

# *Mixture of Experts*

**Adrien Dubois**

02/05/2025

# ***Sparsely Gated Mixture of Experts (SGMoE)***

# Motivation

- Authors:

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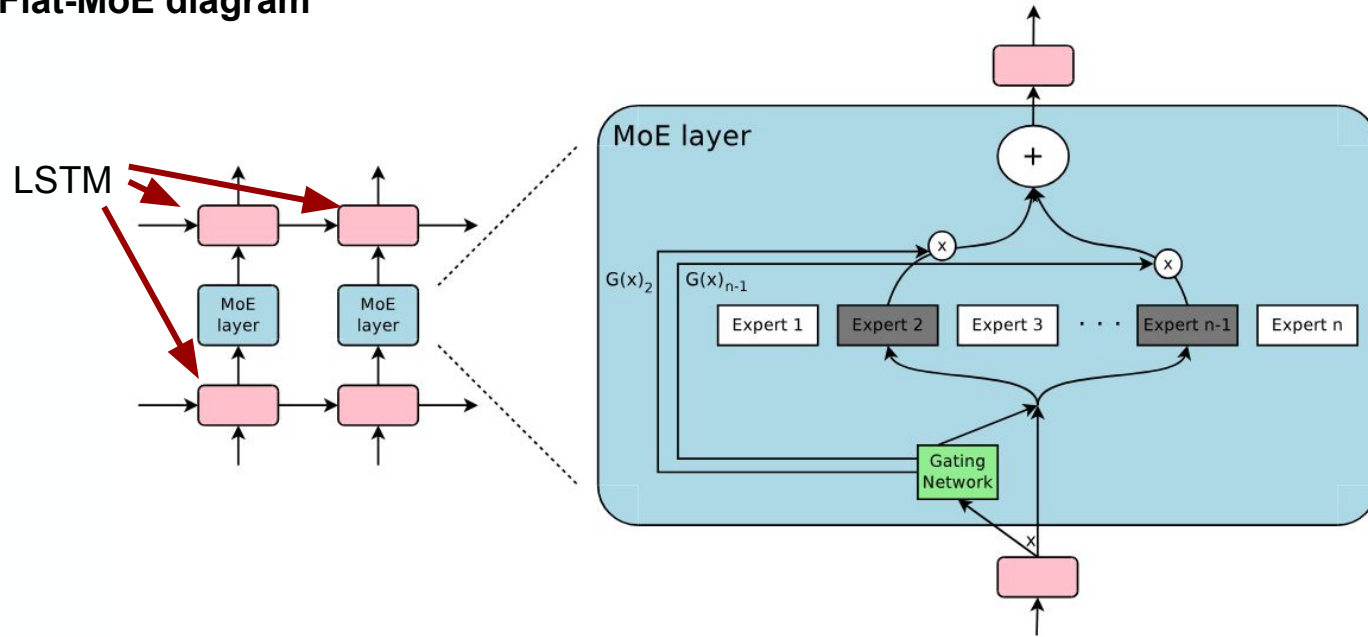
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- Release Date: Jan 2017
- Motivation:
  - Capacity of neural network to absorb information is limited by the number of parameters
    - More parameters = better performance
      - BUT
    - More parameters = more expensive
  - Conditional computation where only parts of the model are active at once allows for more parameters, without increasing computation costs.

# *Their approach*

Flat-MoE diagram



# Challenges with MoE at the time

- Branching computation is expensive, especially model parallelism (dividing 1 large model into subsequent devices)
- Large batches are critical to amortize the costs of parameter transfers and updates. However, conditional computation reduces batch sizes for each expert:

## Note

We select a sparse number of experts for each input feature/token.

Therefore, the input tokens in a batch don't all go through the experts, and one expert may get  $\frac{b}{n}$  input tokens in a batch of size  $b$ . More on this later.

$\ll b$

- Additional loss terms are required to favoring balancing the usage of each expert model
- At the time, the datasets were not large enough to train models with “millions, let alone billions of parameters”

# Gating Architecture

- (Basic, non-sparse) Softmax Gating (**Softmax classifier**):
  - Multiply input  $x$ , by a trainable weight matrix  $W_g$ , then apply softmax to make it a valid probability distribution

$$G_{\sigma}(x) = \text{Softmax}(x \cdot W_g)$$

- (Sparse) Noisy Top-k Gating (**the paper**):
  - The authors add two things: sparsity and noise

$$G(x) = \text{Softmax}(\text{KeepTopK}(H(x), k))$$

$$H(x)_i = (x \cdot W_g)_i + \text{StandardNormal}() \cdot \text{Softplus}((x \cdot W_{\text{noise}})_i)$$

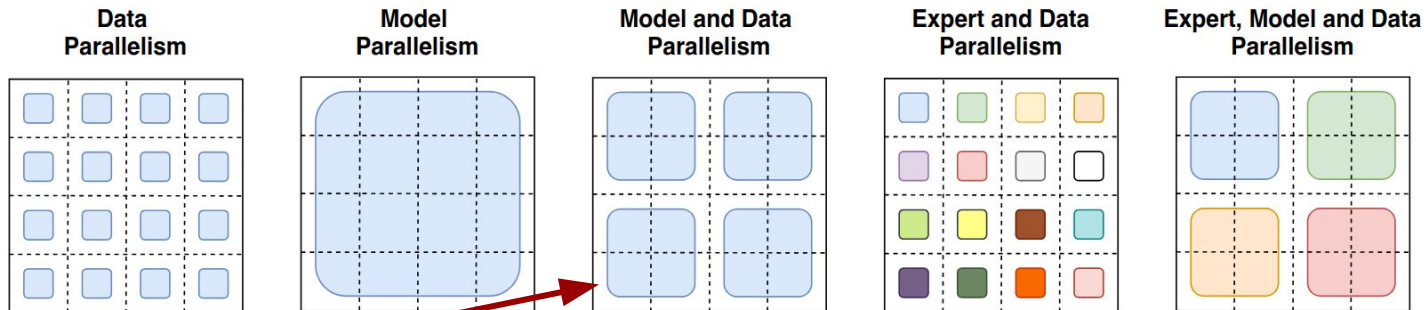
$$\text{KeepTopK}(v, k)_i = \begin{cases} v_i & \text{if } v_i \text{ is in the top } k \text{ elements of } v. \\ -\infty & \text{otherwise.} \end{cases}$$

