Info-gap robustness assessment of reliability evaluations using several OpenTURNS features

OpenTURNS User's Day

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Scientific context

Uncertainty framework

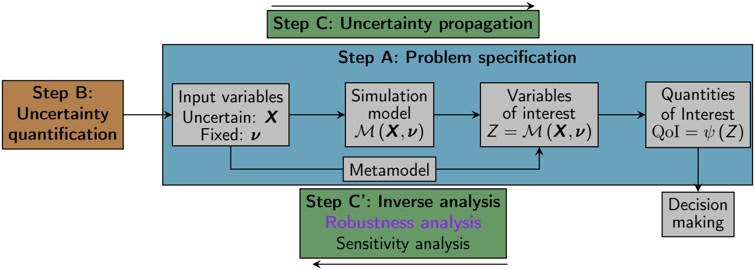

Figure 1: General uncertainty quantification and propagation framework.

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Info-gap applied to reliability analysis

- **Robustness curves** (*Ben-Haïm, 2006*):

decision 1

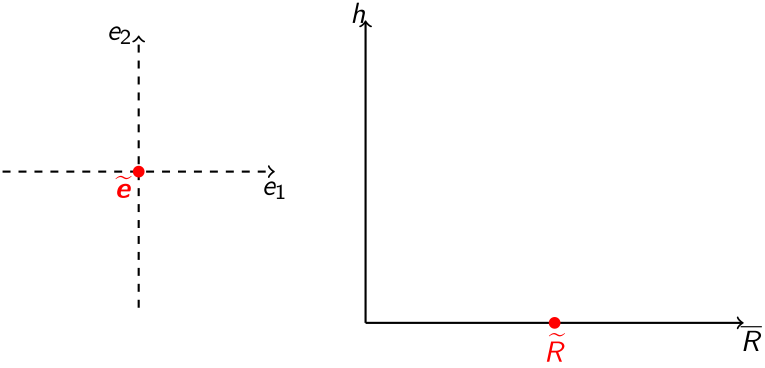

Figure 2: Nested convex sets with robustness curves.

Info-gap applied to reliability analysis

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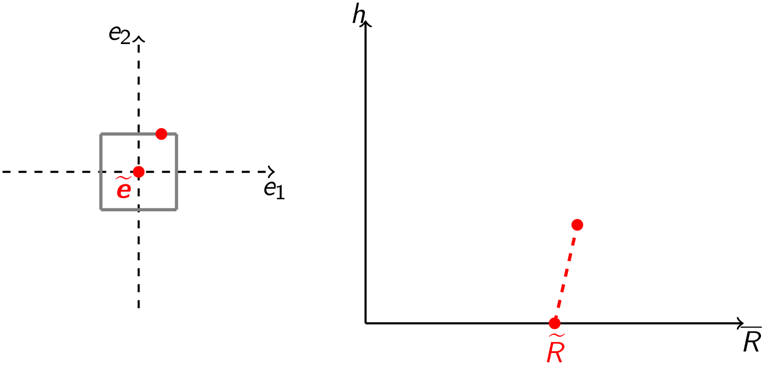

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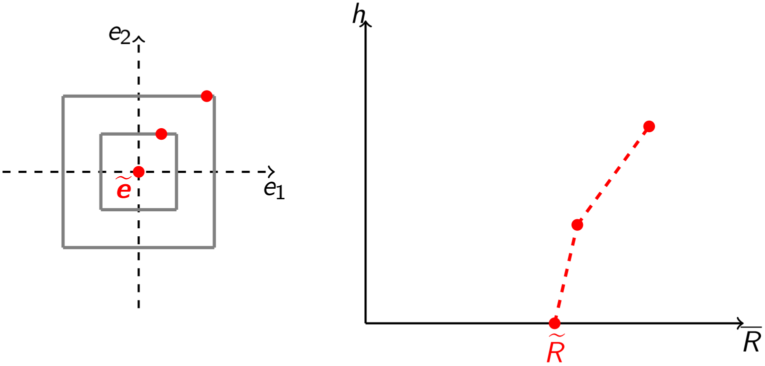

