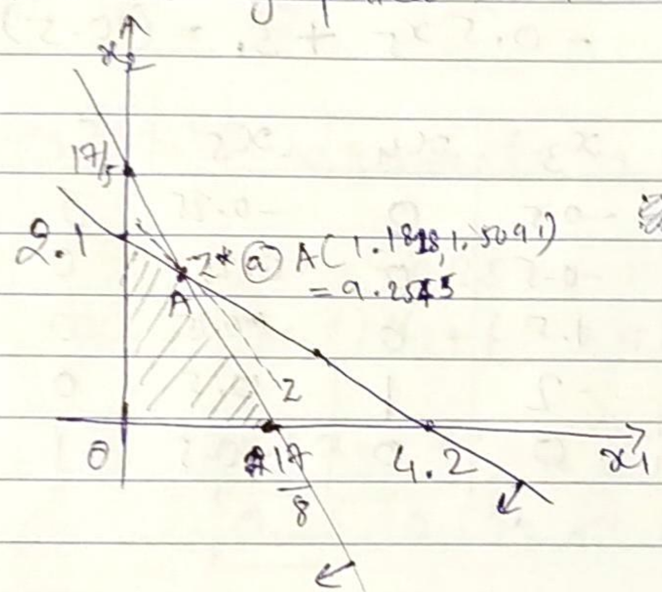
Q2 Use branch and bound to solve the following IP:

$$\text{maximize } z = 4x_1 + 3x_2$$
$$\text{s.t.} \quad 5x_1 + 10x_2 \le 21$$
$$8x_1 + 5x_2 \le 17$$
$$x_1, x_2 \ge 0;$$
$$x_1, x_2 \text{ integer}$$

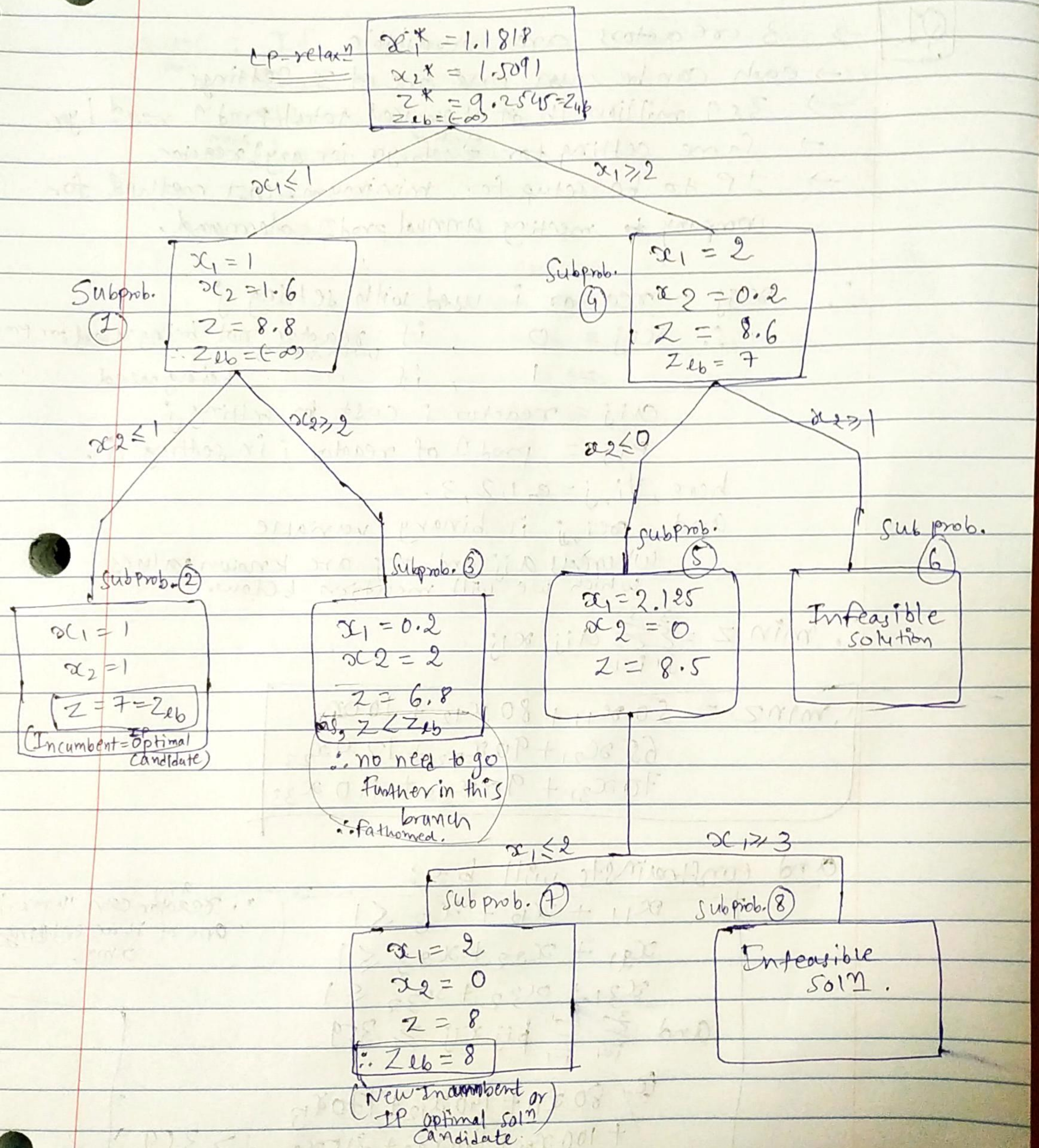
Using LP relaxation, we can first find upperbound of IP problem.
we use graphical method for that:



$\therefore z^* = 9.2545$
$x_1^* = 1.1818$
$x_2^* = 1.5091$

(Also confirmed with MS excel solver)

→ now, we will use branch and bound to update LB = (−∞) to some higher value which has all integer variables as solutions.
→ Using $x_1 \le 1$, $x_2 \ge 2$ as first branch on node 0, we get following tree.
(note: we will use all subproblems using MS Excel solver)



Ans. So, our solution to given IP problem = latest incumbent = $Z_{lb} = 8$

$\therefore \boxed{Z^*(IP) = 8,\ x_1^* = 2,\ x_2^* = 0}$

LP Relaxation	x1	x2	LHS		RHS
Variable values	1.181818	1.509091			
Obj. Fun Coeff	4	3	z	=	9.254545
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	17	<=	17

Sub-problem 1	x1	x2	LHS		RHS
Variable values	1	1.6			
Obj. Fun Coeff	4	3	z	=	8.8
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	16	<=	17
const3 Coeff	1	0	1	<=	1

Sub-problem 2	x1	x2	LHS		RHS
Variable values	1	1			
Obj. Fun Coeff	4	3	z	=	7
const1 Coeff	5	10	15	<=	21
const2 Coeff	8	5	13	<=	17
const3 Coeff	1	0	1	<=	1
const4 Coeff	0	1	1	<=	1

Sub-problem 3	x1	x2	LHS		RHS
Variable values	0.2	2			
Obj. Fun Coeff	4	3	z	=	6.8
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	11.6	<=	17
const3 Coeff	1	0	0.2	<=	1
const4 Coeff	0	1	2	>=	2

Sub-problem 4	x1	x2	LHS		RHS
Variable values	2	0.2			
Obj. Fun Coeff	4	3	z	=	8.6
const1 Coeff	5	10	12	<=	21
const2 Coeff	8	5	17	<=	17
const3 Coeff	1	0	2	>=	2

Sub-problem 5	x1	x2	LHS		RHS
Variable values	2.125	0			
Obj. Fun Coeff	4	3	z	=	8.5
const1 Coeff	5	10	10.625	<=	21
const2 Coeff	8	5	17	<=	17
const3 Coeff	1	0	2.125	>=	2
const4 Coeff	0	1	0	<=	0

Sub-problem 6	x1	x2	LHS		RHS
Variable values	0	0			
Obj. Fun Coeff	4	3	z	=	0
const1 Coeff	5	10	0	<=	21
const2 Coeff	8	5	0	<=	17
const3 Coeff	1	0	0	>=	2
const4 Coeff	0	1	0	>=	1

Sub-problem 7	x1	x2	LHS		RHS
Variable values	2	0			
Obj. Fun Coeff	4	3	z	=	8
const1 Coeff	5	10	10	<=	21
const2 Coeff	8	5	16	<=	17
const3 Coeff	1	0	2	>=	2
const4 Coeff	0	1	0	<=	0
const5 Coeff	1	0	2	<=	2

Sub-problem 8	x1	x2	LHS		RHS
Variable values					
Obj. Fun Coeff	4	3	z	=	0
const1 Coeff	5	10	0	<=	21
const2 Coeff	8	5	0	<=	17
const3 Coeff	1	0	0	>=	2
const4 Coeff	0	1	0	<=	0
const5 Coeff	1	0	0	>=	3