Communication against restricted adversaries: between Shannon and Hamming

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Reliable communication, revisited

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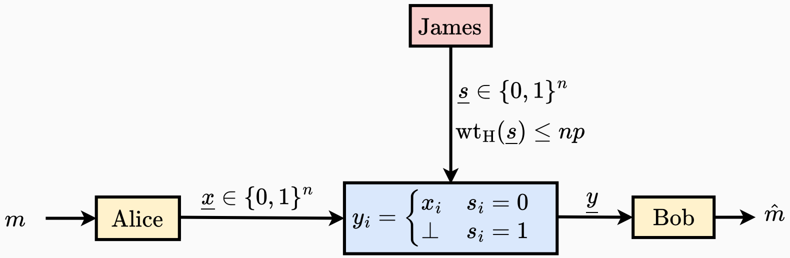
Let's zoom in on binary channels with erasures.

Binary input channels with erasures



- Alice encodes a message $m \in \{1, 2, \ldots, 2^{nR}\}$ into a codeword $\underline{x} = \{0, 1\}^n$.
- The channel *erases* bits: s_i indicates whether $y_i = x_i$ or is erased. Only np erasures can happen during the block.
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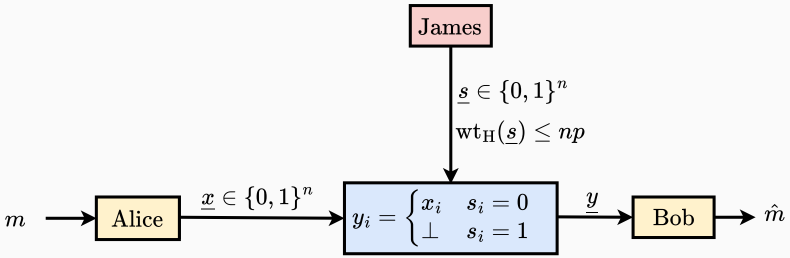

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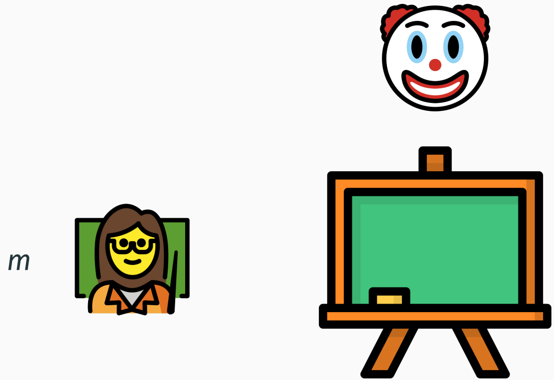
How does James choose $\underline{s}$?

Shannon theory: James is oblivious



Shannon theory studies the binary erasure channel (BEC): $\underline{s} \overset{\text{i.i.d.}}{\sim}$ Bernoulli(p) so with high probability $\approx pn$ positions are erased.

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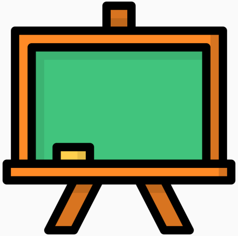

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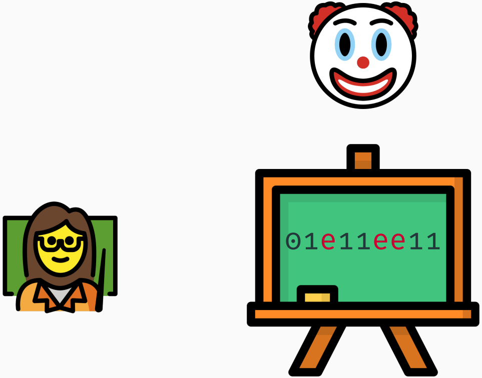

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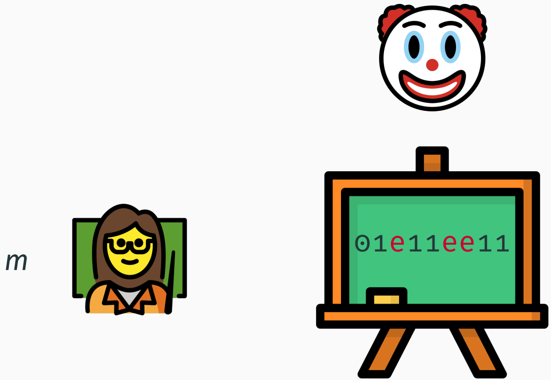

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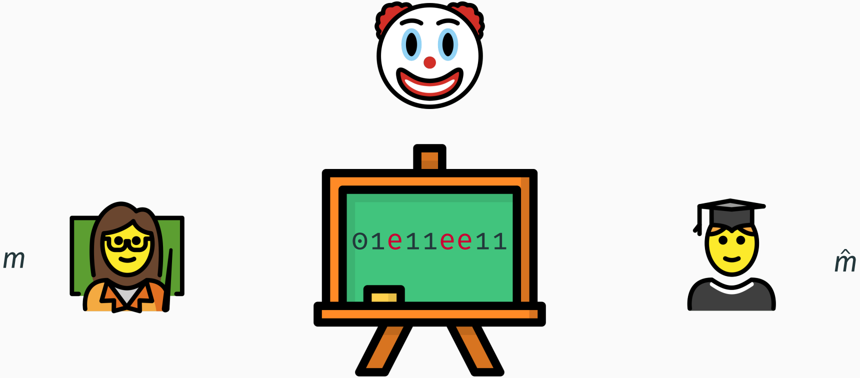

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Coding theory (“Hamming”): James is omniscient

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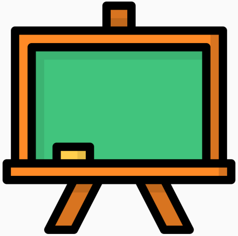

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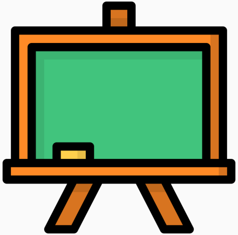

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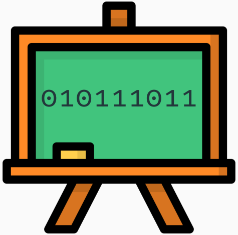
m  

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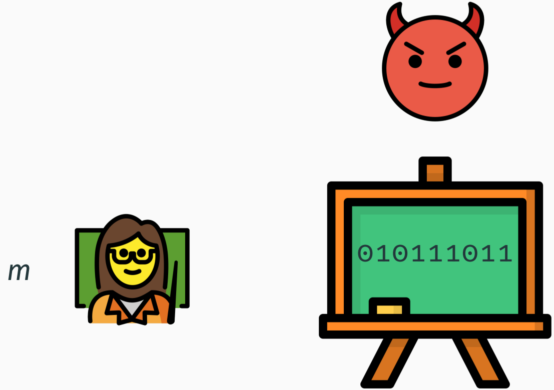

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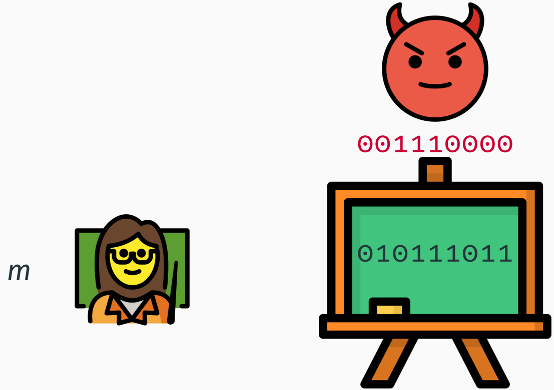

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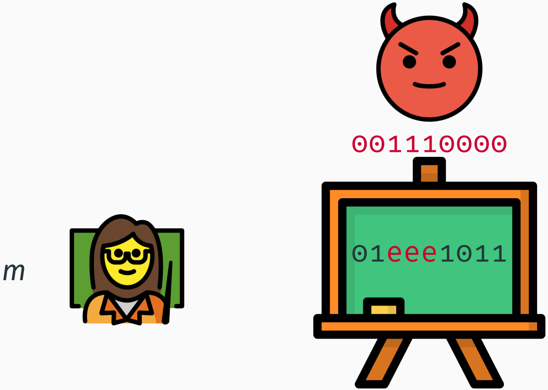

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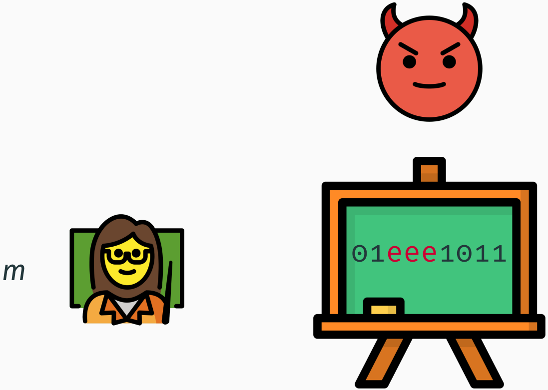

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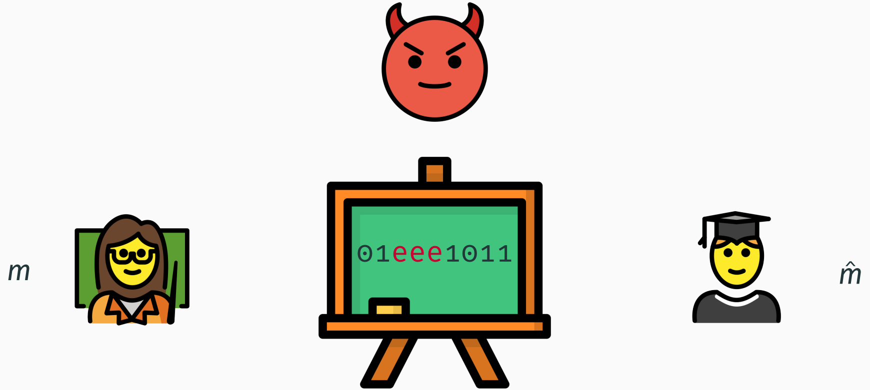

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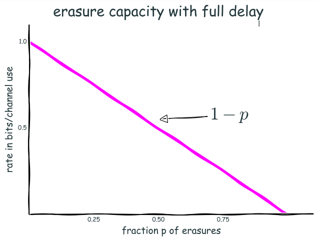
The erasure channel: adversarial vs. random



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There are many different ways to achieve this rate.

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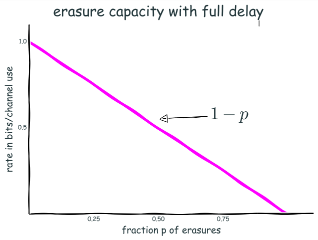

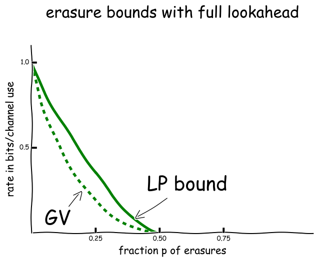

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Lower bound: Gilbert-Varshamov (random) codes.

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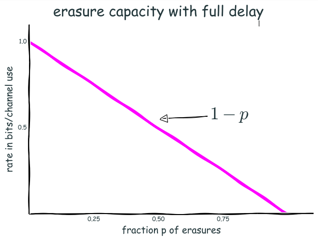

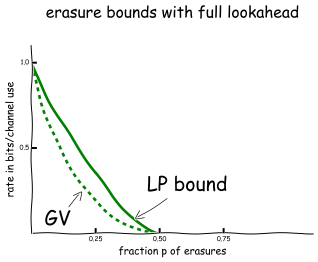

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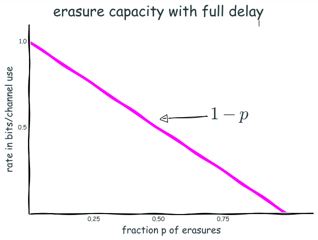

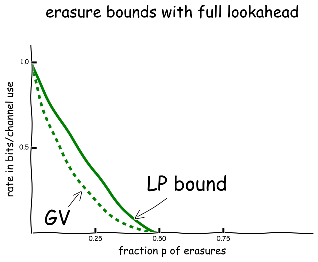

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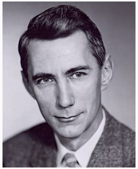
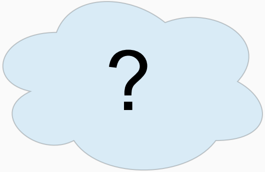
That's a big gap... where does it come from?

Why more is there to explore here?

