CS 477/677 Analysis of Algorithms

Fall 2020

Homework 1

Due date: September 8, 2020

Note: Students in the 400 section can solve the graduate problem or the extra credit problem for additional points. If both problems are attempted, the highest score will be kept (not both).

1. (U & G-required) [30 points] Arrange the following list of functions in ascending order of growth rate. That is, if function g(n) immediately follows function f(n) in your list, then f(n) should be O(g(n)).

$f_1(n) = \frac{n^3+2n+1}{n+3} logn$

$f_2(n) = \sqrt[4]{n^8 + 4}$

$f_3(n) = (logn)^3 + 2n^2 + 3$

$f_4(n) = 5^{n+1}$

$f_5(n) = 10^n$

$f_6(n) = (n + 1)logn$

2. (U & G-required) [30 points] Using mathematical induction, show that the following relations are true for every n ≥1:

a) $\sum_{i=1}^{n} i(i!) = (n + 1)!-1$

b) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

3. (U & G-required) [40 points] Using the formal definition of the asymptotic notations, prove the following statements:

a) $n^3 + 10n^2 \in O(n^3)$

b) $5n^3 + 2000n \in \Omega(n^2)$

c) $n! \in O(n^n)$

d) $10n^2 + 2 \notin O(n)$

4. (G-Required) [20 points] For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. Use the simplest g(n) possible in your answers.

a) $(n^2 + n + 7)^5$

b) $\frac{\sqrt{n^4+2n+5}}{n+1}$

c) $2nlg(2n + 1)^4 + (2n + 1)^4 lgn$

d) $\frac{10^n}{5^{n+1}}$

5. [Extra credit - 20 points] Find the order of growth of the following sums:

a) $\sum_{i=0}^{n-1}(i^2 + 1)^2$

b) $\sum_{i=2}^{n-1} lgi^2$