# *Training the gates and related issues*

- Training the gating network:
  - The gating network is trained through normal backprop
  - Use  $k > 1$  experts, (explained why in future slides)
- Shrinking batch problem:
  - Reminder: As you increase num-experts, batch size per expert decreases which is inefficient
  - Solution: Mixing data parallelism and model parallelism
    - Normal: Model copies are run asynchronously, with synchronous
    - Here: each expert is only on one device as a shared resource. The rest of the model is parallelised. Batches are run synchronously, and relevant tokens are grouped at the expert device.
    - Increases expert batch from  $(kb/n \ll b)$  to  $(kdb/n)$  for:
      - $k$  = num experts from topk
      - $b$  = batch size
      - $n$  = n experts
      - $d$  = d devices for the experts

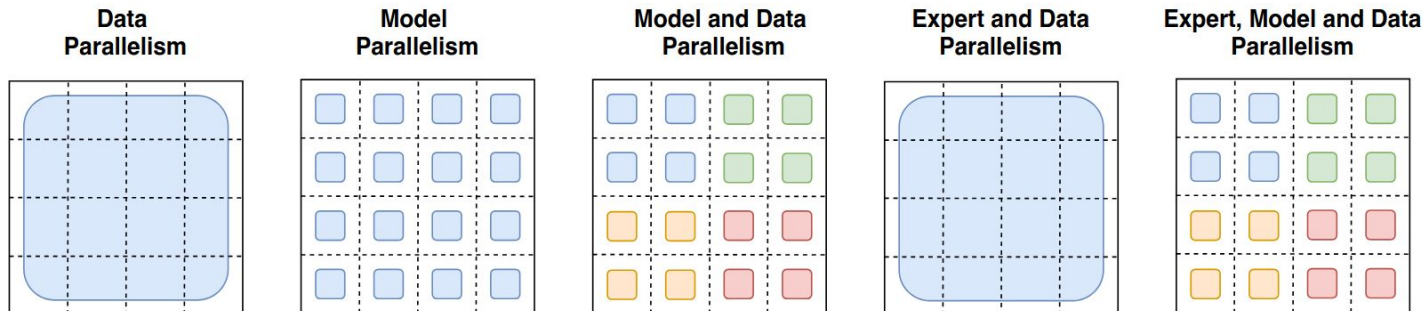
# Switch Transf. on Parallelism Strategies

How the *model weights* are split over cores



SGMoE

How the *data* is split over cores





# Updated loss function for balancing experts

- Without additional loss terms, the network tends to converge to only using the same few experts.
  - This imbalance is self-reinforcing
- So they introduce 2 auxiliary losses that provide a soft-constraint on the batch-wise average of each gate, favoring a uniform distribution.
- Importance loss: Encourages each expert to receive  $\sim$  num tokens within each batch

$$Importance(X) = \sum_{x \in X} G(x)$$

$$L_{importance}(X) = w_{importance} \cdot CV(Importance(X))^2$$

- Load loss: measures the total load of expert  $i$  over the entire dataset  $X$

$$P(x, i) = Pr\left((x \cdot W_g)_i + StandardNormal() \cdot Softplus((x \cdot W_{noise})_i) > kth\_excluding(H(x), k, i)\right) = P(x, i) = \Phi\left(\frac{(x \cdot W_g)_i - kth\_excluding(H(x), k, i)}{Softplus((x \cdot W_{noise})_i)}\right)$$

$$Load(X)_i = \sum_{x \in X} P(x, i)$$

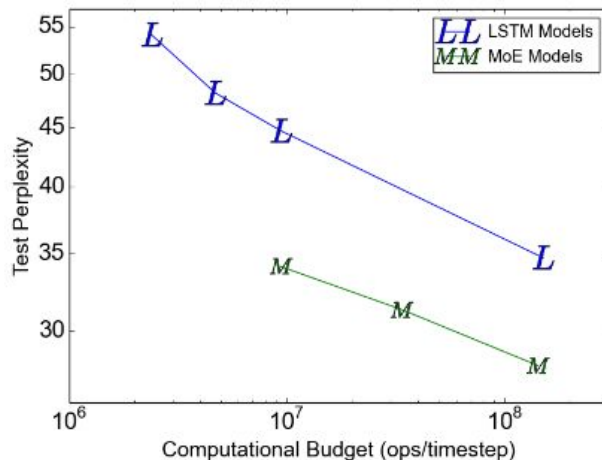
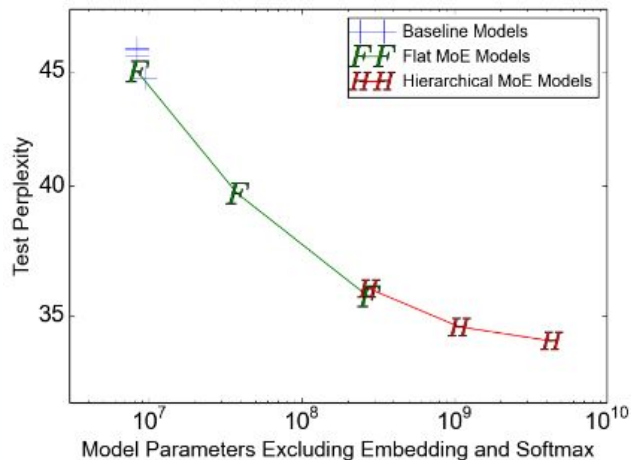
$$L_{load}(X) = w_{load} \cdot CV(Load(X))^2$$

