Vectors

Physics

- direction and magnitude

Maths

- Combines

Computer

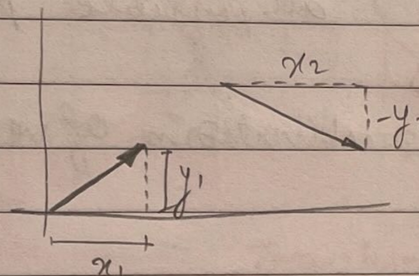
- List of numbers.

→ In linear algebra

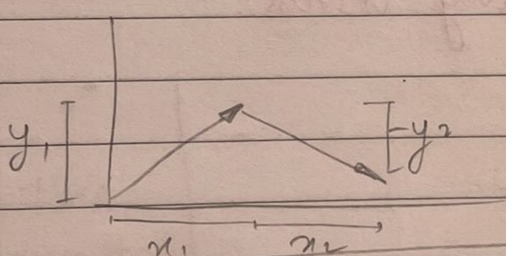
① vectors are always rooted to the origin

vector addition

① * How physics vector addⁿ = computer vector addⁿ ?



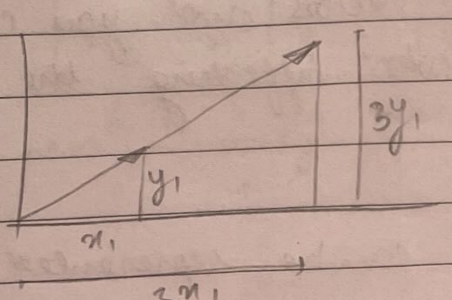
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ -y_2 \end{bmatrix}$$



$$\begin{bmatrix} x_1 + x_2 \\ y_1 - y_2 \end{bmatrix}$$

Scalar multiplication

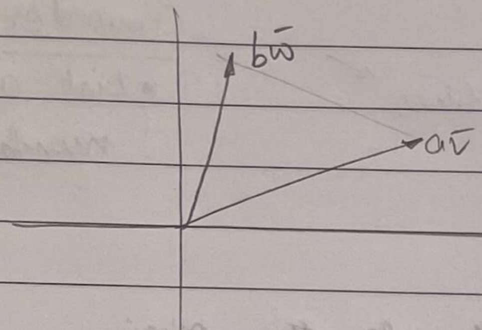
ⓧ * physics multiplication computer multiplication



$$3 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 3x_1 \\ 3y_1 \end{bmatrix}$$

→ $\hat{i}$ and $\hat{j}$ are the basis vectors

→



→ If we kepp changing a, we will get a st. line

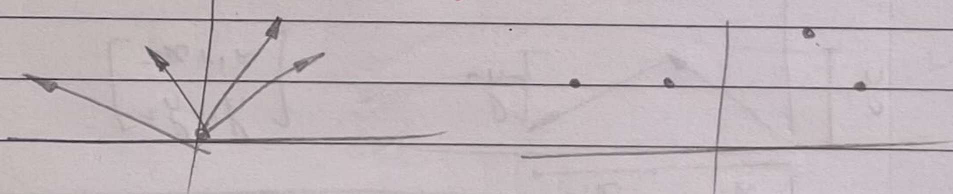
→ If we keep changing b, we will get a st. line.

→ change both → we will get all possible points in the plane

→ Span of all vectors

Set of all linear combinations of vectors $a\bar{v}+b\bar{w}$.

→ Collection of many vectors:



→ Linearly dependant

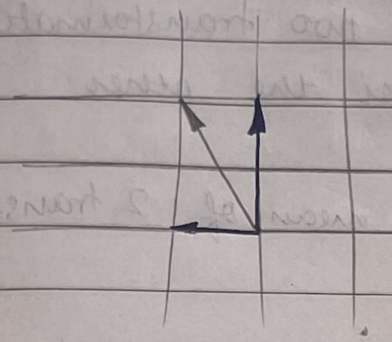
→ When you have multiple vectors and you can remove one of them w/o affecting the span of them.

