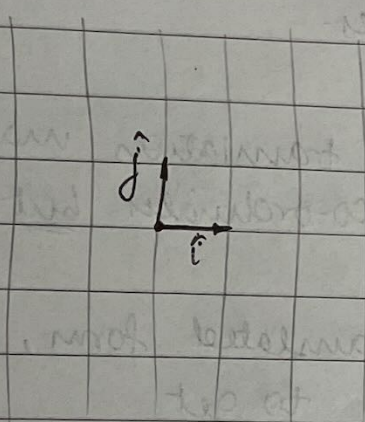
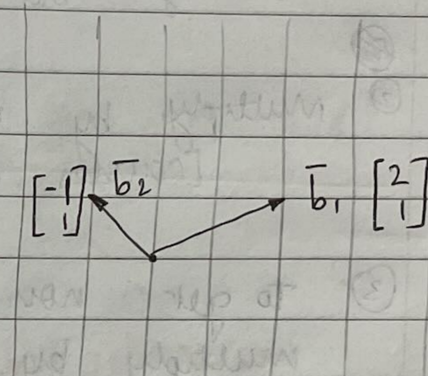
Basis

What is a basis vector?

- It is like a (unit) vector taken as a reference.
- Different sets of grids can have different basis vectors.



Standard



Different set of basis vectors

→ The transformation given for the basis (new) vectors is $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

Standard grid $\xrightarrow{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}$ New grid

Inverse

Co-ordinates of new grid $\xrightarrow{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}$ Co-ordinates of old grid

→ How to ~~app~~ translate a matrix of non-standard grid.

① Convert non-std co-ordinates to std co-ordinates using [transformation of grid] matrix (M)

$$M \times [\text{non-std co-ordinates}]$$

We get std- co-ordinates.

② Multiply by ~~reverse~~ the translation matrix
[causes change in co-ordinates but gives std]

③ To get non-std of translated form, multiply by ~~its~~ inverse to get final co-ordinates of non-std form.

In short:

$$A^{-1}MA$$

→ M = represents transformation meant for grid ~~in~~ std ~~co-ordinates~~

→ the outer A^{-1} ... A help convert the transformation for non-std use.

Then, when we multiply result ($A^{-1}MA$) to non-std co-ordinates,
we will get correct ans for the transformed co-ordinates in non-std grid.

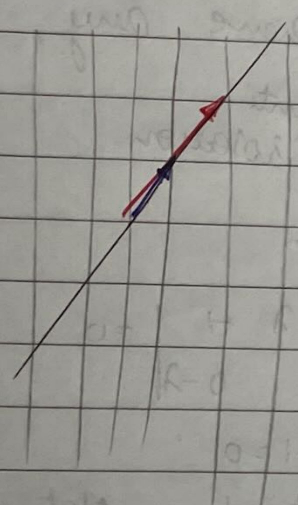
Eigenvectors

→ Vectors, which after undergoing transformation land on the same line as the original vector, are eigen vectors.

→ They only get stretched or compressed but do not undergo rotation on transformation.

Eigenvalue.

The factor by which the vector gets stretched / compressed is known as Eigenvalue.



• - original vector

• - transformed vector.

① some vector, same vector.

$$\boxed{A\vec{V} = (\lambda I)\vec{V}}$$

Transformation matrix, Eigen value

Here $\lambda \equiv \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

ie $\underline{\lambda I}$

② $(A - \lambda I)\vec{V} = \vec{0}$

Hence, $|A - \lambda I| = 0$

eg. Transformation matrix : $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

For knowing the eigenvalues, we do

$$A\vec{v} = \lambda\vec{v}$$

$$(A - \lambda I)\vec{v} = \vec{0}$$

$$\begin{vmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0 \rightarrow \text{Reqd.}$$

→ 90° rotation will never have any eigenvalues

Why? $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow$ 90° anti-rotation

For eigenvalues, $\begin{vmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{vmatrix} = 0$

$\therefore \lambda^2 + 1 = 0$

$\lambda^2 = -1$ Not possible.

New eg.

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow$ stretches every vector by '2' w/o any rotation

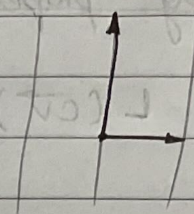
Hence, each vector in the plane is an eigenvector with eigenvalue = 2.

→ When a matrix is a diagonal matrix, its basis vectors (all of them) are eigenvectors

Think! Why?

Because there is only stretching of the vectors
No rotation.

eg. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$



→ Additivity property:
Let L be some transformation

$$L(\vec{v}+\vec{w}) = L(\vec{v}) + L(\vec{w})$$

→ Scaling property:

$$L(c\vec{v}) = c \cdot L(\vec{v})$$

→ Basis functions:

To represent polynomials using matrices

eg. 1) $7x+6$ $\begin{bmatrix} 6 \\ 7 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$ 2) $6+3x+5x^2$ $\begin{bmatrix} 6 \\ 3 \\ 5 \\ 0 \\ \vdots \end{bmatrix}$ 3) $7x^3+1$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 7 \\ 0 \\ \vdots \end{bmatrix}$

Derivative matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 2 & 0 & \cdots \\ 0 & 0 & 0 & 3 & \cdots \\ 0 & \cdots & & & \\ \vdots \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \\ 0 \end{bmatrix}$$

eg. polynomial is $7x^2+1$ $0+14x+0x^2$