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Homework 01

1. Calculating the Area and Approximation of an Oblate Spheroid

1.1 Introduction

In this assignment, the task is to develop a script that will calculate the area of an oblate spheroid with a given formula and calculate the approximation using another given formula. Then, using actual data of the radii of Earth, the area and the approximation is also calculated. The final results will be printed and used for further analysis and discussion, specifically regarding the difference in the area and the approximation.

1.2 Model Theory

The applicable equations for this particular problem are:

$$A(r_1, r_2) = 2\pi(r_1^2 + \frac{r_2^2}{sin(\gamma)} ln(\frac{cos(\gamma)}{1 - sin(\gamma)}))$$

$$A_2(r_1, r_2) = 4\pi(\frac{r_1 + r_2}{2})^2$$

$$\gamma = arccos(\frac{r_2}{r_1})$$

Where

- A = Area
- A_2 = Area Approximation
- r_1 = Equatorial Radius
- r_2 = Polar Radius
- γ = Geocentric Latitude

1.3 Methods and Pseudocode

1. Prompt user to enter equatorial (r_1) and polar (r_2) radi
    a. Error check to make sure the inputs are viable (requirements: $r_2 > r_1$, has to be a nonzero value)
2. Define formulas for Gamma, Area (A), and Area Approximation (A_2)
3. Calculate the Area and Area Approximation using the inputted radii
4. Display the results of each calculation up to 10 decimals to reveal the discrepancies between the Area and Area Approximation equations

1.4 Calculations and Results

With an example radius of 6378.137 (r_1) and 6356.752 (r_2) (Earth's radii), the resulting output from the script is:

```
Please enter a value for r1:
6378.137
Please enter a value for r2:
6356.752

For a radius of r1 = 6378.137
and for a radius of r2 = 6356.752
Area = 510065604.9
Approx = 509495321.6
```

With an example of radius of -1 and 5, the resulting output from the script is:

```
Please enter a value for r1:
-1
Please enter a value for r2:
5
Error: The radii must be greater than 0.
```

1.5 Discussions and Conclusions

There is a noticeable difference between the area and the approximation formula using the Earth's radii as an example. The amount of difference (500,000km) between the two answers further highlights the difference in accuracy as well. After running the calculations, it is clear that the approximation formula is not to be used instead of the oblate spheroid area formula.

2. Calculating the Perimeter of an Ellipse

2.1 Introduction

In this assignment, the task is to develop a script that will calculate the perimeter of an ellipse with 8 possible formulas, dependent on variables a and b. The script will also take into the calculations regarding the departure from "circle-hood" for the formulas that require it. Given a certain range of input values, the script will then be designed to print the results which will be used for comparing each formula to another and to discuss the discrepancies among each formula.

2.2 Model Theory

The applicable equations for this particular problem are

$P = 2\pi r$

$P_1 = \pi(a + b)$

$P_2 = \pi\sqrt{2(a^2 + b^2)}$

$$P_3 = \pi\sqrt{2(a^2 + b^2) - \frac{(a-b)^2}{2}}$$

$$P_4 = \pi(a + b)(1 + \frac{h}{8})^2$$

$$P_5 = \pi(a + b)(1 + \frac{3h}{10+\sqrt{4-3h}})$$

$$P_6 = \pi(a + b)\frac{64-3h^2}{64-16h}$$

$$P_7 = \pi(a + b)\frac{256-48h-21h^2}{256-112h+3h^2}$$

$$P_8 = \pi(a + b)(\frac{3-\sqrt{1-h}}{2})$$

$$h = (\frac{a-b}{a+b})^2$$

Where

- P = Perimeter
- $P_1 \ldots P_8$ = Perimeter Approximation
- h = Departure from “Circle-Hood”
- a = Length of Semi-Major Axis
- b = Length of Semi-Minor Axis

