How GEMSEO uses OpenTURNS for multidisciplinary problems

Matthias DE LOZZO on behalf the GEMSEO Team

OpenTURNS Users'Day, Friday 23 June 2023, EDF Lab Saclay

IRT Saint Exupéry

Accelerate science, technology research
& transfers to industry

What is IRT Saint Exupéry?

The IRT Saint Exupery is a collaborative and integrated technological research center bridging the public research to the industrial one.

Our missions:

- **Promote** the French technological research for the benefit of the industry established on the national territory
- **Develop** the ecosystem for the aeronautics, space and critical systems sectors by providing access to our research projects, technological platforms & expertise.
- **Create** a link between public and private research in order to facilitate transfer by mobilizing resources from the academic world for the implementation of research in industry.
- **Carry out** collaborative and integrated research projects based on industrial needs with an upstream contribution from the academic community, supported and financed by the French government and industrial members

FOUNDER MEMBERS

cnrs	cnes CENTRE NATIONAL D'ÉTUDES SPATIALES
Université Fédérale Toulouse Midi-Pyrénées	ACTIA
université de BORDEAUX	AIRBUS
LIEBHERR	THALES
SAFRAN	

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IRT Saint Exupéry competences and targets

12 competences
@IRT Saint Exupéry

- High voltage energy >
- High Reliability Energy >
- High density energy >
- Metallic materials and processes >
- Surfaces / assemblies >
- Composite materials >
- Advanced Learning >
- AI for critical systems >
- Smart Connectivity and Sensing >
- Systems Engineering >
- Multi Discipline Optimization >
- Critical Embedded Systems >

4 Technological Axes
@the service of industrialists

- Greener Technologies
- Advanced Manufacturing Technologies
- Smart Technologies
- Methods and tools for the development of complex systems

Enable digital and collaborative system engineering
Develop and transfer robust multidisciplinary optimization
Design efficient & secure hardware and software architectures

Target Markets

- Aeronautics
- Space
- Defence

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Agenda.

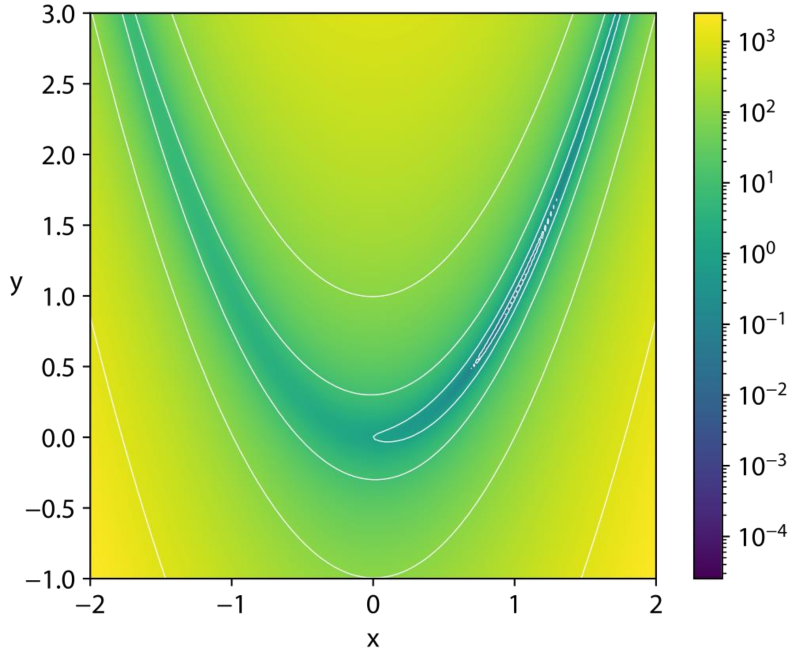
Multidisciplinary design optimization (MDO)

28/06/2023

Minimizing a cost function is an optimization problem.

Initial questions

- Are there constraints?
- Are there local minima?
- Are the variables continuous? discrete? categorical?
- Do I have the gradient?
- Is the evaluation of the function expensive?
- …



Contour plot of the Rosenbrock function (2D) – *Nschloe, CC BY-SA 4.0*

Final questions

- Which optimizer?
  - and which options?

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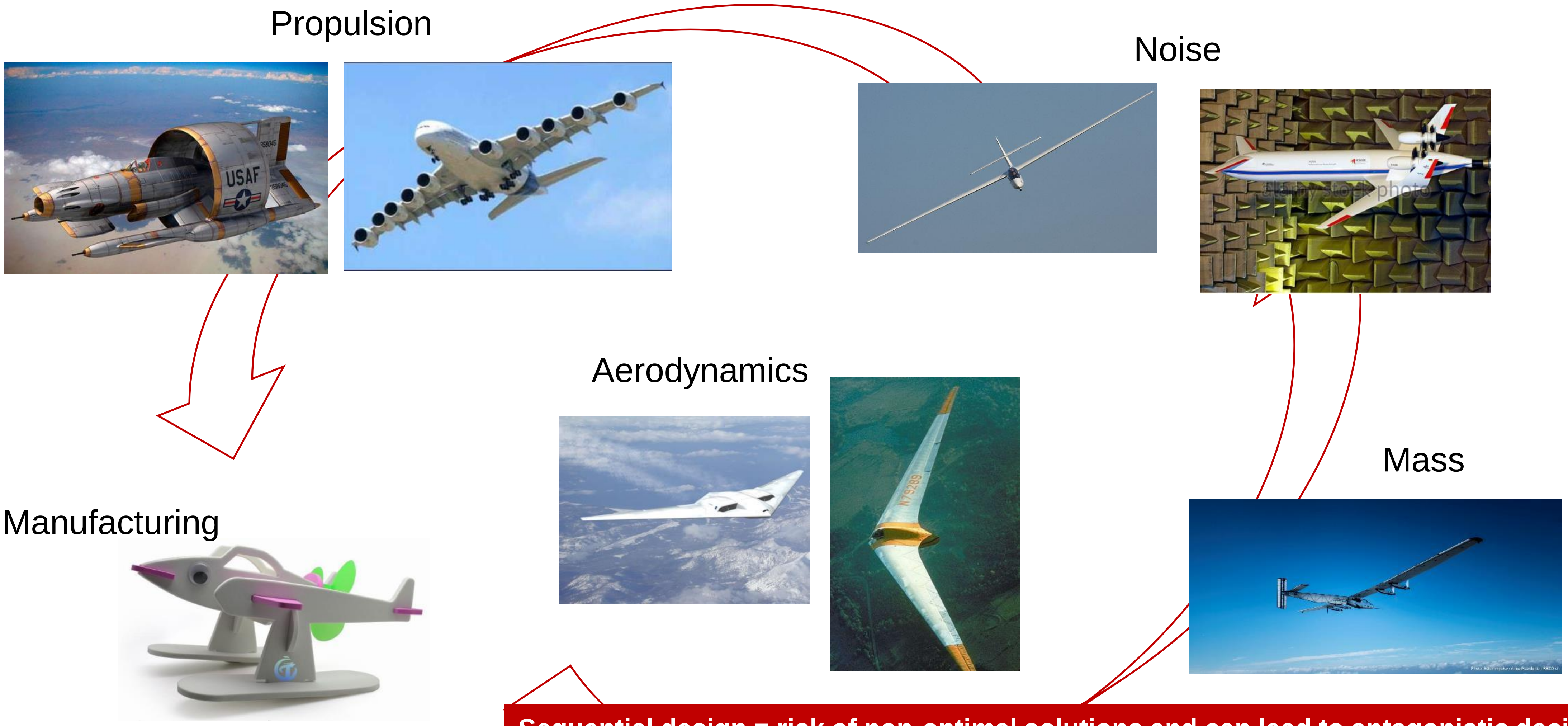
Designing a vehicle is multi-disciplinary challenge.



Sequential design = risk of non-optimal solutions and can lead to antagonistic decisions.

*[Courtesy M. Meaux, Airbus, 2017, « How can **M**ulti-disciplinary **D**esign **O**ptimization (MDO) support R&T Portfolio management ?»]*

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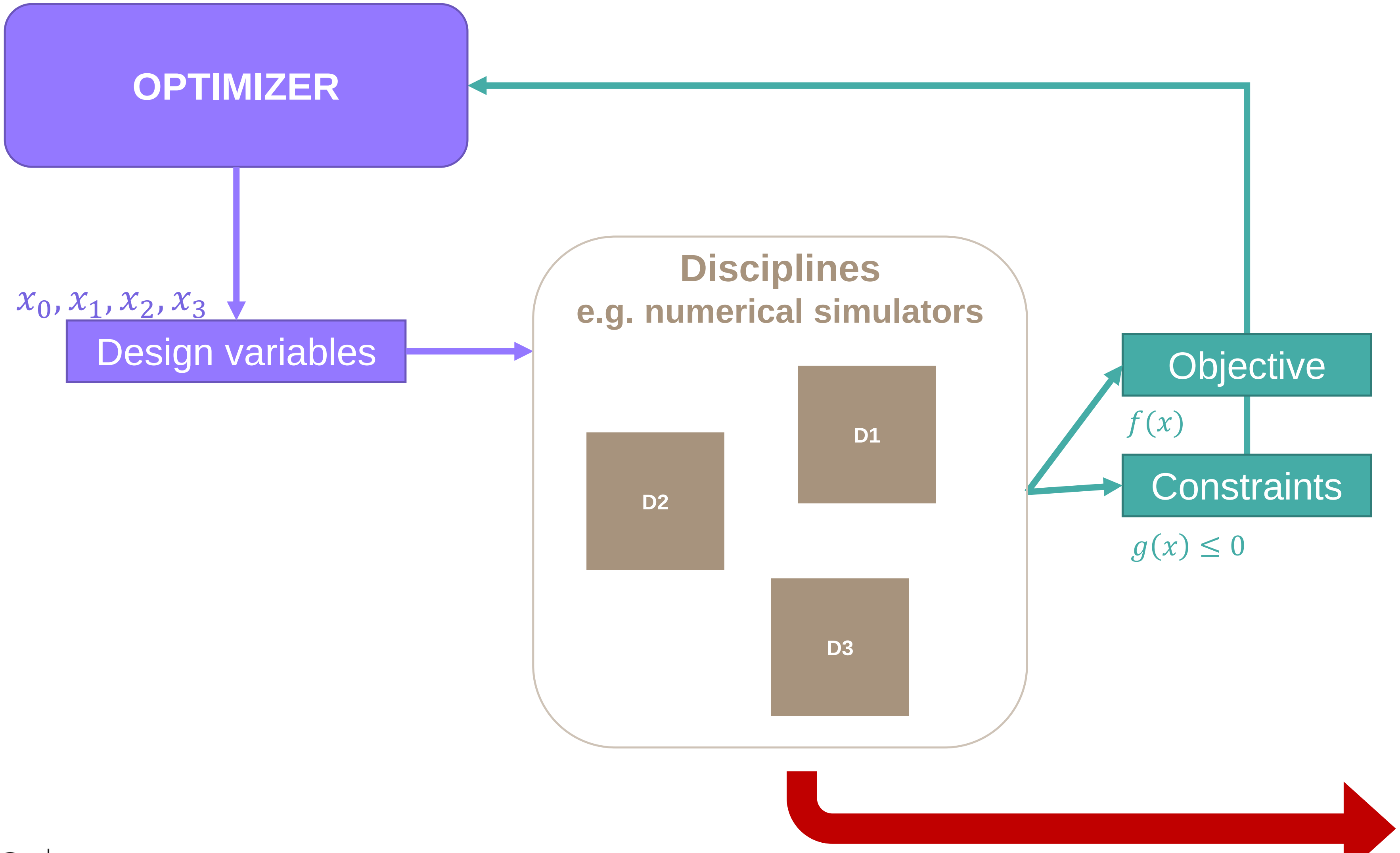
Multidisciplinary design optimization (MDO)