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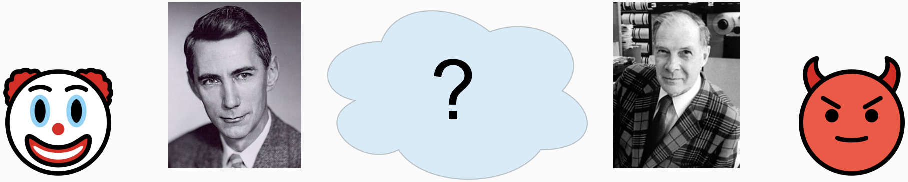

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1. Use **arbitrarily varying channels (AVCs)** to develop a **unified framework** for both the Shannon and Hamming models.
2. Explore **intermediate models** to see **what causes the gap**.

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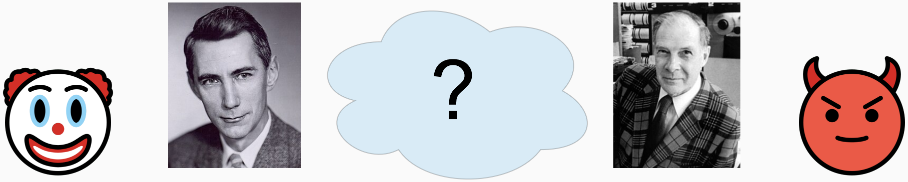

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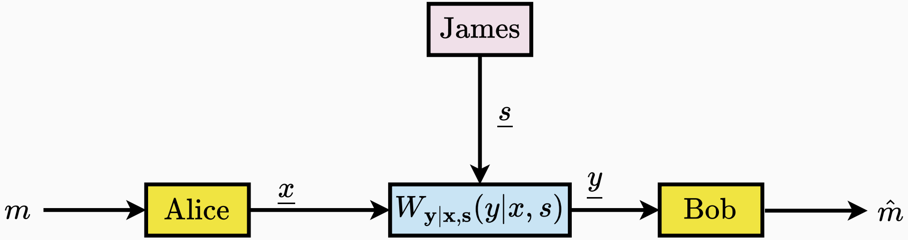
1. Use **arbitrarily varying channels (AVCs)** to develop a **unified framework** for both the Shannon and Hamming models.
2. Explore **intermediate models** to see **what causes the gap**.
3. Discover **coding strategies** to see what **resources are needed** to communicate reliably.

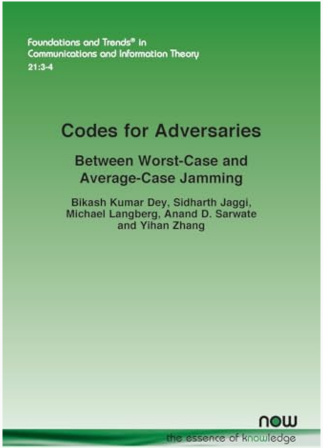
AVCs model channel "noise" as a state variable



In an **adversarial channel model**, **Alice** wants to communicate with **Bob** over a channel whose time-varying state is controlled by an adversarial **jammer** James.

- Alice and James may be **constrained** in how they communicate.
- Capacity depends on **what James knows** about m and $\underline{x}$.

Shameless self-promotion



This talk is based on a recent (December 2024) monograph: check it out!

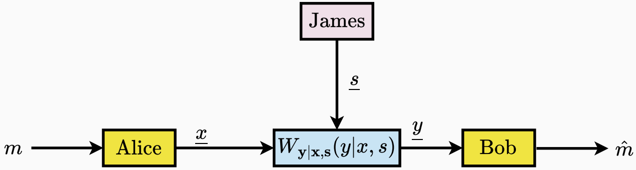
- ✔ Unified treatment of random noise (Shannon-theoretic) and worst-case noise (coding-theoretic).
- ✔ Intermediate models for jammers who can eavesdrop: online and myopic.
- ✔ Examples, open problems, and more!

What’s coming up next

1. Arbitrarily varying channels (AVCs)
2. Some key ingredients
3. Causal adversarial models
4. Myopic adversarial models
5. Computationally efficient codes for causal adversaries
6. Looking forward

Arbitrarily varying channels (AVCs)

The basic channel model



Let $\mathcal{X}$, $\mathcal{S}$, and $\mathcal{Y}$ be discrete alphabets. An AVC is a discrete channel $W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y|x,s)$ such that

$$W_{\underline{\mathbf{y}}|\underline{\mathbf{x}},\underline{\mathbf{s}}}(\underline{y}|\underline{x},\underline{s}) = \prod_{i=1}^{n} W_{\mathbf{y}|\mathbf{x},\mathbf{s}}(y_i|x_i,s_i)$$

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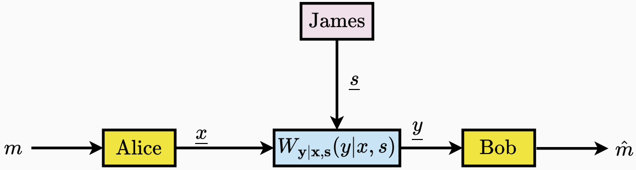

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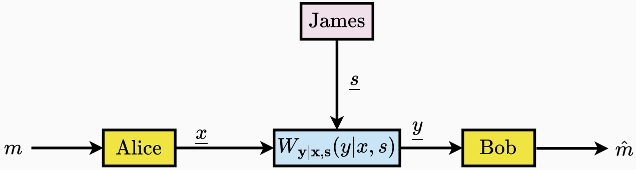

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The **state** $\underline{s} \in \mathcal{S}^n$ is controlled by an adversarial **jammer** (James).
Examples: For binary channels $\underline{s}$ could be an error or erasure pattern.

Input and cost constraints for AVCs

We impose that the types $T_{\underline{x}}$ and $T_{\underline{s}}$ of the codeword $\underline{x}$ and the state $\underline{s}$ lie be in convex subsets of the probability simplices $\Delta(\mathcal{X})$ and $\Delta(\mathcal{S})$:

$$T_{\underline{x}} \in \Gamma \subseteq \Delta(\mathcal{X})$$

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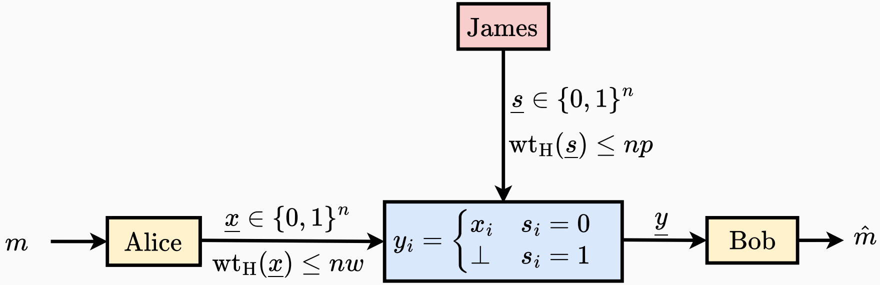
Example: For binary channels $\underline{x}$ and $\underline{s}$ have bounded Hamming weight.



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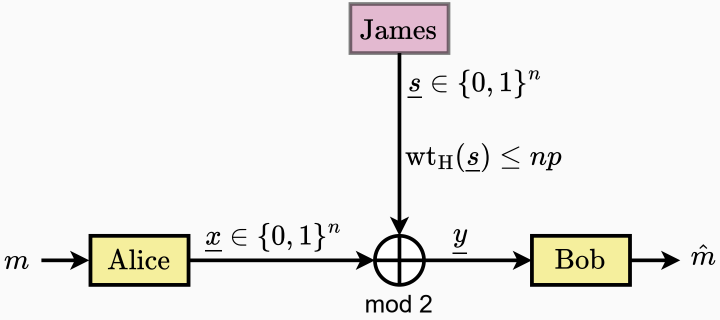
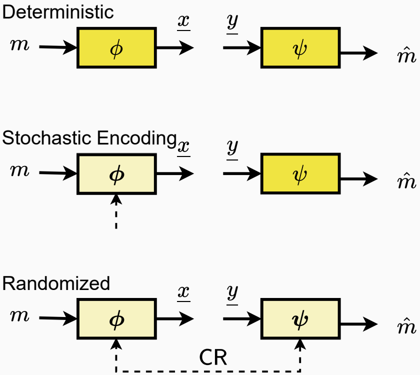


Defining codes and input constraints



An (n, M, Γ) code is

$$\begin{aligned} \phi &: [M] \to \mathcal{X}^n && \text{(encoder)} \\ \psi &: \mathcal{Y}^n \to [M] && \text{(decoder)} \end{aligned}$$

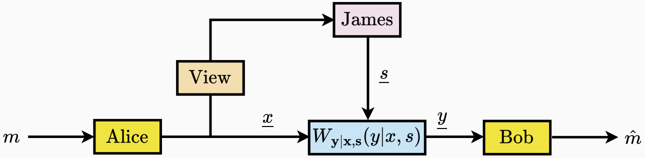
such that

$$T_{\phi(m)} \in \Gamma$$

The rate is $R = \frac{1}{n} \log_2(M)$.

A **randomized code** lets Alice and Bob choose their code in secret. If Alice and Bob do not share common randomness, Alice can still use **stochastic encoding**.

What James knows: Shannon, Hamming, and in between



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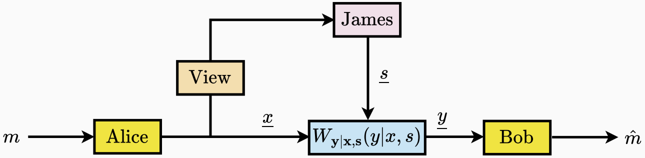

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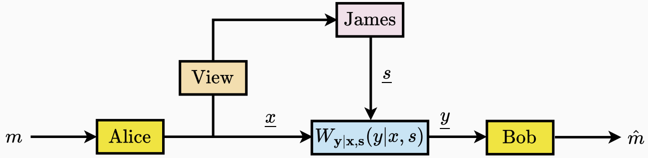

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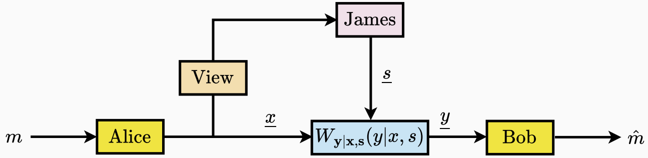

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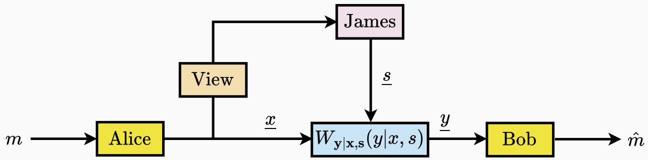

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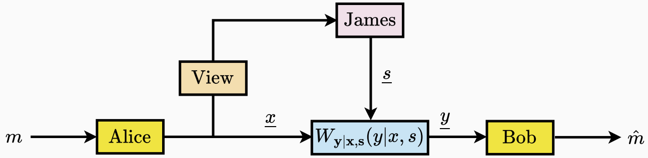

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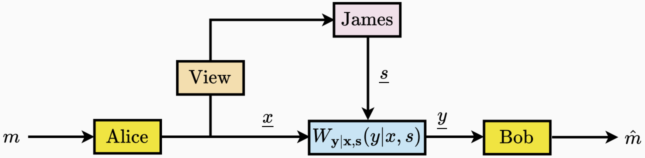

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- **Omniscient** (Hamming): the message and the codeword.

Maximal error and capacity

The **error** for a particular message m is

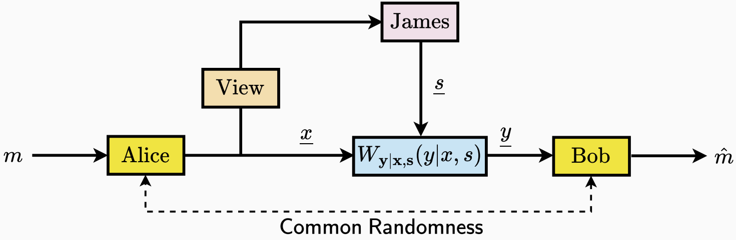
$$P_{\mathrm{err}}(m, \phi, \psi) = \max_{\text{jamming strategies}} \sum_{\mathbf{x} \in \mathcal{X}^n} \mathbb{P}\left(\psi(\mathbf{y}) \neq m \mid \mathbf{x}\right) \mathbb{P}_{\phi}\left(\phi(m) = \mathbf{x}\right)$$

A rate R is **achievable** if for any $\epsilon > 0$ there exists an infinite sequence of rate R codes ($n \to \infty$) such that $P_{\mathrm{err}}(m, \phi, \psi) < \epsilon$ for all m.

