MSCI 719: Operations Analytics

Assignment 5: JOANNalytics: Inventory Demand Prediction and Optimal Dynamic Allocation

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1. Prediction models

In the model we discussed in class, we assumed that the demand for each week in 2019 is equal to the demand for the same week in 2018. Suppose that the demand of week n in 2019 is,

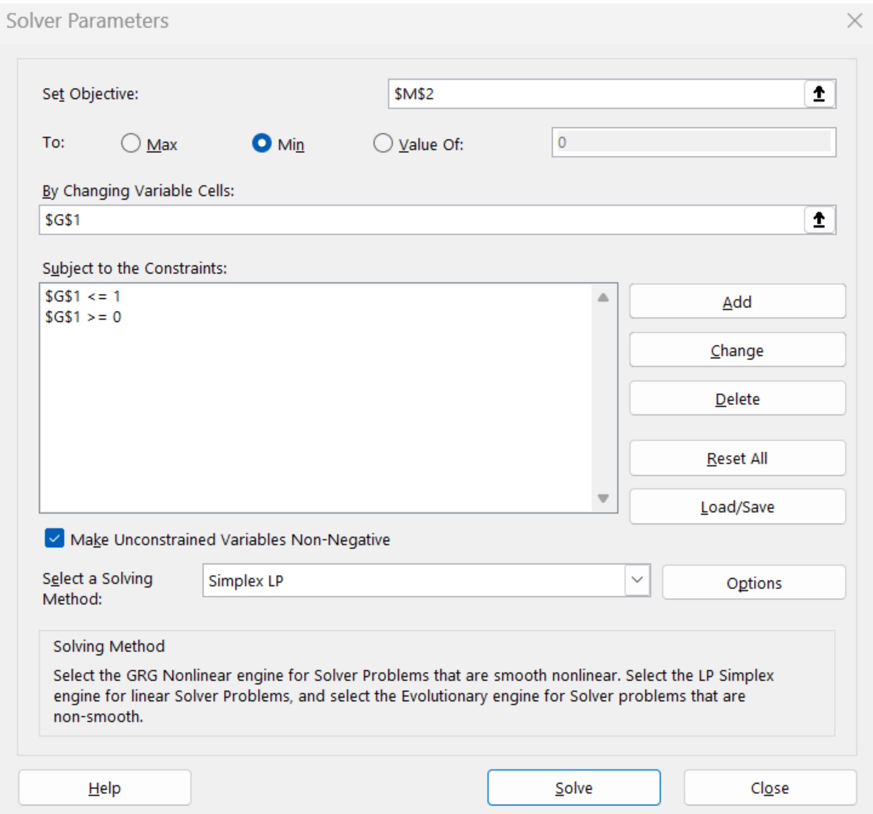
$$\begin{aligned}\text{demand of week } n \text{ in 2019} = &\alpha(\text{demand of week } n \text{ in 2018} - \text{Av. demand in 2018 up to week } n-1)\\ &+ (1-\alpha)(\text{demand of week } n \text{ in 2017} - \text{Av. demand in 2017 up to week } n-1)\\ &+ \text{Av. demand in 2019 up to week } n-1\end{aligned}$$

for a given $\alpha \in [0, 1]$.

1.1. Use the first 5 weeks of 2019 as the testing data. Calculate MSE on test data for different values of α, which value minimizes the error?

We get following results when predictions are made using above model and trained for optimal alpha such that it minimizes combined SSE and MSE of all 3 stores demand predictions for first five weeks:

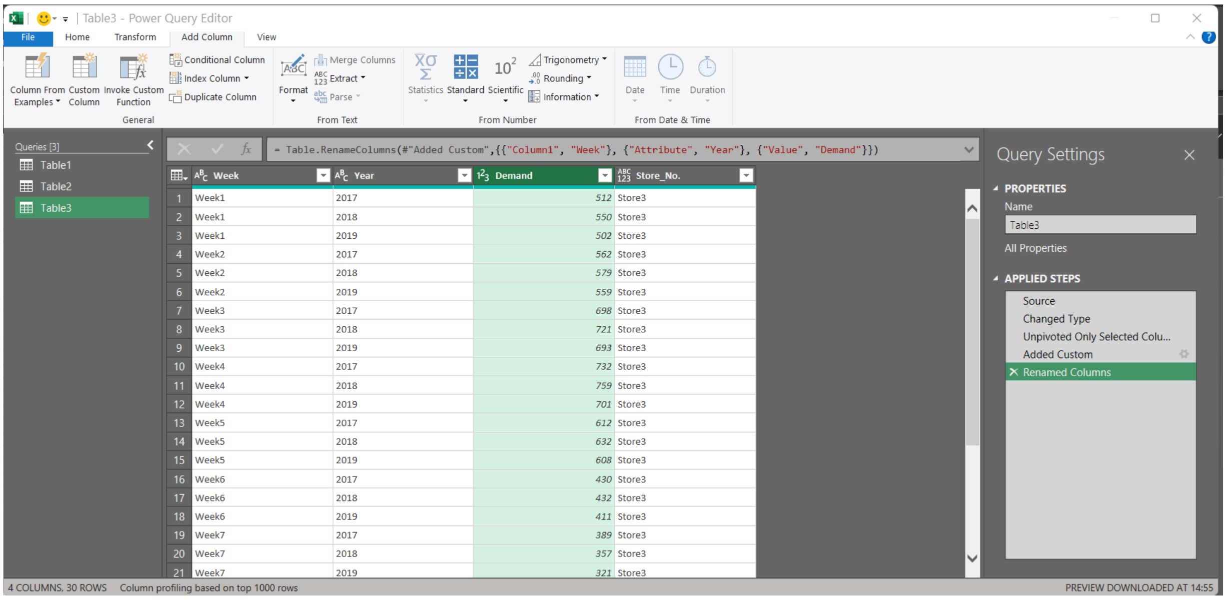
	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Model 1:**	**D11_2019**	**D12_2019**	**D13_2019**		**Alpha=**	0.439545		**SE (Squared Error)**				**MSE**
2	**Week1**	694.78363	728.08546	528.7027				**Week1**	15679.14	2200.974	713.0357		4907.51
3	**Week2**	906.85773	827.24182	542.7695				**Week2**	7769.06	688.633	263.4277		
4	**Week3**	740.42114	1040.8777	689.522				**Week3**	417.023	21280.31	12.09617		
5	**Week4**	1003.2741	815.495	726.4395				**Week4**	45.23783	7141.095	647.1705		
6	**Week5**	594.66558	858.96397	597.0028				**Week5**	15960.39	674.1278	120.9375		
7	**Week6**	657.39473	787.60754	409.2905									
8	**Week7**	695.28652	637.64355	353.6308									
9	**Week8**	661.51266	737.60493	524.2866									
10	**Week9**	852.85677	904.55295	741.5285									
11	**Week10**	1275.9881	1188.4984	1023.988									



So, we get optimal alpha of 0.4395 which gives minimum MSE of 4907.51 on the test data for this model.

1.2. Use 2017 and 2018 data as the training data. Consider the number of periods and the number of weeks as predictors for demand in the year 2019. Fit a linear regression model to predict demand for 2019. Calculate MSE on the same testing data set as part 1.

If we want to create a single linear regression model for demand prediction for given store, week and year, we can pre-process given train dataset in power query and run model in R programming.



Head of processed dataset:

Week	Year	Demand	Store_No.
Week1	2017	795	Store1
Week1	2018	567	Store1
Week1	2019	820	Store1
Week2	2017	899	Store1
Week2	2018	632	Store1
Week2	2019	995	Store1
Week3	2017	623	Store1
Week3	2018	505	Store1
Week3	2019	720	Store1
Week4	2017	912	Store1
Week4	2018	750	Store1
Week4	2019	1010	Store1