2.3 Methods and Pseudocode

1. Prompt user to enter the lengths of the semi-major axis (a) and semi-minor axis (b)
    a. Error check to make sure the inputs are viable (requirements: a and b are greater than 0)
2. Define formulas for Perimeter Approximation ($P_1 \ldots P_8$), Perimeter (P), and Departure from “Circle-Hood” (h)
3. Calculate the Perimeter Approximations and Departure from “Circle-Hood” (and Perimeter if applicable)
4. Display the results of each calculation to reveal the discrepancies between each of the perimeter approximation formulas (and Perimeter if applicable)

2.4 Calculations and Results

With the example a=1 and b=1, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 1.000000
h = 0.000000
P = 6.283185
P1 = 6.283185
P2 = 6.283185
P3 = 6.283185
P4 = 6.283185
P5 = 6.283185
P6 = 6.283185
P7 = 6.283185
P8 = 6.283185
```

With the example a=1 and b=0.9, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.900000
h = 0.002770
P1 = 5.969026
P2 = 5.977288
P3 = 5.973158
P4 = 5.973160
P5 = 5.973160
P6 = 5.973160
P7 = 5.973160
P8 = 5.973163
```

With the example a=1 and b=0.8, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.800000
h = 0.012346
P1 = 5.654867
P2 = 5.689666
P3 = 5.672293
P4 = 5.672334
P5 = 5.672334
P6 = 5.672334
P7 = 5.672334
P8 = 5.672374
```

With the example a=1 and b=0.7, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.700000
h = 0.031142
P1 = 5.340708
P2 = 5.423230
P3 = 5.382127
P4 = 5.382368
P5 = 5.382369
P6 = 5.382369
P7 = 5.382369
P8 = 5.382616
```

With the example a=1 and b=0.6, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.600000
h = 0.062500
P1 = 5.026548
P2 = 5.181247
P3 = 5.104484
P4 = 5.105395
P5 = 5.105400
```

```
P6 = 5.105400
P7 = 5.105400
P8 = 5.106355
```

With the example a=1 and b=0.5, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.500000
h = 0.111111
P1 = 4.712389
P2 = 4.967294
P3 = 4.841519
P4 = 4.844198
P5 = 4.844224
P6 = 4.844224
P7 = 4.844224
P8 = 4.847142
```

With the example a=1 and b=0.4, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.400000
h = 0.183673
P1 = 4.398230
P2 = 4.785131
P3 = 4.595754
P4 = 4.602508
P5 = 4.602622
P6 = 4.602619
P7 = 4.602622
P8 = 4.610427
```

With the example a=1 and b=0.3, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.300000
h = 0.289941
P1 = 4.084070
P2 = 4.638506
P3 = 4.370090
P4 = 4.385470
P5 = 4.385910
P6 = 4.385889
P7 = 4.385908
P8 = 4.405385
```

With the example a=1 and b=0.2, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.200000
h = 0.444444
P1 = 3.769911
```

```
P2 = 4.530869
P3 = 4.167794
P4 = 4.200426
P5 = 4.202005
P6 = 4.201880
P7 = 4.201993
P8 = 4.249904
```

With the example a=1 and b=0.1, the resulting output from the script is:

```
For an entered value of a = 1.000000
For an entered value of b = 0.100000
h = 0.669421
P1 = 3.455752
P2 = 4.465042
P3 = 3.992419
P4 = 4.058288
P5 = 4.063927
P6 = 4.063151
P7 = 4.063794
P8 = 4.190169
```

With the example a=-1 and b=0, the resulting output from the script is:

```
Please enter a value for a.
-1
Please enter a value for b.
0

a and/or b cannot be less than or equal to 0.
```

2.5 Discussions and Conclusions

As the ellipse becomes more oblong, the perimeter approximation formulas become more and more varied. This variation from one another indicates a decrease in the precision of these approximations, and thus, the validity of these approximations. When using these approximations, it is important to take into account their validity as they don't always hold for all ellipses.