Vol 15 No 7, July 2006
1009-1963/2006/15(07)/1471-06

Chinese Physics

The CA model for traffic-flow at the grade roundabout crossing*

Chen Rui-Xiong(陈瑞熊), Bai Ke-Zhao(白克钊), and Liu Mu-Ren(刘慕仁)†

College of Physics and Information Technology, Guangxi Normal University, Guilin 541004, *China*

(Received 7 October 2005; revised manuscript received 20 February 2006)

The cellular automaton model is suggested to describe the traffic-flow at the grade roundabout crossing. After the simulation with computer, the fundamental properties of this model have been revealed. Analysing this kind of road structure, this paper transforms the grade roundabout crossing with inner-roundabout-lane and outer-roundabout-lane into a configuration with many bottlenecks. Because of the self-organization, the traffic flow remains unblocked under a certain vehicle density. Some results of the simulation are close to the actual design parameter.

Keywords: grade roundabout crossing, cellular automaton, bottleneck, self-organization
PACC: 0550

1. Introduction

With the developing economy, the inconsistency between the increasing traffic-flow and the existing roads becomes more and more of a concern. Scientists have made a great deal of investigation on traffic issues, and obtained abundant results. There are many research methods for traffic-flow. One of them is the cellular automaton (CA) model, which is a dynamic method developed in the last decade to deal with traffic-flow. It describes time, space and status variables (including vehicles, persons and traffic facilities) in a traffic system by means of a discrete set. As a computer is well able to make operation on a discrete set, thus it makes possible the unification of research object and manner. Moreover, a CA model has the advantage that it can be easily modified to deal with the effects of different kinds of realistic conditions, such as the style of the vehicle, driver's behaviour, and road status. It has been adopted to simulate many traffic-flow situations: for example, Li and Gao[1] investigated the subject of improvement of driving safety in road system by the CA model. To date, CA models have demonstrated a great many useful results.

Actual urban traffic is a two-dimensional network. In 1992, Biham, Middleton and Levine[2] (BML) first introduced a simple CA model on a two-dimensional lattice to simulate the traffic flow phase transition from free flow to jam with the increasing of vehicle density in city traffic networks. After that, a large amount of generalized BML models were set up to simulate city traffic-flow. Nagatani[3] developed one to take into account the effects of overpass (or two-level crossing). Cuesta *et al*[4] and Lü *et al*[5] considered the turning probability and presented the character of phase transition of the CA model for the traffic-flow in their papers. Tan *et al* [6,7] also discovered the influence of traffic lights on traffic-flow at crossroads. Huang *et al*[8] studied the main road traffic flow on two-dimension and obtain the evolution equation.

There are many crossings in an urban traffic network. One of the commonest crossing configurations is the crossing of two roads, and in general, there are two patterns for it: one is the square crossing where the traffic light is used to control the traffic-flow and the other is the grade roundabout crossing, where the traffic light is absent, the traffic flow is more optimally and in a safe manner. Fouladvand *et al*[9] investigated the characteristics of traffic at an isolated roundabout in the framework of cellular automata and car following models. They also compared their results with traffic light signalization schemes. Wang and Ruskin[10] have investigated a number of properties of single-lane roundabouts using a CA ring model under the offside-

*Project supported by the National Natural Science Foundation of China (Grant Nos 70371067, 10362001, 10347001 and 10562001), the National Natural Science Foundation of Guangxi Zhuang Autonomous Region, China (Grant No 04470307), and the Special Fund for the New Century Trained Talents Program of Guangxi Zhuang Autonomous Region, China (Grant No 20011204).

†E-mail: lmrlmr@mailbox.gxnu.edu.cn

priority rule. This paper will discuss the characters of the traffic-flow at the grade roundabout crossing with a CA model and at the joining-up points the inner-lane-priority is considered, in a more actual manner we not only consider the inner-roundabout-lane but also the outer-roundabout-lane; the traffic flow properties are discussed from a viewpoint of the total construction of the roundabout. Section 2 introduces our model. Section 3 will discuss the computational results and explain why this crossing configuration is often adopted in the actual traffic network. Section 4 is the summary and outlook, it compares the computational results with the actual design parameter.

