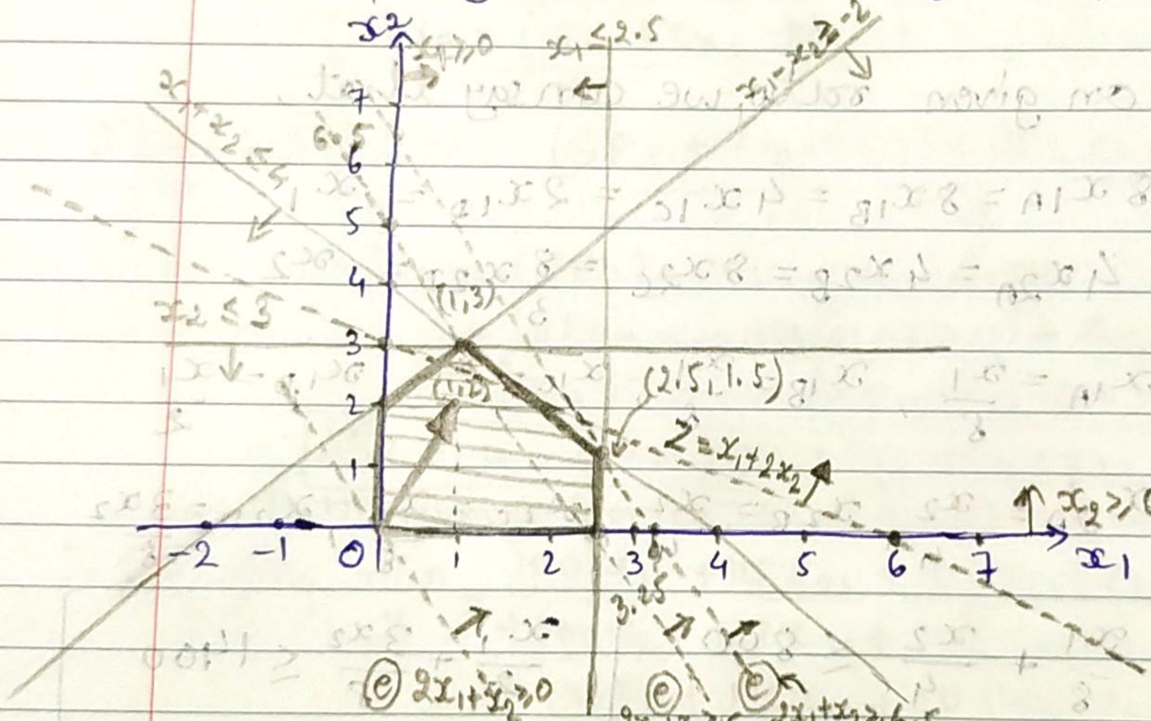
Q5

$$\max.\ Z = x_1 + 2x_2$$

s.t.

$$x_1 - x_2 \geq -2$$
$$x_1 + x_2 \leq 4$$
$$x_1 \leq 2.5$$
$$x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(a) Graphing the feasible region of LP:



→ As, we can say from the graph, the feasible region is bounded, as shaded with hatch lines.

(b) Yes, $x_2 \leq 3$ is a redundant constraint, as it does not change feasible region.

(c) Solving LP using the graphical method:

Obj. fun^n : $Z = x_1 + 2x_2$ (max^n)

here to plot Z in graph we take,

$$Z = 6 = x_1 + 2x_2$$

∴ we can plot it as shown in graph.

∴ gradient of $Z \Rightarrow \left(\frac{\partial Z}{\partial x_1}, \frac{\partial Z}{\partial x_2}\right) = (1, 2)$ ⇒ It gives dirⁿ of increasing Z value.

∴ we can say that, corner point (1, 3) will give us maxᵐ Z value from this feasible region points.

∴ $\boxed{\max_Z = x_1 + 2x_2 = 1 + 2(3) = 7}$

Note that we found corner point (1,3) by solving $x_1 - x_2 = (-2)$ and $x_1 + x_2 = 4$ for their intersection point.

note: If we check, Z value for all corner-points then also, we can cross-verify that only (1,3) point give us maximum Z value.

(d) No, there is only one optimal solution. The reason is as we can see from graph, optimal point is where constraints intersects and objective funⁿ touches that point.

→ Note that we could have a multiple optimal solⁿ only when objective fuⁿ is overlapping with one of the constraint, which is not the case here ∴ we have unique cornerpoint solⁿ here.

(e) Suppose we add the constraint $2x_1 + x_2 \geq \alpha$ to (LP).

(i) for $\alpha \leq 0$ constraint will be redundant.

∵ as seen in graph it will not change feasible region in that case.

(ii) for $\alpha > 5$ and $\alpha \leq 6.5$, optimal solⁿ found above as point (1,3) will no longer be optimal but some other point will become optimal solution, as for $5 < \alpha \leq 6.5$ will make $2x_1 + x_2 \geq \alpha$ constraint go past (1,3) corner point and (1,3) point will no longer be in the feasible region.

(iii) for $\alpha > 6.5$, there will be no feasible region and problem will become infeasible.

note: we found, 6.5 value from point of intersection of (2.5, 1.5) for α. ∵ which gives $2x_1 + x_2 = 5 + 1.5 = 6.5$.

∴ and we found, 5 value from point (1,3) : $2x_1 + x_2 = 2(1) + 3 = 5$.