The capacities C_{obl} and C_{omni} for oblivious and omniscient cases satisfy:

$$(\textit{Hamming}) \qquad C_{\mathrm{omni}} \qquad \leq \qquad C_{\mathrm{obl}} \qquad (\textit{Shannon})$$

Common randomness makes the problem easier



Blackwell et al. (1960) proposed the AVC model and studied **randomized codes**, where Alice and Bob share common randomness. James just minimizes the mutual information over equivalent DMCs:

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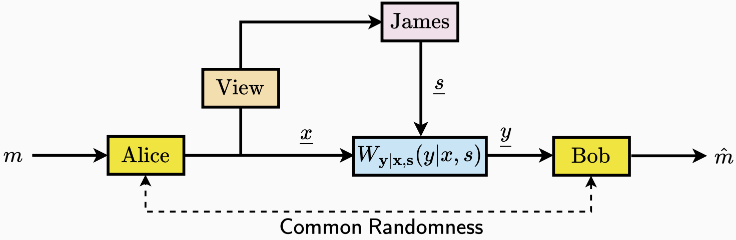

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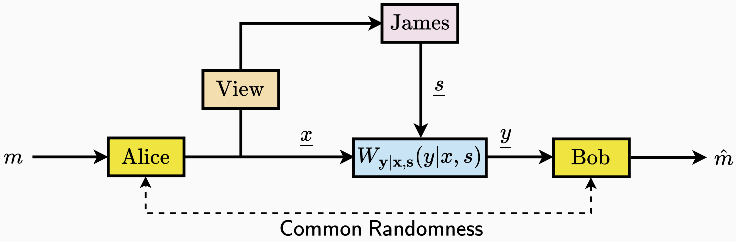

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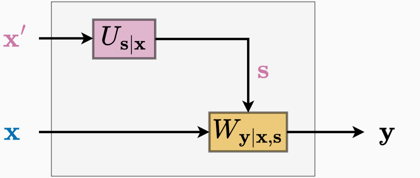
Without Common Randomness: Symmetrization Attacks

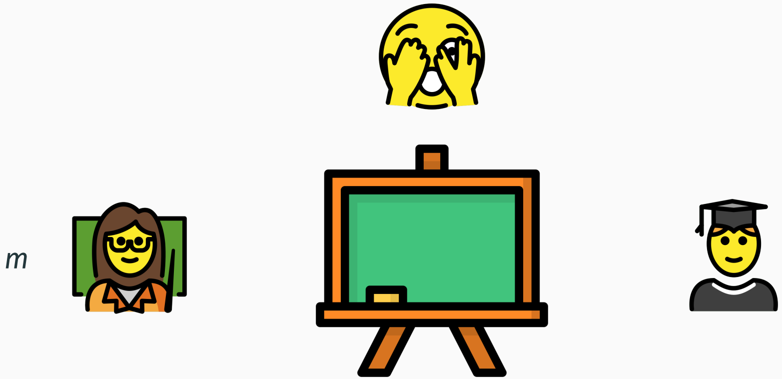


An AVC is **Ericson-Csiszár-Narayan (ECN) symmetrizable** if James can spoof Alice's codeword. That is, for all $(\mathbf{y}, \mathbf{x}, \mathbf{x}')$, we have

$$\sum_{s} U_{\mathbf{s}|\mathbf{x}'} W_{\mathbf{y}|\mathbf{x},\mathbf{s}} = \sum_{s} U_{\mathbf{s}|\mathbf{x}} W_{\mathbf{y}|\mathbf{x}',\mathbf{s}}.$$

Without common randomness, the capacity of a symmetrizable AVC $C_{\mathrm{obl}} = 0$.

Intermediate model 1: delayed information for James



Delayed: James gets a delayed view of the transmitted codeword.

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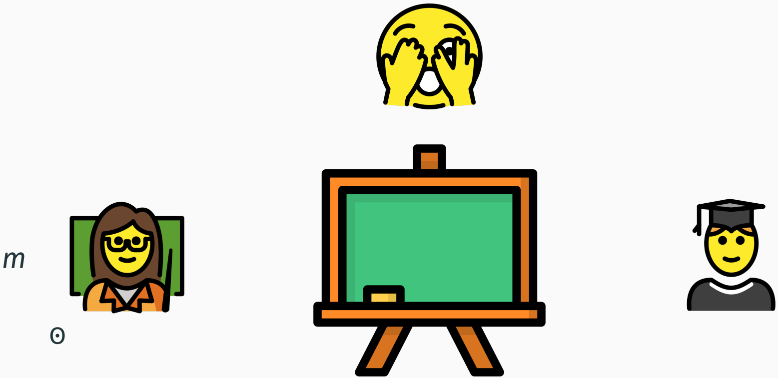

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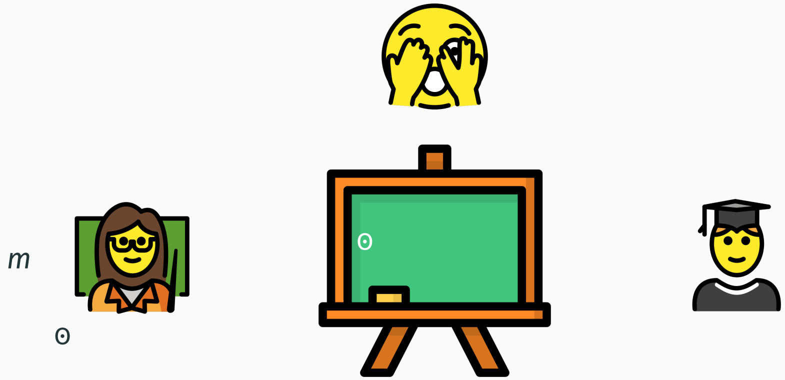

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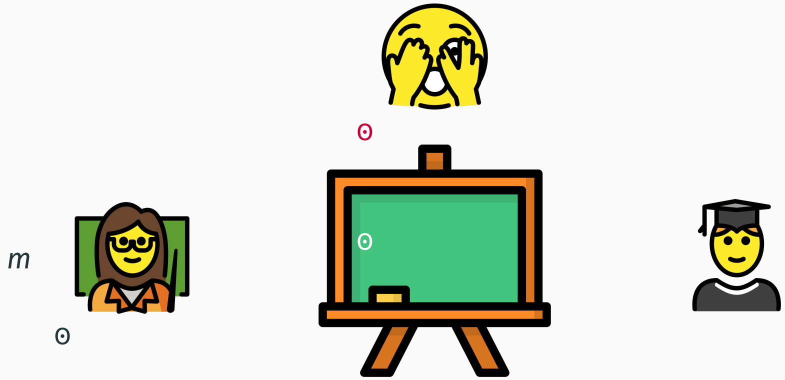

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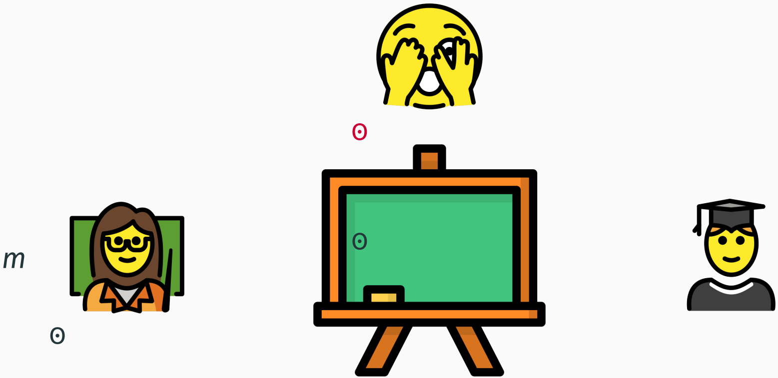

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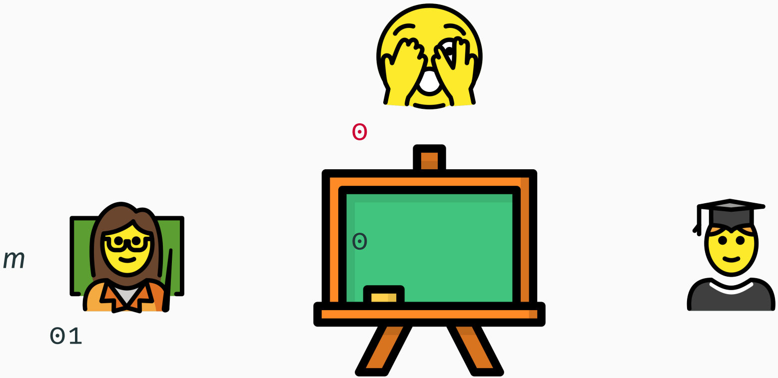

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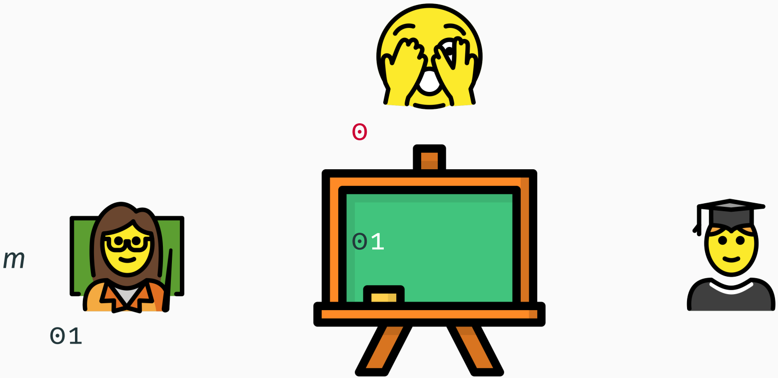

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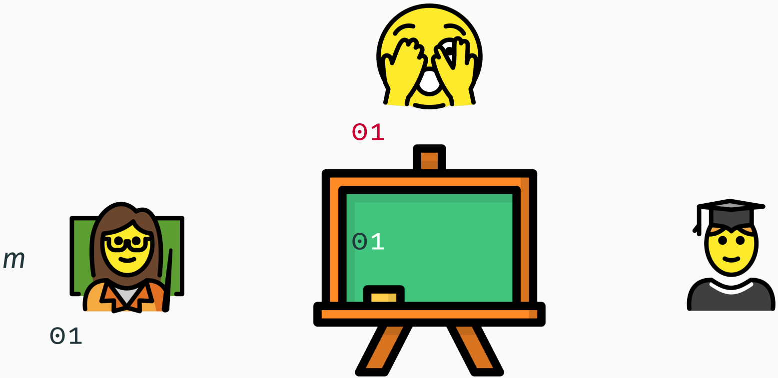

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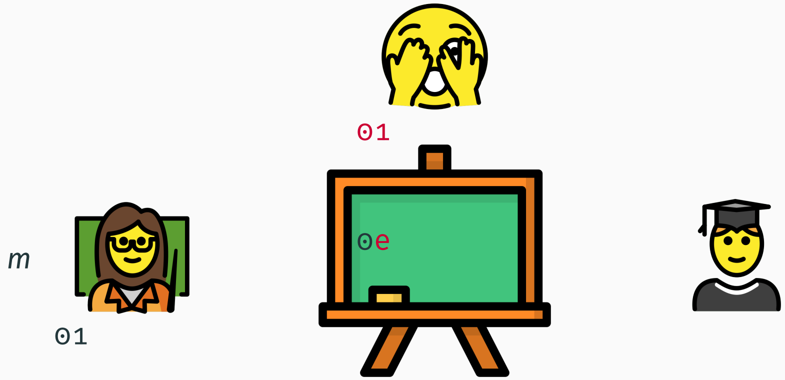

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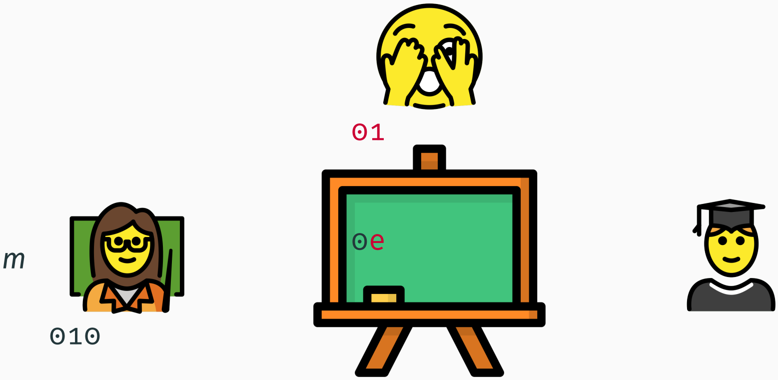