2. Model

In general, there exist three or four lanes in an actual grade roundabout crossing, and they look like concentric circles. According to the traffic rule of China, the vehicles drive on the right. The inner lane is designated for vehicles rounding the traffic island, the outer lane is designated for vehicles turning right. We have modified the roundabout to a polygon so that a CA model can be used to treat it. The schematic map of the grade roundabout crossing is showed in Fig.1. For simplification, the whole grade roundabout crossing merely consists of one inner roundabout lane and four separate outer-roundabout-lane segments. The inner roundabout lane is a closed route containing eight segments ($P_{\rm si} \to P_{\rm eo} \to P_{\rm ei} \to P_{\rm no} \to P_{\rm ni} \to P_{\rm wo} \to P_{\rm wi} \to P_{\rm so} \to P_{\rm si}$), where the arrows represent the driving direction of the vehicles. The four outer-roundabout-lane segments are distributed in different orientations. They are $S_{\rm iR} \to U_{\rm S} \to E_{\rm oR}$ in the southeast, $E_{\rm iR} \to U_{\rm E} \to N_{\rm oR}$ in the northeast, $N_{\rm iR} \to U_{\rm N} \to W_{\rm oR}$ in the northwest and $W_{\rm iR} \to U_{\rm W} \to S_{\rm oR}$ in the southwest respectively.

$S_{\rm i}, E_{\rm i}, N_{\rm i}$ and $W_{\rm i}$ represent the straight lanes for vehicles from the south, east, north, and west respectively to enter on the roundabout crossing (hereafter referred to as driving-in lane). $S_{\rm o}$, $E_{\rm o}$, $N_{\rm o}$ and $W_{\rm o}$ represent the straight lanes for vehicles to go out of the roundabout crossing and run south, east, north and west respectively (hereafter referred to as driving-out lane).

In view of the symmetry of the four straight lanes for vehicles to enter on the roundabout and for the sake of clarity, we simply discuss and demonstrate the situation for a vehicle coming from lane $S_{\rm i}$ to pass through the roundabout crossing. There are three choices for the vehicle: 1) to the west, in this case it will get at point $S_{\rm iR}$, go straight forward along the lane S and arrive at point $P_{\rm si}$ of the inner roundabout lane, counterclockwise pass through points of $P_{\rm eo}$, $P_{\rm ei}$, $P_{\rm no}$, $P_{\rm ni}$ and get at $P_{\rm wo}$, then turn west, go through point $W_{\rm oR}$ and enter on the west straight lane $W_{\rm o}$; 2) to the north, in this case it will get at point $S_{\rm iR}$, go straight forward and arrive at point $P_{\rm si}$ of the inner roundabout lane, counterclockwise pass through points of $P_{\rm eo}$, $P_{\rm ei}$ and get at $P_{\rm no}$, then turn north, go through point $N_{\rm oR}$ and enter on the north straight lane $N_{\rm o}$; 3) to the east, in this case it will arrive at point $S_{\rm iR}$, get over segment $U_{\rm s}$ of the outer-roundabout-lane and at point $E_{\rm oR}$, then turn east and enter on the straight lane $E_{\rm o}$. On reviewing Fig.1, it is obvious that there are four joining-up points i.e. $P_{\rm si}$, $P_{\rm ei}$, $P_{\rm ni}$ and $P_{\rm wi}$ on the inner roundabout lane, and there also exist four joining-up points i.e. $E_{\rm oR}$, $N_{\rm oR}$, $W_{\rm oR}$ and $S_{\rm oR}$ on the outer roundabout lane, their structure is similar to that of the on-ramp system. Points of $P_{\rm so}$, $P_{\rm eo}$, $P_{\rm no}$, $P_{\rm wo}$, $E_{\rm iR}$, $N_{\rm iR}$, $W_{\rm iR}$, and $S_{\rm iR}$ are the taking-off points, their structure is similar to that of the off-ramp system. According to the viewpoint of Ref.[11], on-ramp and off-ramp are both bottlenecks, we can transform the grade roundabout crossing into a configuration with many bottlenecks.

