HOMEWORK 3

1.) Decision trees:

Multiway - split

Compare degree of impurity of parent node with children nodes

$$Gini(t) = 1 - \sum_{i=0}^{c-1} [p(i/t)]^2$$

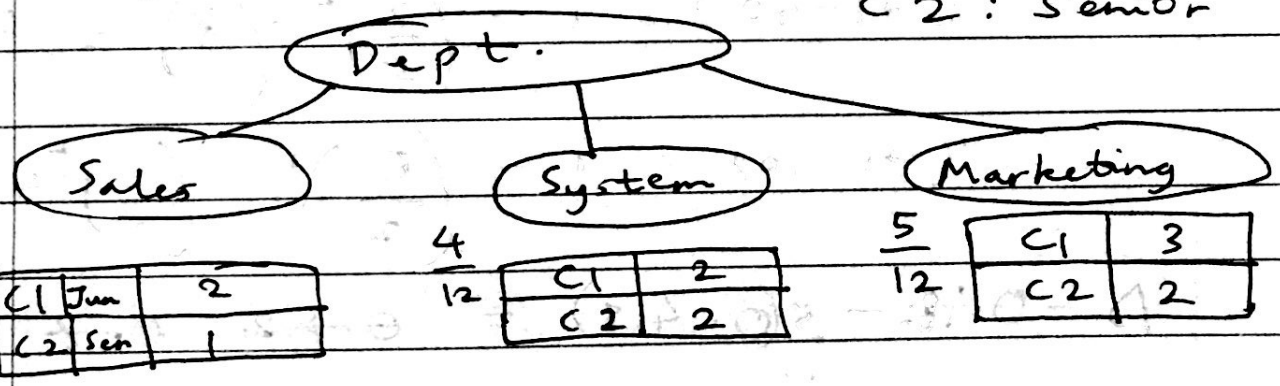
$$\Delta = I(parent) - \sum_{j=1}^{k} \frac{N(v_j)}{N} I(v_j)$$

Choose case with higher Δ gain

a.) Split with Dept:

C1: Junior

C2: Senior



Gini index sales $= 1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = 0.444$

system $= 1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2 = 0.5$

marketing $= 1 - \left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2 = 0.48$

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Gini Parent $= \frac{12}{12}$

C1	7
C2	5

$= 1 - \left(\frac{7}{12}\right)^2 - \left(\frac{5}{12}\right)^2$

$= 0.486$

$\Delta = 0.486 - \frac{3}{12}(0.444) - \frac{4}{12}(0.5) - \frac{5}{12}(0.48)$

$= \cancel{0.523}\ 0.009$

Split with Age:

Age

21-30

$\frac{5}{12}$

C1	5
C2	0

31-40

$\frac{5}{12}$

C1	2
C2	3

41-50

$\frac{2}{12}$

C1	0
C2	2

Gini $(21-30) = 1 - \left(\frac{5}{5}\right)^2 - \left(\frac{0}{5}\right)^2 = 0$

$(31-40) = 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = 0.48$

$(41-30) = 1 - \left(\frac{0}{2}\right)^2 - \left(\frac{2}{2}\right)^2 = 0$

$\Delta = 0.486 - \frac{5}{12}(0.48) = \cancel{0.52}\ \cancel{0.8}$

$= 0.286$

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Split with Salary.

Salary

Low		Medium		High	
$\frac{3}{12}$	C1	3	$\frac{6}{12}$	C1	4	$\frac{3}{12}$	C1	0
	C2	0		C2	2		C2	2

$$\text{Gini (low)} = 1 - \left(\frac{3}{3}\right)^2 - \left(\frac{0}{3}\right)^2 = 0$$

$$\text{(Medium)} = 1 - \left(\frac{4}{6}\right)^2 - \left(\frac{2}{6}\right)^2 = 0.444$$

$$\text{(High)} = 1 - \left(\frac{0}{2}\right)^2 - \left(\frac{2}{2}\right)^2 = 0$$

$$\Delta = 0.486 - \frac{6}{12}(0.444) = \cancel{0.277}\ 0.264$$

Highest Gain with Split w.r.t. Age So we choose that.