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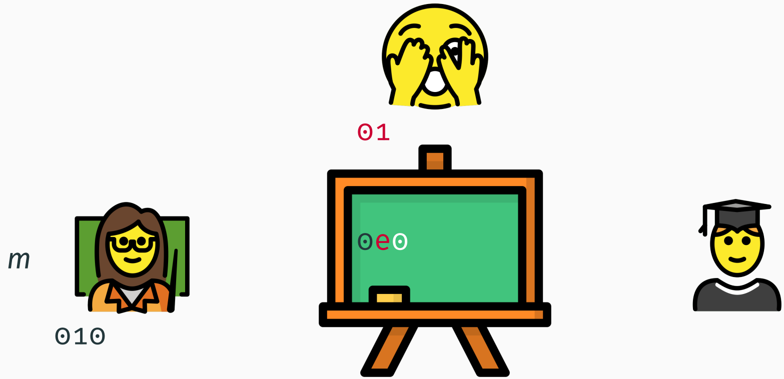

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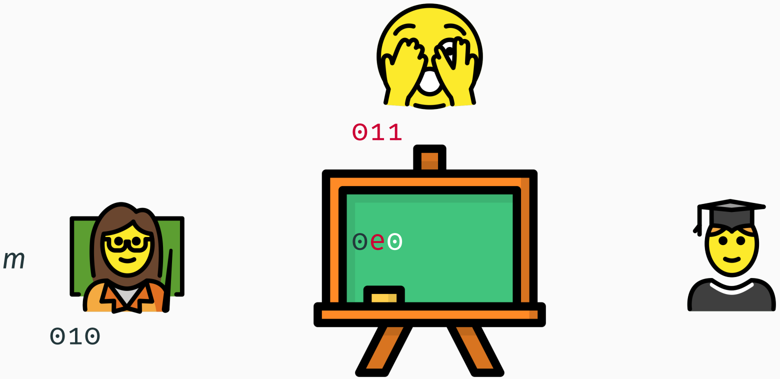

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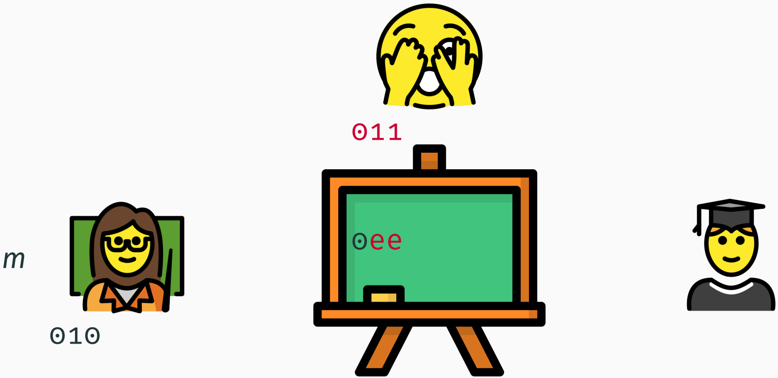

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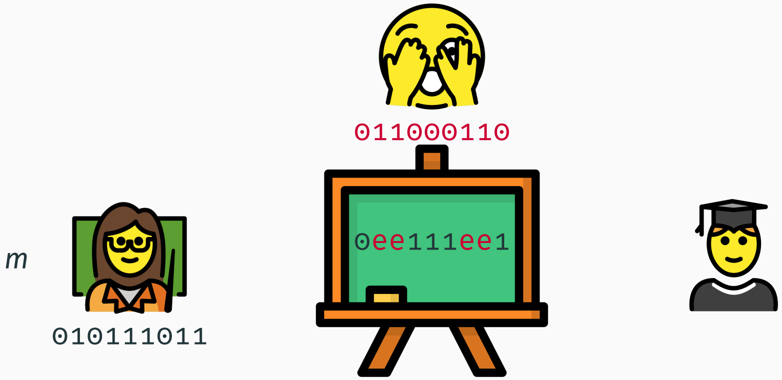

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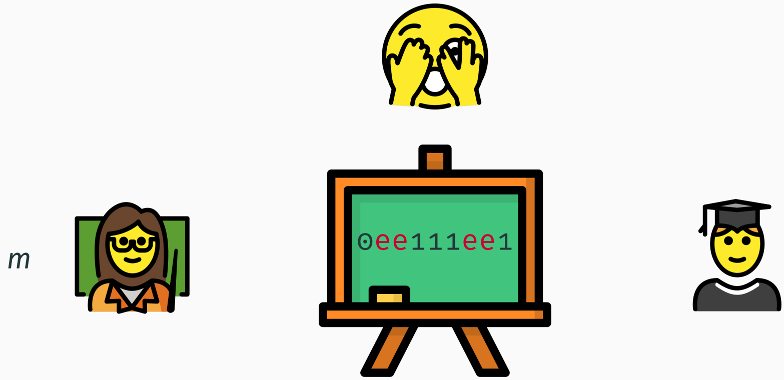

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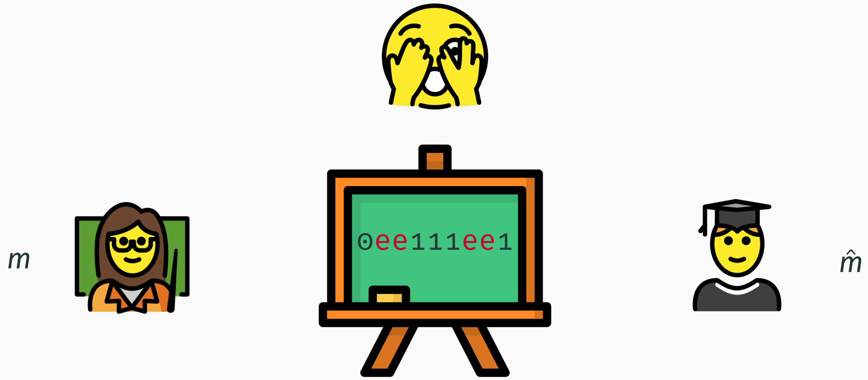

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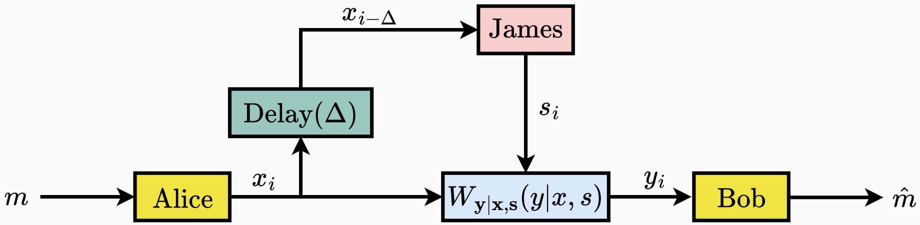
Delay interpolates between oblivious and omniscient



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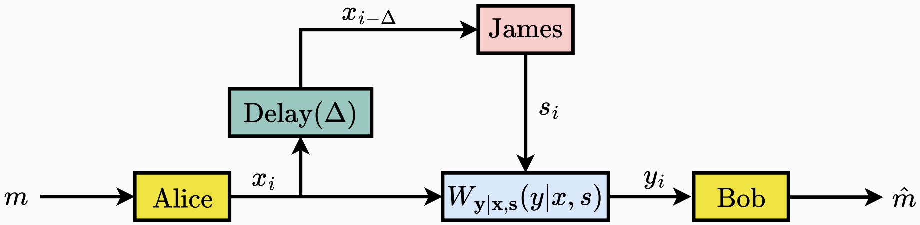

- $\Delta = n$ **(oblivious):** capacity $= 1 - p$ ("Shannon")

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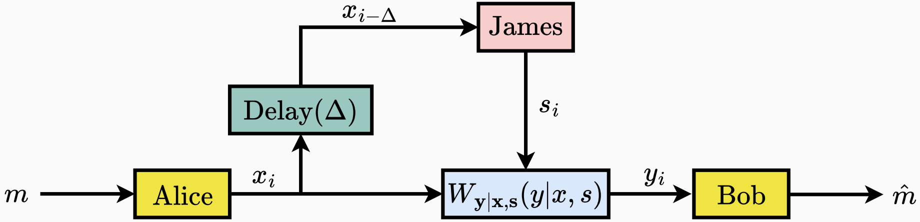

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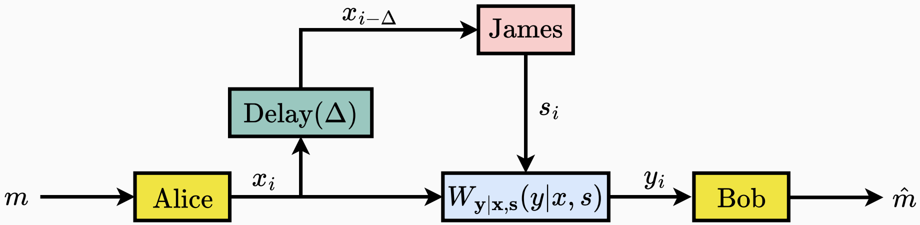

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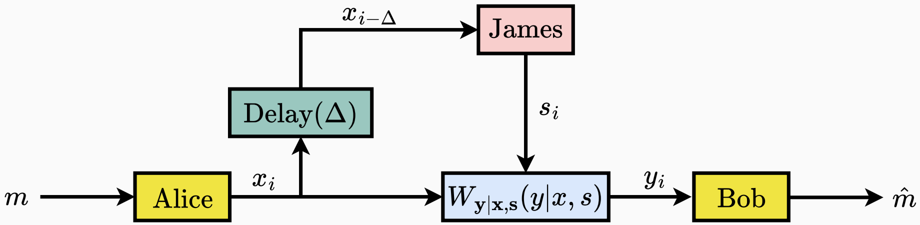

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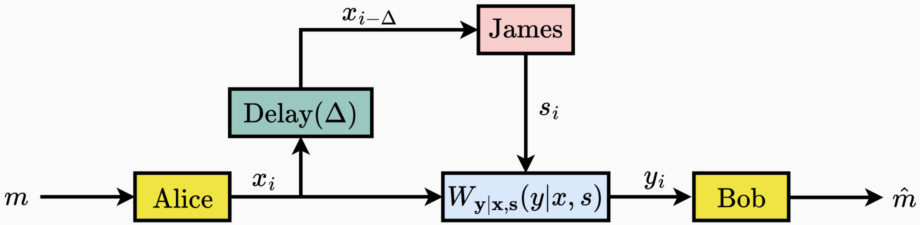

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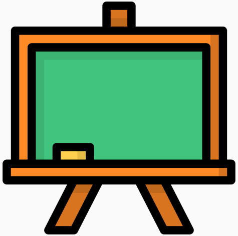
Knowing just the current input gives James a lot of power!

Intermediate model 2: Myopic adversarial models

m 



Myopic: James gets a noisy view of the transmitted codeword.