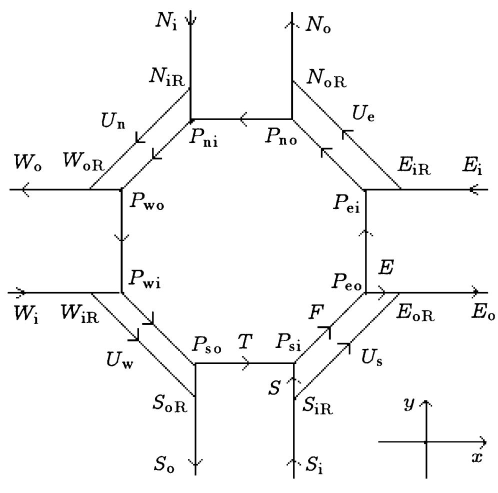

Fig.1. The sketch map of grade roundabout crossing.

The updating regulation is based on the evolution procedure of the NaSch model,[12] and at the joining-up points the inner-lane-priority rule must be considered. There are four steps in the NaSch model i.e. step 1: acceleration, $V_i \to \min(V_{\max}; V_i + 1)$; step 2: braking, $V_i \to \min(d_i; V_i)$; step 3: braking at random with the probability of p, $V_i \to \max(0; V_i - 1)$; step 4: vehicles going ahead and status updating, the result is the vehicle i moving V_i (taking its value) lattice. Where V_i and d_i denote the velocity of vehicle i and the number of empty cells in front of vehicle i respectively. The maximum velocity and the slowdown parameter are denoted as $V_{\max}$ and p respectively. In this paper, we focus our attention on the determinate case with the parameter of p equal to zero, so that step 3 can be ignored. In respect of the inner-lane-priority rule for the vehicles to get at the joining-up points, those set up in the Ref.[13] are followed, i.e. a vehicle entering gives the joining-up point to one already on the roundabout, and a vehicle going out of the roundabout takes priority of occupying the joining-up point. We take the point of $P_{\rm si}$ in Fig.1 as an example to explain the rule, vehicles getting at this point may come from T direction of the inner roundabout lane or from S direction of the straight lane. We denote the leading vehicles on lane T and lane S as $T_{\rm lead}$ and $S_{\rm lead}$ respectively, and the last vehicle leaving from this point and running on lane F as $F_{\rm last}$. Their dynamic positions will be checked one by one time step for collision. If vehicle $T_{\rm lead}$ or vehicle $S_{\rm lead}$ does not get at the point of $P_{\rm si}$ during a certain time step, the position updating of vehicles on lane T and lane S will be independent of each other during this time step.

If both vehicle $T_{\rm lead}$ and vehicle $S_{\rm lead}$ get at the point of $P_{\rm si}$ during a certain time step, the time t_T and t_S needed for vehicles $T_{\rm lead}$ and $S_{\rm lead}$ to get at the point of $P_{\rm si}$ will be calculated:

$$t_T = \frac{x_{P_{\rm si}} - x_{T_{\rm lead}}}{\min(V_{\max}, x_{F_{\rm last}} - x_{T_{\rm lead}} - 1, V_{T_{\rm lead}} + 1)} \qquad (1)$$

$$t_S = \frac{x_{P_{\rm si}} - x_{S_{\rm lead}}}{\min(V_{\max}, x_{F_{\rm last}} - x_{S_{\rm lead}} - 1, V_{S_{\rm lead}} + 1)} \qquad (2)$$

where x denotes the position of the vehicle or the lattice and V denotes the velocity of the vehicle.

If $t_T < t_S$, we can suppose that vehicle $T_{\rm lead}$ has the priority to occupy lattice $P_{\rm si}$. For this case, the update of the vehicles on lane T will not be affected by the vehicles on lane S during this time step. However, the update of the vehicles on lane S will be affected by vehicles on lane T. For this reason, the update is carried out in two sub-steps: 1) $T_{\rm lead}$, the leading vehicle on lane T, passes through point $P_{\rm si}$ and becomes the new last vehicle $F_{\rm last}$ on lane F; 2) the update of vehicles on lane S.

For $t_T > t_S$, the situation is similar, just the roles of lane T and S interchange.

As for $t_T = t_S$, it is reasonable to endow the vehicle nearer to the lattice $P_{\rm si}$ (provided that the distance to $P_{\rm si}$ is different for vehicles $T_{\rm lead}$ and $S_{\rm lead}$) with priority, and the problem becomes the same as the case $t_T > t_S$ or $t_T < t_S$.

