Physics 2211 - Lab 4

Oscillations

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Outline

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Aim

Purpose of this lab assignment

- Analyze the motion of a mass oscillating under the effect of spring force and gravity
- Verify the energy principle for this system.

Outline

Newton's Second Law

Quantitative analysis of motion

"The net force acting on a body is defined as the change in its momentum per unit time."

$$\vec{F}_{net} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \tag{1}$$

where,

- $\vec{F}_{net}$ = Net force acting on a body
- m = Mass of the body
- $\Delta\vec{p} = m \times \Delta\vec{v}$ = Change in momentum

For this experiment, we turn this into the **update** form:

$$\vec{p_f} = \vec{p_i} + \vec{F}_{net} \times \Delta t$$

Dividing by m on both sides:

$$\vec{v_f} = \vec{v_i} + \frac{\vec{F}_{net}}{m} \cdot \Delta t \tag{2}$$

Hooke's Law

Measuring spring forces

"The magnitude of the force exerted by a stretched spring is directly proportional to its change in length."

$$\vec{F}_s = -k\,(L - L_0)\,\hat{L} \tag{3}$$

where,

- $\vec{F}_s =$ The vector force exerted by the spring.
- $L =$ The current length of the spring (scalar).
- $\hat{L} =$ The unit vector pointing from the fixed end of the spring to the free end.
- $L_0 =$ The natural length of the spring.

Energy Principle

"The change in energy of a system is equivalent to the external work done on it."

$$\Delta E = W_{ext} \tag{4}$$

where,

- $\Delta E =$ Change in energy of the system.
- $W_{ext} =$ The external work done by the surroundings.

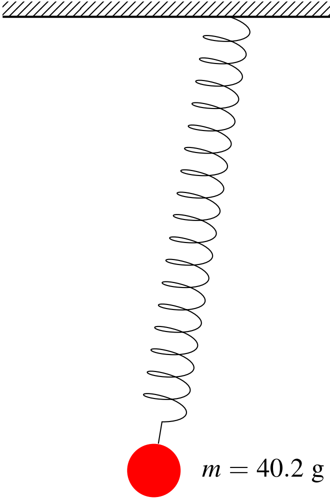
Initial Conditions

- Mass of ball, $m = 4.02 \times 10^{-1}$ m
- Initial position of ball,
  $\vec{r}_{ball} = \left\langle -1.20 \times 10^{-1}, -6.33 \times 10^{-1}, 0 \right\rangle$ m
- Initial velocity of ball, $\vec{v}_{ball} = \langle 0, 0, 0 \rangle$ ms^{-1}
- Stiffness of spring, $k = 6.83 \times 10^{0}$ Nm
- Relaxed length of spring, $L_0 = 1.23 \times 10^{-1}$ m

System and Surroundings

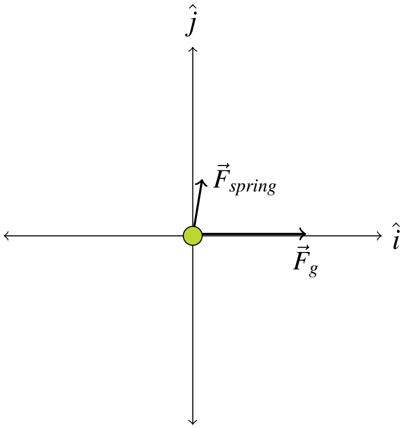
- **System:** Ball + Spring + Earth
- **Surroundings:** Everything else

Diagram



Free-Body Diagram of Spring

Just after it's released



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Formulae

- $\vec{F}_{grav} = \langle 0, -m \cdot g, 0 \rangle$
- $\vec{L} = \vec{r}_{ball} - \langle 0, 0, 0 \rangle$
- $s = |\vec{L}| - L_0$
- $\vec{F}_{spring} = -k_s s \hat{L}$
- $\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{grav}$
- Velocity and position update

Formulae

Continued

- $K = \frac{1}{2} m_{ball} |\vec{v}_{ball}|^2$
- $U_g = m_{ball} g \left(\vec{r}_{ball} \bullet \hat{j}\right)$
- $U_s = \frac{1}{2} k_s s^2$

Simulation code

Conditions alone

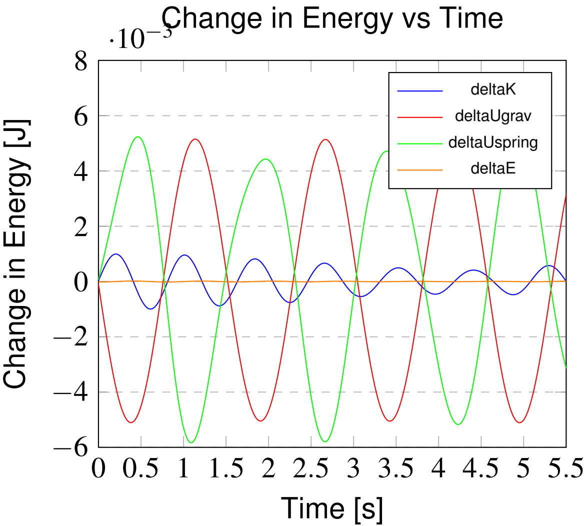
```
# System mass
ball.m = .402

deltat = 1/210  #choose this

ball.pos = vector(X[0],Y[0],0)
ball.vel = vector(0, 0, 0)

# Spring constant
k_s = 6.83

# Relaxed length of spring
L0 = .123
L = ball.pos - spring.pos
Lhat = L/mag(L)
s = mag(L) - L0

# compute the system energies
K = 0.5 * ball.m * mag(ball.vel) ** 2   # kinetic energy
Ug = ball.m * g * ball.pos.y # gravitational potential energy
Us = 0.5 * k_s * s**2  # spring potential energy
E = K + Us + Ug   # total energy

# Calculate gravitational force
Fgrav = vector(0,-ball.m * g, 0)

# Calculate spring force on mass by spring
Fspring = -k_s * s *Lhat
```

Simulation code

Continued

```
# Calculate the net force
Fnet = Fspring + Fgrav

# Apply the Momentum Principle
ball.vel = ball.vel + Fnet / ball.m * deltat
ball.pos = ball.pos + ball.vel * deltat

# Update the spring
L = ball.pos - spring.pos
Lhat = hat(L)
s = mag(L) - L0

spring.axis = L
trail.append(pos=ball.pos)

# Calculate energy changes
K = 0.5 * ball.m * mag(ball.vel) **2
deltaK = K - K_i;print(deltaK)
Ug = ball.m * g * ball.pos.y
deltaUg = Ug - Ug_i
Us = 0.5 * k_s * s**2
deltaUs = Us - Us_i
E = K + Ug + Us
deltaE = E - E_i
```

Energy graphs



Outline

What does it mean?

Validity of Energy Principle

- **Q**: For your model system, is the energy principle satisfied? Justify your answer by discussing briefly your plots of energy changes.
- $W_{ext} = 0 \implies \Delta E = 0$, hence valid

What does it mean?

Oscillation Period

- **Q**: Using the data you obtained in Tracker, make two separate estimates of oscillation periods: first, by estimating the period of oscillation from the x position data and second, by estimating the period of oscillation from the y position data. Compare the two estimates and discuss.
- *Time period is the difference between two successive crests/trough in the x-t graph* - stems from the definition “the time taken for one complete to-and-fro oscillation.
- $T_x = 1.425$ s
- $T_y = 1.330$ s
- Difference because there are two different SHM's in two directions - x and y.
- Other causes could be drag and tracker inaccuracy.