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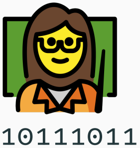
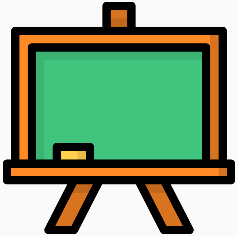
m





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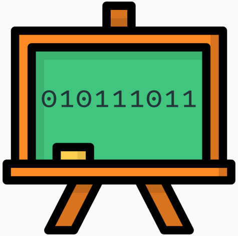

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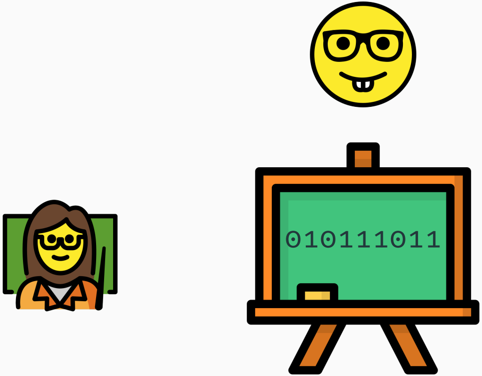

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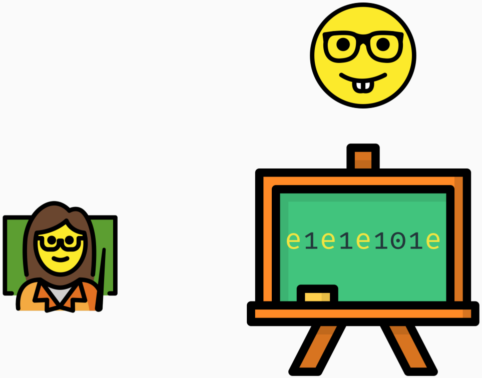

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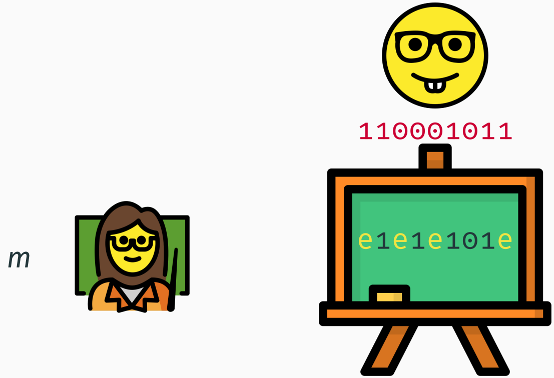

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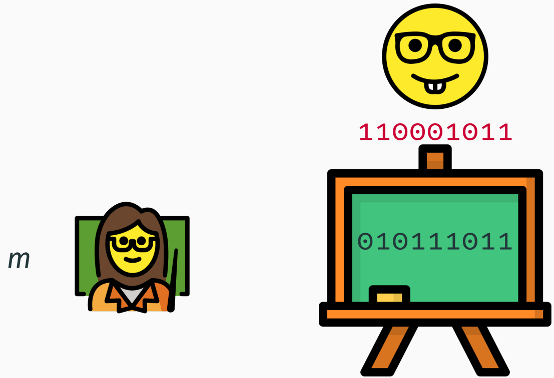

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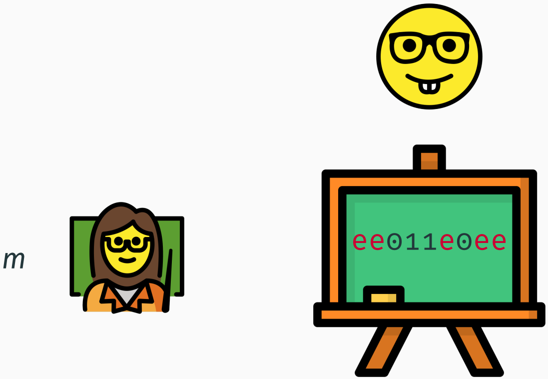

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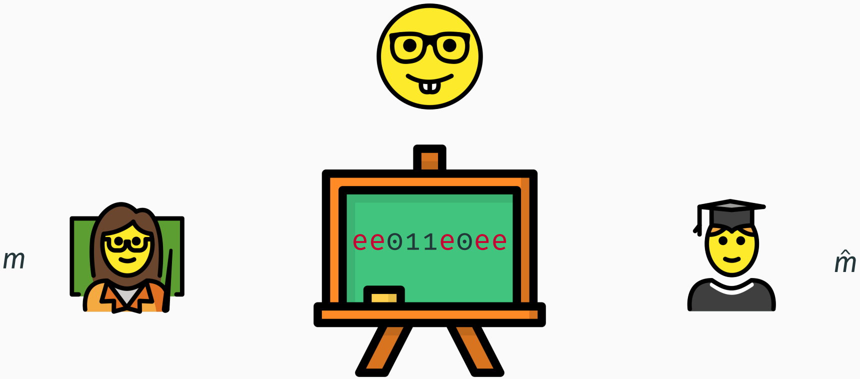

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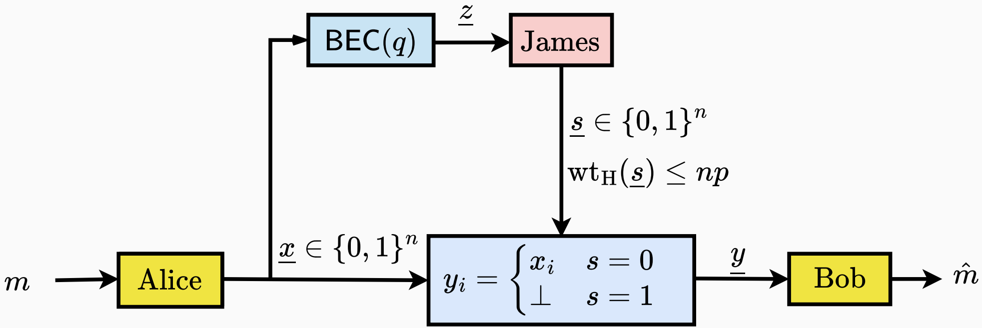
The impact of myopia in the erasure setting



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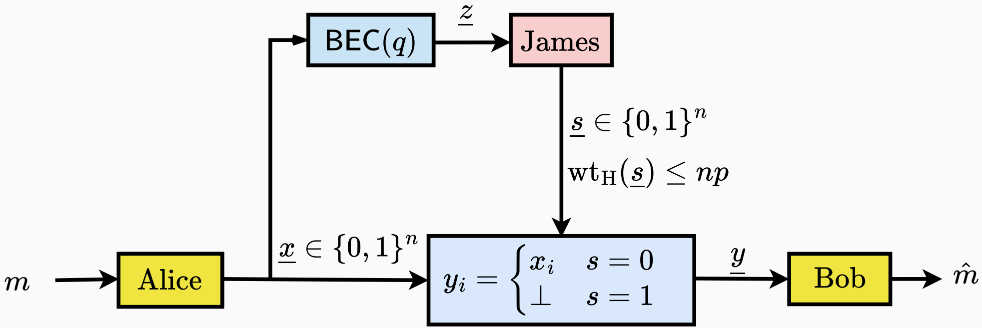

- **Sufficiently myopic:** ($p < q$): capacity $= 1 - p$

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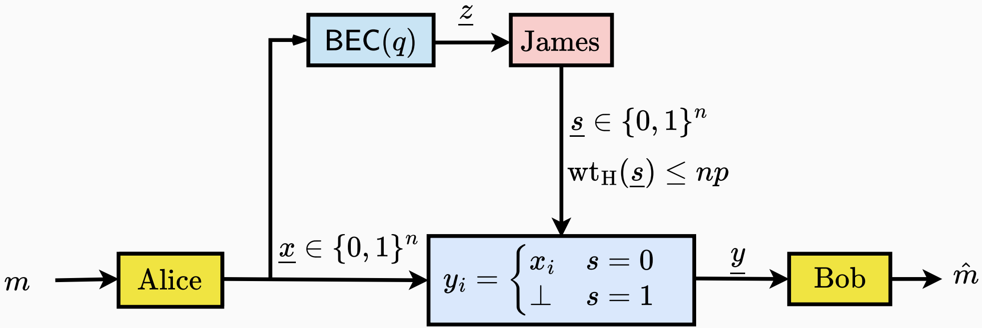
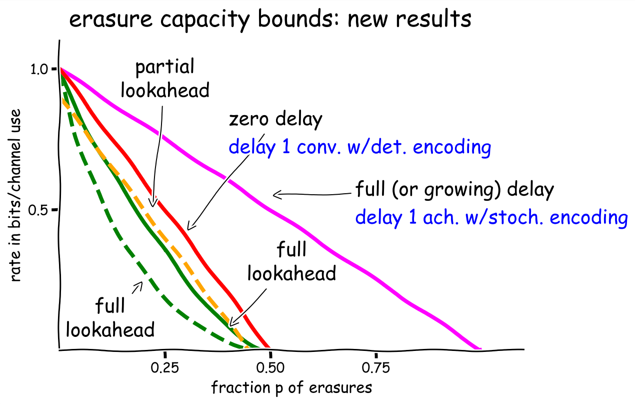


- **Sufficiently myopic:** ($p < q$): capacity $= 1 - p$
- **Otherwise:** ($p > q$): it's more complicated...

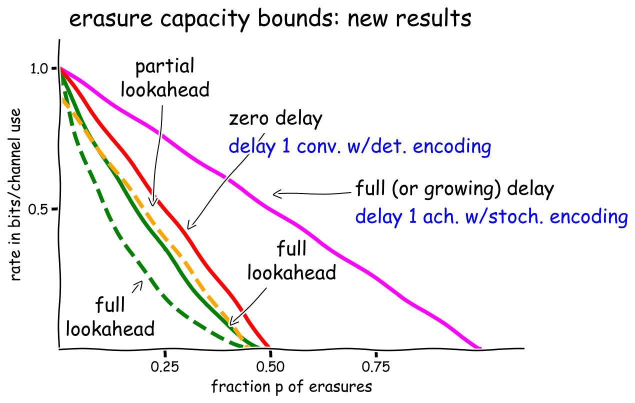
Some key ingredients

Ingredient 1: stochastic encoding



In **stochastic encoding**, Alice uses private randomness to create uncertainty for James

Ingredient 1: stochastic encoding



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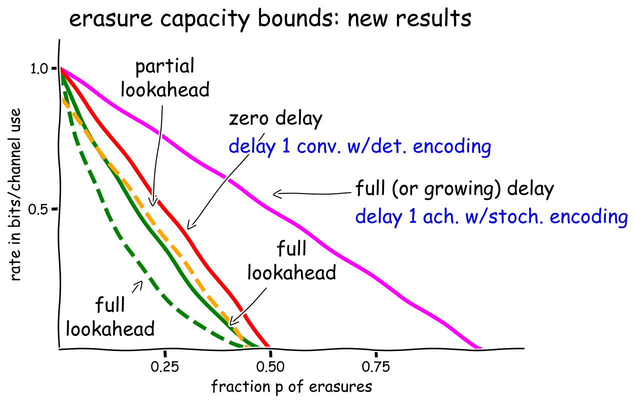

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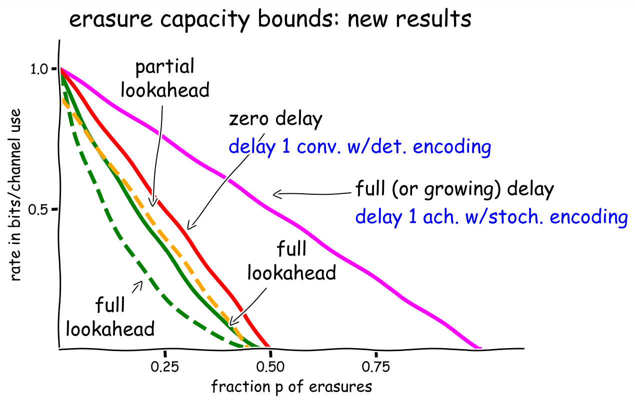

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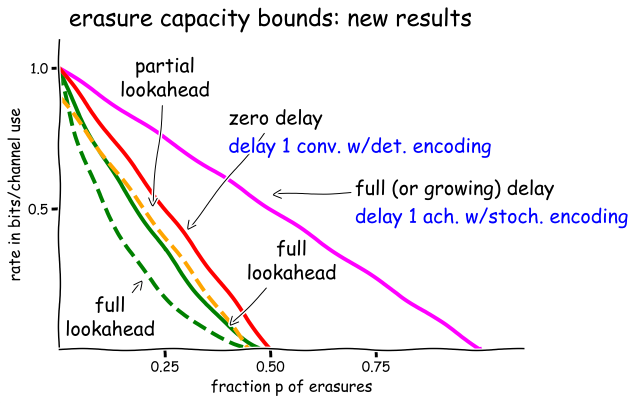

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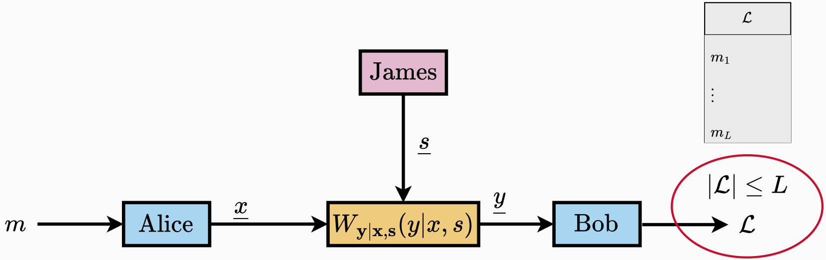
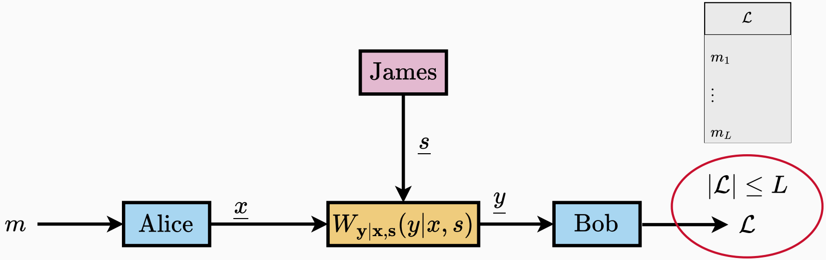
It can be **necessary**: deterministic erasure codes cannot do better than $1 - 2p$ against a James who has a single bit of delay.