If $t_T = t_S$ and vehicles of $T_{\rm lead}$ and $S_{\rm lead}$ have the same distance away from point $P_{\rm si}$, $T_{\rm lead}$ has priority over $S_{\rm lead}$ to occupy lattice $P_{\rm si}$ in accordance the priority rule and the update are similar to those in the case of $t_T < t_S$.

For the vehicles from a straight lane toward the corresponding taking-off point subscribed with iR, the probability for them to pass through the roundabout crossing and run on one of the other three straight roads is specified as one third. In order to make sure that a smooth and unblocked traffic flow appears in the computer simulation, we also establish the following stipulations for the four taking-off points on the outer-roundabout-lane. For a vehicle intending to drive into the inner roundabout lane and change its direction, the evolution will be made in accordance with the above regulations, and in case of $V_i = 0$ at that time step, it will drive out of the outer-roundabout-lane with a certain probability. For the four taking-off points on the inner roundabout lane, where the evolutional result shows $V_i = 0$ at that time step prior to its arrival at the intended outlet, the vehicle will also in advance drive out of the inner roundabout lane with a certain probability. Obviously, the vehicle will take priority of occupation of the corresponding joining-up point at the intended outlets and go out of the roundabout, because of the inner-lane-priority rule.

Although this model may be somewhat different from actual traffic flow, it embodies the nature of 'bottleneck' concentration at the grade roundabout crossing. Some results of the simulation are close to the actual design parameter.

3. Numerical simulation results and analysis

In order to reduce the boundary influence, we adopt the periodic boundary condition. The model simulation starts up with the vehicles distributed random at lattices. The macro-parameters are defined as follows. The ith lane is divided into a certain numbers of lattices and their number is indicated as L_i (the length of each lattice is 7.5m), the number of vehicles on the lane at t time is $N_i\,(t)$, the mean density $\rho_i\,(t) = N_i\,(t)\,/L_i$, the mean velocity

$V_i(t) = \sum_{j=1}^{N_i(t)} V_i(j,t)/N_i(t)$, and the mean flow rate $J_i(t) = \rho_i(t)\, V_i(t) = \sum_{j=1}^{N_i(t)} V_i(j,t)/L_i$. For the whole system, the mean density $\rho_0(t) = \sum_{i=1}^{16} N_i(t)/\sum_{i=1}^{16} L_i$, the mean velocity $V_0(t) = \sum_{i=1}^{16}\sum_{j=1}^{N_j(t)} V_i(j,t)/\sum_{i=1}^{16} N_i(t)$, mean flow rate $J_0(t) = \rho_0(t)\, V_0(t)$. The number of lattices for each straight lane (east, south, west and north) is 500, and for the outer roundabout lane, is 20 with 5 lattices on each segment; and for inner roundabout lane, $\overline{P_{\rm si}P_{\rm eo}} = \overline{P_{\rm ei}P_{\rm no}} = \overline{P_{\rm ni}P_{\rm wo}} = \overline{P_{\rm wi}P_{\rm so}} = 4$ and $\overline{P_{\rm so}P_{\rm si}} = \overline{P_{\rm eo}P_{\rm ei}} = \overline{P_{\rm no}P_{\rm ni}} = \overline{P_{\rm wo}P_{\rm wi}} = 1$, there are 16 lattices in total. The maximal velocity is $V_{\max} = 3$ for simulation, $V = 1$ is equivalent to 27km/h. Each sample runs 65,000 time steps, the average of the numerical results versus time is taken from the last 10,000 time steps, and 25 samples are taken for the assembly average to eliminate the influence of randomness.

As periodic boundary condition is adopted for the whole model, the density of vehicles on a driving-in lane is the same as that of vehicles on a driving-out lane, when the system is in the steady non-equilibrium state. However, the inner roundabout lane is a subsystem with the open boundary condition. At the joining-up points marked with the subscript of i, a certain number of vehicles drive in, and at the taking-off points marked with the subscript of o, a certain number of vehicles drive out. These two kinds of lattices are distributed alternatively in the inner roundabout lane. In addition, the distribution of density is not even: the density at the lane segments parallel to x or y axis is less than that at the lane not in parallel with x or y axis. According to the definition of bottleneck, there exist eight bottlenecks. The interaction between the vehicles for this crossing is stronger than that for other kinds of road structure. For each bottleneck, it is downstream of its above bottleneck as well as upstream of its below bottleneck. The distance between two neighbouring bottlenecks is so short that once a congested traffic flow is on the inner roundabout lane, the expanded congested pattern occurs (see Refs.[14] and [15]). Especially in this case, of the eight bottlenecks, where one is upstream or downstream of another, the traffic pattern will propagate fast from downstream to upstream. For a particular density, there will appear a corresponding steady self-organization structure.