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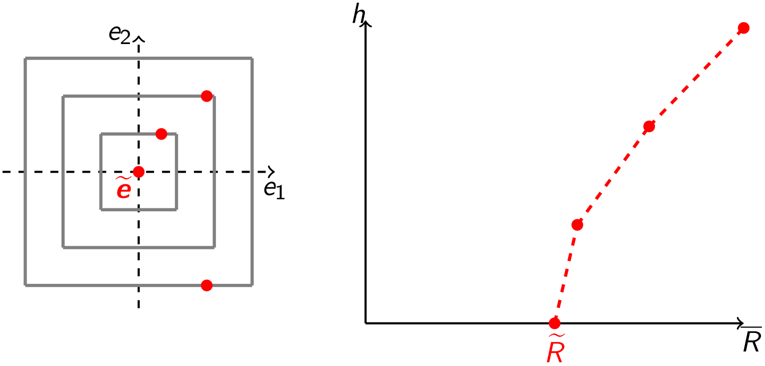

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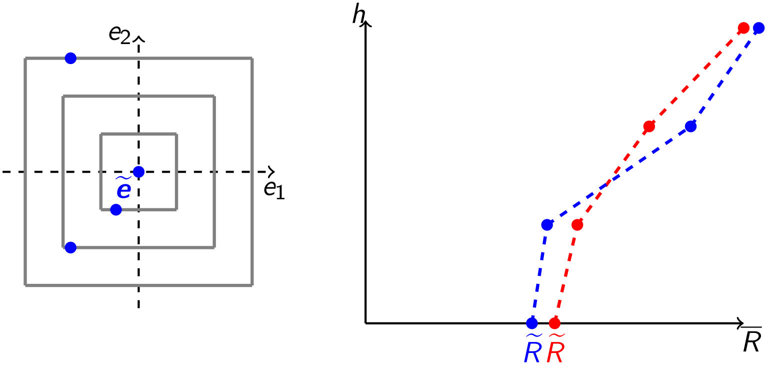

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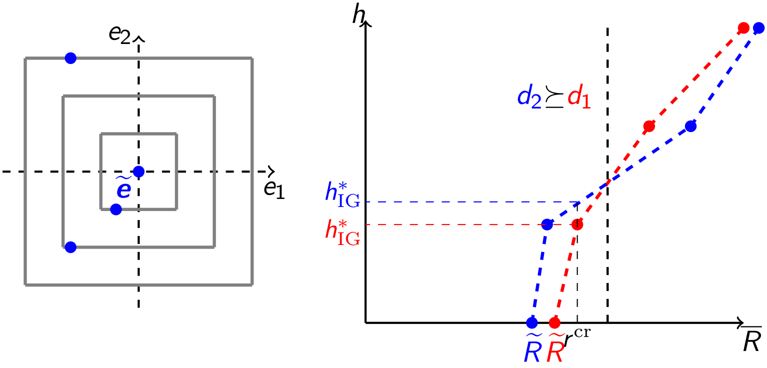

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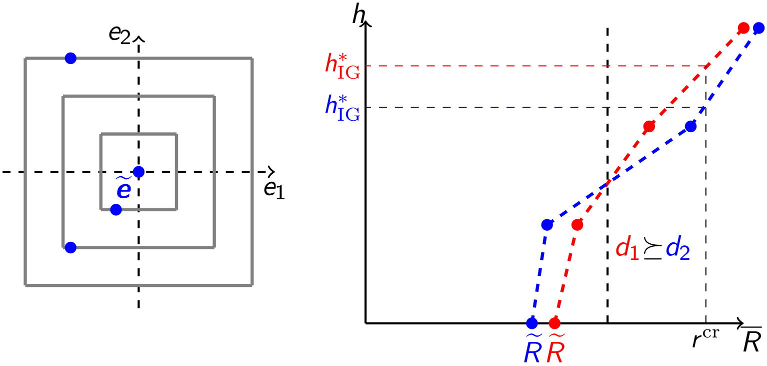