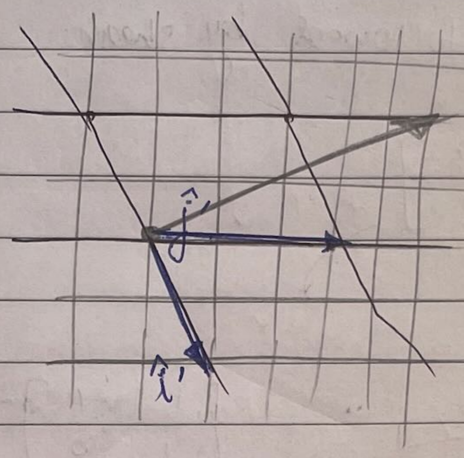
→ one of the (three) vectors can be represented as linear combination of others

→ If each vector adds another dimension to the span → linearly independant.

Matrix Transformations.

$\bar{v} = -\hat{i} + 2\hat{j}$





$\therefore$ Transformed $\bar{v} = -1[\text{Transformed } \hat{i}] + 2[\text{Transformed } \hat{j}]$

$$-1\begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}3\\0\end{bmatrix}$$

$$\begin{bmatrix}5\\2\end{bmatrix}$$

→ How is this transformation depicted?

original vector

$$\begin{bmatrix}+1 & 0\\-2 & 3\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$

new $\hat{i}$ new $\hat{j}$

Matrix Multiplication

→ Multiplication : • Applying two transformations one after the other.

• Geometric mean of 2 transformation matrices.

eg. Rotation 1st, followed by shear.

← Read

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotn}}$$

x

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

overall transformation.

Steps:

① Taking $\hat{i}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Apply previous pg.

$$0 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

temp:

② $\begin{bmatrix} 1 & - \\ 1 & - \end{bmatrix}$

③ Taking $\hat{j}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Applying previous pg

$$-\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ \; \end{bmatrix} = \begin{bmatrix} -1 \\ \; \end{bmatrix}$$

eg. Apply to $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

① $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ g \end{bmatrix} = e\begin{bmatrix} a \\ c \end{bmatrix} \cancel{\begin{matrix} b \\ d \end{matrix}} + g \cancel{\begin{matrix} a \\ c \end{matrix}} \begin{bmatrix} b \\ d \end{bmatrix}$

$= \begin{bmatrix} ea + gb \\ ec + gd \end{bmatrix}$

② $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} f \\ h \end{bmatrix} = f\begin{bmatrix} a \\ c \end{bmatrix} + h\begin{bmatrix} b \\ d \end{bmatrix}$

$= \begin{bmatrix} af + bh \\ fc + dh \end{bmatrix}$

③ we get, $\begin{bmatrix} ae+gb & af+bh \\ ec+gd & fc+dh \end{bmatrix}$

3-D linear transformations.

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} + y\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + z\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

→ 3×3 matrix multiplication:

$$\underbrace{\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix}}_{\text{2nd transformation}} \underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{\text{1st transformation}}$$

① $$\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} = 0\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix} + 6\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 33 \\ 6 \end{bmatrix}$$

② $$\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} = 1\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix} + 7\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 44 \\ 10 \end{bmatrix}$$

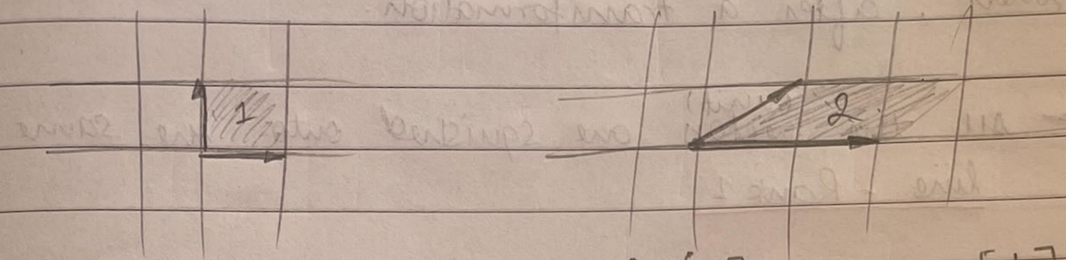
③ $$\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} = 2\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} + 5\begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix} + 8\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 55 \\ 14 \end{bmatrix}$$

$2 + 20 - 8$

∴ Total: $$\begin{bmatrix} 6 & 6 & 6 \\ 33 & 44 & 55 \\ 6 & 10 & 14 \end{bmatrix}$$

Significance of determinant:

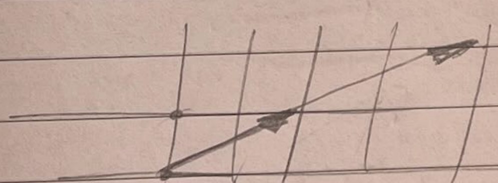
→ 

$\hat{i} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\hat{j} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

Determinant = 2

→ $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$



∴ Area = 0.