Ingredient 2: list decoding



In **list decoding** we allow Bob to output a list $\mathcal{L}$.

Ingredient 2: list decoding



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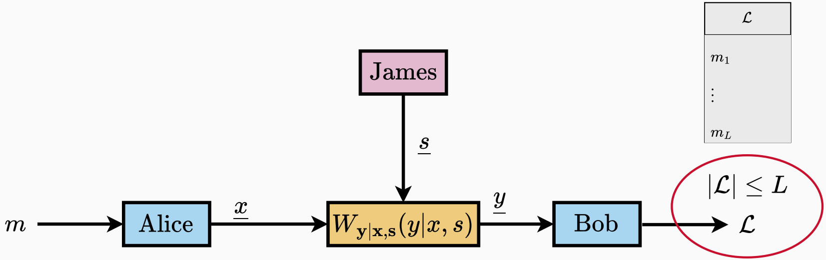

In **list decoding** we allow Bob to output a list $\mathcal{L}$.

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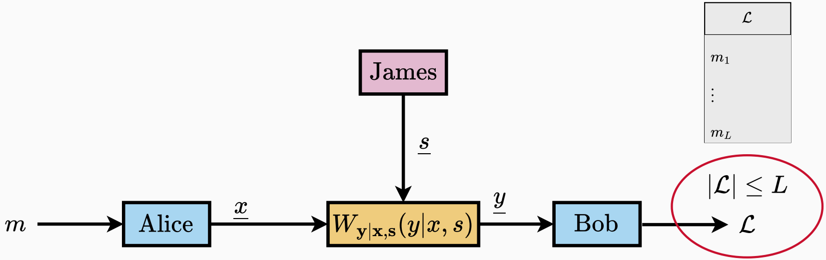
- Require the list size is no larger than than L.

Ingredient 2: list decoding



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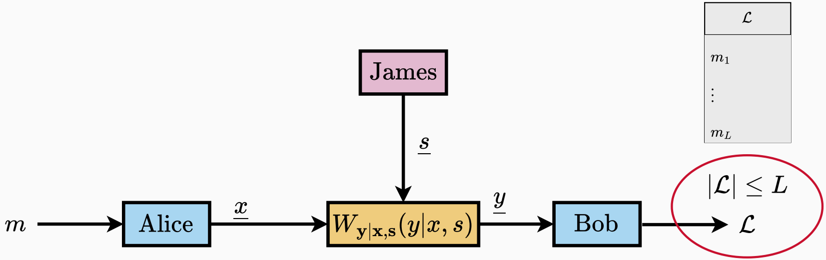

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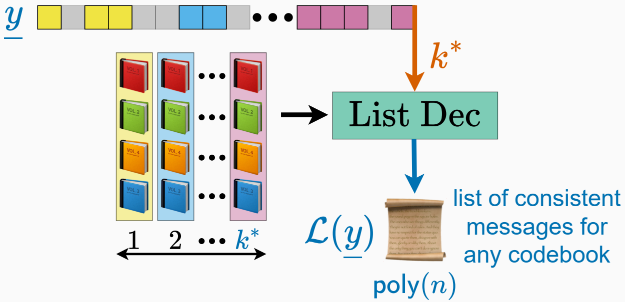
In some cases the list decoding capacity allows **strictly larger** rates:

$$C_{\mathrm{list}}(L) > C_{\mathrm{obl}}.$$

List decoding appears in many ways



List decoding to a “small list” is useful in decoding and jamming:

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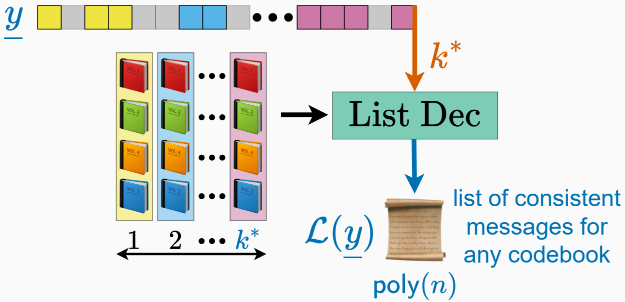

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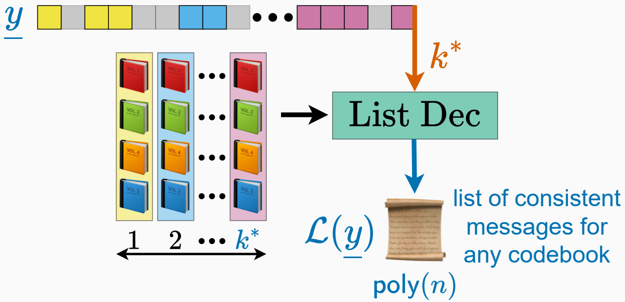

List decoding to a "small list" is useful in decoding and jamming:

- List codes mean $O(\log n)$ bits of common randomness is sufficient.
- James can list decode to jam more effectively.

Ingredient 3: Completely Positive (CP) Couplings and the Plotkin Bound

A more technical ingredient which is particularly useful is the notion of **completely positive couplings**.

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A more technical ingredient which is particularly useful is the notion of **completely positive couplings**.

- Start with a marginal distribution $P_{\mathbf{x}} \in \Delta(\mathcal{X})$.
- A **self-coupling** is a joint distribution $P_{\mathbf{x},\mathbf{x}'}$ where each marginal is $P_{\mathbf{x}}$.
- A self-coupling is **completely positive** if it is a mixture of independent self-couplings:

$$P_{\mathbf{x},\mathbf{x}'}(x, x') = \sum_{i=1}^{|\mathcal{U}|} P_{\mathbf{u}}(i) P_{\mathbf{x}_i}(x) P_{\mathbf{x}_i}(x').$$

Generalizing the Plotkin bound

Question: can we have a codebook where all codewords have pairwise types that are ρ-far from a CP self-coupling?

$$\|T_{\underline{x},\underline{x}'} - P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'}\|_\infty > \rho \qquad \forall \underline{x}, \underline{x}', P^{(\mathrm{CP})}_{\mathbf{x},\mathbf{x}'}$$

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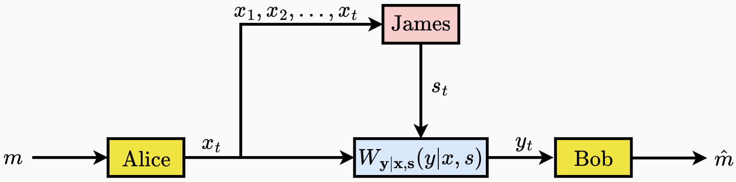
- It turns out that any codes with this property cannot be too large (for large n)!
- Compare this to the Plotkin bound: an upper bound on the size of binary codes with a given distance.
- If our rate is too high, then there will a constant fraction of codeword pairs whose type is close to CP.

Causal adversarial models

Causal adversaries: James can see the current input



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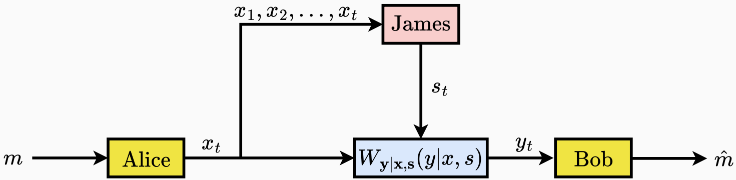

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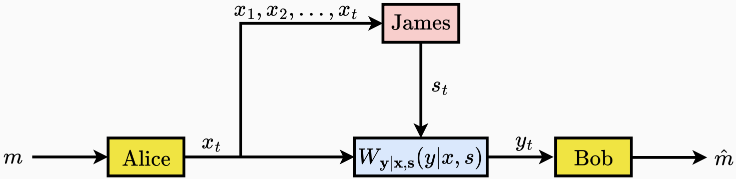
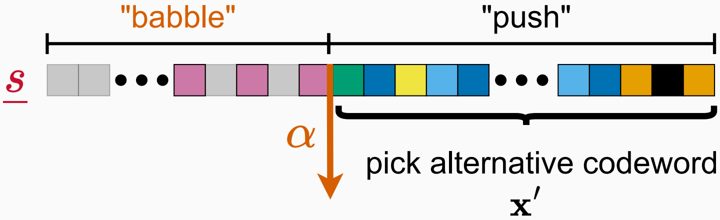


What is "symmetrizability" here and how should James act?

- Spend less power at the beginning to save it up and then push hard in the second half? Bob will get a better initial estimate.
- Spend more power at the beginning in the hope of leading Bob astray? But then the suffix might resolve Bob's uncertainty.

Babble and push: an attack for James



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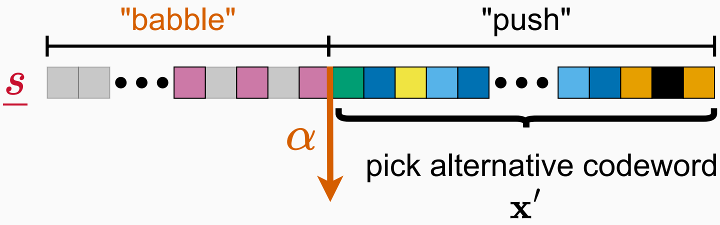

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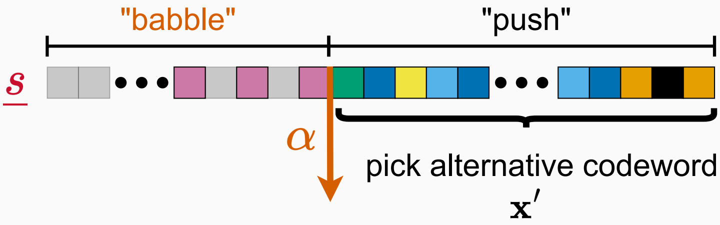

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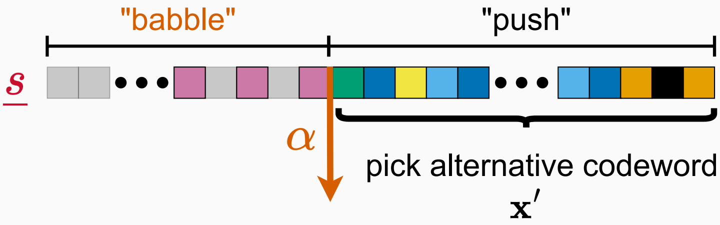

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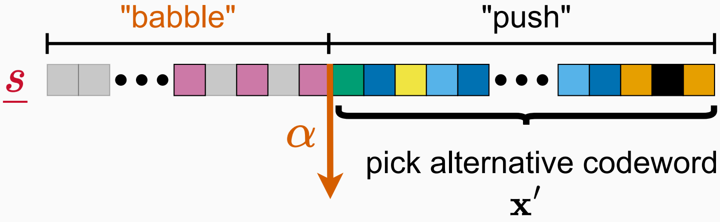

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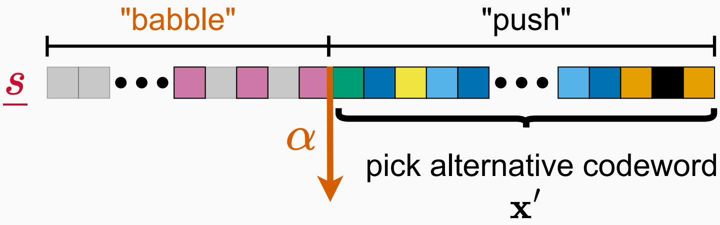

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Use the generalized Plotkin bound (plus more) to show this will work.

