: how many bits do we need to store the code?
- **Encoding**: how many operations are needed to encode a message?
- **Decoding**: how many operations are needed to decode the message?

Main results

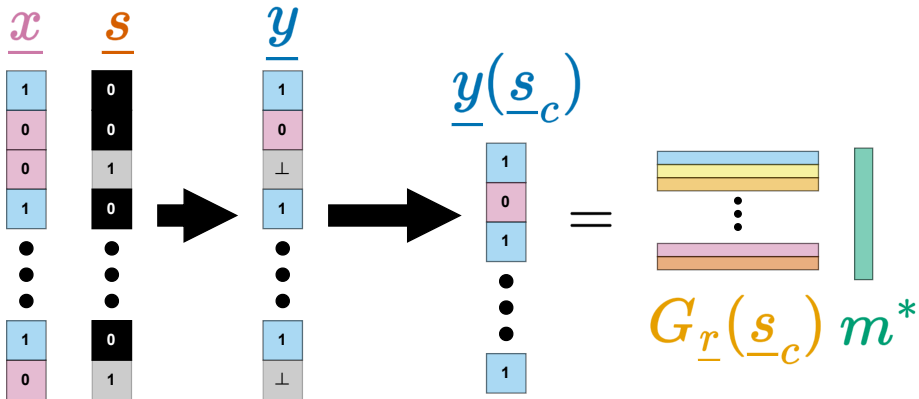
Model rate	Randomness	Enc/Storage	Decoding	P_{error}
Myopic $p < q$ $1 - p - \epsilon$	$\lambda_{SM} \log(n)$	$O(n^{2+\lambda_{SM}})$	$O(n^{3+\lambda_{SM}})$	$O(n^{-\lambda_{SM}})$
Myopic $q < p$ small rate	$O(n \log \log n)$	$O(n^2 \log \log n)$	$O(n^3 \log \log n)$	$O(n^{-4/5})$
Causal $1 - 2p - \epsilon$	$O\left(\frac{\gamma \log n}{\epsilon}\right)$	$O(n^3 \log \log n)$	$O(n^{32/\epsilon})$	$O(n^{-(\gamma-1)})$

Sufficiently myopic adversaries

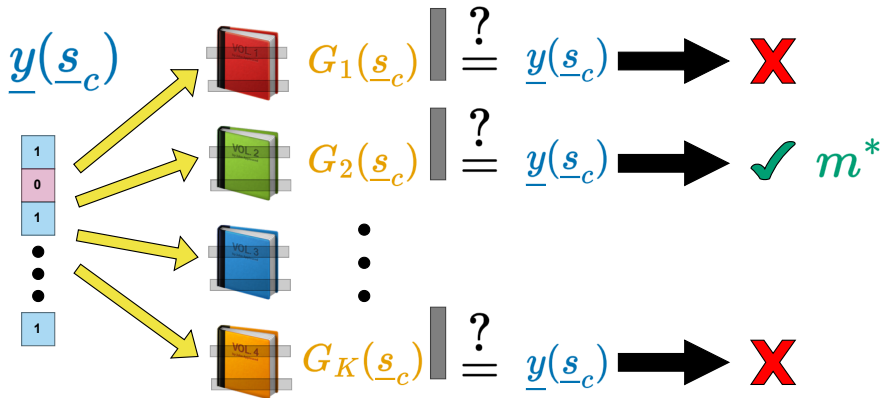
James sees an erased version of the codeword



