

Q2

Use branch and bound to solve the following IP:

$$\text{maximize } z = 4x_1 + 3x_2$$

s.t.

$$5x_1 + 10x_2 \leq 21$$

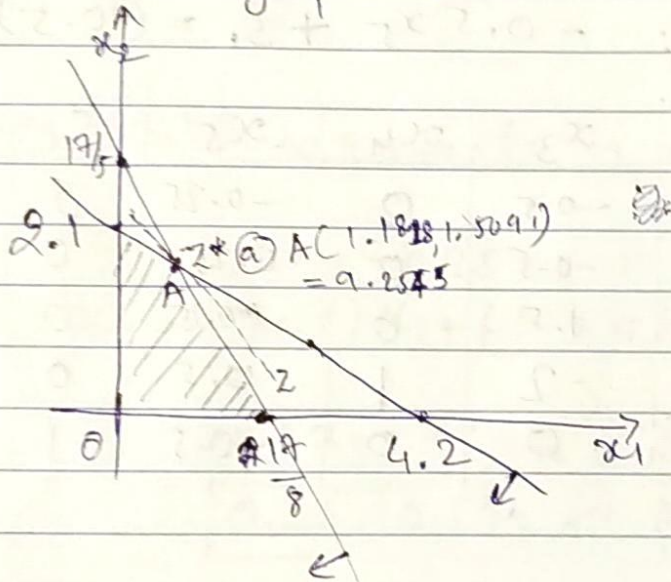
$$8x_1 + 5x_2 \leq 17$$

$$x_1, x_2 \geq 0;$$

$$x_1, x_2 \text{ integer}$$

Using LP relaxation, we can first find upperbound of IP problem.

we use graphical method for that:



$$\therefore z^* = 9.2545$$

$$x_1^* = 1.1818$$

$$x_2^* = 1.5091$$

(Also confirmed with MS Excel solver)

→ now, we will use branch and bound to update $LB = (-\infty)$ to some higher value which has all integer variables as solutions.

→ Using $x_1 \leq 1$, $x_2 \geq 2$ as first branch on node 0, we get following tree.

Note: we will use all subproblems using MS Excel solver

LP-relaxⁿ

$$\begin{aligned}x_1^* &= 1.1818 \\x_2^* &= 1.5091 \\z^* &= 9.2545 = z_{ub} \\z_{lb} &= (-\infty)\end{aligned}$$

$$x_1 \leq 1$$

$$x_1 \geq 2$$

Subprob.
(1)

$$\begin{aligned}x_1 &= 1 \\x_2 &= 1.6 \\z &= 8.8 \\z_{lb} &= (-\infty)\end{aligned}$$

Subprob.
(4)

$$\begin{aligned}x_1 &= 2 \\x_2 &= 0.2 \\z &= 8.6 \\z_{lb} &= 7\end{aligned}$$

$$x_2 \leq 1$$

$$x_2 \geq 2$$

$$x_2 \leq 0$$

$$x_2 \geq 1$$

Subprob. (2)

$$\begin{aligned}x_1 &= 1 \\x_2 &= 1 \\z &= 7 = z_{lb} \\(\text{Incumbent} = \text{Optimal Candidate})\end{aligned}$$

Subprob. (3)

$$\begin{aligned}x_1 &= 0.2 \\x_2 &= 2 \\z &= 6.8 \\z &< z_{lb} \\ \therefore \text{no need to go further in this branch} \\ \therefore \text{fathomed.}\end{aligned}$$

Subprob. (5)

$$\begin{aligned}x_1 &= 2.125 \\x_2 &= 0 \\z &= 8.5\end{aligned}$$

Subprob. (6)

Infeasible solution

$$x_1 \leq 2$$

$$x_1 \geq 3$$

Subprob. (7)

Subprob. (8)

$$\begin{aligned}x_1 &= 2 \\x_2 &= 0 \\z &= 8 \\z_{lb} &= 8\end{aligned}$$

Infeasible soln.

(New incumbent or IP optimal soln candidate)

Ans. So, our solution to given IP problem = latest incumbent = $z_{lb} = 8$
 $\therefore z^*(IP) = 8, x_1^* = 2, x_2^* = 0$

LP Relaxation	x1	x2	LHS		RHS
Variable values	1.181818	1.509091			
Obj. Fun Coeff	4	3	z	=	9.254545
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	17	<=	17

Sub-problem 1	x1	x2	LHS		RHS
Variable values	1	1.6			
Obj. Fun Coeff	4	3	z	=	8.8
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	16	<=	17
const3 Coeff	1	0	1	<=	1

Sub-problem 2	x1	x2	LHS		RHS
Variable values	1	1			
Obj. Fun Coeff	4	3	z	=	7
const1 Coeff	5	10	15	<=	21
const2 Coeff	8	5	13	<=	17
const3 Coeff	1	0	1	<=	1
const4 Coeff	0	1	1	<=	1

Sub-problem 3	x1	x2	LHS		RHS
Variable values	0.2	2			
Obj. Fun Coeff	4	3	z	=	6.8
const1 Coeff	5	10	21	<=	21
const2 Coeff	8	5	11.6	<=	17
const3 Coeff	1	0	0.2	<=	1
const4 Coeff	0	1	2	>=	2

Sub-problem 4	x1	x2	LHS		RHS
Variable values	2	0.2			
Obj. Fun Coeff	4	3	z	=	8.6
const1 Coeff	5	10	12	<=	21
const2 Coeff	8	5	17	<=	17
const3 Coeff	1	0	2	>=	2

Sub-problem 5	x1	x2	LHS		RHS
Variable values	2.125	0			
Obj. Fun Coeff	4	3	z	=	8.5
const1 Coeff	5	10	10.625	<=	21
const2 Coeff	8	5	17	<=	17
const3 Coeff	1	0	2.125	>=	2
const4 Coeff	0	1	0	<=	0

Sub-problem 6	x1	x2	LHS		RHS
Variable values	0	0			
Obj. Fun Coeff	4	3	z	=	0
const1 Coeff	5	10	0	<=	21
const2 Coeff	8	5	0	<=	17
const3 Coeff	1	0	0	>=	2
const4 Coeff	0	1	0	>=	1

Sub-problem 7	x1	x2	LHS		RHS
Variable values	2	0			
Obj. Fun Coeff	4	3	z	=	8
const1 Coeff	5	10	10	<=	21
const2 Coeff	8	5	16	<=	17
const3 Coeff	1	0	2	>=	2
const4 Coeff	0	1	0	<=	0
const5 Coeff	1	0	2	<=	2

Sub-problem 8	x1	x2	LHS		RHS
Variable values					
Obj. Fun Coeff	4	3	z	=	0
const1 Coeff	5	10	0	<=	21
const2 Coeff	8	5	0	<=	17
const3 Coeff	1	0	0	>=	2
const4 Coeff	0	1	0	<=	0
const5 Coeff	1	0	0	>=	3