Now for every node we try and split w.r.t. Dept and salary.

Dept.

21-30

Sales		Systems		Marketing	
$\frac{1}{5}$	C1	1	$\frac{2}{5}$	C1	2	$\frac{2}{5}$	C1	2
	C2	0		C2	0		C2	0

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Salary

21-30

Low		Medium		High	
C1	2	C1	3	C1	0
C2	0	C2	0	C2	0
$\frac{2}{5}$		$\frac{3}{5}$		0	

Gini (Sales) = 1 −

We can see here that Gini gain Δ will be same in both cases.

Also since we know all in age group 21-30 are Juniors.

Same is case with 41-50 all Seniors. So we can stop further iterations for them.

For 31-40

Dept

31-40

Sales		Systems		Marketing	
C1	1	C1	0	C1	1
C2	1	C2	1	C2	1

Gini (sales) $= 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0.5$

(Systems) $= 1 - \left(\frac{1}{1}\right)^2 = 0$

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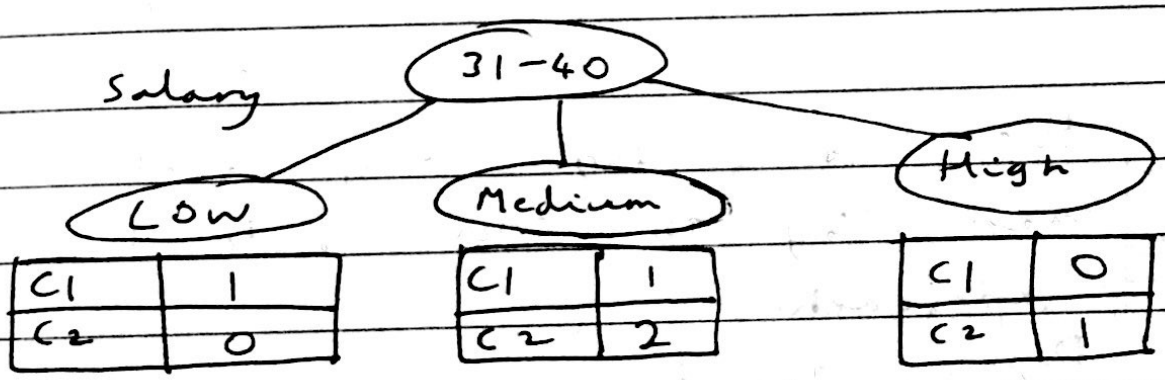
$$(\text{Marketing}) = 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0.5$$

$$\text{Parent Gini} = 1 - \left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = 0.48$$