In order to study the traffic flow on the inner roundabout lane, the relations between mean density and mean flow rate, and those between mean density and mean velocity are shown in Fig.2 and Fig.3 respectively. The relations of the mean system density versus the variables (density, flow rate, and velocity) of the inner roundabout lane are shown in Fig.4, where the abscissa stands for the system density, the ordinate represents the values for variables of the lane respectively. From Fig.3 and Fig.4, it is known that a very low density will make the lane get into the congested pattern and in case of $\rho_0 > 0.15$, there exists the velocity platform, $V = 1$. The interval where the mean velocity keeps unchangeable is $0.15 < \rho_0 < 0.56$, the flow rate of the traffic is also unchanged. Following increment of the density, the velocity and flow rate decrease. There also exists a velocity platform, $V = 1$, on the outer roundabout lane (see Fig.5), we consider that the strong interactions between the vehicles induce the velocity correlation effect.

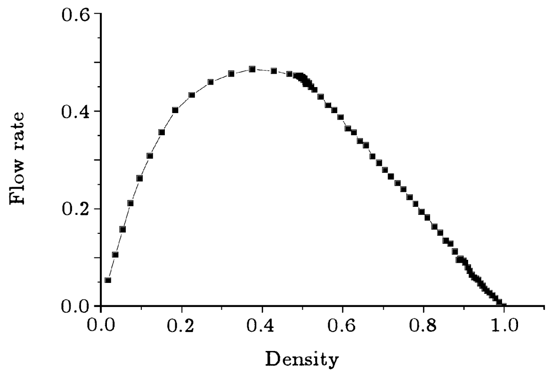

Fig.2. The relations between density-flow rate on inner roundabout lane.

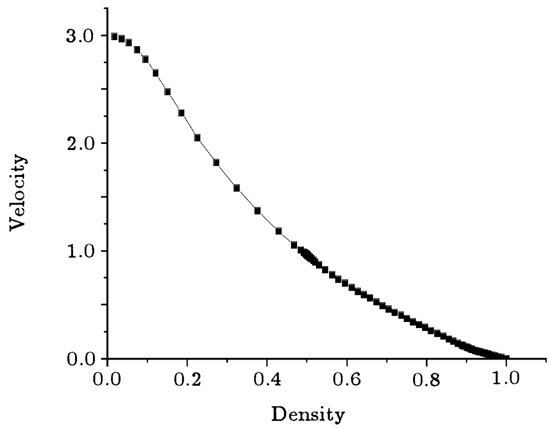

Fig.3. The relations between density-velocity on inner roundabout lane.

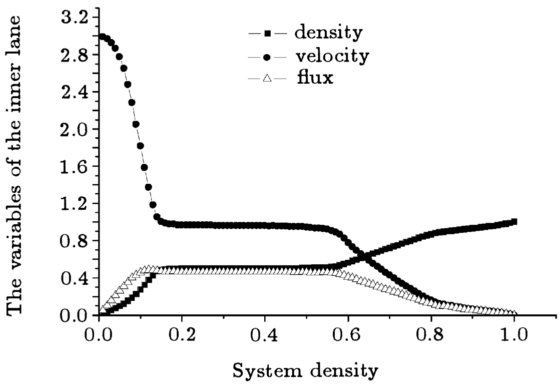

Fig.4. The traffic flow on inner roundabout lane.

For the outer roundabout segments, we take the segment of $S_{iR} \to U_{\rm s} \to E_{\rm oR}$ as an example, the computer simulation results of its traffic flow is shown in Fig.5. It is obvious that each outer roundabout segment is located between two bottlenecks, similar to the off-ramp at the end near the lattice $S_{\rm iR}$; and at the end near the lattice $E_{\rm oR}$ it is like the on-ramp. The traffic flow also appears as the congested pattern in the lower density. In the case of $0.15 < \rho_0 < 0.55$, the mean speed platform $V = 1$ occurs.