# *Determining the number of experts*

- When training the MoE, the authors state that  $k > 1$  (for topk experts) is ideal due to:
  - **Improved load balancing**: Gating mechanism becomes too deterministic otherwise which means that the load balancing loss cannot effectively encourage a uniform distribution utilization
  - **Better gradient flow**: Backprop goes through multiple experts
    - Gradients are less likely to vanish/explode
    - Combining the outputs of multiple experts creates more robust representations.

# SGMoE Evaluation

- Flat MoE are tested for 4, 32 and 256 experts
- Hierarchical MoEs are tested for 256, 1024, 4096 experts



# SGMoE Evaluation 2

Table 2: Results on WMT'14 En→Fr newstest2014 (bold values represent best results).

Model	Test Perplexity	Test BLEU	ops/timestep	Total #Parameters	Training Time
MoE with 2048 Experts	2.69	40.35	85M	8.7B	3 days/64 k40s
MoE with 2048 Experts (longer training)	<b>2.63</b>	<b>40.56</b>	85M	8.7B	6 days/64 k40s
GNMT (Wu et al., 2016)	2.79	39.22	214M	278M	6 days/96 k80s
GNMT+RL (Wu et al., 2016)	2.96	39.92	214M	278M	6 days/96 k80s
PBMT (Durrani et al., 2014)		37.0			
LSTM (6-layer) (Luong et al., 2015b)		31.5			
LSTM (6-layer+PosUnk) (Luong et al., 2015b)		33.1			
DeepAtt (Zhou et al., 2016)		37.7			
DeepAtt+PosUnk (Zhou et al., 2016)		39.2			

Table 3: Results on WMT'14 En → De newstest2014 (bold values represent best results).

Model	Test Perplexity	Test BLEU	ops/timestep	Total #Parameters	Training Time
MoE with 2048 Experts	<b>4.64</b>	<b>26.03</b>	85M	8.7B	1 day/64 k40s
GNMT (Wu et al., 2016)	5.25	24.91	214M	278M	1 day/96 k80s
GNMT+RL (Wu et al., 2016)	8.08	24.66	214M	278M	1 day/96 k80s
PBMT (Durrani et al., 2014)		20.7			
DeepAtt (Zhou et al., 2016)		20.6			

Table 4: Results on the Google Production En→Fr dataset (bold values represent best results).

Model	Eval Perplexity	Eval BLEU	Test Perplexity	Test BLEU	ops/timestep	Total #Parameters	Training Time
MoE with 2048 Experts	<b>2.60</b>	<b>37.27</b>	<b>2.69</b>	<b>36.57</b>	85M	8.7B	1 day/64 k40s
GNMT (Wu et al., 2016)	2.78	35.80	2.87	35.56	214M	278M	6 days/96 k80s

# ***Switch Transformer (ST)***

# Motivation

- Authors:

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Same as MgMoE



- Release Date: June 2022

# Motivation

- Fun goal:
  - Train a model with 1 trillion params on 1 billion words (people didn't think this was viable at the time)
- True motivation:

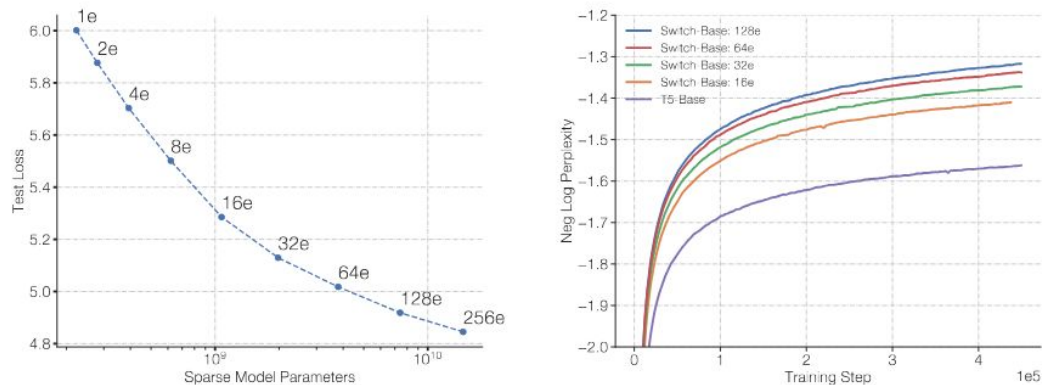
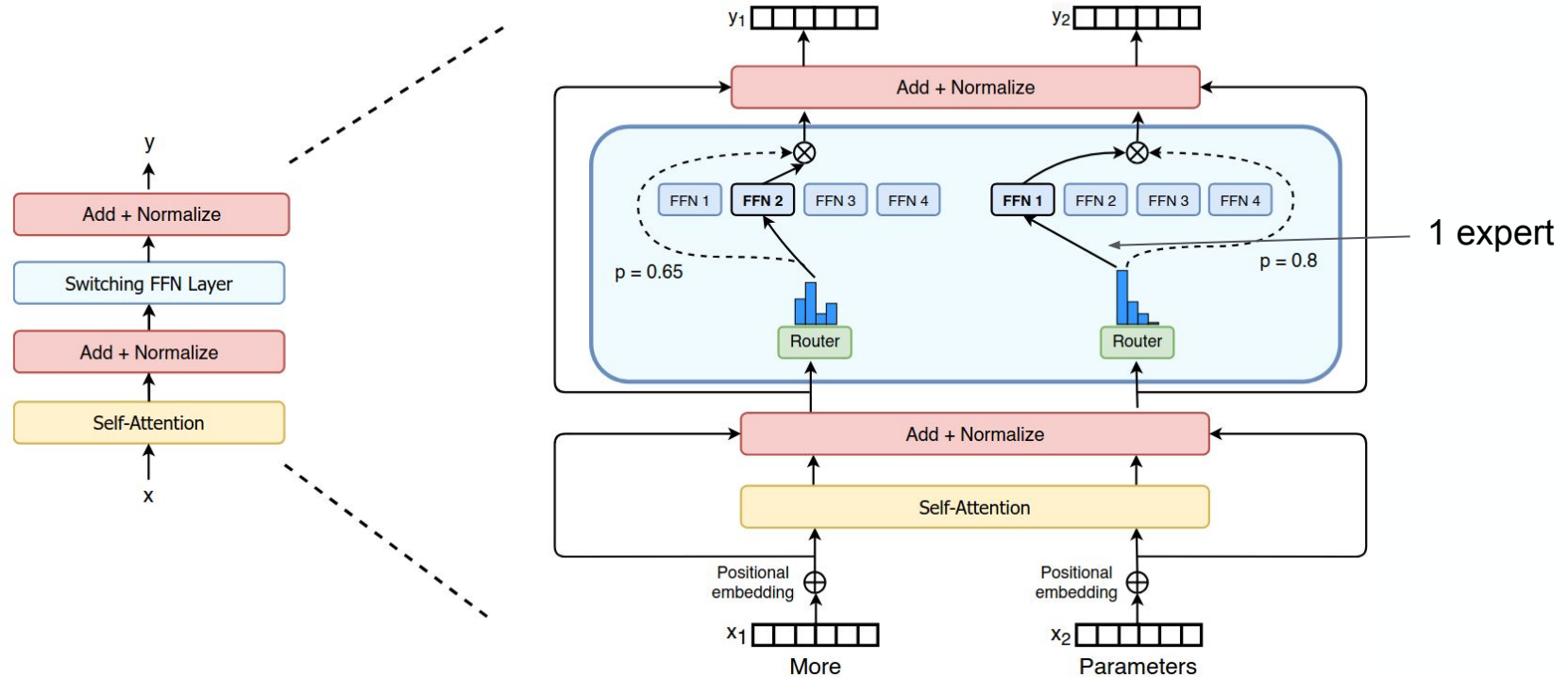


Figure 1: Scaling and sample efficiency of Switch Transformers. Left Plot: Scaling properties for increasingly sparse (more experts) Switch Transformers. Right Plot: Negative log perplexity comparing Switch Transformers to T5 (Raffel et al., 2019) models using the same compute budget.

# Integration of MoE with transformers

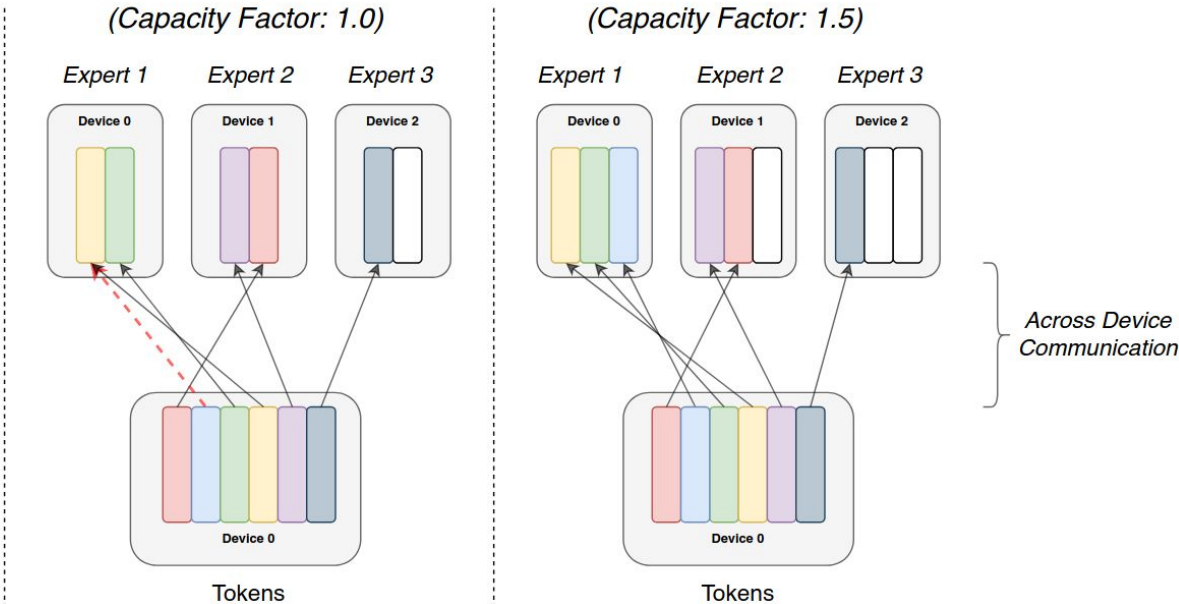




# Expert routing with capacity

## Terminology

- **Experts:** Split across devices, each having their own unique parameters. Perform standard feed-forward computation.
- **Expert Capacity:** Batch size of each expert. Calculated as  $(\text{tokens\_per\_batch} / \text{num\_experts}) * \text{capacity\_factor}$
- **Capacity Factor:** Used when calculating expert capacity. Expert capacity allows more buffer to help mitigate token overflow during routing.