$$\Delta = 0.48 - \frac{2}{5}(0.5) - \frac{2}{5}(0.5)$$

$$= 0.08$$



$$\text{Gini (low)} = 0$$

$$(\text{Medium}) = 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2$$

$$= 0.444$$

$$(\text{High}) = 0$$

$$\Delta = 0.48 - \frac{3}{5}(0.444)$$

$$= 0.213$$

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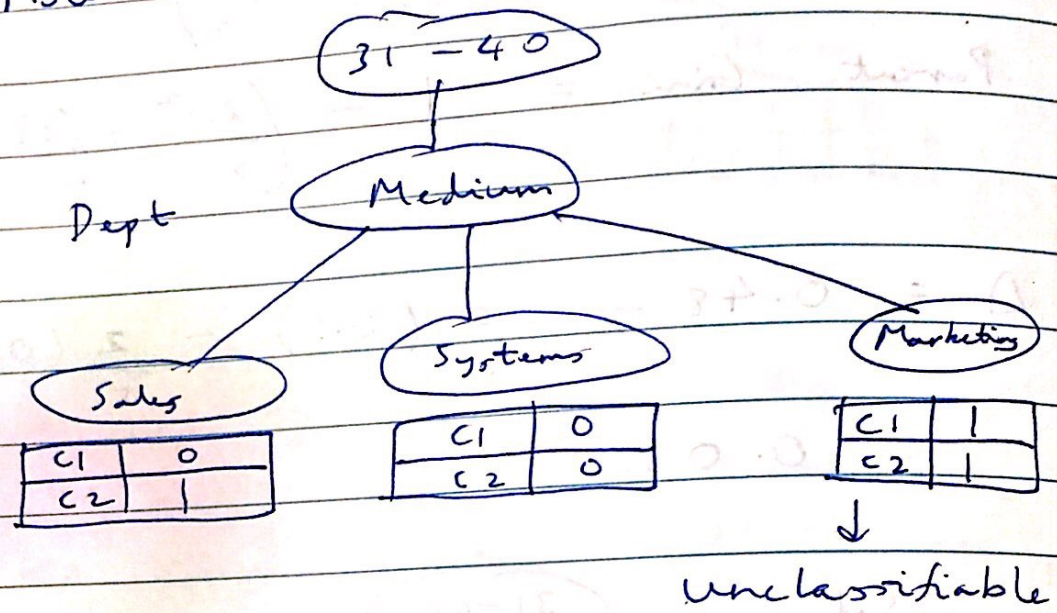
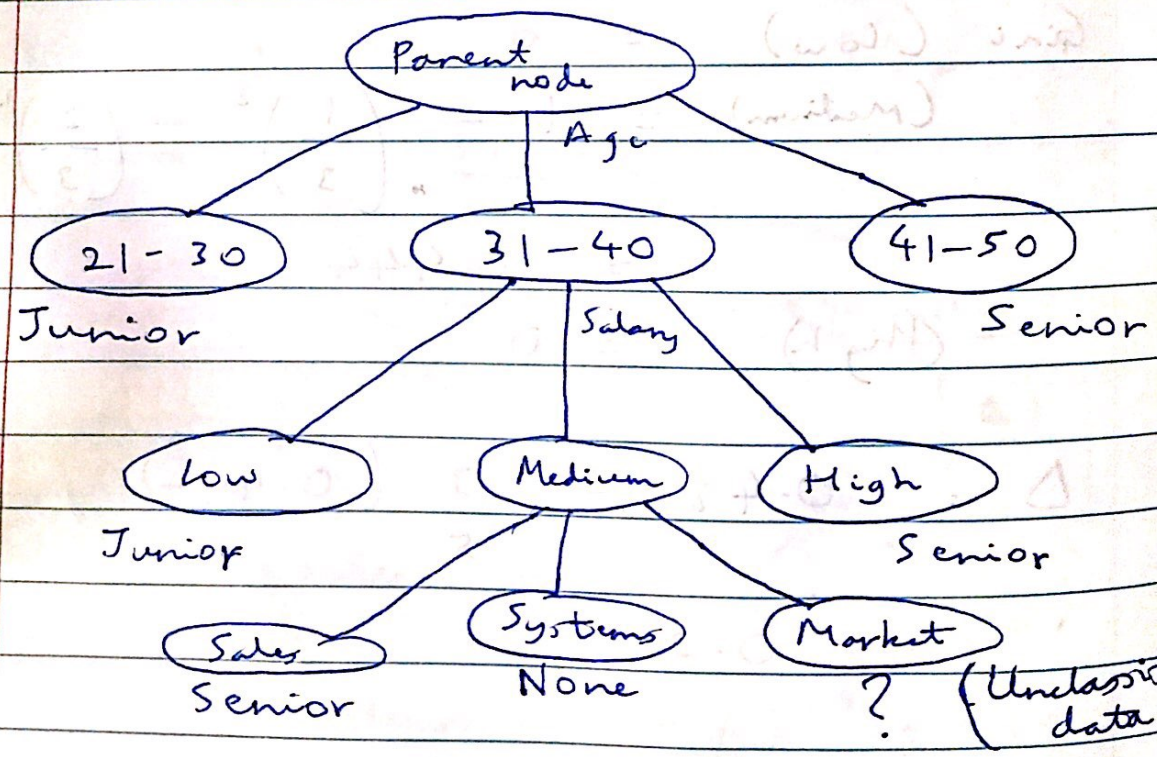
Now we come to



We can assign any label here according to some rule-based approach.

Decision tree:



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b.) Assuming unclassified as error

i.) Individual class accuracy :

Junior $= \frac{6}{7}$

Senior $= \frac{4}{5}$

ii.) Overall accuracy $= \frac{10}{12}$

c.)