Look at “unerased” rows of codebook

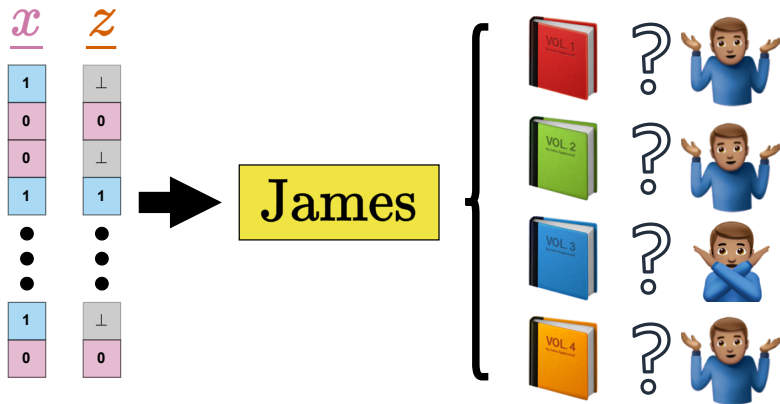


Decoder just tries every codebook



Complexity: n^3 per codebook, $K = n^{n\lambda_{SM}}$ codebooks.

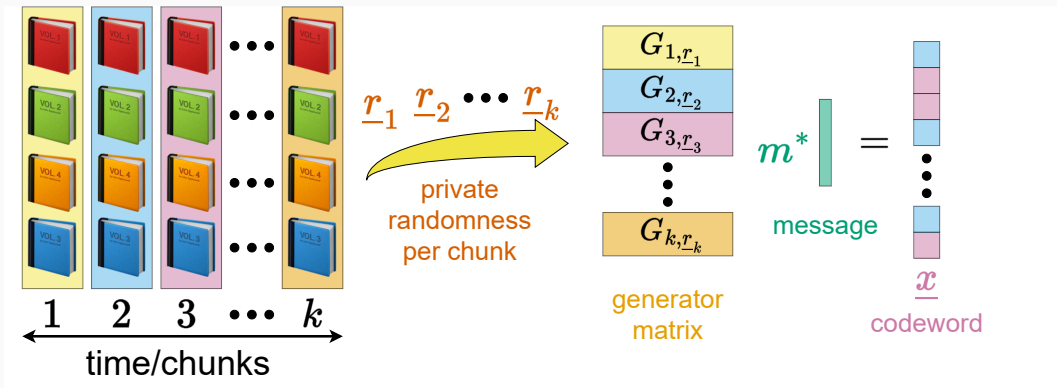
Myopia helps: if $q > p$, James cannot guess the correct codebook



$$P_{\text{error}} = O\left(\mathbf{n}^{-\lambda \mathbf{SM}}\right)$$

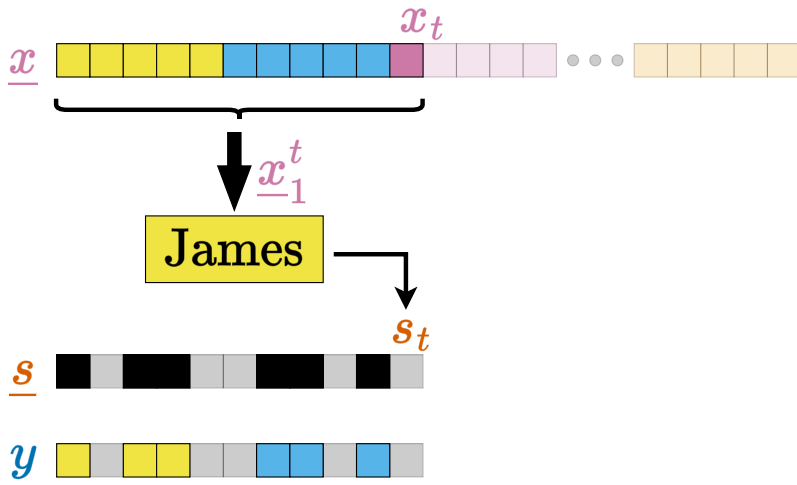
Causal adversaries

Encode splits block into a constant $k = \lceil \frac{n}{\epsilon} \rceil$ chunks

