Q1

(a)

Checks	Site(¢) 1	2
Vendor	5	3
Salary	4	4
Personal	2	5

→ As sites can process limited daily checks, it indicates supply in our transportation problem.
∴ ~~a~~ type of checks to be processed each day indicates demand.
∴ Putting them in supply vs. demand form; and assigning vendor, salary and personal 1, 2 and 3 resply, we get,

(site)	Checks category 1	2	3	Supply ('1000)
1	5	4	2	10
2	3	4	5	6
demand → ('1000)	5	5	5	

here, supply = 16000 whereas,
demand = 15000
∴ Unbalanced problem.
∴ We will add a dummy checks category with 1000 demand to balance supply and demand and our transportation problem.

∴

	1	2	3	4	
1	5	4	2	0	10
2	3	4	5	0	6
	5	5	5	1	

~~hen~~

and our minimum cost will be,

$$\min. \text{"}w\text{"} = 5x_{11} + 4x_{12} + 2x_{13} + 0x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 0x_{24}$$

and,
$$x_{11} + x_{12} + x_{13} + x_{14} = 10$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 6$$
$$x_{11} + x_{21} = 5$$
$$x_{12} + x_{22} = 5$$
$$x_{13} + x_{23} = 5$$
and $x_{14} + x_{24} = 1$

and $\forall x_{ij}$ (as integers and ≥ 0.), $\forall i = 1, 2$, $\forall j = 1, 2, 3, 4$.

→ Note, we have considered x_{ij} in thousand numbers and obj. fun unit is cents.

Now,

(b) → Solving this problem using transportation method.

→ First we will find initial BFS using minimum cost method:

by allocating max^m possible units to least cost cells; we get from previous table:

	1	2	3	4	
1	$\boxed{5}$	4 $\boxed{4}$	5 $\boxed{2}$	1 $\boxed{0}$	~~10~~ ~~7~~ ~~4~~ 0
2	5 $\boxed{3}$	1 $\boxed{4}$	$\boxed{5}$	~~1~~ $\boxed{0}$	~~6~~ ~~5~~ 1 0
	~~5~~ 0	~~5~~ ~~4~~ 0	~~5~~ 0	~~1~~ 0	

now, calculating $\bar{c}_{ij}$ (reduced costs) for NBV of initial BFS based table, after assigning $u_1 = 0$ and calculating u_2, v_1, v_2, v_3, v_4, we will get following results :→ [where u_i and v_j are dual variables]

[note that $\bar{c}_{ij} = u_i + v_j - c_{ij}$ and it will be zero for Basic variables, so we will find $\bar{c}_{ij}$ only for NBV variables]

	$v_1=3$	$v_2=4$	$v_3=2$	$v_4=0$
$u_1=0$	5	4 4	5 2	1 0
$u_2=0$	5 3	1 4	5	0

$\therefore$ $\bar{C}_{11} = 3-5 = (-2)$
$\bar{C}_{23} = 2-5 = (-3)$
$\bar{C}_{24} = 0-0 = 0$

so, as, all NBV reduced costs ≤ 0,
our current solution is optimal
one:

$$\boxed{\begin{array}{l} x_{12}=4,\ x_{13}=5,\ x_{21}=5,\ x_{22}=1 \\ \text{and, } x_{11} = x_{23} = x_{24} = 0 \end{array}}$$

$\therefore$ min "w" $= (4\times 4) + (5\times 2) + (5\times 3) + (1\times 4)$
$= 16 + 10 + 15 + 4$
$= 45$

(in thousands ~~cents~~ checks unit)
($i \to$ site
$j \to$ check type)

$\therefore$ In thousands terms,

$$\boxed{\text{minimum cost} = 45{,}000 \text{ cents}}$$