We can load this csv data in R programming and create a Linear regression model fit on it, whose summary can be seen below:

```
model1 = lm(Demand~Week+factor(Year)+Store_No.,df)
summary(model1)
```

```
Call:
lm(formula = Demand ~ Week + factor(Year) + Store_No., data = df)

Residuals:
     Min       1Q   Median       3Q      Max 
-197.900  -55.283    6.317   46.250  268.233 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)       572.10      48.72  11.744 1.40e-15 ***
WeekWeek10        464.67      60.42   7.690 7.42e-10 ***
WeekWeek2          58.67      60.42   0.971   0.3366    
WeekWeek3         115.33      60.42   1.909   0.0624 .  
WeekWeek4         155.50      60.42   2.573   0.0133 *  
WeekWeek5         -12.00      60.42  -0.199   0.8434    
WeekWeek6         -87.83      60.42  -1.454   0.1527    
WeekWeek7        -142.17      60.42  -2.353   0.0229 *  
WeekWeek8         -63.17      60.42  -1.045   0.3012    
WeekWeek9         133.67      60.42   2.212   0.0319 *  
factor(Year)2018   15.47      27.02   0.572   0.5698    
Store_No.Store2   217.10      33.10   6.560 3.81e-08 ***
Store_No.Store3    -6.60      33.10  -0.199   0.8428    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 104.7 on 47 degrees of freedom
Multiple R-squared:  0.8145,	Adjusted R-squared:  0.7671 
F-statistic: 17.19 on 12 and 47 DF,  p-value: 2.615e-13
```

However, we can do this model preparation in excel as well. And here we will create store-wise LR models with year and week as one hot encoded variables, as shown below:

Sample store 1 demand data with one hot encoded week and year categorical variables.

is_week1	is_week2	is_week3	is_week4	is_week5	is_week6	is_week7	is_week8	is_week9	is_week10	is_2017	is_2018	is_2019	Demar
1	0	0	0	0	0	0	0	0	0	1	0	0	795
0	1	0	0	0	0	0	0	0	0	1	0	0	899
0	0	1	0	0	0	0	0	0	0	1	0	0	623
0	0	0	1	0	0	0	0	0	0	1	0	0	912
0	0	0	0	1	0	0	0	0	0	1	0	0	376
0	0	0	0	0	1	0	0	0	0	1	0	0	435
0	0	0	0	0	0	1	0	0	0	1	0	0	455
0	0	0	0	0	0	0	1	0	0	1	0	0	510
0	0	0	0	0	0	0	0	1	0	1	0	0	632
0	0	0	0	0	0	0	0	0	1	1	0	0	1150
1	0	0	0	0	0	0	0	0	0	0	1	0	567
0	1	0	0	0	0	0	0	0	0	0	1	0	632
0	0	1	0	0	0	0	0	0	0	0	1	0	505
0	0	0	1	0	0	0	0	0	0	0	1	0	750
0	0	0	0	1	0	0	0	0	0	0	1	0	500
0	0	0	0	0	1	0	0	0	0	0	1	0	510
0	0	0	0	0	0	1	0	0	0	0	1	0	550
0	0	0	0	0	0	0	1	0	0	0	1	0	382
0	0	0	0	0	0	0	0	1	0	0	1	0	673
0	0	0	0	0	0	0	0	0	1	0	1	0	986
1	0	0	0	0	0	0	0	0	0	0	0	1	820
0	1	0	0	0	0	0	0	0	0	0	0	1	995
0	0	1	0	0	0	0	0	0	0	0	0	1	720
0	0	0	1	0	0	0	0	0	0	0	0	1	1010

On creating separate models and calculating squared errors, we get following MSE result:

Model 2:	D11_2019	D12_2019	D13_2019		SE (Squared Error)			MSE
Week1	869	716.53333	502.0667	**Week1**	2401	3418.351	0.004444	1375.73
Week2	983.66667	759.86667	547.4	**Week2**	128.4444	1691.951	134.56	
Week3	757.66667	946.2	684.7333	**Week3**	1418.778	2621.44	68.33778	
Week4	1032.3333	826.2	711.4	**Week4**	498.7778	5446.44	108.16	
Week5	674	813.2	598.0667	**Week5**	2209	392.04	98.67711	

1.3. Calculate the MSE for the model discussed in class on the testing data set. Which one of these three models performs best for predicting demand?

In the model we discussed in class, we assumed that the demand for each week in 2019 is equal to the demand for the same week in 2018. So, we get MSE of 29273.8 in this prediction model, which is quite high compared to previous two models.

Model 3:	D11_2019	D12_2019	D13_2019		SE (Squared Error)			MSE
Week1	567	820	550	Week1	64009	2025	2304	29273.8
Week2	632	870	579	Week2	131769	4761	400	
Week3	505	1150	721	Week3	46225	65025	784	
Week4	750	920	759	Week4	67600	400	3364	
Week5	500	865	632	Week5	48841	1024	576	

As we can see from the results of previous sections, Model 2 is performing the best in our demand prediction task, which gave MSE of just 1375.73, so we will use it in our subsequent sections.