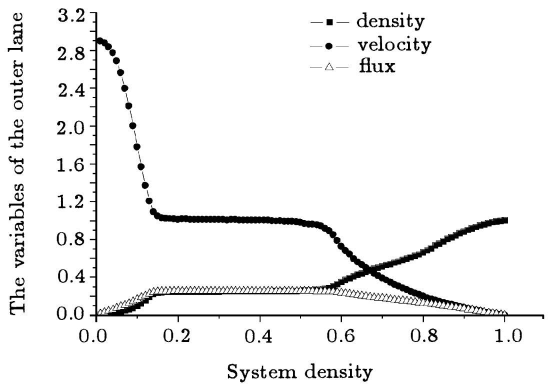

Fig.5. The traffic flow on outer roundabout lane.

The fundamental diagram for the driving-in and driving-out lanes is shown in Fig.6 and Fig.7 respectively. It is obvious that the impact of the roundabout lane on the straight lanes simply happens at the boundary, and it does not affect the fundamental diagram. Similar to Ref.[7], the flow rate remains unchanged in a certain density interval.

On the basis of the characteristics of a particular lane, the grade roundabout crossing as a whole will be discussed. In the actual traffic, no matter whether it goes straight forward or turns left, or turns right, a vehicle passing through the roundabout crossing must go through some bottlenecks. Once the congested pattern occurs in the traffic flow, the sixteen bottlenecks in the roundabout area with diameter of about 50–60m make the interactions of the vehicles become stronger. There are some differences between the two cases of focus on the whole system and on a single lane: 1) the bottleneck lattice related to the driving-out lane is downstream of the traffic flow; 2) the bottleneck lattice related to the driving-in lane is upstream of the flow; 3) in the case that a vehicle out of the inner roundabout and one on the outer roundabout lane segment to compete to occupy the joining-up points. In order to keep the inner roundabout unblocked, the inner-lane-priority rule shall be followed. For example, for the lattice $E_{\rm oR}$, the segment of $P_{\rm eo} \to E \to E_{\rm oR}$ is taken as the main lane. When the flow rate in the going-out lane decreases following the increase of the density, the vehicles in the district of roundabout cannot drive out to the downstream, and jams occur in the district. Generally, the grade roundabout crossing is used in those places where the traffic flow rate is less than 2000 veh/h.[16]

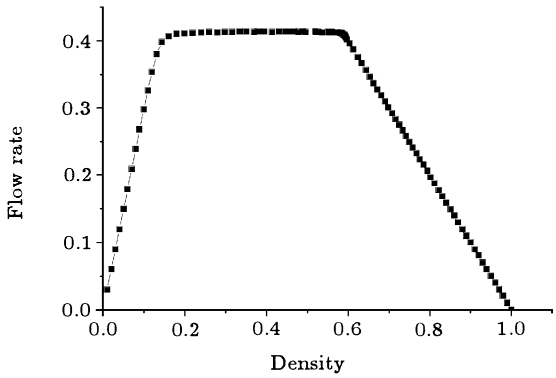

Fig.6. The fundamental diagram of the entering lane.

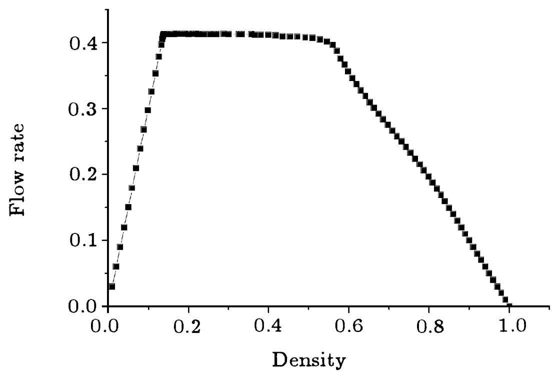

Fig.7. The fundamental diagram of the going-out lane.

Figure 8 demonstrates the traffic status of the whole system, the abscissa represents the mean system density, and the ordinate represents the mean velocity, mean flow rate and mean density respectively. It is obvious that in case of the mean density $\rho_0 < 0.15$, the traffic flow is in the free phase, the mean velocity keeps the maximum and unchanging, and the mean flow rate rises up following the increase of the mean density. In case of $0.15 < \rho_0 < 0.58$, the mean velocity gradually reduces and the flow rate keeps a more stable value and appears as a slightly declined platform. In case of $\rho_0 > 0.58$, mean velocity and flow rate will rapidly reduce, and jams appear over the whole system. From Figs.4, 5 and 8, it is known that near the point of $\rho_0 = 0.58$ there appear the turn points of the curves of velocity and flow rate. It proves that this work is self-consistent.