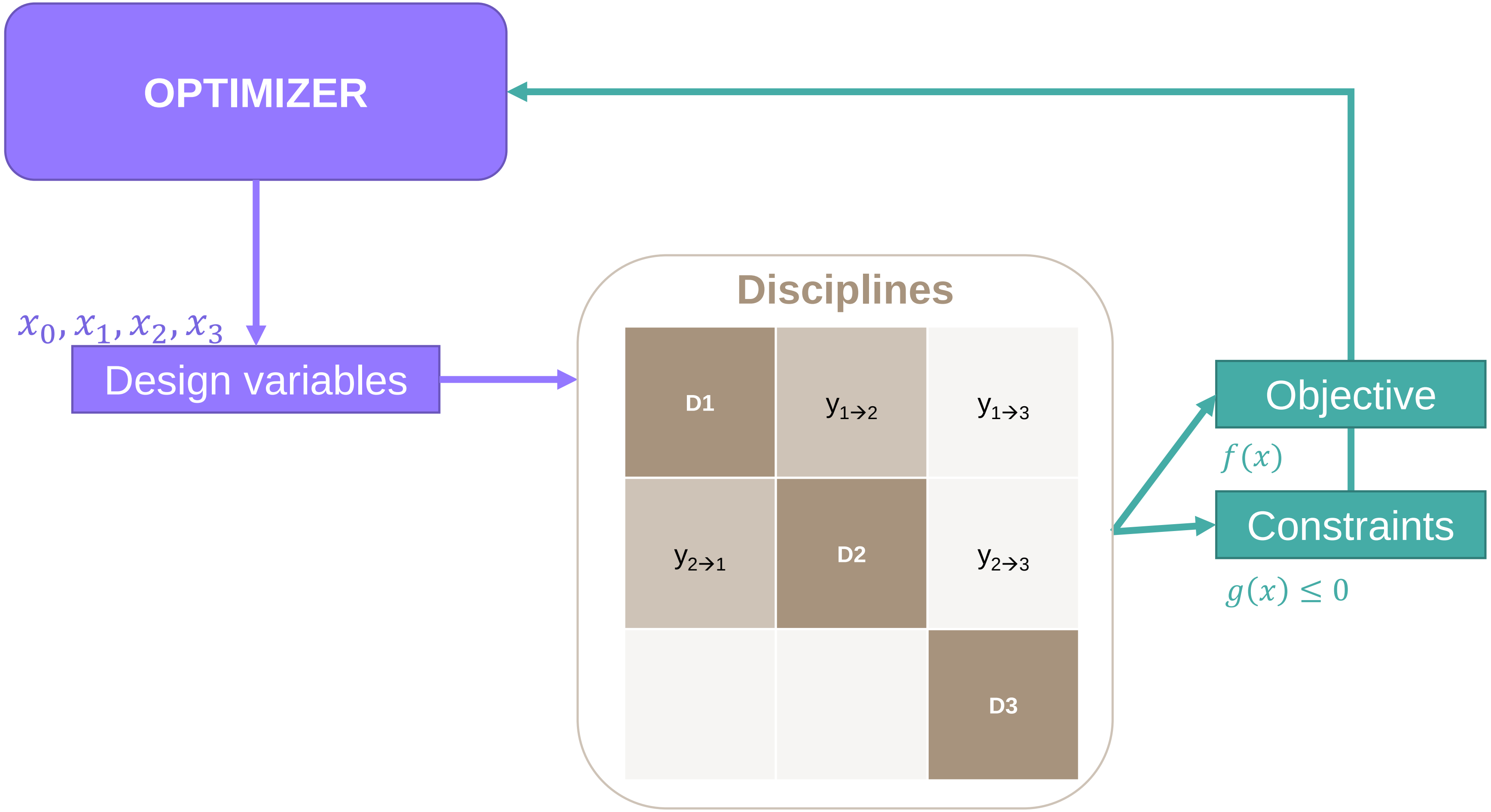
Coupling matrix

D1	$y_{1\to2}$	$y_{1\to3}$
$y_{2\to1}$	D2	$y_{2\to3}$
		D3

Looking for MDO formulations handling the multidisciplinary coupling
i.e. ensuring y = D(x_0, x_1, x_2, x_3,y)

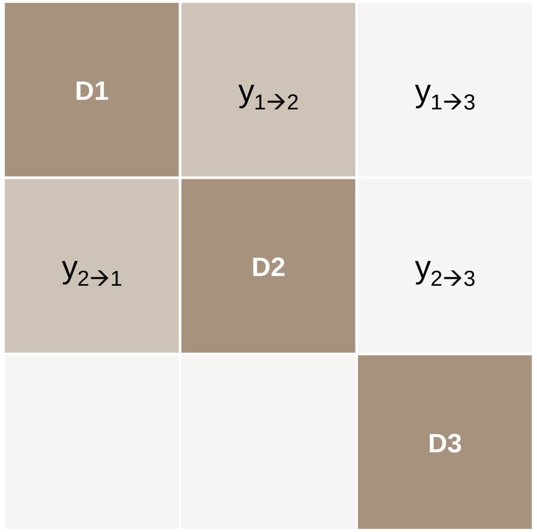
Multidisciplinary design optimization (MDO)

MDF formulation



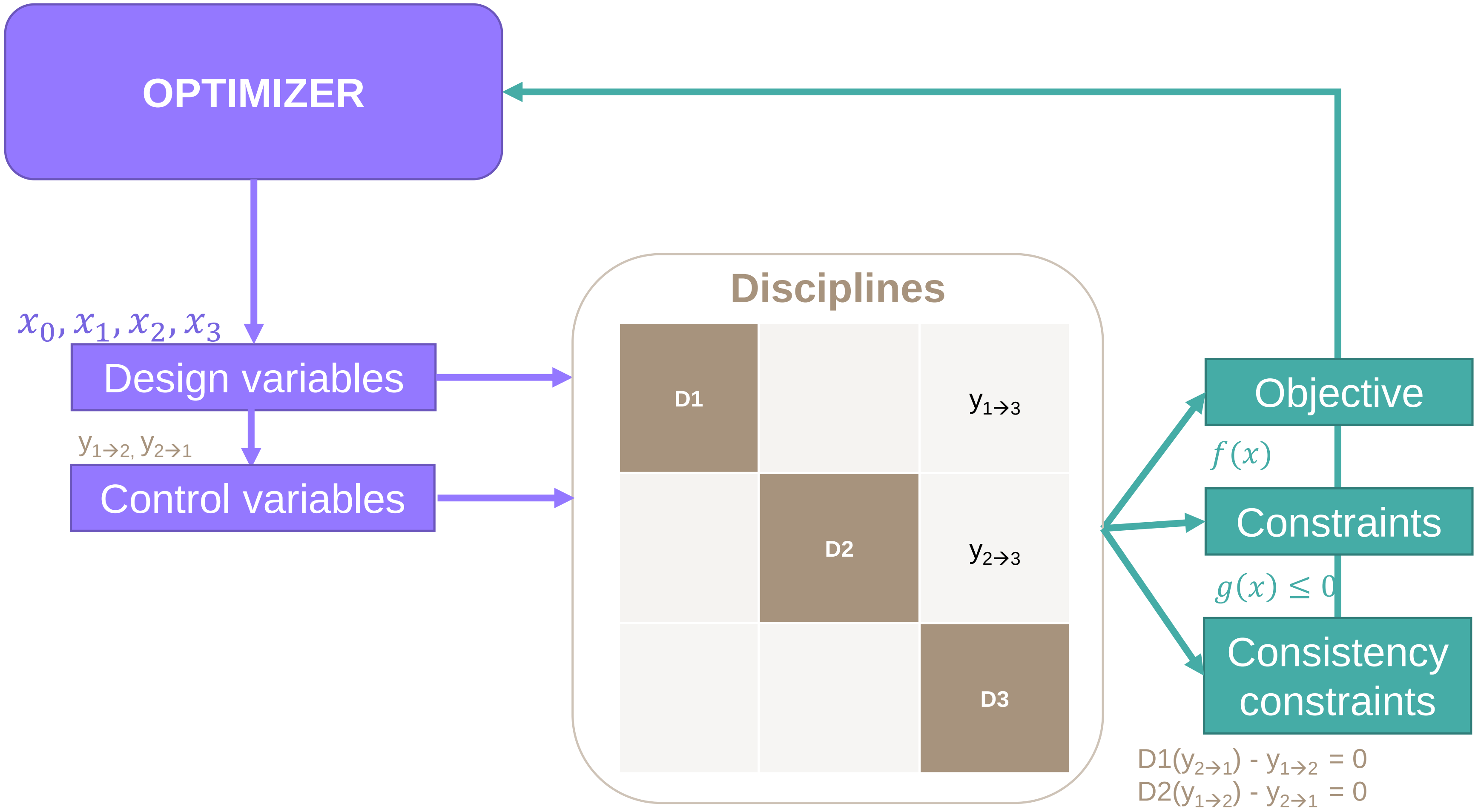
Multi-Disciplinary Analysis (MDA)

Coupling matrix



Multidisciplinary design optimization (MDO)