2. Weekly allocation vs. 10-week allocation

We are using the best prediction model developed in Part 1 to predict the demand for 2019. So, below are our predicted demands for the product 1 in year 2019 for store 1,2 and 3 respectively for the weeks 1 to 10:

	Rounded-Up Predicted Demand:		
Week	D11_2019	D12_2019	D13_2019
1	869	717	503
2	984	760	548
3	758	947	685
4	1033	827	712
5	674	814	599
6	694	722	406
7	730	597	337
8	647	707	533
9	847	879	747
10	1313	1120	1003

Consider the following two scenarios:

1. At the beginning of week 1 in 2019, an allocation plan is created for the whole 10 weeks, and at the beginning of each week (week 1-10) inventory is allocated to stores based on the plan, independent of overstocking/understocking during each week.

2. At the beginning of each week, the allocation plan is updated by considering the available inventory at each store. Assume that the allocation to each store should not exceed the demand minus the available inventory at the store at the beginning of the week.

Suppose the following assumptions hold.

• The holding and shortage costs are 3 and 5 per unit, respectively.

• Each item that remains in the distribution center at the end of week 10, incurs 2-dollar cost.

• Target inventory at the beginning of week 1 (available inventory at the distribution center) is 20000.

• The inventory allocated to each store at the beginning of each week should be between 150 and 1000.

• Available inventory at the beginning of week 1, at stores 1, 2, and 3 are 55, 83, and 110, respectively.

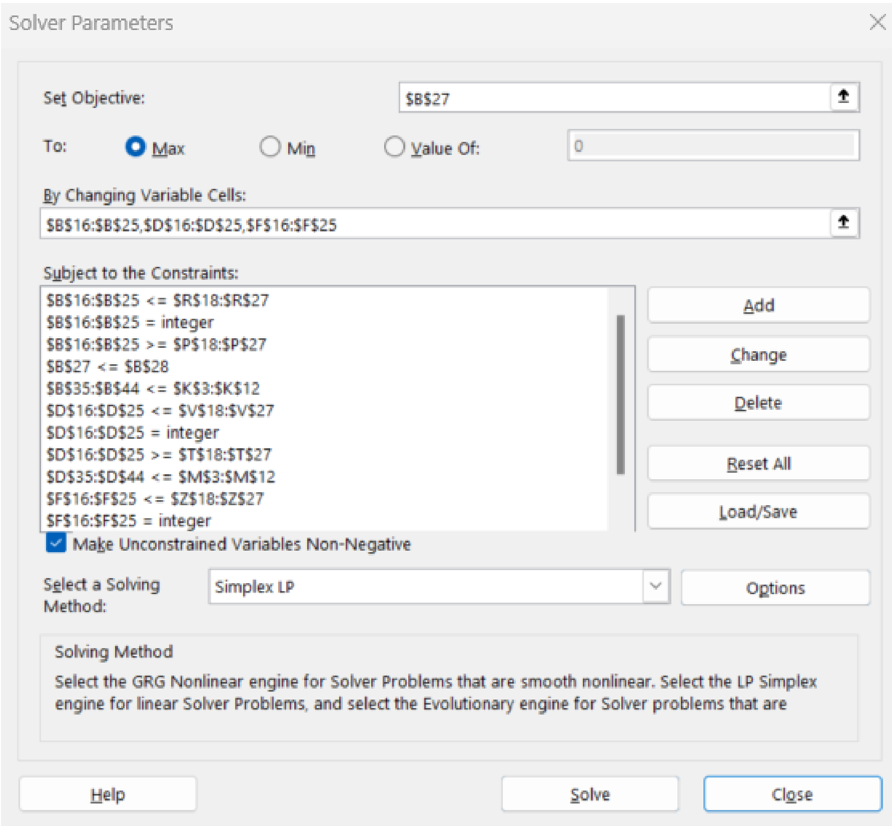
2.1. What is the total cost under each scenario?

