RoboMechanics Lab Semester Report

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Summary: Over the past semester, my research mainly revolved around modelling and simulating the direct drive gripper hand. The hand itself was composed of two 5-bar linkages which came together to create a gripper. The work was both in creating a valid workspace for the linkages and modeling a possible spring-dampener system for the fingers of the gripper. The codebase can be found **here**.

1. Forward Kinematics

We use a model of a pantograph, which is originally a 5-bar linkage, but we set the a_5 to be of length zero. To find the end effector position, we consider the positions and angles of the two other joints as well. [1]

$$P_2 = [a_1 cos(\theta_1), a_1 sin(\theta_1)] \tag{1}$$

$$P_4 = [a_4 cos(\theta_5) - a_5, a_4 sin(\theta_5)] \tag{2}$$

Now, we can find quantities for the end effector.

$$\|P_2 - P_h\| = \frac{a_2^2 - a_3^2 + \|P_4 - P_2\|^2}{2\|P_4 - P_2\|} \tag{3}$$

$$P_h = P_2 + \frac{\|P_2 - P_h\|}{\|P_2 - P_4\|}(P_4 - P - 2) \tag{4}$$

$$\|P_3 - P_h\| = \sqrt{a_2^2 - \|P_2 - P_h\|^2} \tag{5}$$

Finally, the end effector position is given by:

$$x_3 = x_h + \frac{\|P_3 - P_h\|}{\|P_2 - P_4\|}(y_4 - y_2) \tag{6}$$

$$y_3 = y_h - \frac{\|P_3 - P_h\|}{\|P_2 - P_4\|}(x_4 - x_2) \tag{7}$$

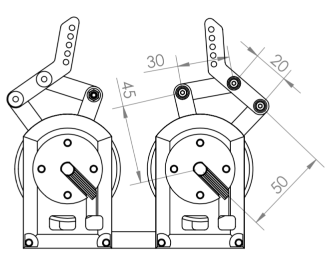

Fig. 1: DD Hand Linkage Diagram

1.1. Workspace and Singularities

The Jacobian was then found using Matlab's Symbolic Toolkit. The Jacobian was used to identify points of singularity, as we know when $det(J) = 0$, the system is singular. The green points are valid workspace coordinates, the red points are singularities. The main singularity occurs in this linkage when the two higher links are parallel, causing the arm to go into a configuration that looks like a triangle. The specifics of the plotting are covered in DD_hand _workspace.m.

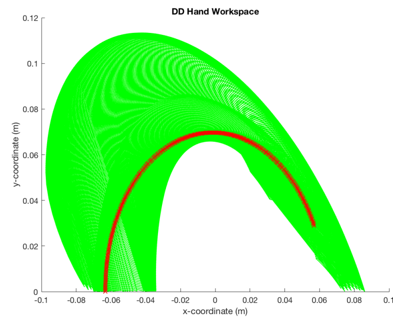

Fig. 2: 5-Bar Linkage Workspace and Singularities

2. Dynamics and Force Controller

The original goal was to get varying response in the x and y axes. This way, if the arm were to hit something in the y-axis, it could respond differently (have less give) than if it were resisting in the x-axis. To do this, I first began by understanding how dynamics related to a simple PID controller. Since the kinematics of a spring-dampener system are essentially a PID controller loop, it was easy to get a position controller up and running. (in PID_test.m)

However, to consider the dynamics of, for example, of a two-link robot arm, we must set up a state space model of the arm, then use functions such as $ODE45$ within Matlab to solve for the next state. The dynamics equations are as follows (m_x is the mass assumed to be at the center of mass of a joint x, l_x is the length of joint x):

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{8}$$

$$\alpha = I_1 + I_2 + m_1 r_1^2 + m_2(l_1^2 + r_2^2) \tag{9}$$

$$\beta = m_2 l_1 r_2 \tag{10}$$

$$\delta = I_2 + m_2 r_2^2 \tag{11}$$

$$M = \begin{bmatrix} \alpha + 2\beta\cos\theta_2 & \delta + \beta\cos\theta_2 \\ \delta + \beta\cos\theta_2 & \delta \end{bmatrix} \tag{12}$$

$$C = \begin{bmatrix} -\beta\sin\theta_2\dot{\theta}_2 & -\beta\sin\theta_2(\dot{\theta}_1 + \dot{\theta}_2) \\ \beta\sin\theta_2\dot{\theta}_1 & 0 \end{bmatrix} \tag{13}$$

Setting these equal to the generalized forces (torque), we get:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = M\ddot{q} + C\dot{q} + G \tag{14}$$

$$\tag{15}$$

The second order ODE we solve is then:

$$\ddot{q} = M^{-1}(-C\dot{q} - G + \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}) \tag{16}$$

A Force controller was implemented in Matlab, solving for each timestep using $ODE45$, then torques were commanded in order to return to a goal position. The Jacobian was used to translate the force to a torque: $\tau = J^T F$. This torque was fed back into the state-space system to get the new torques to be commanded. The "spring" constants can be adjusted by changing the update forces within the control loop.

References

[1] Campion, Gianni Wang, Qi Hayward, Vincent. (2005). The Pantograph Mk-II: A haptic instrument. 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS. 193 - 198. 10.1109/IROS.2005.1545066.