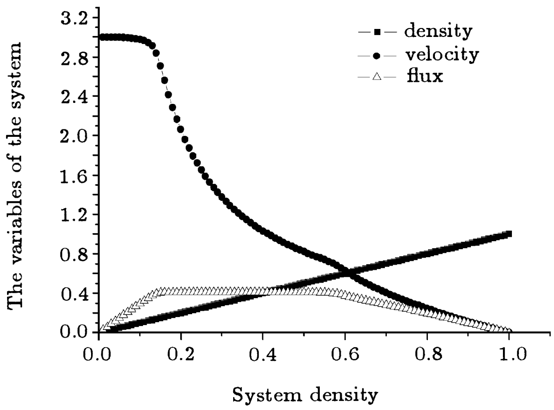

Fig.8. The traffic flow of the system (average).

4. Summaries and outlook

In order to research urban traffic flow in a more realistic manner, a model for traffic flow at the grade roundabout crossing is suggested and simulation carried out. Because of concentrated bottlenecks in this kind of road structure, it is indicated that when the traffic flow gets into the congested pattern, the expanded congested pattern will play an important role. Due to the self-organization, the mean velocity on the roundabout is not too high and the correlation effect is obvious. The simulation results of velocity in this work are close to the design data required[16] (the diameter of inner roundabout lane in this paper is about 50m, $V = 1$ is equivalent 27km/h. Reference [16] shows that if the velocity on the roundabout lane is given (suppose $V = 25$km/h), 48m is required for the design diameter of the roundabout). Especially, in the case that the congested pattern occurs in the traffic flow near the bottlenecks, its spatial–temporal structure is predictable and reproducible. Therefore, for its specific application to an actual traffic project, the design will be simplified.

As a beginning of this kind of work, this paper appears somewhat simple and coarse. Abundant phenomena of traffic flow at the grade roundabout crossing are being discussed, and the comparison of traffic flow in this structure with those in other kinds of other structures will be taken into account. Especially, we can combine Refs.[9] and [17] to research into more qualitative problems. Because of the low construction cost, the grade roundabout crossing as a common crossing configuration is adopted in the traffic network, especially in areas with a middle traffic flow. We will make more actual and detailed study in the future.

References

[1] Li K P and Gao Z Y 2005 *Chin. Phys.* **14** 1128
[2] Biham O, Middleton A A and Levine D 1992 *Phys. Rev.* A **46** 6124
[3] Nagatani T 1993 *Phys. Rev.* E **48** 3290
[4] Cuesta J, Martine F, Molera J and Sanchez A 1993 *Phys. Rev.* E **48** R4175
[5] Lü X Y, Chen R H, Liu M R and Kong L J 1997 *Acta Phys. Sin.* **46** 435 (in Chinese)
[6] Tan H L, Huang P H, Li H B, Liu M R and Kong L J 2003 *Acta Phys. Sin.* **52** 1127 (in Chinese)
[7] Tan H L, Zhang C Y, Kong L J and Liu M R 2004 *International Journal of Modern Physics* B **18** 2568
[8] Huang P H, Kong L J and Liu M R 2002 *Chin. Phys.* **11** 678
[9] Fouladvand M E, Sadjadi Z and Shaebani M R 2004 *Phys. Rev.* E **70** 046132
[10] Wang R and Ruskin H J 2002 *Comp. Phys. Comm.* **147** 570
[11] Kerner B S 2004 *Physica* A **333** 379
[12] Nagel K and Schrenckenberg M 1992 *J. Phys.* I **2** 2221(in France)
[13] Jang R, Wu Q S and Wang B H 2002 *Phys. Rev.* E **66** 036104
[14] Kerner B S 2002 *Phys. Rev.* E **65** 046138
[15] Kerner B S and Klenov S L 2003 *Phys. Rev.* E **68** 036130
[16] Chen H R 1991 *The Design of Crossing Road* (Beijing: People's Traffic Press) (in Chinese)
[17] Xue Y 2002 *Chin. Phys.* **11** 1128