IDF formulation



Coupling matrix

D1	$y_{1\to2}$	$y_{1\to3}$
$y_{2\to1}$	D2	$y_{2\to3}$
		D3

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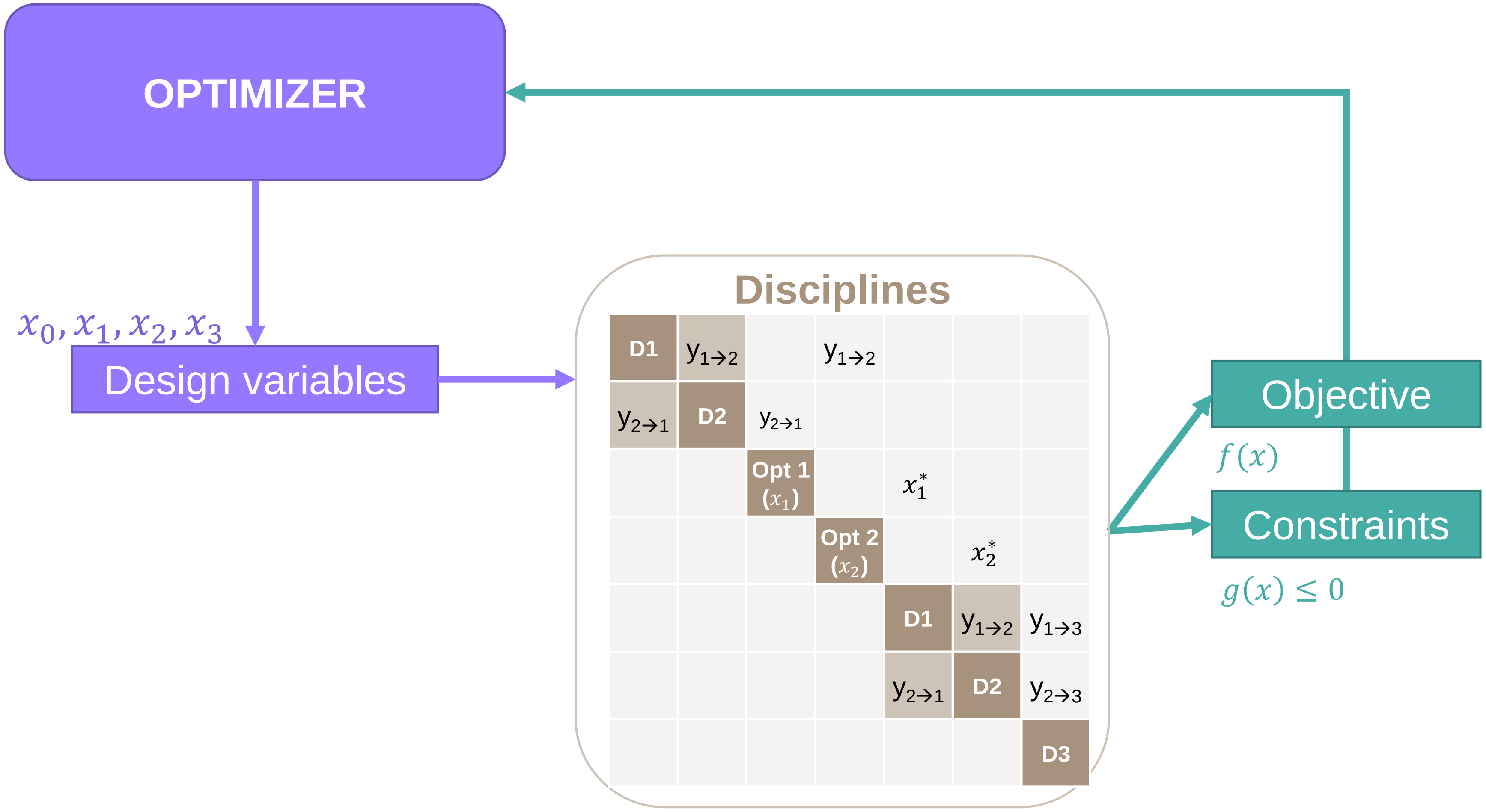
Multidisciplinary design optimization (MDO)

Bi-level formulation



Coupling matrix

D1	$y_{1\to2}$	$y_{1\to3}$
$y_{2\to1}$	D2	$y_{2\to3}$
		D3

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Initial questions

- Are there constraints?
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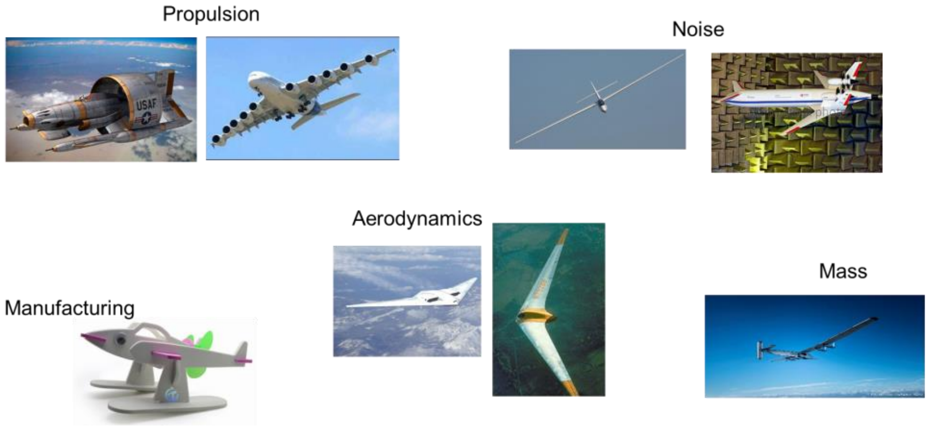

Remind:
An MDO formulation is
a mathematical strategy
to define
the optimization problem(s)
to be solved.

Final questions

- Which disciplines?
- Which MDO formulation?
  - and which options?
- Which optimizer(s)?
  - and which options?

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GEMSEO,
an open source Python library

28/06/2023

GEMSEO, an open source Python library for MDO and more

created in 2015 at IRT Saint Exupéry within the MDO competence center

Generic Engine
for Multidisciplinary Scenarios,
Exploration and Optimization

gemseo.org

User guide
Notebooks

open source

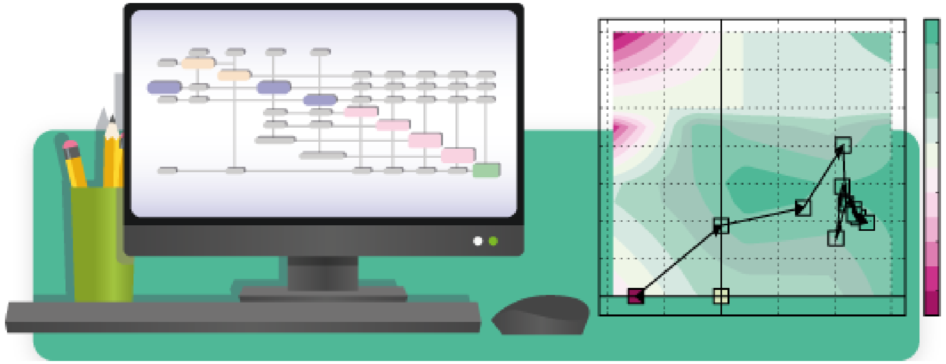
GNU LGPL v3.0

Python

Code quality
Testing
Documentation
CI

Offers

- Automation of MDO processes based on MDO formulations
- Features: coupling, optimization, design of experiments, visualization, surrogate modeling, machine learning, uncertainty quantification, ...
- Easy to embed in simulation platforms or to use as a standalone software
- Can use tools in Python, Matlab, Excel, Scilab, executables, ...



GEMSEO and its plug-ins are on GitLab
gitlab.com/gemseo/dev

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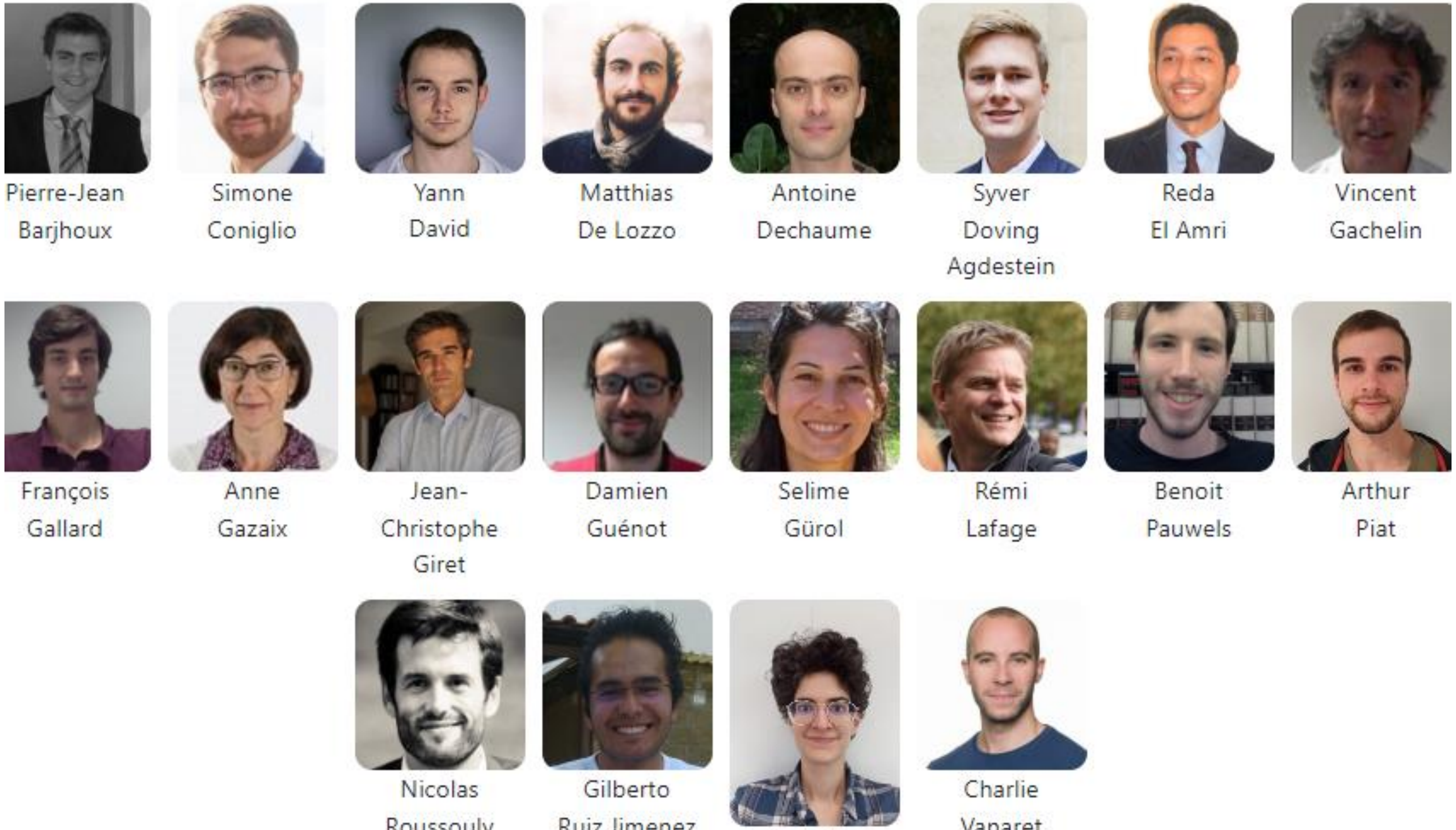
Contributors



