A2

Q1 Using Simplex method we need to solve following linear program:

$$\max\ z = x_1 + 2x_2 - x_3$$

$$\text{s.t.} \quad x_1 + x_2 - 2x_3 \le 4$$

$$2x_1 - x_2 - 2x_3 \le 5$$

$$x_1, x_2, x_3 \ge 0$$

$\Rightarrow$ Converting this problem into standard LP form:

$$z - x_1 - 2x_2 + x_3 = 0$$

$$x_1 + x_2 - 2x_3 + s_1 = 4$$

$$2x_1 - x_2 - 2x_3 + s_2 = 5$$

where, $x_1, x_2, x_3, s_1, s_2 \ge 0$

$\therefore$ Simplex tableau for the given eqns is following:

	Z	x_1	x_2	x_3	S_1	S_2	RHS	BV	Ratio
R0:	1	-1	-2	1	0	0	0	Z	
R1:	0	1	1	-2	1	0	4	S_1	
R2:	0	2	-1	-1	-2	0	1	(	

max	z	x_1	x_2	x_3	s_1	s_2	RHS	BV	Ratio
R0:	1	-1	-2	1	0	0	0	z	
R1:	0	1	1	-2	1	0	4	$s_1=4$	$4/1$
R2:	0	2	-1	-2	0	1	5	$s_2=5$	none

Initial ~~opt~~ basic feasible solution is,
$\underbrace{s_1 = 4,\ s_2 = 5}_{\text{(B.V. → Basic variables)}}$, and $z = 0$, and $\underbrace{x_1 = x_2 = x_3 = 0}_{\text{(non basic variables = NBV)}}$

Now analyzing the tableau, we see that x_2 non-basic variable has the minimum value (i.e. max^m negative) value) ∴ It becomes entering variable ∴ looking at pivot column and having ratio test we get ~~a~~ that s_1 has least ratio and it becomes the leaving variable.

→ Dividing and updating x_2 column for identity matrix we get following table:

	z	x_1	x_2	x_3	s_1	s_2	RHS	BV	Ratio
$2R_1 + R_0 \to R0$:	1	1	0	-3	2	0	8	z	
$R_1 \to R1$:	0	1	1	-2	1	0	4	x_2	none
$R_1 + R_2 \to R2$:	0	3	0	~~-0~~ (-4)	1	1	9	s_2	none

→ Now, we have one column with all ~~neg~~ non-positive elements ∴ It suggest unbounded feasible region.

→ now, choosing most negative coefficient to enter the basis, here, x_3 would be our entering variable.

→ However, we can't determine the leaving variable because the elements of the pivot column are all non-positive ∴ No results in ratio test.

Ans. → ∴ optimal solution is unbounded.