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title: "Solution to Class 2 in-class problem"

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Question: The Hoover Dam contains  $7 \times 10^9$  kg concrete, and produces  $2 \times 10^9$  W of energy. I take 1 MJ of energy to produce 1 kg concrete. How long did the dam's power plant have to run to "pay back" the energy cost of manufacturing its concrete?

Solution:

$$\frac{7 \times 10^9 \text{ kg concrete}}{1 \text{ kg concrete}} \cdot \frac{10^6 \text{ J}}{1 \text{ kg concrete}} \cdot \frac{1 \text{ W s}}{1 \text{ J}} \cdot \frac{1}{2 \times 10^9 \text{ W}} = 3.5 \times 10^6 \text{ s}$$

That looks funny - we don't usually see a quantity in the denominator of a fraction when there's nothing in the numerator - but it is totally legal. We could just as well start the equation off with the fraction

$$\frac{7 \times 10^9 \text{ kg concrete}}{2 \times 10^9 \text{ W}}$$

The only reason I didn't, is that it is tough to know from the start that  $2 \times 10^9$  W will end up in the denominator, rather than the numerator.

We now have the correct answer, but it is perhaps polite to convert to a more intuitive unit of time:

$$\frac{3.5 \times 10^6 \text{ s}}{3600 \text{ s}} \cdot \frac{1 \text{ hr}}{24 \text{ hr}} = 40.5 \text{ days}$$

An important thing to understand is that there are two ways to solve the part of this problem that I think is the hardest, which is realizing that  $1 \text{ W} \cdot \text{s} = 1 \text{ J}$ :

- You can just look at the units: 1 J is  $1 \text{ kg m}^2 / \text{s}^2$ , and 1 W is  $1 \text{ kg m}^2 / \text{s}^3$ , so  $1 \text{ W} \cdot \text{s} = 1 \text{ J}$ .
- (Better): You can think about what these mean. A Watt is a rate of energy delivery. If you deliver 1 W for 1 s, you have delivered 1 J of energy. This is a little like saying that if you travel at 1 mile per hour for 1 hour, you have traveled 1 mile - so  $1 \text{ mph} \cdot \text{h} = 1 \text{ mile}$ .