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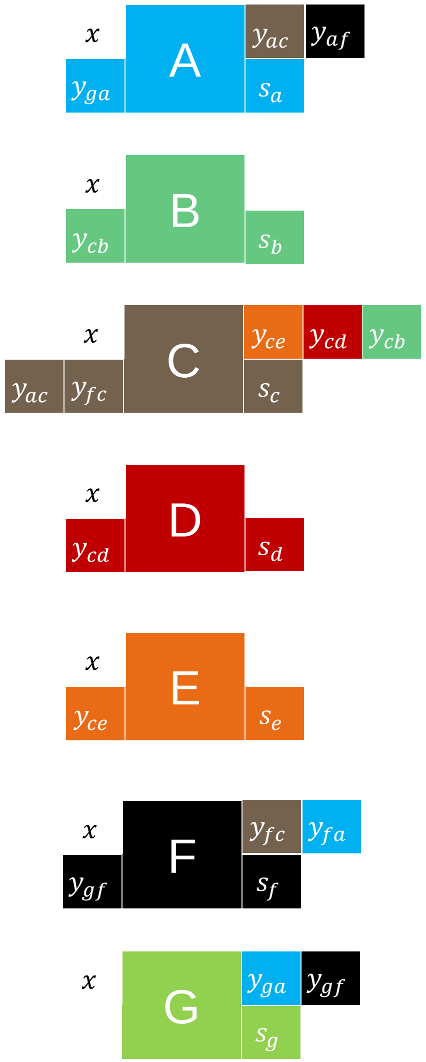
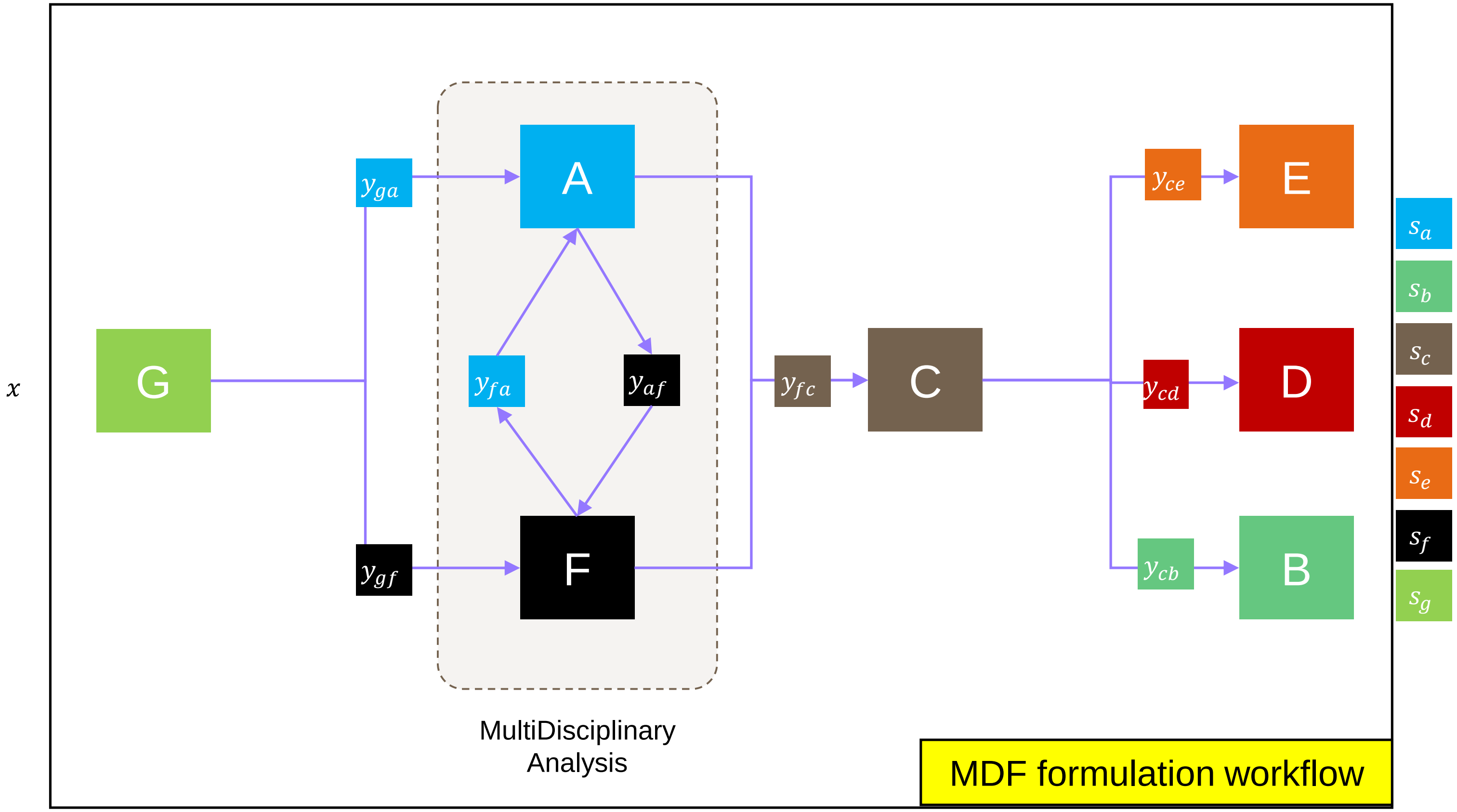
GEMSEO automates the generation of workflows

1. The user defines the IO disciplines:



2. GEMSEO generates an executable workflow:

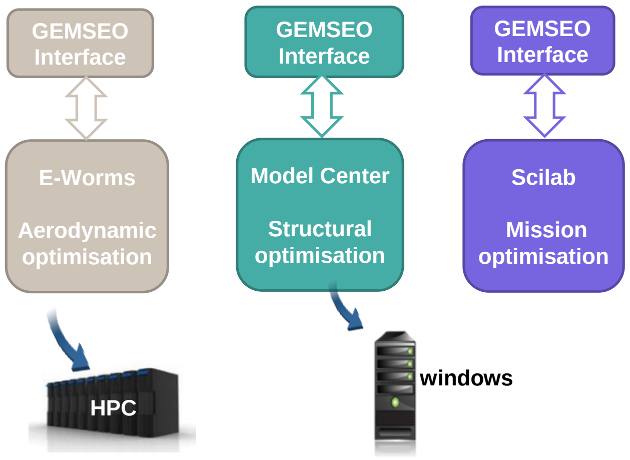


➔ GEMSEO can generate workflows from thousands of disciplines, which would be very costly to do by hand!

GEMSEO automates the creation of MDO processes

Define your optimization problem: $\min_{x} f(x) \quad \text{s.t.} \quad g(x) \leq 3 \quad \text{and} \quad h(x) = 0$

Define your disciplines.



GEMSEO saves a lot of programing time by automatically generating MDO processes according to a catalog of MDO formulations.

Hence, GEMSEO reduces maintenance issues and enables to easily reconfigure MDO processes.

Select an MDO formulation (or architecture), i.e. a mathematical rewriting of the optimization problem from the disciplines.

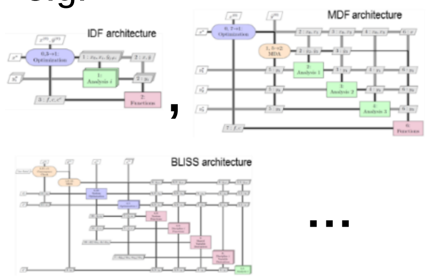
An MDO formulation is

- a mathematical strategy to define the optimization problem(s)
- a template of the MDO process organization,
- problem-independent.

- Optimization problem
- Disciplines
- MDO formulation names and options

→ **MDO formulation engine** → **MDO process**

e.g.



, , …

Solve the optimization problem with an optimizer, based on this MDO process.

API

```
scenario = MDOScenario([●,●,●], "MDF", "f", design_space)
scenario.add_constraint("h")
scenario.add_constraint("g", "ineq", value=3.)
scenario.execute({"algo": "SLSQP" , "max_iter": 100})
```

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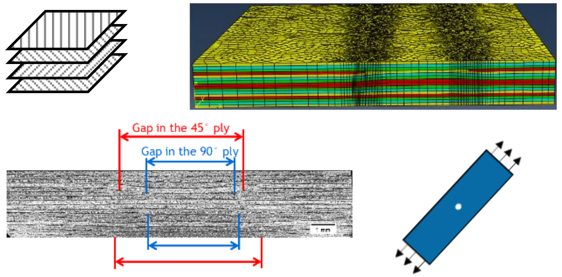
GEMSEO facing uncertainties

28/06/2023

IRT project / VITAL / UQ for composite damage model

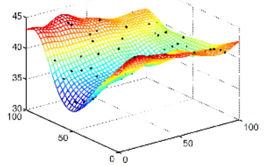
VITAL developed VIMS, a platform dedicated to Virtual Testing Integration for Decision Making Support.
It uses and develops the UQ part of GEMSEO

Coupon-level use cases



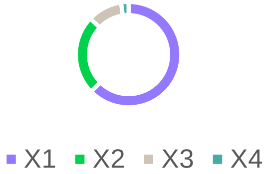
Numerical simulators

Surrogate models



Replace costly simulators

Model exploration

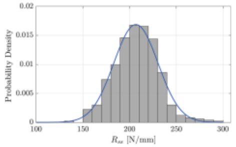


Make a sensitivity analysis
to filter out the non-influential parameters

Model calibration

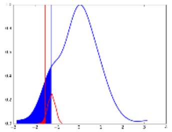
Update the model parameters
by using data-fitting techniques

Uncertainty propagation



Use probability distributions
either data-based or knowledge-based

Reliable design



Evaluate a failure probability
e.g. B-value

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J. Camacho Casero, L. Barriere, S. Miot and M. De Lozzo (2021)
Virtual testing applied to composite material modelling,
SIAM-MS21