- If the expert is already at capacity (red), that input token is dropped.

# How to stabilize the learning with $k=1$

Model	Parameters	FLOPs/seq	$d_{model}$	$FFN_{GEGLU}$	$d_{ff}$	$d_{kv}$	Num. Heads
T5-Base	0.2B	124B	768	✓	2048	64	12
T5-Large	0.7B	425B	1024	✓	2816	64	16
T5-XXL	11B	6.3T	4096	✓	10240	64	64
Switch-Base	7B	124B	768	✓	2048	64	12
Switch-Large	26B	425B	1024	✓	2816	64	16
Switch-XXL	395B	6.3T	4096	✓	10240	64	64
Switch-C	1571B	890B	2080		6144	64	32
Model	Expert Freq.	Num. Layers	Num Experts	Neg. Log Perp. @250k	Neg. Log Perp. @ 500k		
T5-Base	–	12	–	-1.599	-1.556		
T5-Large	–	24	–	-1.402	-1.350		
T5-XXL	–	24	–	-1.147	-1.095		
Switch-Base	1/2	12	128	-1.370	-1.306		
Switch-Large	1/2	24	128	-1.248	-1.177		
Switch-XXL	1/2	24	64	<b>-1.086</b>	<b>-1.008</b>		
Switch-C	1	15	2048	-1.096	-1.043		

# Dropout in gating mechanism

- Instead of adding random noise similar to SGMoE, the authors introduce dropout to the gating system
- This is mainly done at the expert level (ed = expert dropout).
  - This is possible since the experts are already sparse so even removing an entire expert may not reduce model capability by a huge amount.

Model (dropout)	GLUE	CNNDM	SQuAD	SuperGLUE
T5-Base (d=0.1)	82.9	<b>19.6</b>	83.5	72.4
Switch-Base (d=0.1)	84.7	19.1	<b>83.7</b>	<b>73.0</b>
Switch-Base (d=0.2)	84.4	19.2	<b>83.9</b>	<b>73.2</b>
Switch-Base (d=0.3)	83.9	19.6	83.4	70.7
Switch-Base (d=0.1, ed=0.4)	<b>85.2</b>	<b>19.6</b>	<b>83.7</b>	<b>73.0</b>

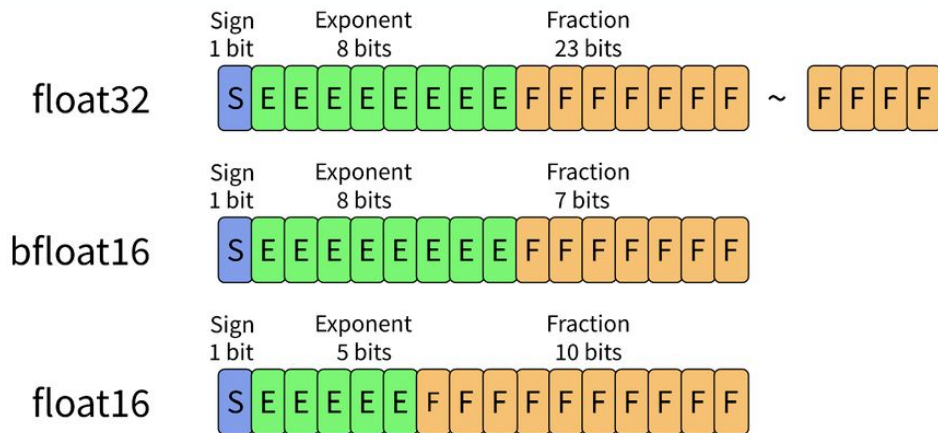
Table 4: Fine-tuning regularization results. A sweep of dropout rates while fine-tuning Switch Transformer models pre-trained on 34B tokens of the C4 data set (higher numbers are better). We observe that using a lower standard dropout rate at all non-expert layer, with a much larger dropout rate on the expert feed-forward layers, to perform the best.

# ***Smaller Parameter Initialization***

- The authors initialize the model parameters from a truncated Normal distribution with:
  - mean = 0
  - $\text{stdev} = \sqrt{s/n}$ 
    - $s$  = scale hyper-parameter (they reduce it by a factor of 10 as compared to default transformer)
    - $n$  = number of input units in the weight tensor (fan-in)

# Selective Precision

- Model instability hinders training fully at BF16, so previous papers required full float32 computation (default in Pytorch).
- This has expensive communication costs.
- So they “selectively cast” input tokens to float32 when entering each expert’s device, then convert back to BF16 before leaving the GPU.
- This truncation is done by truncation of the “mantissa” (fraction) bits.



# Load balancing Loss

- As in SGMoE, the authors encourage expert balancing through auxiliary loss. However, they are both combined into one simplified loss term here.