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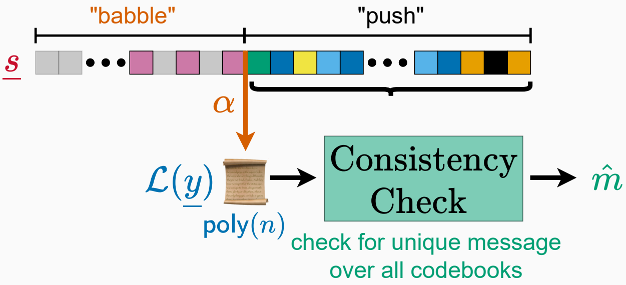

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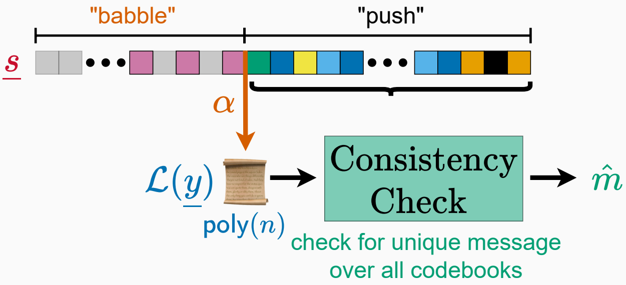

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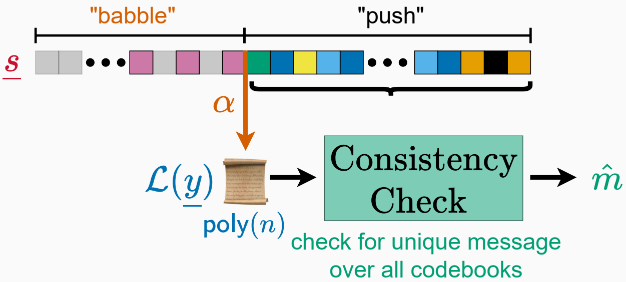

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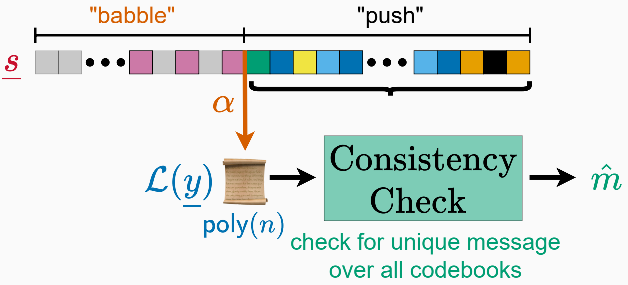

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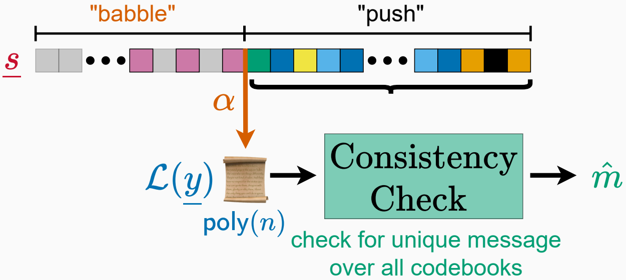

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Basically have to define what “feasible” means in this setting (quite involved).

A multi-letter block characterization

Pros and cons:

A multi-letter block characterization

$$C := \limsup_{K\to\infty} \max_{\substack{P_{\mathbf{x}|\mathbf{u}}\in\Delta(\mathcal{X}|[1:K]) \\ \left[\mathrm{Unif}([K])P_{\mathbf{x}|\mathbf{u}}\right]_{\mathbf{x}}\in\Lambda_{\mathbf{x}}}} \min\left\{ \min_{V_{\mathbf{s}|\mathbf{x},\mathbf{u}}\in\mathcal{F}(P_{\mathbf{x}|\mathbf{u}})} I(P_{\mathbf{x}|\mathbf{u}}, V_{\mathbf{s}|\mathbf{x},\mathbf{u}}), \right.$$

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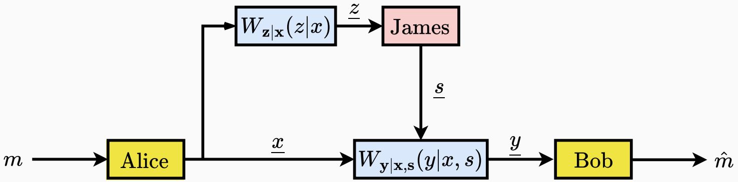
Pros and cons:

- ✗ We end up with a multi-letter expression for the capacity.
- ✔ Significantly generalizes prior arguments to general channels.
- ✔ Plotkin results may be useful elsewhere.

Myopic adversarial models

Myopic adversaries: James sees the whole codeword in noise



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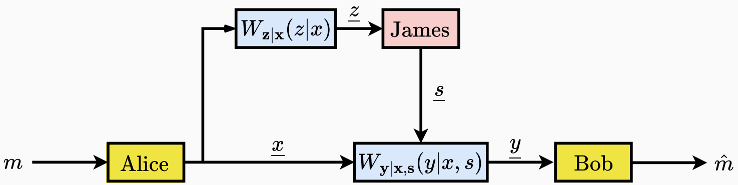

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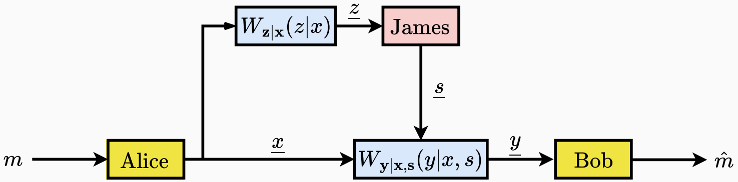

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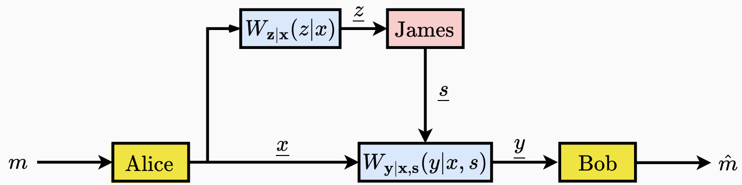

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- By changing $W_{\mathbf{z}|\mathbf{x}}$ we can recover the oblivious and omniscient settings.

Symmetrizability for myopic AVCs

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This means **some input distributions are disallowed**:

$$\mathcal{P}_{\mathrm{Sym}} = \{P_{\mathbf{x}} \in \Gamma : P_{\mathbf{x}} \text{ is symmetrizable}\}.$$

Sufficient myopia and achievability

James can create an "effective DMC"

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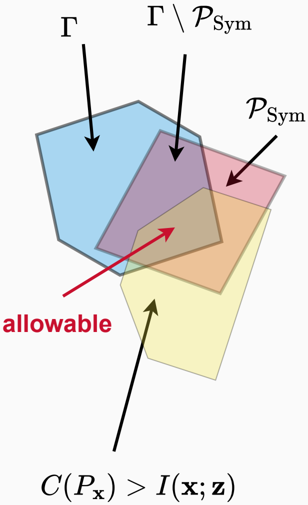

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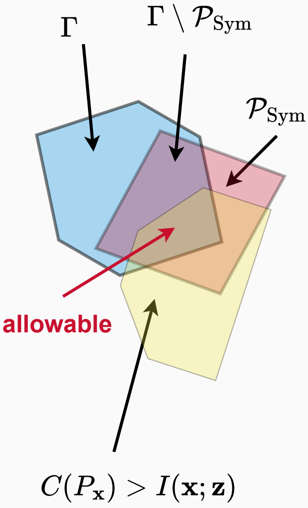
Alice/Bob cannot use any $P_{\mathbf{x}} \in \mathcal{P}_{\text{Sym}}$. If they choose $P_{\mathbf{x}} \in \Gamma \setminus \mathcal{P}_{\text{Sym}}$ they could target the mutual information

$$C(P_{\mathbf{x}}) = \min_{\mathcal{W}} I(\mathbf{x};\mathbf{y}).$$



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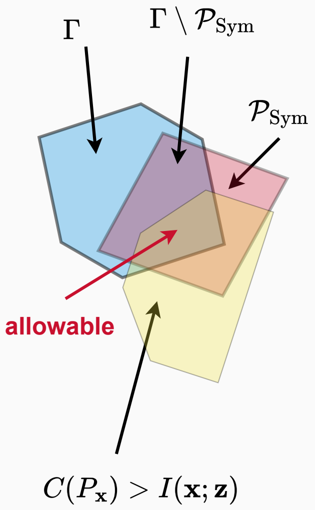
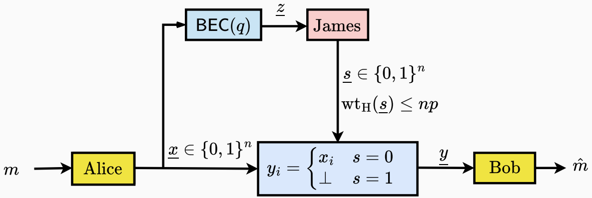
If $I(\mathbf{z};\mathbf{x}) < C(P_{\mathbf{x}})$ we say James is **sufficiently myopic**. In that case we can achieve any rate

$$R < \max_{P_{\mathbf{x}} \in \Gamma \setminus \mathcal{P}_{\mathrm{Sym}}} C(P_{\mathbf{x}}).$$



Myopic adversaries in the erasure setting



In the erasure setting the eavesdropping channel is a BEC(q) and James can erase at most pn bits. If $p < q$, James is **sufficiently myopic**.

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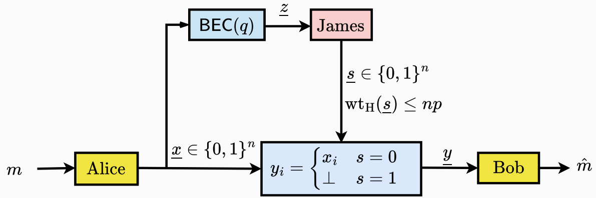
$$C = 1 - p.$$



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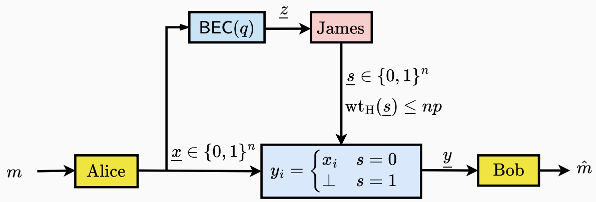

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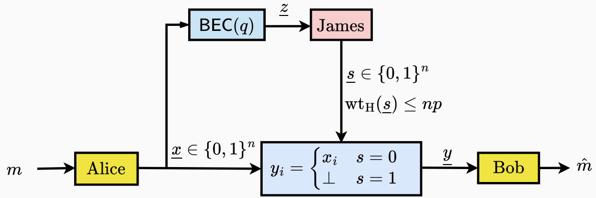

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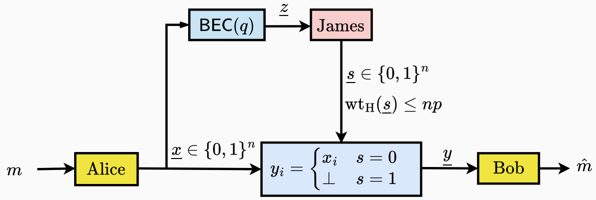
1. If $q > 2p - 1$,

$$C \in \left(0, (1-q)\bar{\alpha}\left(\frac{p-q}{1-q}\right)\right],$$

where $\bar{\alpha}$ is the LP bound for normalized distance.

Myopic adversaries in the erasure setting



In the erasure setting the eavesdropping channel is a BEC(q) and James can erase at most pn bits. If $p < q$, James is **sufficiently myopic**.

If $p < q$ (sufficiently myopic),

$$C = 1 - p.$$

If $p > q$ we have two cases:

1. If $q > 2p - 1$,

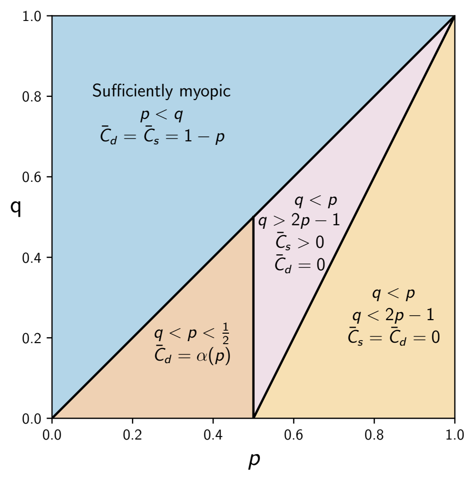
$$C \in \left(0, (1-q)\bar{\alpha}\left(\frac{p-q}{1-q}\right)\right],$$

where $\bar{\alpha}$ is the LP bound for normalized distance.

2. If $q < 2p - 1$,

$$C = 0.$$

Myopic adversaries in the erasure setting



Computationally efficient codes for causal adversaries

Main questions we address

Can we design **efficient codes for causal and myopic models?**

By **efficient** we mean that they take **polynomial time** to encode, decode, and store.

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  $\longrightarrow$ use a **library of linear codebooks**.
- **common randomness** is unrealistic.
  $\longrightarrow$ use **limited encoder randomization** to **confuse the adversary**.
- **minimum distance coding** is not efficient in general.
  $\longrightarrow$ use **list decoding** to permit **efficient decoding**.

