Zad. 4. Konsument - 2 okresy

$u(c_1, c_2) = \ln(c_1) + 0{,}6 \ln(c_2)$

$m_1 = 100$

$m_2 = 0$

$r = 10\%$

S - oszczędności w okresie 1

a) ograniczenie budżetowe → czyli ile może mieć konsument w okresie 2

$c_1 + S = m_1 \Rightarrow S = m_1 - c_1$

$c_2 = m_2 + (1+r) \cdot S$

$c_2 = m_2 + (1+r) \cdot (m_1 - c_1)$

$c_2 = 0 + (1+0{,}1)(100 - c_1)$

$c_2 = 110 - 1{,}1 c_1$

b) $c_1 = ?$ $c_2 = ?$

$\max_{c_1, c_2}$, ale $c_2 = 110 - 1{,}1 c_1$

$\max \{ \ln(c_1) + 0{,}6 \ln(c_2) \}$

p.w. $c_2 = 110 - 1{,}1 c_1 = (1+r)(m_1 - c_1)$

$L = \ln(c_1) + 0{,}6 \ln(c_2) - \lambda\left((1+r)(m_1 - c_1) - c_2 \right)$

$L : \ln(c_1) + 0{,}6 \ln(c_2) - \lambda(m_1 + m_1 \cdot r - c_1 - c_1 \cdot r - c_2)$

$$\begin{cases} \frac{\partial L}{\partial c_1} = 0 \Leftrightarrow \frac{1}{c_1} + \lambda(1+r) = 0 \Leftrightarrow \lambda = -\frac{1}{c_1(1+r)} \\ \frac{\partial L}{\partial c_2} = 0 \Leftrightarrow \frac{0{,}6}{c_2} + \lambda = 0 \Leftrightarrow \lambda = -\frac{0{,}6}{c_2} \rightarrow 0{,}6 c_1 (1+r) = c_2 \\ \frac{\partial L}{\partial \lambda} = 0 \Leftrightarrow -[m_1 + m_1 \cdot r - c_1 - c_1 \cdot r - c_2] = 0 \\ m_1(1+r) - c_1(1+r) - 0{,}6 c_1 (1+r) = 0 \end{cases}$$

verte

$$1{,}6\,c_1 = m_1$$

$$c_1 = \frac{m_1}{1{,}6} = \frac{100}{1{,}6} = \underline{62{,}5}$$

$$c_2 = 0{,}6\,(c_1(1+r)) = 0{,}6 \cdot 62{,}5 \cdot 1{,}1 = \underline{41{,}25}$$

sprawdzenie:

$$s = 100 - 62{,}5 = 37{,}5$$
$$s(1+r) = 37{,}5 \cdot 1{,}1 = 41{,}25$$

c) podatek $T = 20\%$

od dochodu

$$c_1 + s = (1-T)m_1 \Rightarrow s = (1-T)m_1 - c_1$$

$$c_2 = (1+r)\left[(1-T)m_1 - c_1\right]$$

$$c_2 = 88 - 1{,}1\,c_1$$

$\max_{c_1, c_2}$, ele $c_2 = 88 - 1{,}1\,c_1$

Analogicznie do b)

$$L: \ln(c_1) + 0{,}6\ln(c_2) - \lambda\left((1+r)\left((1-T)m_1 - c_1\right) - c_2\right)$$

$$\begin{cases} \frac{\partial L}{\partial c_1} = 0 \Leftrightarrow \frac{1}{c_1} + \lambda(1+r) = 0 \\ \frac{\partial L}{\partial c_2} = 0 \Leftrightarrow \frac{0{,}6}{c_2} + \lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \Leftrightarrow -\left[(1+r)(1-T)m_1 - (1+r)c_1 - c_2\right] = 0 \end{cases}$$

$\rightarrow \quad c_2 = 0{,}6\,c_1(1+r)$

$$\cancel{(1+r)}(1-T)m_1 - \cancel{(1+r)}c_1 - 0{,}6\cancel{(1+r)}c_1 = 0$$

$$1{,}6\,c_1 = (1-T)m_1$$

$$c_1 = \frac{(1-T)m_1}{1{,}6} = \frac{80}{1{,}6} = \underline{50} \qquad c_2 = 0{,}6 \cdot 50 \cdot 1{,}1 = \underline{33}$$

sprawdzenie

$$s = 80 - 50 = 30$$
$$s(1+r) = 30 \cdot 1{,}1 = 33$$