Sales	21 - 30	High	Junior
Systems	21 - 30	Medium	Junior
Marketing	41 - 50	High	Senior
Marketing	31 - 40	Low	Junior

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2.) Naïve Bayes Classification :

a.) Assumption:

Attributes are conditionally independent given the class label y.

$$P(X \mid Y=y) = \prod_{i=1}^{d} \left(P(X_i \mid Y=y) \right)$$

Attribute set $X = \{X_1, X_2 \ldots\ldots X_d\}$

b.) $$P(Y \mid X) = \frac{P(Y) \prod_{i=1}^{d} P(X_i \mid Y)}{P(X)}$$

$$P(A=0 \mid +) = \frac{P(A=0) \cdot P(+ \mid A=0)}{P(+)}$$

$$= \frac{1/2 \cdot 2/5}{1/2}$$

$$= \frac{2}{5}$$

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$$P(A=0 \mid -) = \frac{P(A=0) \cdot P(- \mid A=0)}{P(-)}$$

$$= \frac{1/2 \cdot 3/5}{1/2}$$

$$= \frac{3}{5}$$

$$P(A=1 \mid +) = \frac{P(A=1) \cdot P(+ \mid A=1)}{P(+)}$$

$$= \frac{1/2 \cdot 3/5}{1/2}$$

$$= \frac{3}{5}$$

$$P(A=1 \mid -) = \frac{P(A=1) \cdot P(- \mid A=1)}{P(-)}$$

$$= \frac{1/2 \cdot 2/5}{1/2}$$

$$= \frac{2}{5}$$

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$$P(B=0|+) = \frac{P(B=0) \cdot P(+|B=0)}{P(+)}$$

$$= \frac{7/10 \cdot 4/7}{1/2}$$

$$= \frac{4}{5}$$

$$P(B=0|-) = \frac{7/10 \cdot 3/7}{1/2}$$

$$= \frac{3}{5}$$

$$P(B=1|+) = \frac{3/10 \cdot 1/3}{1/2}$$

$$= \frac{1}{5}$$

$$P(B=1|-) = \frac{3/10 \cdot 2/3}{1/2}$$

$$= \frac{2}{5}$$

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$$P(C=0|+) = \frac{1/10 \cdot 1}{1/2}$$

$$= \frac{1}{5}$$

$$P(C=0|-) = \frac{1/10 \cdot 0}{1/2} = 0$$

$$P(C=1|+) = \frac{9/10 \cdot 4/9}{1/2} = \frac{4}{5}$$

$$P(C=1|-) = \frac{9/10 \cdot 5/9}{1/2} = 1$$

$P(A=0|+) = 2/5$

$P(A=1|+) = 3/5$

$P(B=0|+) = 4/5$

$P(B=1|+) = 1/5$

$P(C=0|+) = 1/5$

$P(C=1|+) = 4/5$

$P(A=0|-) = 3/5$

$P(A=1|-) = 2/5$

$P(B=0|-) = 3/5$

$P(B=1|-) = 2/5$

$P(C=0|-) = 0$

$P(C=1|-) = 1$

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c.) $A = 0,\ B = 1,\ C = 0$

Method - I :

Let's check for $-$ & $+$

Considering the occurrence of this instance

If negative $-$; $P(-)$: A, B, C.

All are independent so.

$$P(-|A=0) \cdot P(-|B=1) \cdot P(-|C=0)$$

$$= \frac{P(-) \cdot P(A=0|-)}{P(A=0)} \cdot \frac{P(-) \cdot P(B=1|-)}{P(B=1)} \cdot \frac{P(-) \cdot P(C=0)}{P(C=0)}$$

$$= \frac{1/2 \cdot 3/5}{1/2} \cdot \frac{1/2 \cdot 2/5}{3/10} \cdot \frac{1/2 \cdot 0}{1/10}$$

$$= \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{0}{9} = \frac{0}{9} = 0$$

If $+$; $P(+)$

$$P(+|A=0) \cdot P(+|B=1) \cdot P(+|C=1)$$

$$= \frac{2}{5} \cdot \frac{1}{3} \cdot \frac{4}{5} = \frac{8}{25}$$

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Since $P(+) > P(-)$

we can estimate that class label is "+".

d.) m- estimate approach:

$p = 0.5, \; m = 4$

$$p(x_i | y_j) = \frac{n_c + mp}{n + m}$$

$n \rightarrow$ total instances from class y_j

$n_c \rightarrow$ training examples from y_j that take on value x_i.

