

Q2] Pre-tax cost function,

$$C(q) = 100 + 10q - q^2 + \frac{1}{3}q^3$$

with a specific tax of $(10=t)$

$$C(q) = 100 + 10q - q^2 + \frac{q^3}{3} + (10 \times q)$$

$$= 100 + 20q - q^2 + \frac{q^3}{3}$$

for max^m profit,

$$MC = MR = P$$

$$\therefore 20 - 2q + q^2 = P$$

now, we can solve for a single, profit-maximizing q in terms of P by using roots of quadratic eqⁿ as

$$q = \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right),$$

$$\text{where, } q^2 - 2q + (20 - P) = 0$$

$$\text{and, } a = 1, b = (-2), c = (20 - P)$$

$$\therefore q = \frac{+2 \pm \sqrt{4 - 4(1)(20 - P)}}{2(1)}$$

$$= 1 \pm \sqrt{1 - 20 + P}$$

$$\boxed{q^* = 1 \pm \sqrt{P - 19}} \quad \text{--- (1)}$$

but, for max^m profit, at q^*

$$\frac{d(MC)}{dq} > 0 \quad \therefore -2 + 2q^* > 0$$

$$\therefore \boxed{q^* > 1} \quad \text{--- (2)}$$

based on eqⁿ (1) (2), we have to drop $q^* = 1 - \sqrt{P - 19}$ from our solⁿ.

$\Rightarrow \therefore \boxed{q^* = 1 + \sqrt{P - 19}}$ is our required answer.