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Info-gap applied to reliability analysis

Robustness of reliability evaluations

$$h^*_{\mathrm{IG}} = \max_{h} \left\{ \max_{\mathsf{e} \in U\left(h, \widetilde{\mathsf{e}}\right)} \mathrm{QoI}\left(\mathsf{d}, \mathsf{e}\right) \leq \mathrm{QoI}^{\mathrm{cr}} \right\} \tag{1}$$

- QoI: (small) failure probability, (high-order) quantile, superquantile ...
- ***e***: input variables, hyperparameters of a probability distribution, model errors, ...

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Q1 How to apply IG uncertainty models to the reliability framework?

Info-gap applied to reliability analysis

Robustness of reliability evaluations

$$h^*_{\mathrm{IG}} = \max_{h} \left\{ \max_{\mathrm{e} \in U\left(h, \widetilde{\mathrm{e}}\right)} \boxed{\mathrm{QoI}\,(\mathrm{d}, \mathrm{e})} \leq \mathrm{QoI}^{\mathrm{cr}} \right\} \tag{1}$$

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Q2 How to propagate hybrid uncertainty (aleatory and epistemic) in order to estimate the bounds of the QoI?

Info-gap applied to reliability analysis

Robustness of reliability evaluations

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- QoI: (small) failure probability, (high-order) quantile, superquantile ...
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Q3 How to efficiently reduce the computational burden induced by both the QoI estimator and the IG formulation?

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2 – Hybrid reliability analysis

- **Failure probability estimation under aleatory uncertainty**

$$P_{\mathrm{f}} = \Pr\left[g\left(\boldsymbol{X}\right) \leq 0\right] = \int_{g(\boldsymbol{x}) \leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = \int_{\mathbb{R}^{n_{\boldsymbol{X}}}} 1_{g(\boldsymbol{x}) \leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \qquad (2)$$

- **Failure probability estimation under hybrid uncertainty**

$$P_{\mathrm{f}} = \Pr\left[g\left(\boldsymbol{X}, \boldsymbol{Y}\right) \leq 0\right] \qquad (3)$$

- Random vector $\boldsymbol{X} \sim f_{\boldsymbol{X}}$
- Epistemic vector $\boldsymbol{Y}$: intervals, convex sets, possibility distribution, Dempster-Shafer structures, p-boxes, probability distribution ...
- Only the bounds $\left[\underline{P_{\mathrm{f}}}, \overline{P_{\mathrm{f}}}\right]$ of the failure probability can be estimated
- Need for a common framework for estimating and comparing these bounds

2 – Hybrid reliability analysis

- **Random sets as a unifying framework** (*Alvarez, 2006*)
  - With $\Omega = [0,1]^{n_X + n_Y}$ and $\alpha_{Y_i} \sim U(0,1)$, one retrieves several uncertainty representations:

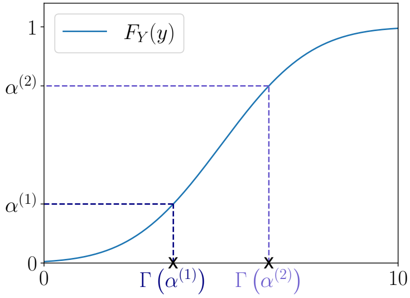

Figure 3: Random set with a probabilistic cdf.

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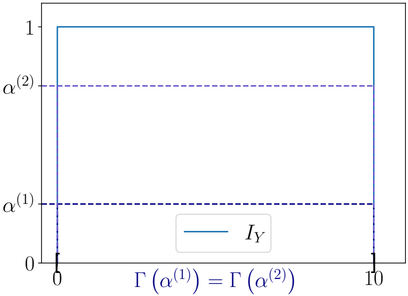

Figure 4: Random set with an interval.