IRT project / R-EVOL / UQ for MDO

In an uncertain frame,

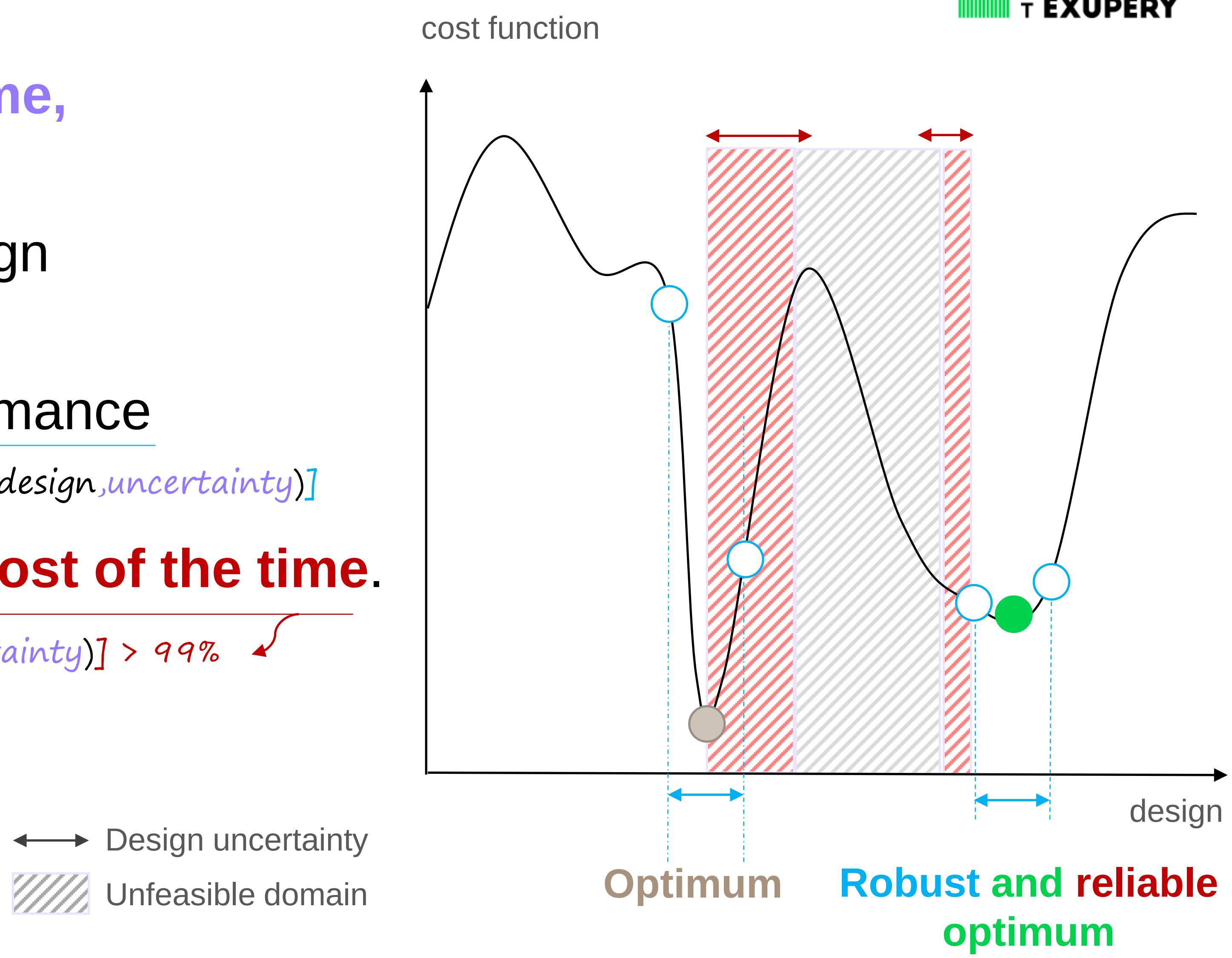
we look for the design

ensuring a **mean** performance

mean[cost(design,uncertainty)]

whilst satisfying the constraints **most of the time**.

probability[satisfied_constraints(design,uncertainty)] > 99%



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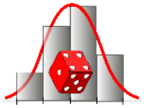
work in progress...

IRT project / R-EVOL / UQ for MDO

Cost function $\mathbb{K}_f[f(x, U)]$
Constraints $\mathbb{K}_g[g(x, U)]$

Design variables
$x = [x_0, x_1 \dots x_N]$



Uncertainty variables
$U = [U_0, U_1 \dots U_N]$

UNCERTAINTY PROPAGAT

Disciplines

ϕ_1

ϕ_3

What mathematical strategy?
Which MDO formulation?
How to estimate the statistics?

Cost function statistics
$\mathbb{K}_f[f(x, U)]$
e.g. $\mathbb{E}[f(x, U)]$

Constraints statistics
$\mathbb{K}_g[g(x, U)]$
e.g. $\mathbb{P}[g(x, U) \geq 0$
or
$\mathbb{E}[g(x, U)] + \kappa\sigma[g(x, U)]$

Keep in mind
the coupling matrix!

D1	$y_{1\to2}$	$y_{1\to3}$
$y_{2\to1}$	D2	$y_{2\to3}$
		D3

work in progress...

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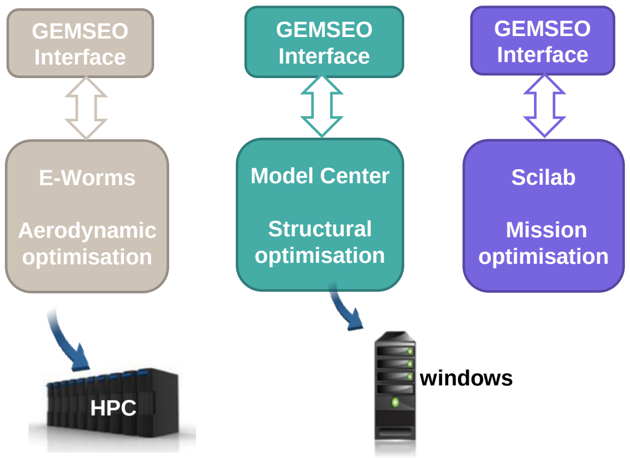
IRT project / R-EVOL / UQ for MDO / Formulation engine

Define your optimization problem: $\min_{x} \mathbb{E}[f(x,U)]$ s.t. $\mathbb{P}[f(x,U) \leq 3] \geq 1-\epsilon_g$ and $\mathbb{P}[-\epsilon \leq h(x,U) \leq \epsilon] = 1-\epsilon_h$

Define your disciplines.



Select an MDO formulation (or architecture), i.e. a mathematical rewriting of the optimization problem from the disciplines.
and estimators of statistics

- Optimization problem
- Disciplines
- MDO formulation names and options
- Statistics estimators names and options

→ **Uncertainty-based** / **MDO formulation engine** → **MDO process**

Solve the optimization problem with an optimizer, based on this MDO process.

API for MDO under uncertainty

```
scenario = MDOScenario([●,●,●], "MDF", "f", design_space, uncertain_space, "Mean")
scenario.add_constraint("h", "Probability", threshold=1-eps_h, tolerance=eps)
scenario.add_constraint("g", "Probability", threshold=1-eps_g, greater=True, constraint_type="ineq", value=3.)
scenario.execute({"algo": "SLSQP" , "max_iter": 100})
```

work in progress...

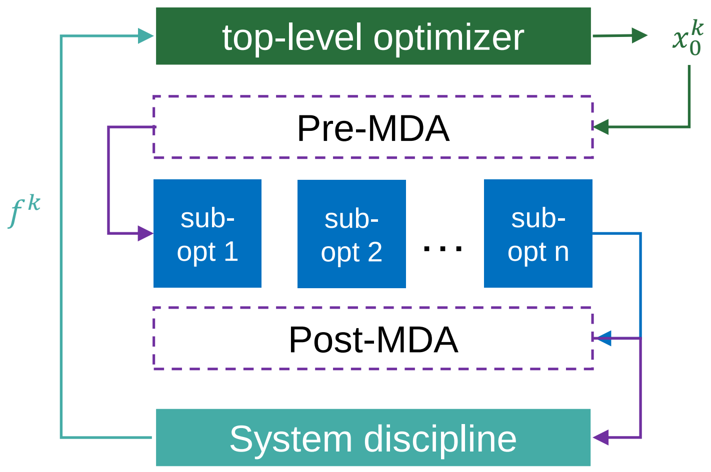
IRT project / R-EVOL / UQ for MDO / Bi-level formulation

Bi-level formulation for MDO available in GEMSEO

$$\min_{x_0} f\big(x_0, y_1\big(x_0, x_1^*(x_0)\big), \ldots, y_n\big(x_0, x_n^*(x_0)\big)\big)$$

with

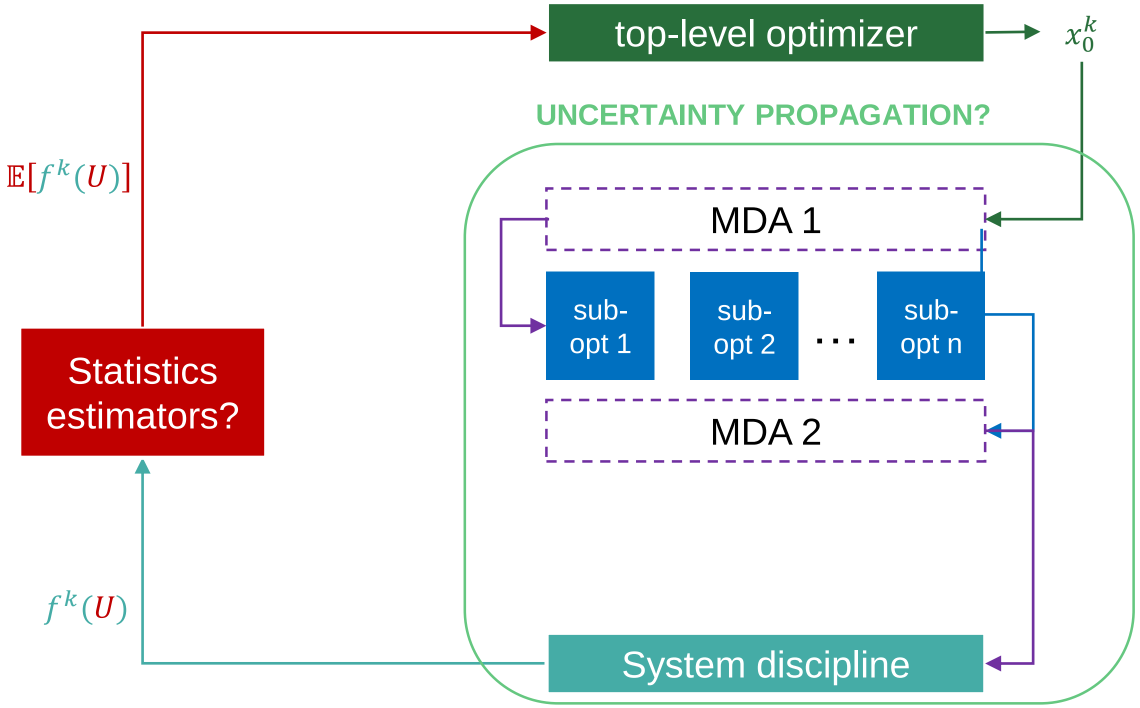
$$x_i^*(x_0) \in \operatorname{argmin}_{x_i} \; f_i\big(x_0, x_i, y_{-i}(x_0, \mu)\big)$$



This formulation meets the industria expectations:
relying on existing optimizers, validated long ago
and on which a great deal of effort has been focused.

Bi-level formulation for MDO under uncertainty

We want to create an analogous version under uncertainty (PhD thesis of Amine Aziz-Alaoui).



work in progress...

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IRT project / R-EVOL / UQ for MDO

A scalable problem to benchmark robust MDO algorithms

A. Aziz-Alaoui, O. Roustant and M. De Lozzo (2023)
A scalable benchmark for multidisciplinary optimization.
Submitted to Optimization and Engineering.

shared design variable

coupling variable outputted by the i-th discipline

margin $\mathbb{E} + \kappa \times \mathbb{S}$ or probability $\mathbb{P}_\varepsilon$

threshold vector

feasibility level

coupling variables

$$\min_{x\in[\mathbf{0},\mathbf{1}]} \mathbb{E}\left[\boldsymbol{x}_0^T\boldsymbol{x}_0 + \sum_{1\le i\le N} \boldsymbol{Y}_i^T\boldsymbol{Y}_i\right] \quad \text{s.t.} \quad \mathbb{K}[\boldsymbol{t}_i(\alpha) - \boldsymbol{Y}_i] \preccurlyeq 0$$

design variable specific to the i-th discipline

$$\text{where}\quad \boldsymbol{Y}_i = \boldsymbol{a}_i - \boldsymbol{D}_{i,0}\boldsymbol{x}_0 - \boldsymbol{D}_{i,i}\boldsymbol{x}_i + \sum_{1\le i\le N} \boldsymbol{C}_{i,j}\boldsymbol{Y}_j + \boldsymbol{U}_i$$

random vectors and matrices with coefficients in [0,1]

U_i can follow any distribution with zero mean and covariance matrix Σ.