Given  $N$  experts indexed by  $i = 1$  to  $N$  and a batch  $\mathcal{B}$  with  $T$  tokens, the auxiliary loss is computed as the scaled dot-product between vectors  $f$  and  $P$ ,

$$\text{loss} = \alpha \cdot N \cdot \sum_{i=1}^N f_i \cdot P_i \quad (4)$$

where  $f_i$  is the fraction of tokens dispatched to expert  $i$ ,

$$f_i = \frac{1}{T} \sum_{x \in \mathcal{B}} \mathbb{1}\{\text{argmax } p(x) = i\} \quad (5)$$

and  $P_i$  is the fraction of the router probability allocated for expert  $i$ ,<sup>2</sup>

$$P_i = \frac{1}{T} \sum_{x \in \mathcal{B}} p_i(x). \quad (6)$$

# Evaluation 1

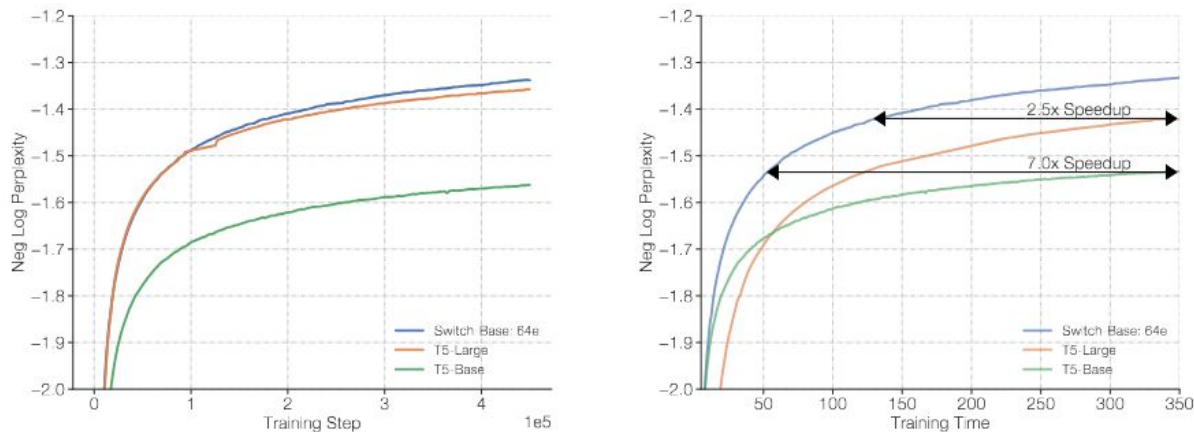


Figure 6: Scaling Transformer models with Switch layers or with standard dense model scaling. Left Plot: Switch-Base is more sample efficient than both the T5-Base, and T5-Large variant, which applies 3.5x more FLOPS per token. Right Plot: As before, on a wall-clock basis, we find that Switch-Base is still faster, and yields a 2.5x speedup over T5-Large.

# Evaluation 2: converges faster

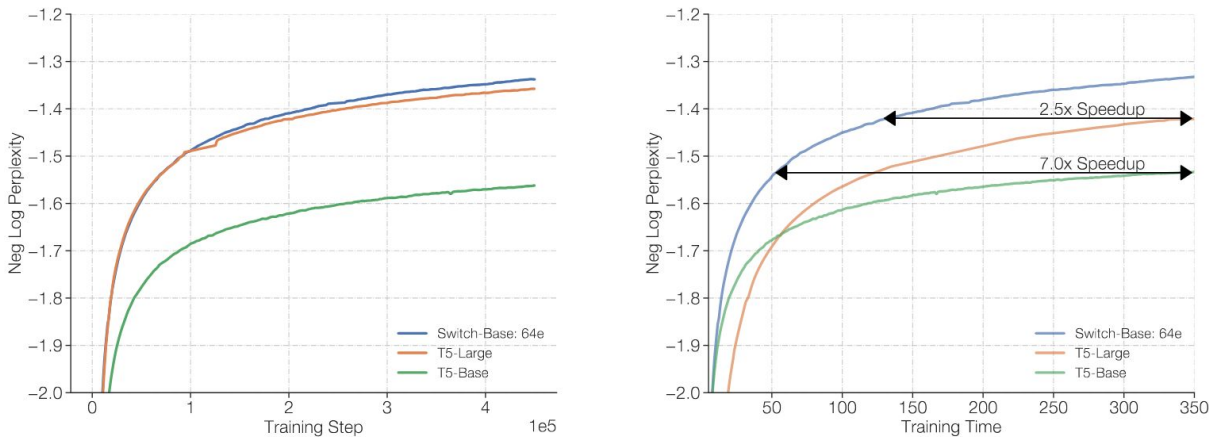


Figure 6: Scaling Transformer models with Switch layers or with standard dense model scaling. Left Plot: Switch-Base is more sample efficient than both the T5-Base, and T5-Large variant, which applies 3.5x more FLOPS per token. Right Plot: As before, on a wall-clock basis, we find that Switch-Base is still faster, and yields a 2.5x speedup over T5-Large.