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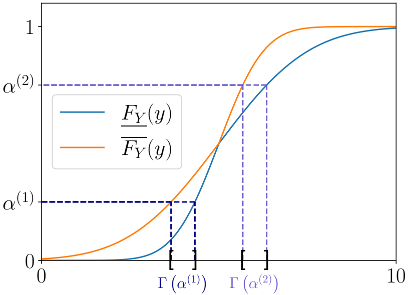


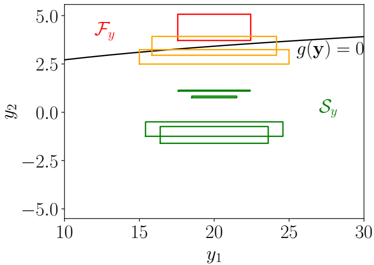
Figure 5: Random set with a p-box.

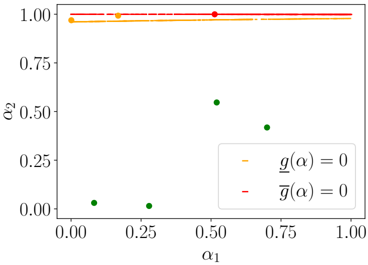
2 – Hybrid reliability analysis

- **Random sets in the reliability framework** (*Alvarez et al., 2018*)

$$\overline{P_\mathrm{f}} = \Pr\left[\min_{\Gamma(\boldsymbol{\alpha})} g\left(\boldsymbol{\alpha}\right) \leq 0\right] = \int_\Omega \mathbb{1}_{\underline{g}(\boldsymbol{\alpha})\leq 0}\mathrm{d}\boldsymbol{\alpha} \quad (5a)$$

$$\underline{P_\mathrm{f}} = \Pr\left[\max_{\Gamma(\boldsymbol{\alpha})} g\left(\boldsymbol{\alpha}\right) \leq 0\right] = \int_\Omega \mathbb{1}_{\overline{g}(\boldsymbol{\alpha})\leq 0}\mathrm{d}\boldsymbol{\alpha} \quad (5b)$$





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3 – Reliability of penstocks

Figure 6: Example of a penstock operated by EDF.

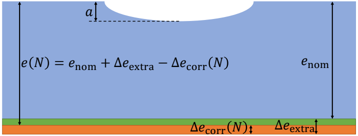

Figure 7: illustration of a penstock presenting a planar flaw of height a.

3 – Reliability of penstocks

- **Main characteristics** (*Ardillon et al., 2022*)
  - **Failure mode** (*Step A*): brittle failure affecting welds due to corrosion and the presence of a default
  - **Simulation model** (*Step A*): analytical expressions → fast to compute
  - **QoI** (*Steps A and C*): small failure probability

$$P_{\mathrm{f}} = \mathrm{Pr}\left(\{G_{N+1}\left(\boldsymbol{X}, \boldsymbol{Y}\right) \leq 0\} \cap \{G_N\left(\boldsymbol{X}, \boldsymbol{Y}\right) > 0\} \cap \{G_{\mathrm{HPT}}\left(\boldsymbol{X}, \boldsymbol{Y}\right) > 0\}\right) \tag{6}$$

  - **Uncertain inputs dimension** (*Step A*): $n_{\boldsymbol{X}} + n_{\boldsymbol{Y}} = 6$
  - **IG motivation** (*Step C'*): are there some nominal configurations of penstocks more robust than others?
  - **Challenge** (*Step C*): complexity of the failure domain and of the failure probability estimation

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4 – Robustness of the reliability assessment of penstocks

IG uncertainty model (*Step B*)

Variable X_i	Distribution	Param. 1	Param. 2	Param. 3
$X_1 = R_m$ (MPa)	Lognormal	480	24	-
$X_2 = \varepsilon$ (MPa)	Normal	0	16.816	-
$Y_1 = \Delta e_{\mathrm{extra}}$ (mm)	Normal	θ_1	0.25	-
$Y_2 = \Delta e_{\mathrm{corr}}$ (mm) (mm)	Normal	θ_2	0.4	-
$Y_3 = a$ (mm)	Uniform	0	θ_3	-
$Y_4 = K_{\mathrm{IC}}$ (MPa.$\sqrt{\mathrm{m}}$)	Weibull Min	θ_4	4	20

Table 1: Input probabilistic modeling of $\boldsymbol{X}$ for the penstock use-case.

