TPO

$$Y = X\beta + \varepsilon$$

$$\begin{cases} \beta \sim N(0, \sigma_\beta^2 I) \\ \varepsilon \sim N(0, \sigma_\varepsilon^2 I) \end{cases} \qquad \begin{aligned} G &= \sigma_\beta^2 I \\ R &= \sigma_\varepsilon^2 I \end{aligned}$$

$$V(Y) = \sigma_\beta^2 X X' + \sigma_\varepsilon^2 I \quad = XGX' + R$$

On a : $$f(\beta \mid Y) \propto \underbrace{f(Y \mid \beta)}_{N(X\beta,\, \sigma_\varepsilon^2 I)} \; \underbrace{f(\beta)}_{N(0,\, \sigma_\beta^2 I)}$$

$$\Rightarrow f(\beta, Y) \propto e^{-\frac{1}{2}(Y - X\beta)' R^{-1} (Y - X\beta) - \frac{1}{2}\beta' G^{-1} \beta}$$

$$= e^{-\frac{1}{2}\{ Y' R^{-1} \cancel{Y} \beta' X' R^{-1} Y - Y R^{-1} X \beta + \beta' X' R^{-1} X \}}$$

$$e^{-\frac{1}{2}\beta' G^{-1} \beta}$$

$$\propto e^{-\frac{1}{2}\{\beta' \underbrace{(X' R^{-1} X + G^{-1})}_{V^{-1}} \beta - \beta' \underbrace{X' R^{-1} Y}_{V^{-1} m} - \underbrace{Y R^{-1} X}_{m' V^{-1}} \beta}$$

$$= N(m, V)$$

avec $$\begin{cases} m = V X' R^{-1} Y \\ V = (X' R^{-1} X + G^{-1})^{-1} \end{cases}$$