Here is our inventory allocation optimization model objective function and constraints:

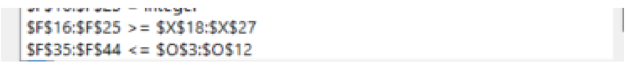
$$Max \sum_i \sum_j \sum_k x_{ijk} \qquad \text{s.t.} \qquad \begin{aligned} x_{ijk} + I_{ijk} &\le \widehat{D}_{ijk} && \forall_{all\ i,j,k} \\ l_{ijk} \le x_{ijk} &\le u_{ijk} && \forall_{all\ i,j,k} \end{aligned}$$

Scenario 1:

Following optimization model was used in excel solver:



Other constraints:



we get following results by optimizing for maximum allocations such that it meets above requirements:

Decision Variables					
Store 1	Value	Store 2	Value	Store 3	Value
X1,1	814	X2,1	634	X3,1	393
X1,2	984	X2,2	760	X3,2	548
X1,3	758	X2,3	947	X3,3	685
X1,4	1000	X2,4	827	X3,4	712
X1,5	674	X2,5	814	X3,5	599
X1,6	694	X2,6	722	X3,6	406
X1,7	730	X2,7	597	X3,7	172
X1,8	647	X2,8	707	X3,8	150
X1,9	847	X2,9	879	X3,9	150
X1,10	1000	X2,10	1000	X3,10	150

Optimized Objective function results:

total Qty allocations = 20,000 units = max available inventory of the product for the year 2019

And below are the total lost sales (shortages) and unsold items under this allocation model:

Lost sales (Shortages)				
	Store 1	Store 2	Store 3	Total
Week 1	0	58	0	58
Week 2	11	41	11	63
Week 3	0	0	8	8
Week 4	10	73	0	83
Week 5	47	19	9	75
Week 6	18	0	5	23
Week 7	30	0	149	179
Week 8	0	0	396	396
Week 9	0	0	611	611
Week 10	376	10	814	1200
Total	492	201	2003	2696

Unsold Items (subject to holding charges)				
	Store 1	Store 2	Store 3	Total
Week 1	49	0	1	50
Week 2	0	0	0	0
Week 3	38	52	0	90
Week 4	0	0	11	11
Week 5	0	0	0	0
Week 6	0	22	0	22
Week 7	0	1	0	1
Week 8	25	7	0	32
Week 9	37	4	0	41
Week 10	0	0	0	0
Total	149	86	12	247

Therefore, below is the total cost for this allocation scenario:

total cost=	14221

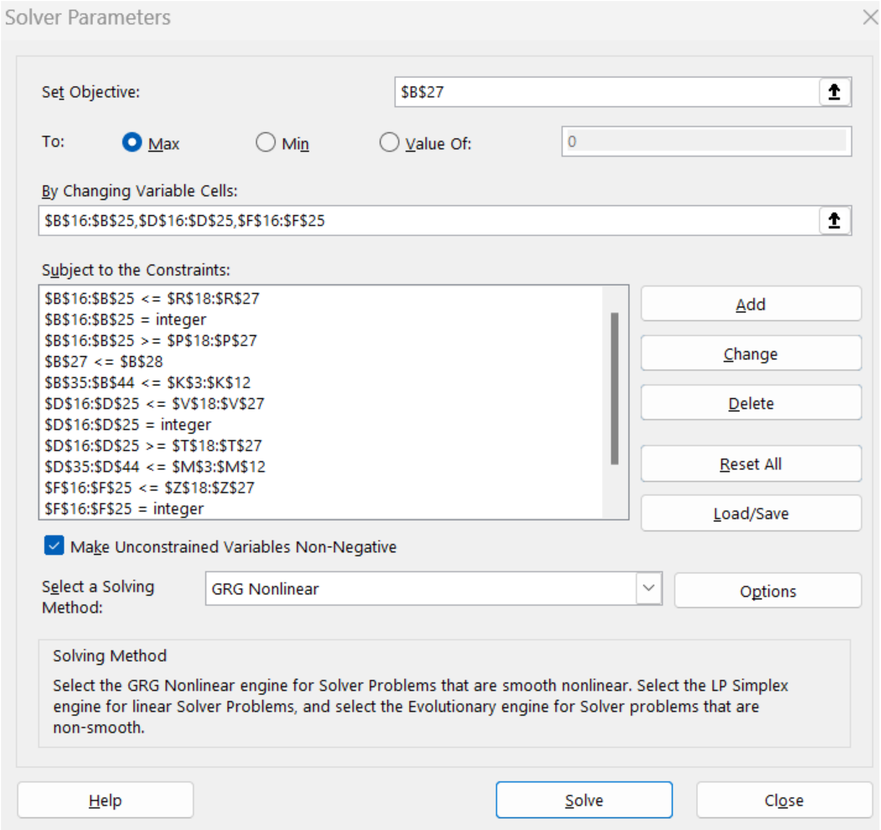
Scenario 2:

In this model, we just use unsold inventory information to adjust each week allocations, and hence it can be called as a dynamic allocation model. We get following allocation results:

Optimized Objective function results:

total Qty allocations = 20,000 units = max available inventory of the product for the year 2019

Decision Variables					
Store 1	Value	Store 2	Value	Store 3	Value
X1,1	814	X2,1	634	X3,1	393
X1,2	935	X2,2	760	X3,2	547
X1,3	745	X2,3	947	X3,3	685
X1,4	1000	X2,4	775	X3,4	712
X1,5	658	X2,5	814	X3,5	588
X1,6	694	X2,6	722	X3,6	406
X1,7	730	X2,7	575	X3,7	232
X1,8	647	X2,8	700	X3,8	196
X1,9	822	X2,9	877	X3,9	196
X1,10	1000	X2,10	1000	X3,10	196



Other constraints:

$F$16:$F$25 >= $X$18:$X$27
$F$35:$F$44 <= $O$3:$O$12

Note that, we had to use here GRG non-linear optimization technique instead of simplex LP to solve this optimization problem in excel solver.

And below are the total lost sales (shortages) and unsold items under this allocation model:

	Lost sales			
	Store 1	Store 2	Store 3	Total
Week 1	0	58	0	58
Week 2	11	41	11	63
Week 3	0	0	8	8
Week 4	0	73	0	73
Week 5	48	19	9	76
Week 6	18	0	5	23
Week 7	30	0	89	119
Week 8	0	0	350	350
Week 9	0	0	565	565
Week 10	339	7	768	1114
Total	446	198	1805	2449

	Unsold Item			
	Store 1	Store 2	Store 3	Total
Week 1	49	0	1	50
Week 2	0	0	0	0
Week 3	25	52	0	77
Week 4	15	0	11	26
Week 5	0	0	0	0
Week 6	0	22	0	22
Week 7	0	1	0	1
Week 8	25	1	0	26
Week 9	37	3	0	40
Week 10	0	0	0	0
Total	151	79	12	242

Therefore, below is the total cost for this allocation scenario:

total cost=	12971

Note: If we keep our objective function as total cost minimization instead of maximum qty. allocation, we get following minimum costs for both scenarios (refer excel sheet 2.1.1/2_extra):

total cost=	12245

2.2. Which scenario is less expensive? How do you explain this?

On comparing results of both scenarios, we can easily notice that there is a significant cost saving in scenario 2 i.e. savings of about $ 14221- $ 12971 = $ 1250. So, scenario 2 less expensive in this aspect. However, we must note that in order to run model as per scenario 2, we would need dynamic store demand data each week. And due to current digitalization trends, getting such dynamic data shouldn't pose any challenge for JOANN.

Now let us understand why scenario 2 is less expensive. The main difference is in scenario 1, we plan inventory for all weeks in the beginning of the year and do not take into consideration the actual demand and extra inventory each store would have at the end of each week in case of less demand then predicted, but we just dump the inventory every week as planned in the beginning of year. However, in scenario 2, we delay inventory allocation and that helps us to be more reactive to in season or weekly demand and learn about unsold quantity each store already has at the end of the week. And this actually help us to refrain from sending additional items which store ultimately would not be able to sell it. So this saves holding and shortage costs significantly in scenario 2 compared to scenario 1, and hence scenario 2 becomes comparatively cheaper and better alternative.

References:

[1] H A Mehrizi, eBook: MSCI 719 Winter 2023 Cases Multiple (ID: 9723713) Accessed: Jan. 22, 2023. [Online].

Available:
https://www.campusebookstore.com/integration/AccessCodes/default.aspx?permalinkId=ee044bf2-fe82-4db0-ad22-088e81954eef&frame=YES&t=permalink&sid=4u2faw45zyslbp45bbqlpc55