- Parametric p-box uncertainty model

$$D_{\boldsymbol{\theta}}\left(h, \widetilde{\boldsymbol{\theta}}\right) = \bigtimes_{i=1}^{4} I_{\theta_i}(h) \text{ with } \begin{cases} I_{\theta_i}(h) = \left[\widetilde{\theta}_i\,(1-h)\,, \widetilde{\theta}_i\,(1+h)\right] \text{ if } \widetilde{\theta}_i > 0 \\ I_{\theta_i}(h) = [-h, h] \text{ if } \widetilde{\theta}_i = 0 \end{cases} \tag{7}$$

- Robustness function

$$h^*_{\mathrm{IG}} = \max_{h} \left\{ \max_{\boldsymbol{\theta} \in D_{\boldsymbol{\theta}}\left(h, \widetilde{\boldsymbol{\theta}}\right)} P_{\mathrm{f}}\left(\boldsymbol{\theta}\right) \leq P_{\mathrm{f}}^{\mathrm{cr}} \right\} \tag{8}$$

4 – Robustness of the reliability assessment of penstocks

- **Failure probability estimator** (*Step C*)
  - **FISSTAR**: FORM-IS-Tested Automatically-Rapid seaRch: current algorithm used for the failure probability estimations
  - **Line sampling**: directional technique investigated in order to better target the restricted failure domain

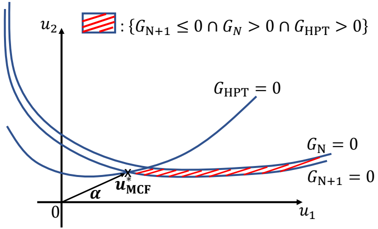

Figure 8: Illustration of the failure domain for the reliability of penstocks.

4 – Robustness of the reliability assessment of penstocks

"Standard" Line Sampling (*Koutsourelakis, 2004*)

- $\boldsymbol{u} = u^{\|}\boldsymbol{\alpha} + \boldsymbol{u}^{\perp}$

$$P_{\mathrm{f}} = \mathbb{E}_{\varphi_{\boldsymbol{u}^{\perp}}}\left[\int_{G\left(u^{\|}\boldsymbol{\alpha}+\boldsymbol{u}^{\perp}\right)\leq 0}\varphi\left(u^{\|}\right)\mathrm{d}u^{\|}\right] = \mathbb{E}_{\varphi_{\boldsymbol{u}^{\perp}}}\left[\Phi\left(-r\left(\boldsymbol{u}^{\perp}\right)\right)\right] \tag{9}$$

- LS estimator:
$\boldsymbol{u}^{\perp,(i)} = \boldsymbol{u}^{(i)} - \left(\boldsymbol{u}^{(i)}\boldsymbol{\alpha}\right)\boldsymbol{\alpha}^{\top}$

$$\widehat{P}_{\mathrm{f}} = \frac{1}{n_{\mathrm{LS}}}\sum_{i=1}^{n_{\mathrm{LS}}}\Phi\left(-r\left(\boldsymbol{u}^{\perp,(i)}\right)\right) = \frac{1}{n_{\mathrm{LS}}}\sum_{i=1}^{n_{\mathrm{LS}}}p_{\mathrm{f}}^{(i)} \tag{10}$$

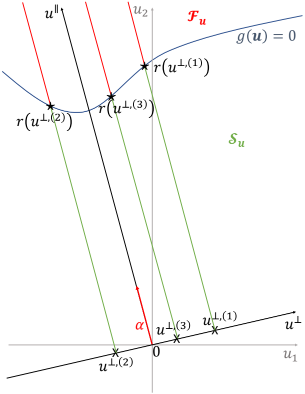

Figure 9: LS illustration.