$m \rightarrow$ equivalent sample size.

$p \rightarrow$ user specified.

$$P(A=0|+) = \frac{2+2}{5+4} = \frac{4}{9}$$

$$P(A=0|-) = \frac{3+2}{5+4} = \frac{5}{9}$$

$$P(A=1|+) = \frac{3+2}{5+4} = \frac{5}{9}$$

$$P(A=1|-) = \frac{2+2}{5+4} = \frac{4}{9}$$

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$$P(B=0|+) = \frac{4+2}{5+4} = \frac{2}{3}$$

$$P(B=0|-) = \frac{3+2}{5+4} = \frac{5}{9}$$

$$P(B=1|+) = \frac{1+2}{5+4} = \frac{1}{3}$$

$$P(B=1|-) = \frac{2+2}{5+4} = \frac{4}{9}$$

$$P(C=0|+) = \frac{1+2}{5+4} = \frac{1}{3}$$

$$P(C=0|-) = \frac{0+2}{5+4} = \frac{2}{9}$$

$$P(C=1|+) = \frac{4+2}{5+4} = \frac{2}{3}$$

$$P(C=1|-) = \frac{5+2}{5+4} = \frac{7}{9}$$

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e.) $A = 0, B = 1, C = 0$

Method – II

We can choose to ignore the denominator $P(x)$ as it is not part of the condition, remaining constant.

If we assume $P(A = 0, B = 1, C = 0) = p$

then.

$$P(+ \mid A=0, B=1, C=0) = P(A=0, B=1, C=0 \mid +) \times P(+)/p$$

$$= P(A=0 \mid +) \times P(B=1 \mid +) \times P(C=0 \mid +) \times P(+)/p$$

$$= \frac{4/9 \times 1/3 \times 1/3 \times 1/2}{p} = 0.0247/p$$

$$P(- \mid A=0, B=1, C=0) = P(A=0 \mid -) \times P(B=1 \mid -) \times P(C=0 \mid -) \times P(-)/p$$

$$= 5/9 \times \frac{4}{9} \times \frac{2}{9} \times 1/2 \,/p = 0.0274/p$$

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$P(-) > P(+)$

$\therefore$ Class label is '−'.

f.) m-estimate probability approach is better as the entire posterior probability of an instance becomes zero if one of the conditional probabilities is zero, in case of basic approach, which may lead to misclassification. As seen in the example above.

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3) Holt's 1-Rule method

Rule for value 0 in A

Most frequent class c is '−'

Clause: If A = 0 class is '−'

Classification accuracy $\frac{4}{5}$

Rule for value 1 in A

Most frequent class c is '+'

Clause: If A = 1 class is '+'.

Classification accuracy $\frac{3}{5}$

Rule for value 0 in B

Most frequent class c is '−'

Clause: If B = 0 class is '−'

Classification accuracy = $\frac{4}{7}$

Rule for value 1 in B

Most frequent clause c is '−'

Clause: If B = 1 class is '−'

Classification accuracy = $\frac{2}{3}$

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Rule for $C = 0$
Most frequent class c is '–'
Clause: If $C = 0$ class is '–'
Classification accuracy $= \frac{3}{5}$

Rule for value 1 in C
Most frequent class c is '–'
Clause: If $C = 1$ class is '–'
Classification accuracy $= \frac{3}{5}$

i.) Rule for attribute 'A' is
If $A = 0$ class is '–', otherwise class is '+' ($A = 1$)

ii.) Rule for attribute 'B' is
If $B = 1$ class is '–' otherwise class is '+' ($B = 0$)

iii.) Rule for attribute 'C' is
If $C = 0$ class is '–' otherwise class is '+' ($C = 1$) [Actually undecided as accuracy is same]

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b.) 'A' is the best attribute.
as it has highest accuracy,
total error is minimum.

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