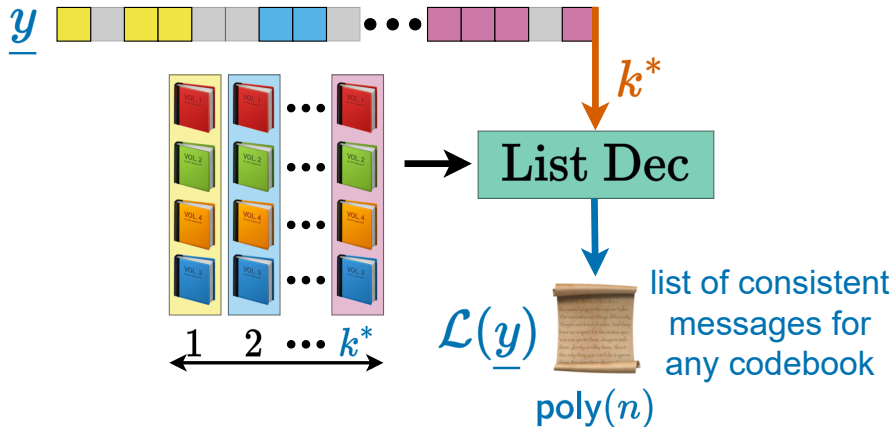


Generate a library of linear codebooks independently for each chunk.

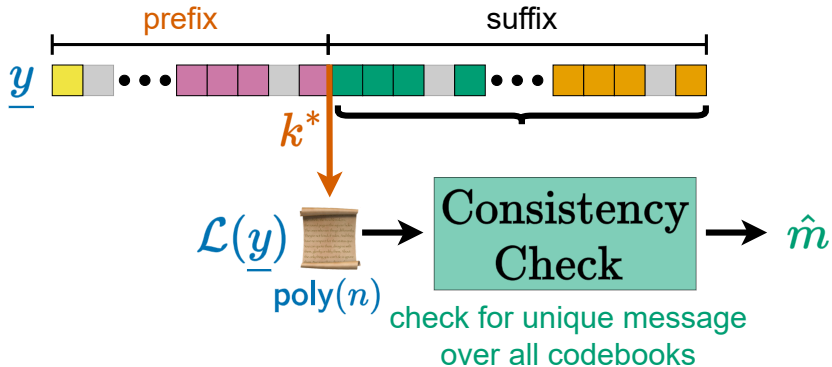
James can erase with causal information only



Bob decodes to a polynomial list after a certain time



Bob uses suffix to disambiguate the list



Why does this work?

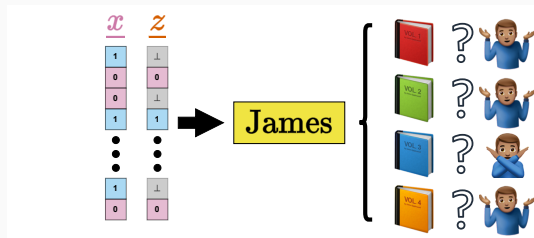
1. Bob can **track James's erasure budget**.
2. List decoding creates **a smaller set of messages** to check for consistency.
3. James has a choice to **make the list larger** (erase more earlier, less later) or **conserve his budget** (erase less earlier, more later).
4. **Poor James, he can't win.**

Recap and next steps

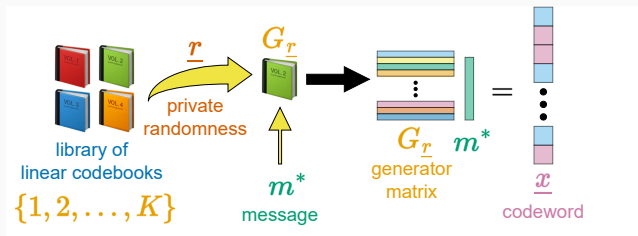
We design efficient (polynomial time) codes for both causal and myopic models.

- Use libraries of linear codebooks for efficient decoding.
- Use limited encoder randomization to confuse the adversary.
- Use list decoding to permit efficient decoding.

Open questions and future directions



- Other adversary structures?
- Better degree for “polynomial”?
- Better error guarantees?
- General AVC models?



Thank you!