We showed that this robust MDO problem reduces to the quadratic optimization (QP) problem

$$\min_{x} \quad \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d} + \mathbb{E}[\boldsymbol{U}^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{U}]$$

$$\text{s.t:} \quad \boldsymbol{A}\boldsymbol{x} \preccurlyeq \boldsymbol{b} + \mathbf{offset}(\mathbb{K})$$

The user can change

- the number of disciplines N,
- the dimension of $\boldsymbol{x}_0, \boldsymbol{x}_1, \dots, \boldsymbol{x}_N$,
- the dimension of $\boldsymbol{y}_1, \dots, \boldsymbol{y}_N$,
- the feasibility level α,
- the distribution.

The user can compare

- MDO formulations,
- optimization algorithms,
- coupling algorithms,
- estimators of statistics,
- mixtures of that,
- …

	$\Delta_x(\%)$	$\Delta_f(\%)$	$\Delta_g(\%)$
MC	0.370 (0.176)	0.592 (0.127)	0.877 (0.278)
TP	0.093	0.005	0.143

Percentage errors of the solution with Monte Carlo estimators and Taylor polynomials

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IRT project / R-EVOL / UQ for MDO / Statistics estimation → MLMC + polynomial chaos expansion (PCE) + control variate (CV)

M. Reda El Amri, P. Mycek, S. Ricci, M. De Lozzo (2023)
Multilevel Surrogate-based Control Variates.
Submitted to SIAM JUQ.

We showed that mixing sampling and surrogate modelling reduce the variance of statistics estimators, ***even with*** poor-quality surrogate models.
Using multi-level simulators for both sampling and control-variates can reduce it even further.

We proposed an MLMC-MLCV algorithm allocating samples to levels according to the variance reduction.

Algorithm 1 Simplified MLMC-* algorithm inspired by [36].

Require: $n_\ell^{\text{init}} > 1$, $r_\ell > 1$, surrogate models (depending on the method), and budget $\mathcal{C}$.
Set consumed budget to $\bar{\mathcal{C}} = 0$ and $\delta n_\ell = n_\ell^{\text{init}}$ samples on levels $\ell \le L$;
while $\bar{\mathcal{C}} \le \mathcal{C}$ **do**
  compute δn_ℓ samples on each level by evaluating f_ℓ and the appropriate surrogates;
  update sample size on each level: $n_\ell \leftarrow n_\ell + \delta n_\ell$;
  update consumed budget: $\bar{\mathcal{C}} \leftarrow \bar{\mathcal{C}} + \sum_{\ell=0}^{L} \delta n_\ell (\mathcal{C}_\ell + \mathcal{C}_{\ell-1})$;
  estimate the optimal CV parameter(s) on each level;
  compute/update CV estimates for $\hat{T}_\ell^{(\ell)}$ and $\mathcal{V}_\ell^{\text{CV}}$ from samples on levels $\ell \le L$;
  select level $\ell^* = \arg\max_{0 \le \ell \le L} \dfrac{\mathcal{V}_\ell^{\text{CV}}}{r_\ell n_\ell^2 (\mathcal{C}_\ell + \mathcal{C}_{\ell-1})}$;
  $\delta n_{\ell^*} \leftarrow \lfloor (r_{\ell^*} - 1) n_{\ell^*} \rfloor$, $\delta n_{\ell \neq \ell^*} \leftarrow 0$;
end while
return $\hat{\theta}_L^{\text{MLMC-*}}$, the MLMC-* estimate of θ_L.

1 Use surrogate-based CV

Model-based MC estimator: high variance | Surrogate-based MC estimator: bias | Surrogate-based CV estimator: no bias / low variance

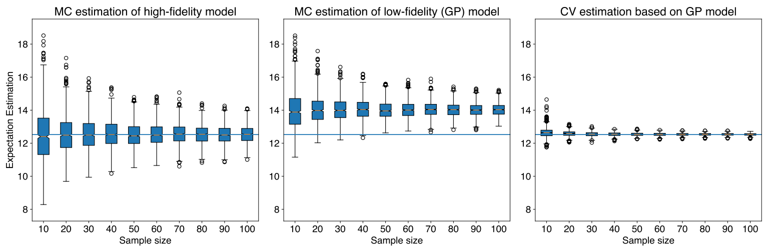


Illustration: stochastic Rosenbrock

2 Combine surrogate-based CVs (SBCVs)

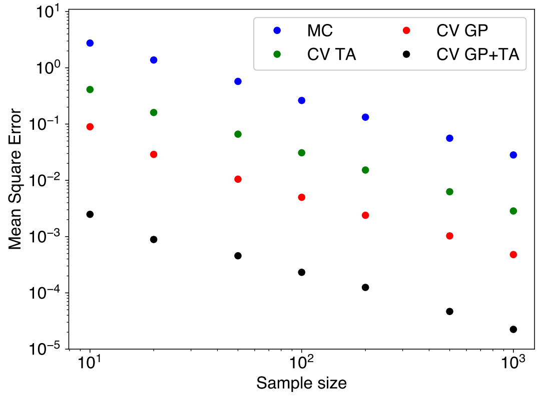
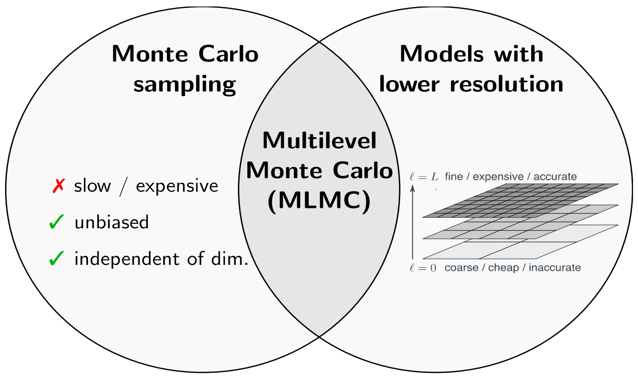


Illustration: stochastic Rosenbrock

3 Use multi-level simulators (MLS)



The is a rich literature on MLMC.

4 Combine MC, MLS and SBCVs

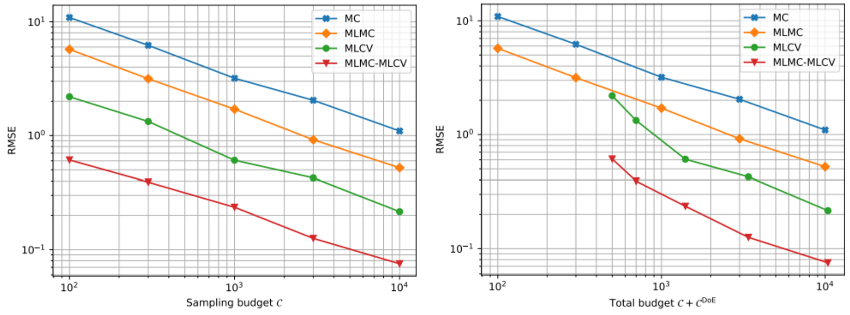


Illustration: heat equation; CV = Polynomial Chaos Expansion with LARS

GEMSEO using OpenTURNS

28/06/2023

Before UQ, GEMSEO allowed the user to …

- create a **DesignSpace**
- solve an **OptimizationProblem**
  - create objective and constraints as **MDOFunction**s,
  - use an optimization or DOE algorithm from a **DriverLibrary** interfacing an algorithm library callable from Python
  - post-process the results
- solve an MDO problem
  - create **MDODiscipline**s (analytic formula, Python function, executable, …)
  - select an **MDOFormulation** (IDF, MDF, BiLevel, …)
  - use an optimization or DOE algorithm from any **DriverLibrary** callable from Python (e.g. SciPy, NLOPT, pymoo, OpenTURNS, …)
  - post-process the results
- automatically couple disciplines:
  - create **MDODiscipline**s… and voilà!
- create **SurrogateDiscipline** from any machine learning or scientific library callable from Python (e.g., SciPy, scikit-learn, …)
- …

The user was familiar with the GEMSEO concepts
and somehow appreciated its abstract layer
that doesn't make it dependent on a single library ,
but allows it to be extended to a multitude of libraries.

So we started the UQ with an abstract layer
independent of OpenTURNS
or any other UQ library.

Probability distributions

- *Distribution*
  - SPDistribution (based on SciPy)
  - OTDistribution (any OpenTURNS distribution)
    - OTExponentialDistribution
    - OTNormalDistribution
    - OTUniformDistribution
    - OTTriangularDistribution

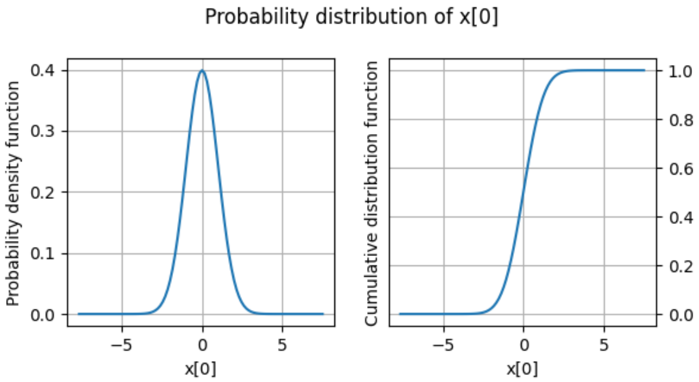
    - OTComposedDistribution (used copula)

```
# First option
normal = OTNormalDistribution("x", 1.0, 2.0)

# Second option (direct interface to OpenTURNS)
normal = OTDistribution("x", "normal", (1.0, 2.0))

normal.mean
normal.standard_deviation
normal.support
normal.range

normal.plot()
```



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Uncertainty propagation

- *AlgorithmLibrary*
  - *LinearSolverLibrary*
  - *DriverLibrary*
    - *OptimizerLibrary*
    - *DOELibrary*
      - PyDOE
      - CustomDOE (from CSV file or arrays
      - **OpenTURNS**
        - "OT_MONTE_CARLO"
        - "OT_FAURE"
        - "OT_HALTON"
        - "OT_HASELGROVE"
        - "OT_REVERSE_HALTON"
        - "OT_SOBOL"
        - "OT_LHS"
        - "OT_OPT_LHS"
        - "OT_AXIAL"
        - "OT_COMPOSITE"
        - "OT_FACTORIAL"
        - "OT_FULLFACT"

```
# Creation of the uncertain space
uncertain_space = ParameterSpace()
uncertain_space.add_random_variable("x", "OTNormalDistribution", …)
uncertain_space.add_random_variable("y", "OTTriangularDistribution", …)

# A scenario creates the OptimizationProblem corresponding to a formulation.
scenario = create_scenario(
    disciplines,
    formulation="MDF",
    objective_name="foo",
    design_space=uncertain_space,
    scenario_type="DOE",
)
scenario.add_constraint("bar")
scenario.add_observable("baz")

# Use any algorithm name.
scenario.execute({"algo": "OT_OPT_LHS“, "n_samples": 30})

scenario.post_process(“ScatterPlotMatrix”)

dataset = scenario.to_dataset()  # A Dataset is an advanced pandas dataframe.

dataset.describe()
```

