Q1 ⇒ First of all, we can formulate LP model based on given data:→

$8000 = available cash

here, $K = 10^3 = 1000$

Beginning of month	Cash + Rev. + Loans	Bill + return of loaned amt. with interest
July 1st	$8K + 1000 + x + x_1$	$12K$
Aug	$x_2 + 4K + S_1$	$11K + 1.1x_1$
Sept	$x_3 + 3K + S_2$	$10K + 1.1x_2$
Oct	$x_4 + 8K + S_3$	$4K + 1.1x_3$
nov.	$x_5 + 17K + S_4$	$5K + 1.1x_4$
Dec. start	$12K + S_5$	$1K + 1.1x_5$
December end	S_6	$1.15x$

$$\min z = 0.15x + 0.1(x_1 + \cdots + x_5)$$

S.t. $8K + 1K + x + x_1 \geq 12K$

$4K + x_2 + S_1 \geq 11K + 1.1x_1$

$3K + x_3 + S_2 \geq 10K + 1.1x_2$

$8K + x_4 + S_3 \geq 4K + 1.1x_3$

$17K + x_5 + S_4 \geq 5K + 1.1x_4$

$12K + S_5 \geq 1K + 1.1x_5$

$S_6 \geq 1.15x$

where, $S_1 = x_1 + 9K + x - 12K$

$S_2 = x_2 + 4K + S_1 - 11K - 1.1x_1$

$S_3 = x_3 + 3K + S_2 - 10K - 1.1x_2$

$S_4 = x_4 + 8K + S_3 - 4K - 1.1x_3$

$S_5 = x_5 + 17K + S_4 - 5K - 1.1x_4$

$S_6 = 12K + S_5 - 1K - 1.1x_5$

and, $x_i \geq 0, \quad \forall i = 1, 2, \cdots, 5.$

$S_j \geq 0, \quad \forall j = 1, 2, \cdots, 6.$

$x \geq 0$

→ Now, adding this optimization problem along with constraints and solving using excel solver, LP method,

→ ∴ we get following solution:

$x_1 = 13363.64$ dollars
$x_3 = 3636.364$ dollars
$x_1, x_2, x_4, x_5 = 0$ dollars
and $Z = 2368.18$ dollars

∴ ABC can minimize cost (total interest paid) by borrowing $13363.64 at the beginning of July for a six-month loan period, and, $3636.36 in the beginning of month of September for a one-month period.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Optimal Values of variables	13363.64	0	0	3636.364	0	0						
2	Coefficients of variables	0.15	0.1	0.1	0.1	0.1	0.1	2368.181818	=Minimum value of 'z' s.t contraints				
3	name of variables in 'z'	x	x1	x2	x3	x4	x5						
4	s.t.							K13:K18	LHS		RHS		
5	=B5*B1+C5*C1	1	1	0	0	0	0	0	13363.64	>=	3000		
6	=C6*C1+D6*D1+H6	0	-1.1	1	0	0	0	10363.63636	10363.64	>=	7000		
7	=D7*D1+E7*E1+H7	0	0	-1.1	1	0	0	3363.636364	7000	>=	7000		
8	=E8*E1+F8*F1+H8	0	0	0	-1.1	1	0	0	-4000	>=	-4000		
9	=F9*F1+G9*G1+H9	0	0	0	0	-1.1	1	-5.29235E-09	-5.3E-09	>=	-12000		
10	=G10*G1+H10	0	0	0	0	0	-1.1	12000	12000	>=	-11000		
11	=B11*B1+H11	-1.15	0	0	0	0	0	23000	7631.818	>=	0		
12													
13									s1	=	10363.63636	This s1 to s6 values calculated in cells K13:K18 are used to fill in cells H6:H11	=x1+x-3000
14									s2	=	3363.636364		=x2+s1-1.1x1-7000
15									s3	=	0		=x3+s2-1.1x2-7000
16									s4	=	-5.29235E-09		=x4+s3-1.1x3+4000
17									s5	=	12000		=x5+s4-1.1x4+12000
18									s6	=	23000		=s5-1.1x5+11000
19													
20									xi	>=	0	for all i= 1,2,..,5	
21									x	>=	0		
22									sj	>=	0	for all j= 1,2,..,6	

Microsoft Excel 16.0 Answer Report
Worksheet: [A1_problem_solving.xlsx]Calc
Report Created: 25-09-2022 15:38:21
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: Simplex LP

Solution Time: 0.031 Seconds.

Iterations: 5 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$H$2		0	2368.181818

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$1	z	0	13363.63636	Contin
$C$1	z	0	0	Contin
$D$1	z	0	0	Contin
$E$1	z	0	3636.363636	Contin
$F$1	z	0	0	Contin
$G$1	z	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$I$5		13363.63636	$I$5>=$K$5	Not Binding	10363.63636
$I$6		10363.63636	$I$6>=$K$6	Not Binding	3363.636364
$I$7		7000	$I$7>=$K$7	Binding	0
$I$8		-4000	$I$8>=$K$8	Binding	0
$I$9		-5.29235E-09	$I$9>=$K$9	Not Binding	12000
$I$10		12000	$I$10>=$K$10	Not Binding	23000
$I$11		7631.818182	$I$11>=$K$11	Not Binding	7631.818182

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [A1_problem_solving.xlsx]Calc
Report Created: 25-09-2022 15:38:21

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$1	z	13363.63636	0	0.15	0.06	0.05
$C$1	z	0	0.115	0.1	1E+30	0.115
$D$1	z	0	0.115	0.1	1E+30	0.115
$E$1	z	3636.363636	0	0.1	0.05	0.06
$F$1	z	0	0.054545455	0.1	1E+30	0.054545455
$G$1	z	0	0.1	0.1	1E+30	0.1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$5		13363.63636	0	3000	10363.63636	1E+30
$I$6		10363.63636	0	7000	3363.636364	1E+30
$I$7		7000	0.104545455	7000	73000	4000
$I$8		-4000	0.045454545	-4000	4000	3700
$I$9		-5.29235E-09	0	-12000	12000	1E+30
$I$10		12000	0	-11000	23000	1E+30
$I$11		7631.818182	0	0	7631.818182	1E+30

Microsoft Excel 16.0 Limits Report
Worksheet: [A1_problem_solving.xlsx]Calc
Report Created: 25-09-2022 15:38:21

Cell	Objective Name	Value
$H$2		2368.181818

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$B$1	z	13363.63636	13363.63636	2368.181818	64242.42424	10000
$C$1	z	0	0	2368.181818	0	2368.181818
$D$1	z	0	0	2368.181818	0	2368.181818
$E$1	z	3636.363636	3636.363636	2368.181818	3636.363636	2368.181818
$F$1	z	0	5.29235E-09	2368.181818	76318.18182	10000
$G$1	z	0	0	2368.181818	76318.18182	10000