4 – Robustness of the reliability assessment of penstocks

- LS applied to the reliability of penstocks
  - **Case 1**: no root $\rightarrow$ $p_{\mathrm{f}}^{(i)} = 0$
  - **Case 2**: two roots $\rightarrow$ $p_{\mathrm{f}}^{(i)} = \Phi\left(-r_1\left(\boldsymbol{u}^{\perp,(i)}\right)\right) - \Phi\left(-r_2\left(\boldsymbol{u}^{\perp,(i)}\right)\right)$

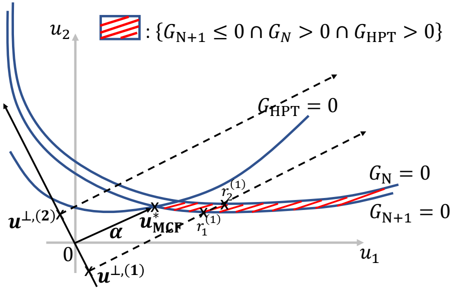

Figure 10: LS applied to the restricted failure domain

4 – Robustness of the reliability assessment of penstocks

- Adapted LS algorithms for each formulation E_i
  - $E_3 = \{G_{N+1} \leq 0 \cap G_N > 0 \cap G_{\mathrm{HPT}} > 0\}$

A_{E_3}
Find r_1 s.t. $G^{\perp}_{N+1}(r_1) = 0$
If $G^{\perp}_{\mathrm{HPT}}(r_1) > 0$ **then**
Find r_2 s.t. $G^{\perp}_{N}(r_2) = 0$
If $G^{\perp}_{\mathrm{HPT}}(r_2) < 0$ **then**
Find r_2 s.t. $G^{\perp}_{\mathrm{HPT}}(r_2) = 0$
Else:
no roots

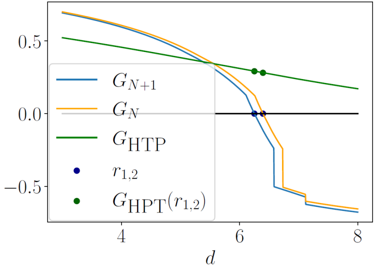

Figure 11: Roots search on E_3

4 – Robustness of the reliability assessment of penstocks

- Application of the methodology for the robustness evaluation (*Step C'*)

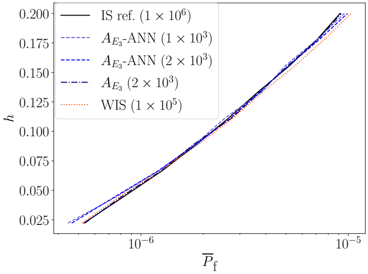

Figure 12: Comparison of robustness curves.

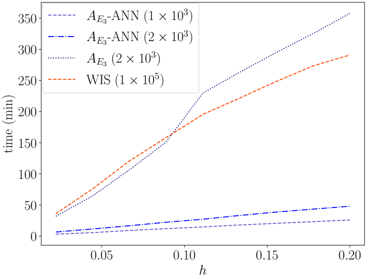

Figure 13: Comparison of the cumulative time needed for the robustness assessment.

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Conclusion : thank you OpenTURNS

- **Data analysis**
  - ...
- **Probabilistic modeling**
  - Standard parametric models
  - Copulas
  - Convex sets, possibility distributions, DS structures, p-boxes, random sets
- **Meta modeling**
  - Artificial neural networks (Keras)
- **Reliability, sensitivity**
  - MC, FORM, IS, SS
  - Line Sampling
- **Numerical methods**
  - Uniform Random Generator
  - Distribution realizations + random set realizations
  - Optimization Algorithms : NLopt (DIRECT) + root search (scipy)
  - Isoprobabilistic transformation

References I

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