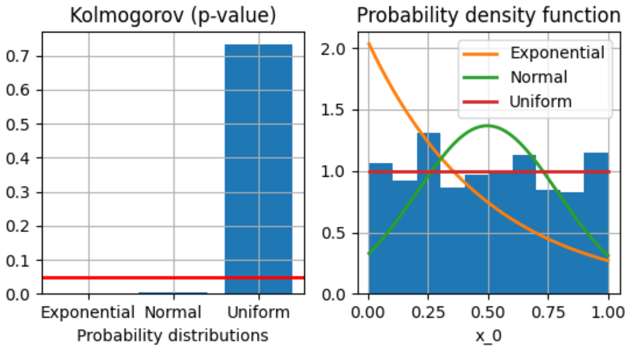
Uncertainty quantification

- *Dataset-based statistics*
  - *Methods:*
    - *compute_mean*
    - *compute_variance*
    - *compute_probability*
    - *…*
  - EmpiricalStatistics (pure NumPy / SciPy)
  - ParametricStatistics
    - ot.DistributionFactory
    - ot.FittingTest.{BIC,ChiSquared,Kolmogorov}

```
statistics = ParametricStatistics(dataset, ["Normal", "Uniform", "Triangular"])
print(statistics.get_fitting_matrix())
```

Variable	Exponential	Normal	Uniform	Selection
x_0	1.602160180879313e-10	0.005823020521403932	0.7338504331264553	Uniform
x_1	2.82659088382179e-53	0.8587721484840084	5.660300987516015e-18	Normal
x_2	1.5387797946575896e-09	0.0016128012413438864	7.748433868335025e-67	Normal
x_3	0.864074427829853	2.0987474708559965e-10	7.782983660200643e-152	Exponential

```
mean = statistics.plot_criteria("x_0")
```

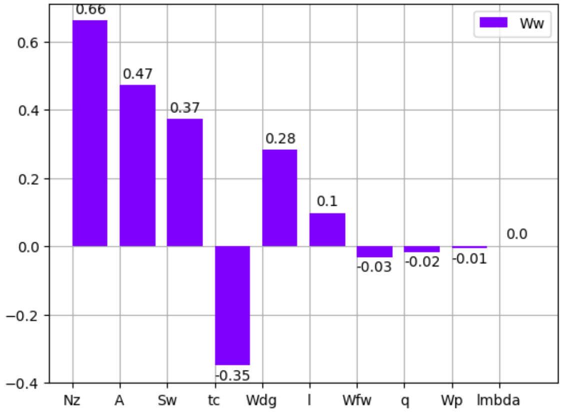


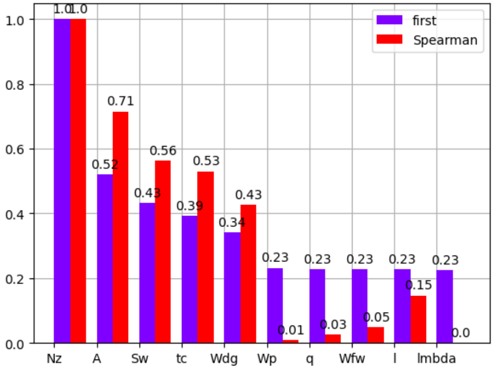

```
mean = statistics.compute_mean()
```

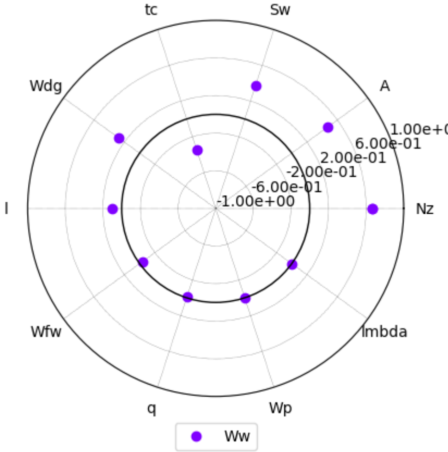
Sensitivity analysis

- *SensitivityAnalysis*
  - MorrisAnalysis
  - CorrelationAnalysis
  - SobolAnalysis
- Methods
  - Visualization (with/without sorting)
    - plot_bar
    - plot_radar → higher scalability
    - plot_field → 1D/2D output
    - plot_comparison (to be used carefully!)
    - plot → specific to a type of sensitivity analysis
  - sort_parameters according to main_method

- Can be applied to
  - 1 discipline
  - 1 multidisciplinary process (disciplines + formulations)







Wing weight function

Sensitivity analysis – Morris method

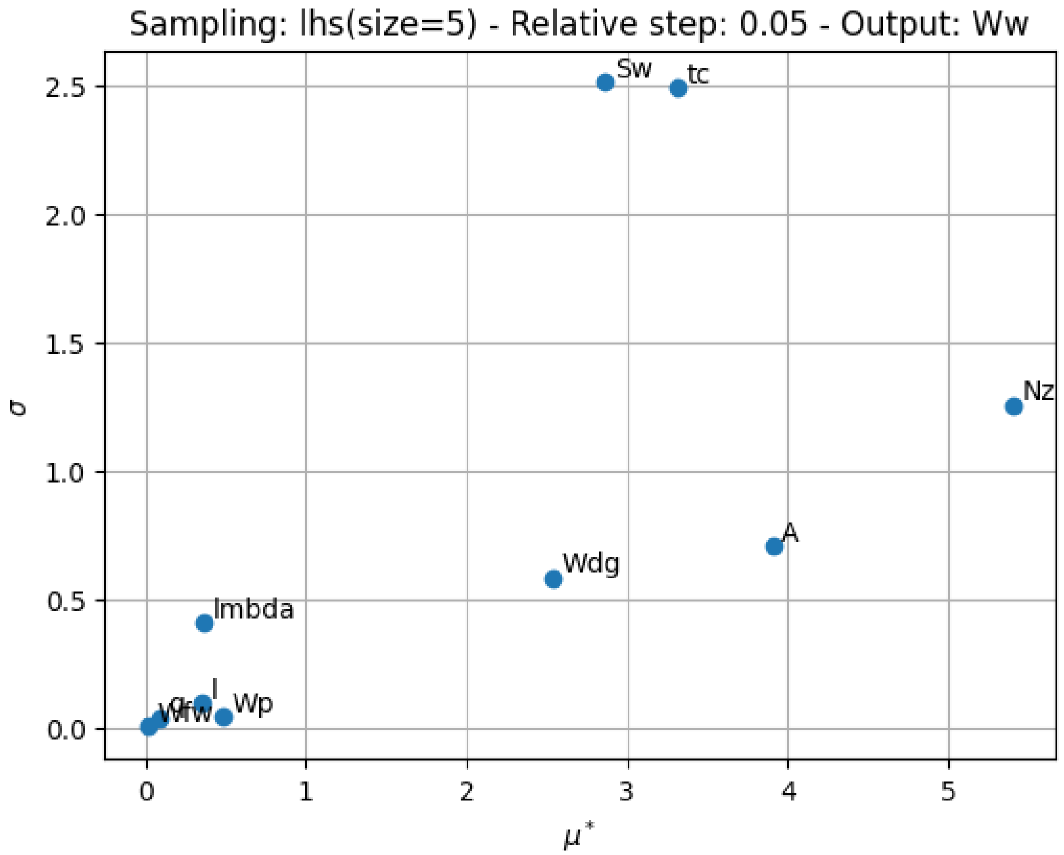
Idea: compare the impact of the uncertain variables on the output based on finite differences (FD)

Properties:

- μ = mean of the FD
- μ^* = mean of the absolute FD
- σ = standard deviation of the FD
- min = minimum value of the output
- max = maximum value of the output

Options:

- Step (5%)
- Number of repetitions (5)
- DOE algo (LHS)
- Filter variables in the plot



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Wing weight function

Sensitivity analysis – Correlation coefficients

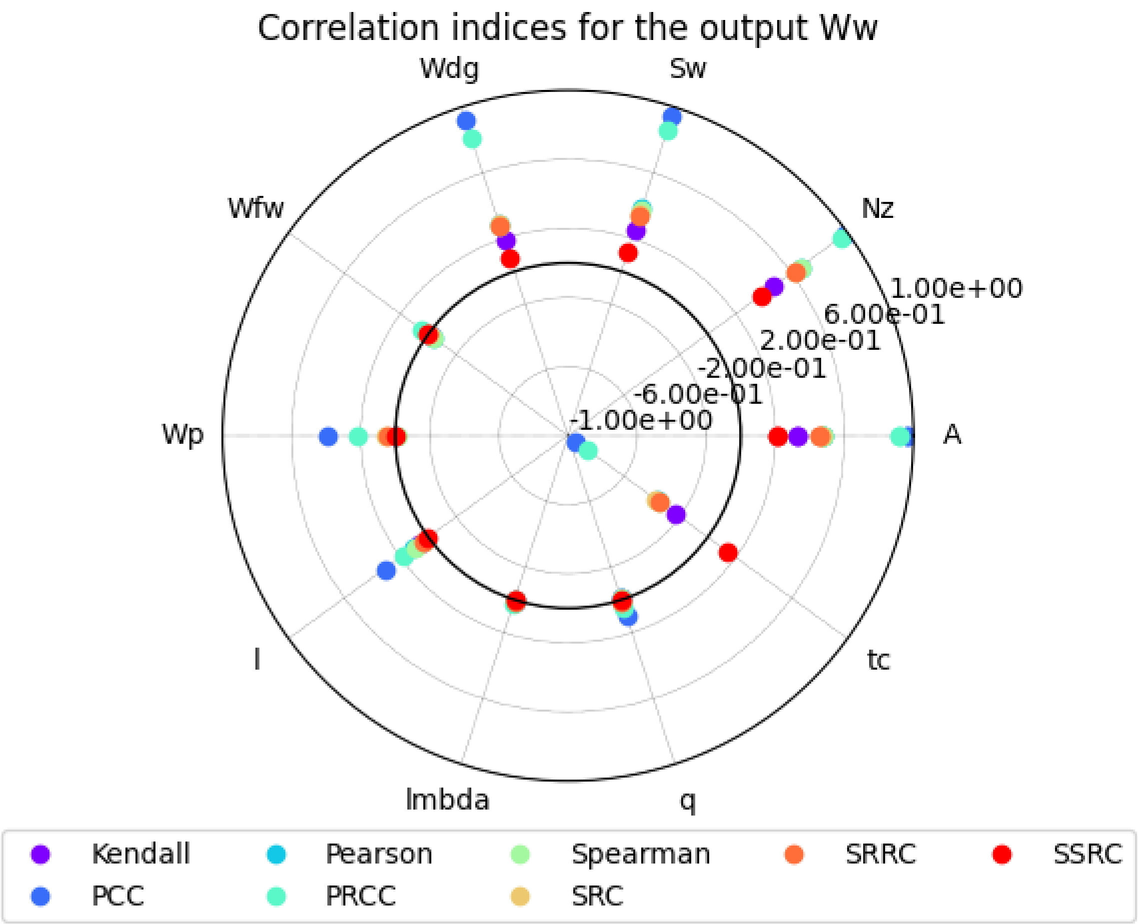
Idea: compare the impact of the uncertain variables on the output based on correlations