# *Evaluation 2: higher accuracy in all langs.*

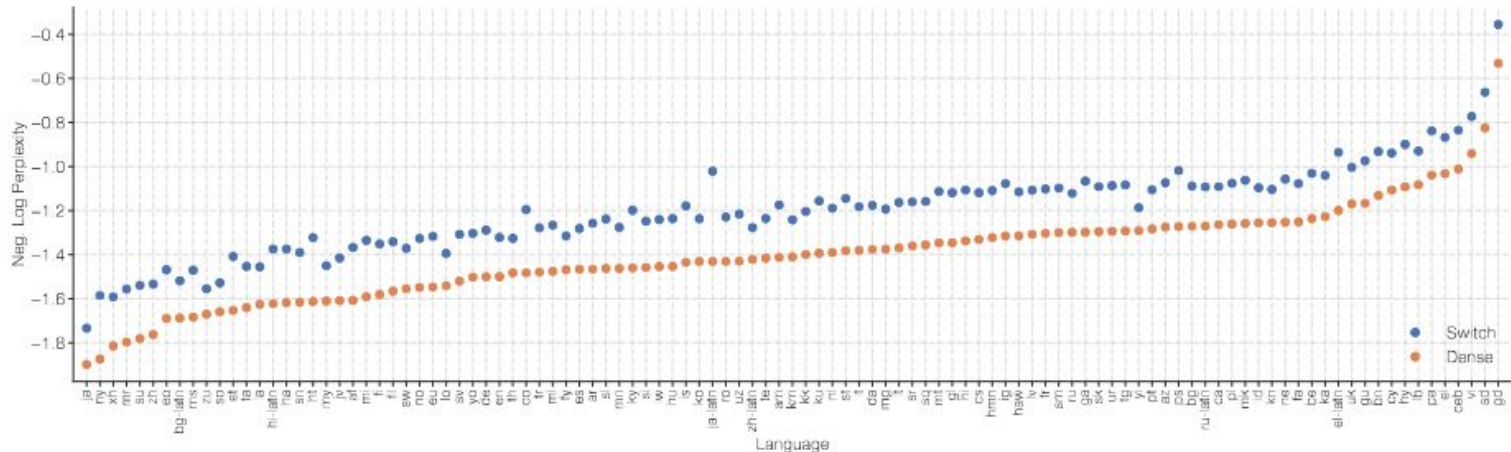


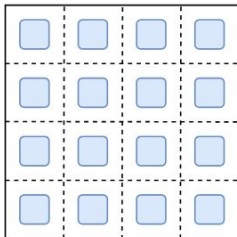
Figure 7: Multilingual pre-training on 101 languages. Improvements of Switch T5 Base model over dense baseline when multi-task training on 101 languages. We observe Switch Transformers to do quite well in the multi-task training setup and yield improvements on all 101 languages.

# Switch Transf. on Parallelism Strategies

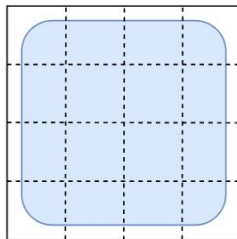
How the *model weights* are split over cores

Switch XXL

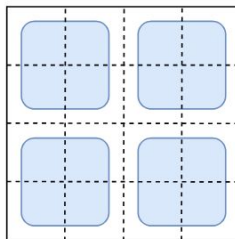
Data Parallelism



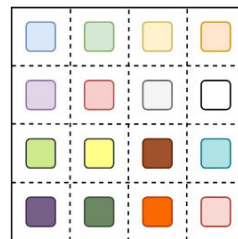
Model Parallelism



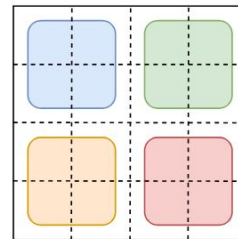
Model and Data Parallelism



Expert and Data Parallelism



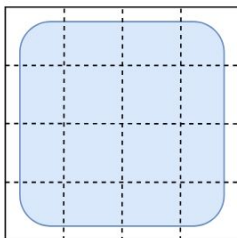
Expert, Model and Data Parallelism



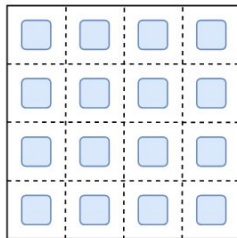
Switch C (1T)

