1.

x	3	4	5
$p(x)$	0.1	0.6	0.3

x = # of walks per day for Leo

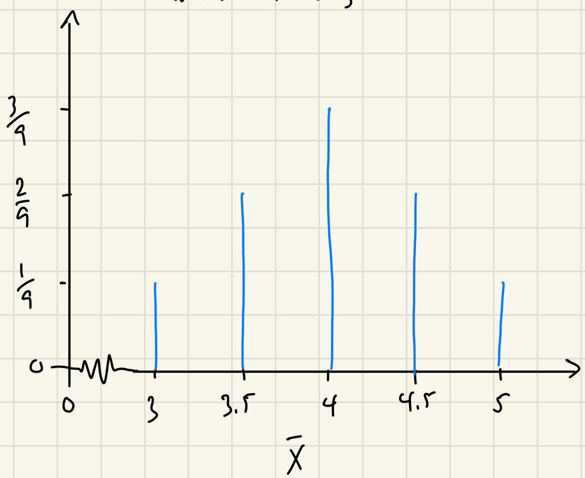
$$E[x] = \mu = (3)(0.1) + (4)(0.6) + (5)(0.3)$$
$$= 4.2$$

$$V[x] = (3^2)(0.1) + (4^2)(0.6) + (5^2)(0.3) - 4.2^2$$
$$= 0.36$$

a)

day 1	day 2	$\bar{x}$
3	3	3
3	4	3.5
3	5	4
4	3	3.5
4	4	4
4	5	4.5
5	3	4
5	4	4.5
5	5	5

sampling distribution of the mean number of long walks in 2 days



b) since $n > 30$ $\quad \bar{x}$ approx. $N\left(4.2, \frac{0.36}{40}\right)$

$\therefore$ for $n = 40$, $\bar{x}$ approx. $N\left(4.2, \frac{0.36}{40}\right)$

c) $P(\bar{x} > 4.1) = P\left(Z > \frac{4.1 - 4.2}{\sqrt{\frac{0.36}{40}}}\right)$

$= P(Z > -1.05)$

$= P(Z < 1.05)$

$= 0.8531$

$\therefore$ The probability of the sample mean of 40 days of walks being greater than 4.1 walks/day is approximately 85 %.

2. Define:
85% of vaccinations are effective

X = # of people with effective vaccinations

$n = 40$ vaccinations

$X \sim B(n = 40, p = 0.85)$

a) $P(X = 30) = \binom{40}{30}(0.15)^{10}(0.85)^{30}$

$= 0.0373$

$\therefore$ The probability of 30 out of 40 people having effective vaccinations is around 3.73%

b) $np = (40)(0.85) = 34$

$nq = (40)(0.15) = 6$

$np > 5$ and $nq > 5$

$\therefore$ X approx. $N(\mu = np = 34,\ \sigma^2 = npq = 5.1)$

$$P(32 < X \leq 38) = P(X \leq 38) - P(X < 32)$$

$$= P\left(Z \leq \frac{38-34}{\sqrt{5.1}}\right) - P\left(Z < \frac{32-34}{\sqrt{5.1}}\right)$$

$$= P(Z \leq 0.56) - P(Z < -0.89)$$

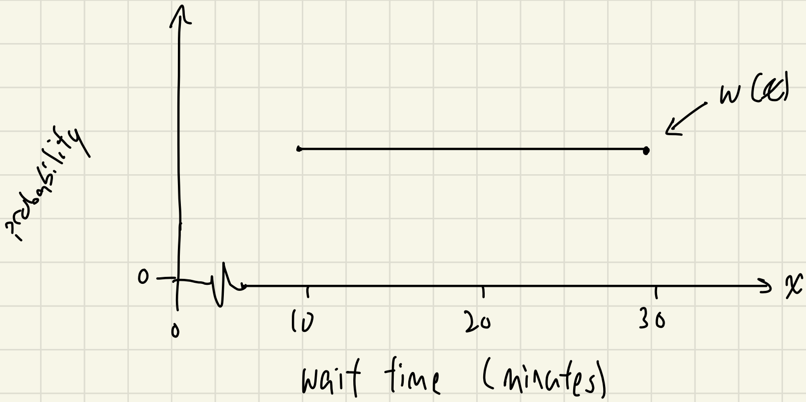
$$= 0.7123 - 0.1867$$

$$= 0.5256$$

$\therefore$ The probability of having between 32 and 38 people out of 40 with effective vaccinations is around 52.56%

3.

PDF of wait times



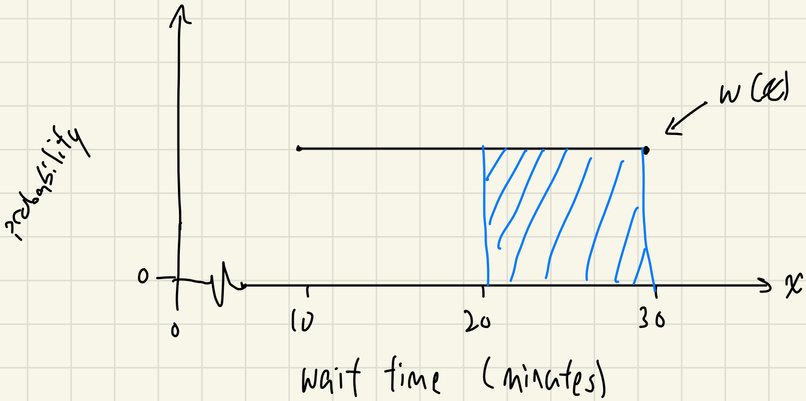
Let $w(x)$ be defined for $10 \leq x \leq 30$.