Properties:

- μ = mean of the FD
- μ^* = mean of the absolute FD
- σ = standard deviation of the FD
- min = minimum value of the output
- max = maximum value of the output

Options:

- Compute 2nd order indices (True)
- Asymptotic distribution (True)
- Algorithm (Saltelli)
- Confidence level (95%)



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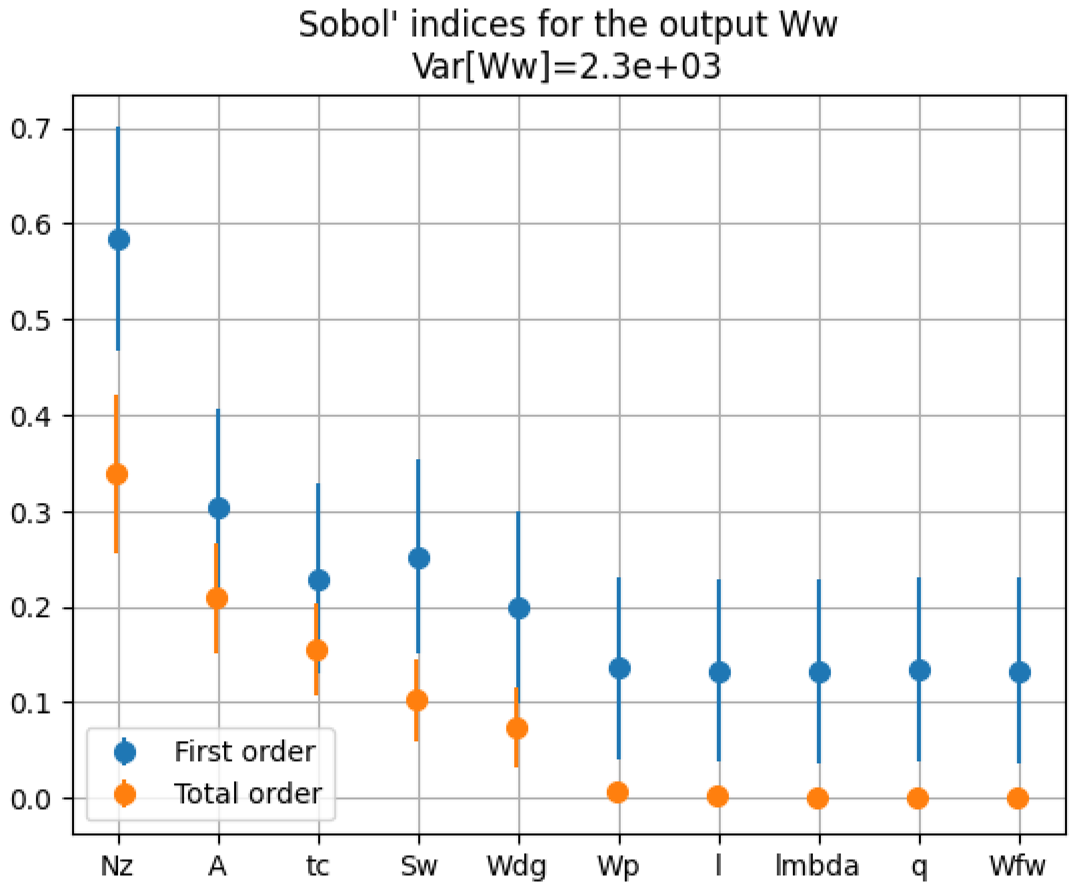
Wing weight function

Sensitivity analysis – Sobol' indices

Idea: compare the impact of the uncertain variables on the output based on variance decomposition

Properties:
- μ = mean of the FD
- μ^* = mean of the absolute FD
- σ = standard deviation of the FD
- min = minimum value of the output
- max = maximum value of the output

Options:
- Compute 2nd order indices (True)
- Asymptotic distribution (True)
- Algorithm (Saltelli)
- Confidence level (95%)



Wing weight function

Sensitivity analysis – Sobol' graph

Idea: plot the first-, second- and total-order indices with a graph.

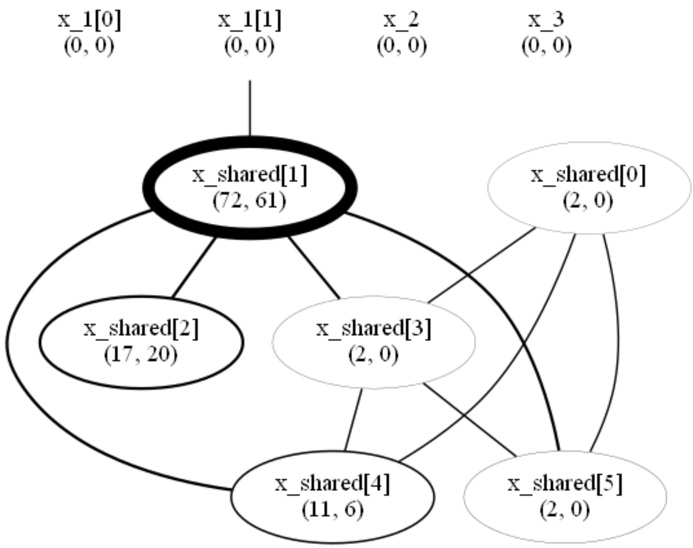
T. Muehlenstaedt, O. Roustant, L. Carraro and S. Kuhnt (2012)
Data-driven Kriging models based on FANOVA-decomposition.
Statistics and Computing

Nodes:
- Name: input variable (total-order in %, first-order in %)
- Thickness: total-order index

Edges:
- Thickness: second-order index

```
sobol_analysis = SobolAnalysis(disciplines, uncertain_space, 1000)
sobol_analysis.compute_indices()
```

```
sobol_graph = SobolGraph.from_analysis(sobol_analysis, output_name="y_4")
sobol_graph.visualize(show=True, file_path="sobol_graph.pdf")
```



Sensitivity analysis – Uncertain coupling graph

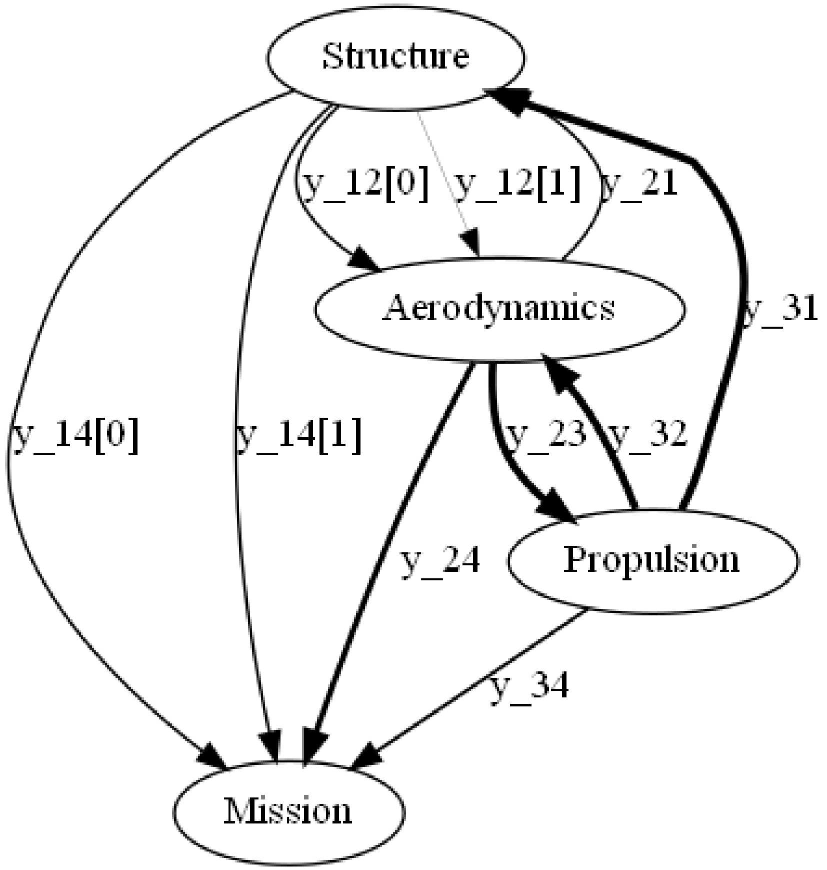
Idea: highlight the most uncertain paths in a multidisciplinary system.

Ingredients:

- a coupling graph
  - nodes = disciplines
  - edges = coupling variables

- a statistic measuring the dispersion, e.g.
  - coefficient of variation σ/μ, a.k.a. relative standard deviation
  - quartile coefficient of dispersion $$(q_{75\%} - q_{25\%})/(q_{25\%} + q_{75\%})$$

- edges proportional to the statistics



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Conclusion

History of GEMSEO’s use of OpenTURNS

2015 | 2019 | 2020-2022 | 2023 >>>

DOE

LHS,

low-discrepancy sequences,

stratified DOEs, …

Surrogates

Polynomial chaos expansion

UQ

Distributions,

distribution fitting,

Sobol’ analysis,

correlation,

…

Surrogates

Kriging
(with LAPACK & HMAT)

UQ

Copula,

iterative statistics,

HSIC, …

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Take-home message

- GEMSEO is made for MDO and can address other multidisciplinary problems.
- It offers MDO features, e.g. optimizers, coupling algorithms, linear solvers, visualization for optimization histories, …
- These features are fully expandable thanks to factory mechanics (*« Derive from the base class and voilà! »*).
- It has also DOE functionalities for trade-off studies (*« MDO with a DOE instead of an optimizer often gives a first idea. »*).
- It proposes a machine learning package to build efficient surrogate models by interfacing specialized libraries.

- For the four past years, we've been adding UQ inside GEMSEO, always starting with abstract classes in pure Python:
  - Probability distributions
  - Probability distribution fitting
  - Sensitivity analysis
- Sometimes we develop functionalities in GEMSEO that would be better placed in a UQ library… if we ever find them in OpenTURNS, we'll surely use them .☺
- The idea is not to create Python wrappers of OpenTURNS functions but to use OpenTURNS in GEMSEO, a Python library for multidisciplinary studies, fully object-oriented and *keen* to be enriched.

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Roadmap

Multi-fidelity

Use of multiple levels of precision for a given discipline to accelerate the computational time

Machine learning

Automate the construction, selection and calibration of surrogate models and extend GEMSEO by plugging specialized ML and IA libraries

Active learning

Improve a model, e.g. surrogate model, by learning new data during a process, e.g. optimization or statistics estimation

Sensitivity analysis

Understand the influence of uncertain inputs on outputs, with a focus on dependency and high dimension for coupling variables

Robust MDO

Use of MDO formulations and efficient estimation of statistics for MDO under uncertainty

Optimizers

Use of global optimization algorithms for cheap disciplines as well as surrogate-based optimization

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