“Efficient” coding schemes

To get **polynomial complexity**, use

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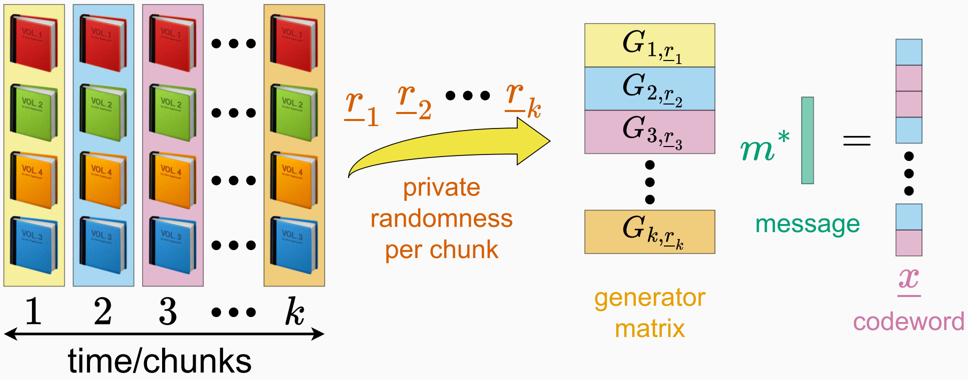
There are different types of complexity we would like to control:

- **Design**: how many bits do we need to generate the code?
- **Storage**: how many bits do we need to store the code?
- **Encoding**: how many operations are needed to encode a message?
- **Decoding**: how many operations are needed to decode the message?

Main results

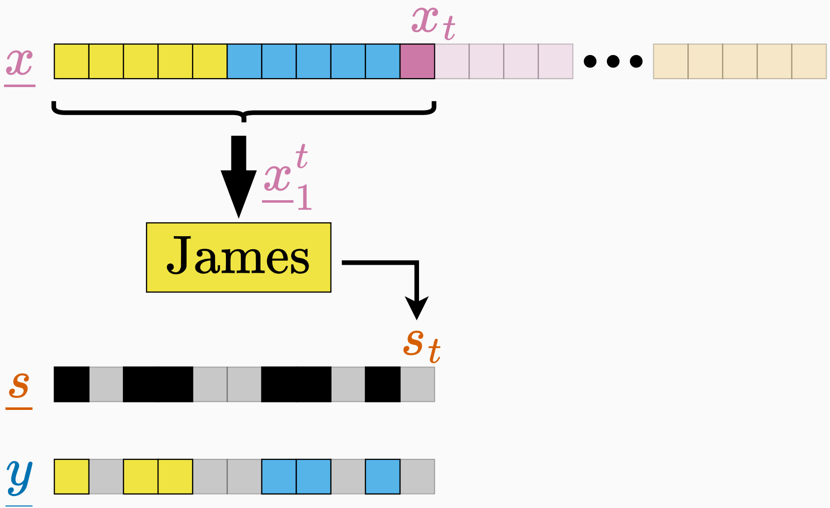
Model **rate**	Randomness	Enc/Storage	Decoding	$\mathbf{P}_{\text{error}}$
Causal $\mathbf{1-2p-\epsilon}$	$O\left(\frac{\gamma \log n}{\epsilon}\right)$	$O(n^3 \log\log n)$	$O(n^{32/\epsilon})$	$O(n^{-(\gamma-1)})$
Myopic $p<q$ $\mathbf{1-p-\epsilon}$	$\lambda_{SM}\log(n)$	$O(n^{2+\lambda_{SM}})$	$O(n^{3+\lambda_{SM}})$	$O(n^{-\lambda_{SM}})$
Myopic $q<p$ **small rate**	$O(n \log\log n)$	$O(n^2 \log\log n)$	$O(n^3 \log\log n)$	$O(n^{-4/5})$

Encode splits block into a constant $k = \lceil \frac{n}{\epsilon} \rceil$ chunks

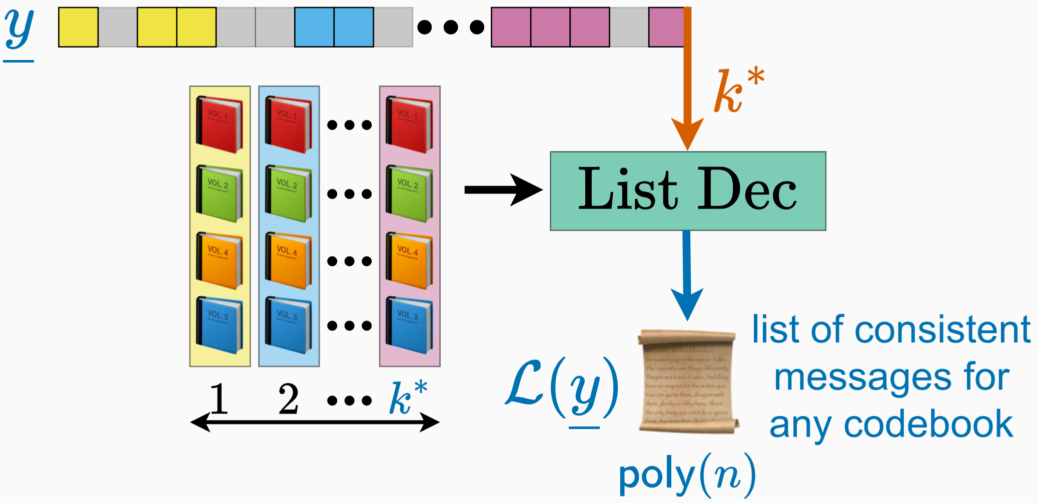


Generate a **library of linear codebooks** independently for each chunk.

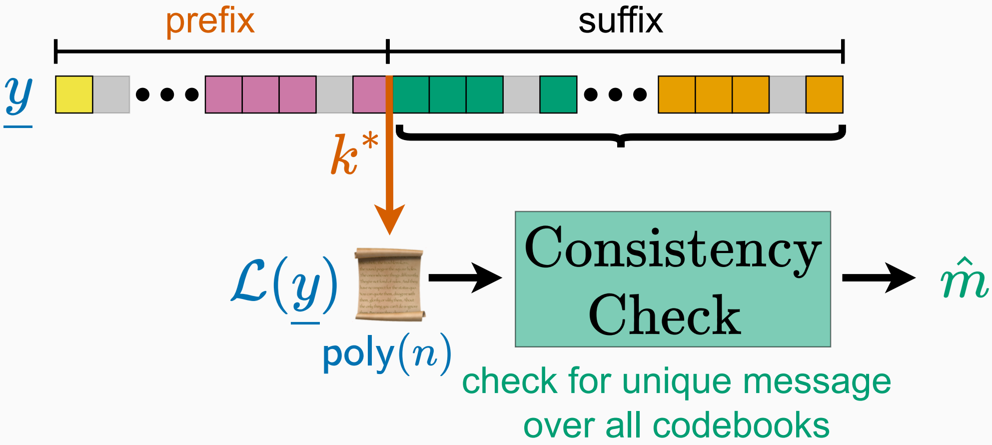
James can erase with causal information only



Bob decodes to a polynomial list



Bob uses suffix to disambiguate the list

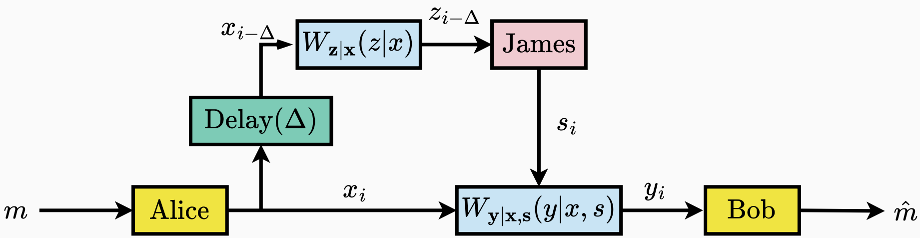


Why does this work?

1. Bob can **track James's erasure budget**.
2. List decoding creates **a smaller set of messages** to check for consistency.
3. James has a choice to **make the list larger** (erase more earlier, less later) or **conserve his budget** (erase less earlier, more later).
4. **Poor James, he can't win.**

Looking forward

Other intermediate models



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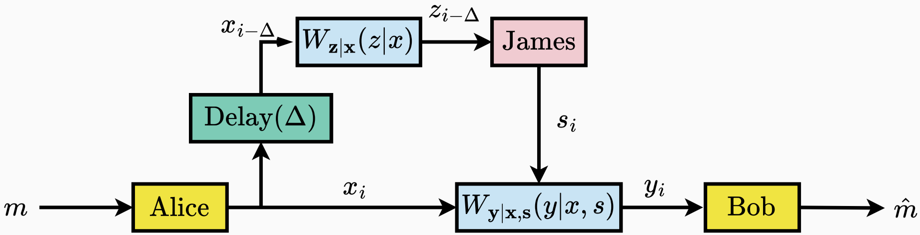

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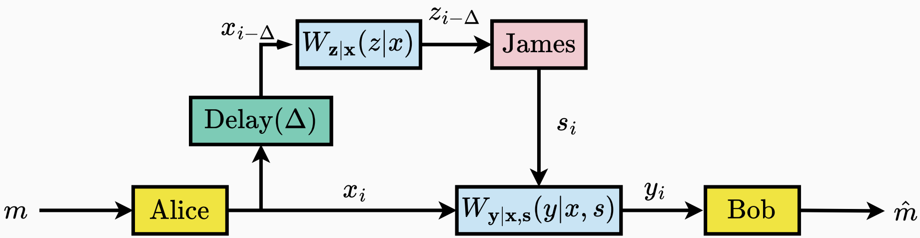

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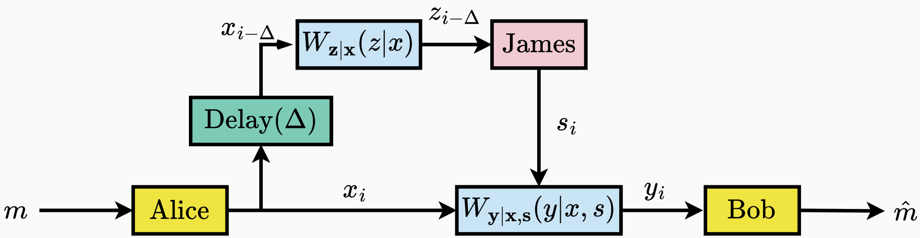

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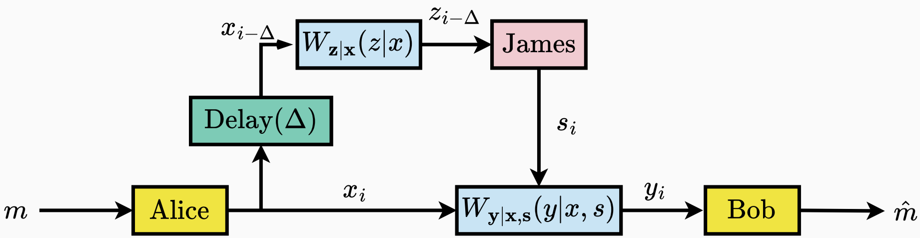

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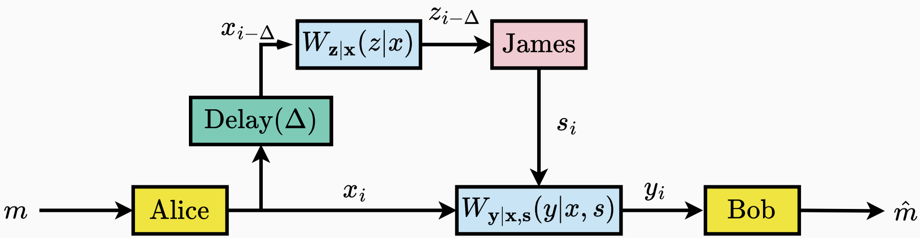

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Each model will reveal something about what the **worst-case channel** looks like.

And for the theory folks...

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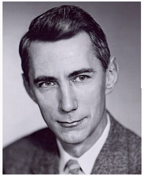
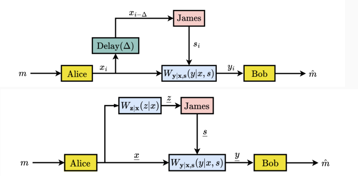
- other fun combinatorial problems

A final recap and takeaways





AVCs can capture models between average and worst-case channels.

- **Causal:** capacity depends on **what James knows about the current input.**
- **Myopic:** capacity depends on **whether James can (partially) "decode."**

Some insights:

- **Stochastic encoding** and **list decoding** can help!
- Worst-case attacks are ones that "push" **at the end of decoding**.

Thank you!