$$\int_{-\infty}^{\infty} w(x)\,dx = 1$$

Let $X = w(x)$ = wait time in minutes

a) $P(X > 20) = \int_{20}^{\infty} w(x)\,dx = 0.5$

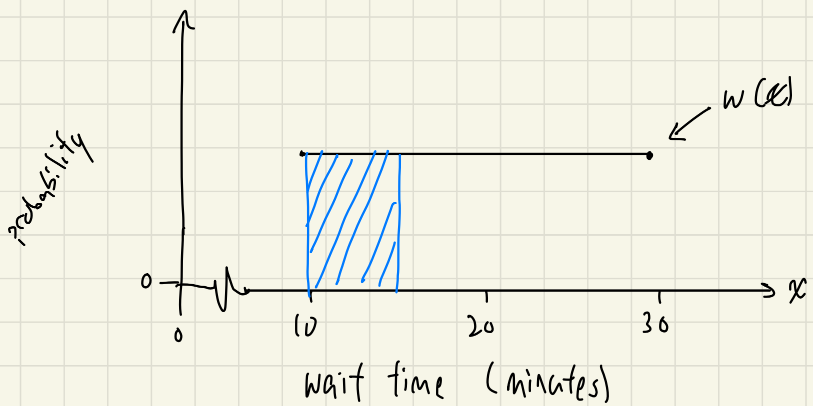
PDF of wait times



$\therefore$ 50% chance of waiting > 20 minutes

b) $P(X < 15) = \int_{-\infty}^{15} w(x)\,dx = 0.25$

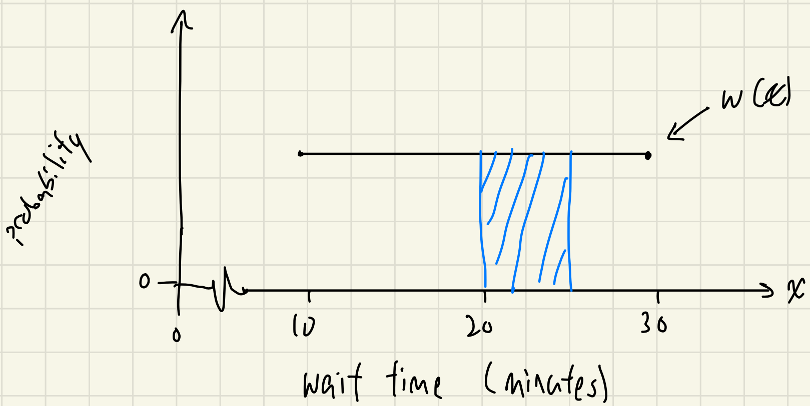
PDF of wait times



$\therefore$ 25% chance of waiting < 15 minutes

c) $P(20 < X < 25) = \int_{20}^{25} w(x)\,dx = 0.25$

PDF of wait times



$\therefore$ 25% chance of waiting between 20 and 25 minutes

4. Define:

X = systolic blood pressure of someone in mmHg

$X \sim N(120 \text{ mmHg}, (25 \text{ mmHg})^2)$

a) $P(110 < X < 125) = P(X < 125) - P(X < 110)$

$$= P\left(Z < \frac{125-120}{\sqrt{25^2}}\right) - P\left(Z < \frac{110-120}{\sqrt{25^2}}\right)$$

$$= P(Z < 0.2) - P(Z < -0.4)$$

$$= 0.5793 - 0.3446$$

$$= 0.2347$$

$\therefore$ The probability of this dude's systolic blood pressure reading between 110 mmHg and 125 mmHg is around 23.5%

b) $P(X < 130) = P\left(Z < \frac{130-120}{\sqrt{25^2}}\right)$

$$= P(Z < 0.4)$$

$$= 0.6554$$

$\therefore$ The probability of this dude's systolic blood pressure reading less than 130 mmHg is around 65.5%

c) $P(X > 140) = P\left(Z > \frac{140-120}{\sqrt{25^2}}\right)$

$= P(Z > 0.8)$

$= P(Z < -0.8)$

$= 0.2119$

$\therefore$ The probability of this person having a hypertensive reading is around 21.2%

5. Define:

X = lifetime of a certain type and brand of TV in years

$X \sim \exp(8 \text{ years})$

a) $P(X > 6) = e^{-\frac{6}{8}} = 0.4723$

$\therefore$ probability of this brand and make of TV lasting >6 years is around 47.2%

b) $P(X < 7) = 1 - e^{-\frac{7}{8}} = 1 - 0.4169 = 0.5831$

$\therefore$ probability of this brand and make of TV lasting <7 years is around 58.3%

c) $0.9 = P(X < x)$

$0.9 = e^{-\frac{x}{8}} \longrightarrow x = -8\ln(0.9)$

$= 0.8429$

$\therefore$ Most TVs of this make and brand last only for around 0.8429 years or 308 days.