How the *data* is split over cores

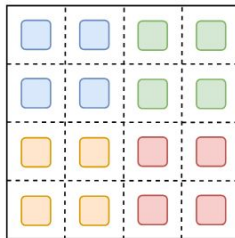
Data Parallelism



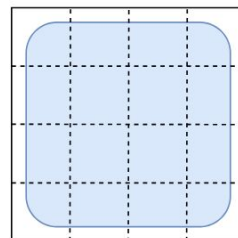
Model Parallelism



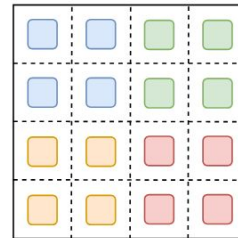
Model and Data Parallelism



Expert and Data Parallelism

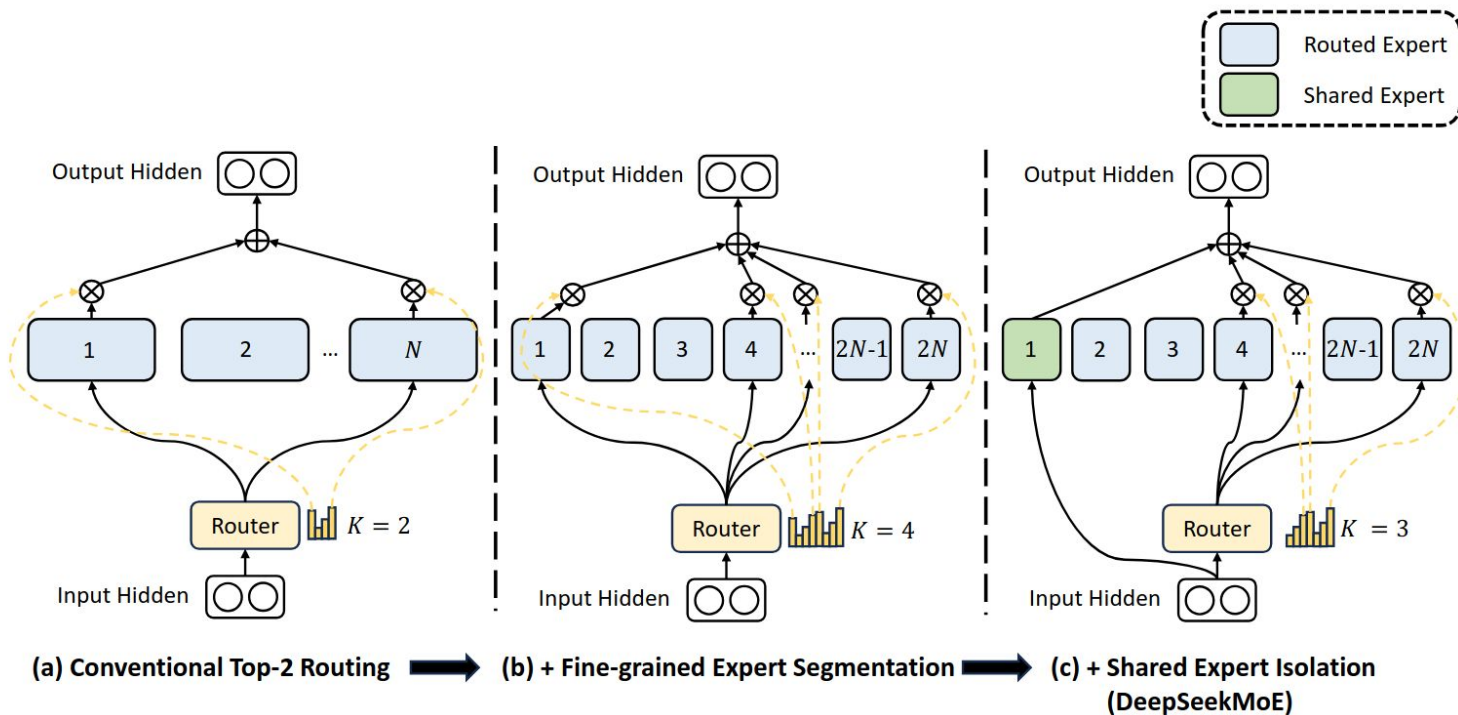


Expert, Model and Data Parallelism



# ***Deepseek Updates***

# ***Fine-Grained Expert Seg./Shared Expert Iso***



# DeepSeek - Removal of Auxiliary Loss

- Drops auxiliary losses and instead uses a bias term:
  - If expert  $i$  is overused, decrease the bias
    - $b_i = b_i - \gamma$
  - Else:
    - $b_i = b_i + \gamma$
- The expert is thus chosen through this formula:

$$g'_{i,t} = \begin{cases} s_{i,t}, & s_{i,t} + b_i \in \text{Topk}(\{s_{j,t} + b_j | 1 \leq j \leq N_r\}, K_r), \\ 0, & \text{otherwise.} \end{cases}$$

# *Thank You*

Purdue RVL



# ***Challenges with MoE at the time***

- Authors:

# Deepseek:

- Hierarchical MoE structure. The tokens go through the following hierarchies:
  - **Global Selection:** inputs are routed to an initial pool of experts using Softmax Affinity Scoring
  - **Cluster-Level Pruning:** within each pool, a secondary gating mechanism prunes experts based on entropy constraints
  - **Final expert assignments:** Top-k experts are chosen using either Entropy Aware Gating, or RL agent in Deepseek R1.
- Auxiliary Losses:
  - **Load balance loss:** we saw in switch transformer. Balances the usage of each individual expert.
  - **Device balance loss:** the experts are split into groups and assigned to devices. They want all devices to be used relatively equally.
  - **Communication balance loss:** balances the communication load across experts.
- Token-Dropping Strategy:
  - Capacity factor = 1
  - When overflowing, drop the token with the lowest affinity score
  - Randomly sample 10% of the input sequences to have no dropped tokens.



# DeepseekV3:

- Drops auxiliary losses and instead uses a bias term:
  - If expert  $i$  is overused, decrease the bias
    - $b_i = b_i - \text{gamma}$
  - Else:
    - $b_i = b_i + \text{gamma}$
- Mostly engineering improvements on routing experts in distributed learning setting.

# DeepseekR1:

- Introduces RL based expert routing
- Instead of using a learned linear layer with softmax activation for expert routing, Deepseek R1 utilizes a learned RL policy to dynamically assign tokens to experts.
- The policy is as follows
  - The expert selection function is formulated as an RL policy optimization problem, where the probability of selecting expert  $e_i$  for token  $t$  is adjusted dynamically based on token embeddings  $u_t$ :

$$g_{i,t} = \pi_{\theta}(e_i|u_t)$$

- where  $\pi_{\theta}$  is the policy network that selects experts based on contextual embeddings. The optimization objective follows GRPO:

$$J_{\text{GRPO}}(\theta) = \mathbb{E}_{q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}} \left[ \frac{1}{G} \sum_{i=1}^G \min \left( \frac{\pi_{\theta}(o_i|q)}{\pi_{\theta_{\text{old}}}(o_i|q)} A_i, \text{clip}(\cdot) \right) - \beta D_{\text{KL}}(\pi_{\theta} || \pi_{\text{ref}}) \right]$$

- where  $D_{\text{KL}}$  regularizes the policy update to prevent drastic shifts.