* When is determinant negative?
when orientation of $\hat{i}$ and $\hat{j}$ reverses.

→ 3-D [3×3 matrices]

* Determinants give volume.
* Right-hand rule [1st finger: x ; 2nd finger: y
thumb: z]

+ve volume.

$$\boxed{\det(M_1 M_2) = \det(M_1) \times \det(M_2)}$$

why?

Rank:

When, after a transformation:

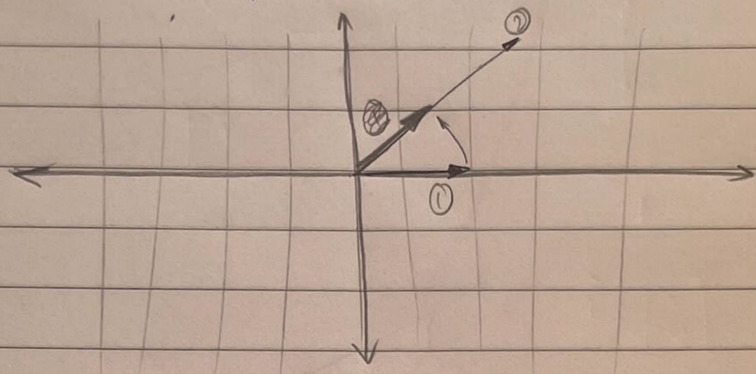
→ All the (unit) vectors are squished onto the same line - Rank 1

→ All the (unit) vectors are squished onto ~~onto~~ one same plane - Rank 2.

Hence Rank means no. of dimensions in the output.

→ For a non-zero determinant,
volume $\neq 0$ (spanned by unit vectors) $\therefore$ Rank $= 3$.

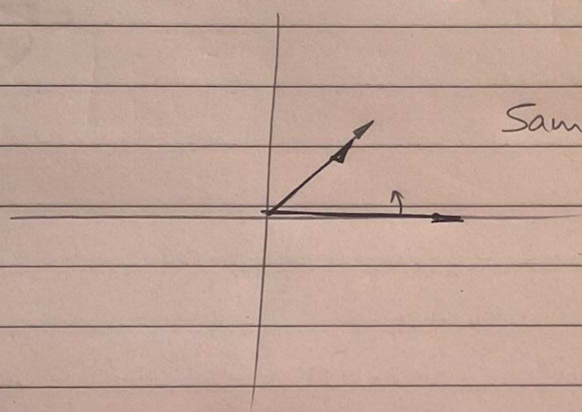
Dot product



Projection of one along other

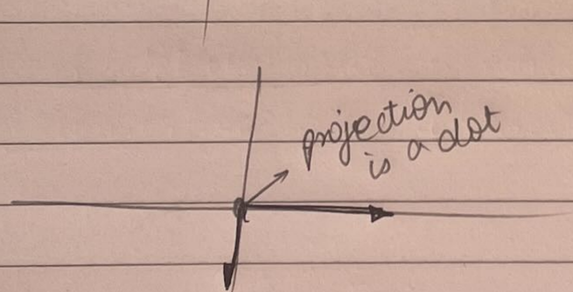
Dot product = (Length of projection of ①) (Length of ②)

①



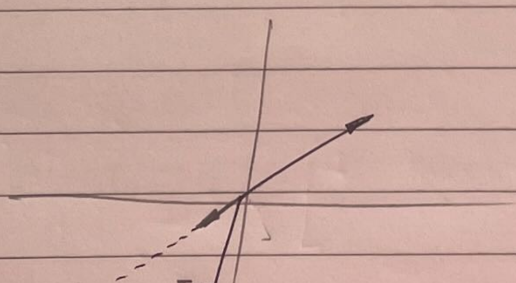
Same dir^n: Dot product +ve.

②



: Dot product = 0

③



~~p~~ Opp direction

Dot product is negative.