Tutorial 6 Solutions

Problem 4:-

$n_1 = 1.46$

$\Delta = 0.01$

Basic parameters (n_1, n_2, Δ, θ_c, θ_{imax} & NA)

* $\Delta = \frac{n_1 - n_2}{n_1} = 0.01 \longrightarrow 1.46 - n_2 = 0.01 \times 1.46 \Longrightarrow n_2 = 1.4454$

* $\theta_c = \sin^{-1} \frac{n_2}{n_1} = 81.89^\circ$

* $\theta_{imax} = \sin^{-1}(n_1\sqrt{2\Delta}) = 11.92^\circ$

* $NA = n_a \sin\theta_{imax} = n_1\sqrt{2\Delta} = 0.206$

Problem 5:-

$n_1 = 1.448$

$NA = 0.21$

$\Delta = ??$

$n_2 = ??$

$NA = 0.21 = n_1\sqrt{2\Delta} \Longrightarrow \Delta = \frac{(NA)^2}{2n_1^2} = \frac{(0.21)^2}{2(1.448)^2} = 0.0105$

$\Delta = \frac{n_1 - n_2}{n_1} \longrightarrow 1.448 - n_2 = 0.0105 \times 1.448$

$\longrightarrow n_2 = 1.433$

Problem 8 :-

$n_1 = 1.446$

$n_2 = 1.44$

$2a = 8\,\mu m$

→ Condition on λ for Single Mode Propagation

For Single mode propagation $\Rightarrow V \leqslant 2.405$

$$\frac{2\pi a NA}{\lambda} \leqslant 2.405$$

$$\therefore \lambda \geqslant \frac{2\pi a NA}{2.405}$$

$$NA = n_1\sqrt{2\Delta} = 1.446\sqrt{2\,\frac{1.446-1.44}{1.446}} = 0.1317$$

$$\therefore \lambda \geqslant \frac{2\pi\, 4\times10^{-6}\times 0.1317}{2.405} \geqslant 1.377\,\mu m$$

Problem 9 :-

SI

$a = 5\,\mu m$

$n_1 = 1.46$

$\Delta = 0.01$

$NA = ??$

$f_{o\,max} = ??$ for single mode propagation.

* $NA = n_1\sqrt{2\Delta} = 1.46\sqrt{2\times0.01} = 0.206$

$\lambda = \frac{c}{f}$

* For single mode prop. & SI $\Rightarrow V \leqslant 2.405$

$$\frac{2\pi a NA f_{o\,max}}{c} = 2.405$$

$$\therefore f_{o\,max} = \frac{2.405\times3\times10^{8}}{2\pi\times5\times10^{-6}\times0.206} = 1.115\times10^{14}\,Hz$$

Problem 10:-

GI

$n_1 = 1.5$

$\alpha = 1.9$

$\Delta = 1.3\%$

$2a = 40\,\mu m$

$M_{GI} = ??$

$\lambda = 1.55\,\mu m$

$V_c = ??$ for Single mode propogation

$$M_{GI} = \frac{V^2}{2}\left(\frac{\alpha}{\alpha+2}\right)$$

$$V = \frac{2\pi a NA}{\lambda} = \frac{2\pi a n_1\sqrt{2\Delta}}{\lambda} = \frac{2\pi\, 20\times10^{-6}\times 1.5\sqrt{2\times0.013}}{1.55\times10^{-6}} = 19.6$$

$$\therefore M_{GI} = \frac{(19.6)^2}{2}\left(\frac{1.9}{1.9+2}\right) = 93.7 = 93 \text{ modes}$$

For Single Mode & GI $\Rightarrow$ $V \leq 2.405\sqrt{1+\frac{2}{\alpha}}$

$$\therefore V_c = 2.405\sqrt{1+\frac{2}{1.9}}$$

$$= 3.4456$$

Problem 11:-

SI

$2a = 80 \mu m$

$\Delta = 1.5\%$

$\lambda = 0.85 \mu m$

$n_1 = 1.48$

$V = ??$

$M_{SI} = ??$

$2a_{max}$ for single mode prop. = ??

" " " " " for $\Delta = 0.15\% = ??$

$$V = \frac{2\pi a NA}{\lambda} = \frac{2\pi 40 \times 10^{-6} \times 1.48\sqrt{2 \times 0.015}}{0.85 \times 10^{-6}} = 75.8$$

$$M_{SI} = \frac{V^2}{2} = 2872 \text{ modes}$$

For Single mode prop. & SI $\Rightarrow$ $V \leqslant 2.405$

$$\frac{2a_{max} \pi NA}{\lambda} = 2.405$$

$$\therefore 2a_{max} = 2.538 \mu m$$

For $\Delta = 0.15\%$ $\Rightarrow$ $2a_{max} = 8.03 \mu m$

Problem 12:-

GI $\alpha = 2$

$2a = 50 \mu m$

$NA = 0.2$

$\lambda = 1 \mu m$

$M_{GI} = ??$

$$M_{GI} = \frac{V^2}{2}\left(\frac{\alpha}{\alpha+2}\right) = \frac{V^2}{4}$$

$$V = \frac{2\pi a NA}{\lambda} = \frac{\pi \times 50 \times 10^{-6} \times 0.2}{1 \times 10^{-6}} = 31.42 \Rightarrow M_{GI} = 246 \text{ modes}$$

Problem 13:-

LED $\rightarrow r_s = 20\,\mu m$ & $P_s = 0.2\,mW$

$P_c = ??$

SI 50 / 125 fiber $\rightarrow$ Diameters in μm

core diameter (2a) — cladding diameter (not used)

$NA = 0.25$

Repeat for Single mode 5/125 with $N = 0.15$

* for $2a = 50\,\mu m$:

$$\frac{P_c}{P_s} = (NA)^2 \min\left[1, \left(\frac{a}{r_s}\right)^2\right]$$

$$P_c = P_s (NA)^2 \min\left[1, \left(\frac{a}{r_s}\right)^2\right]$$ $a > r_s \Rightarrow 1$ is the min

$$\therefore P_c = 0.2 \times 10^{-3} (0.25)^2 = 12.5\,\mu W$$

* for $2a = 5\,\mu m$:

$$P_c = P_s (NA)^2 \min\left[1, \left(\frac{a}{r_s}\right)^2\right]$$ $a < r_s \rightarrow \left(\frac{a}{r_s}\right)^2$ is the min

$$\therefore P_c = 0.2 \times 10^{-3} (0.15)^2 \left(\frac{2.5}{20}\right)^2 = 0.07\,\mu W$$

Problem 14:-

LED $\rightarrow r_s = 25\,\mu m$

$2a = 5\,\mu m$ SM

$NA = 0.15$

$\eta = ??$

$$\eta_{SI} = (NA)^2 \min\left[1, \left(\frac{a}{r_s}\right)^2\right]$$ $a < r_s \Rightarrow \left(\frac{a}{r_s}\right)^2$ is the min.

$$\therefore \eta_{SI} = (0.15)^2 \left(\frac{2.5}{25}\right)^2 = 2.25 \times 10^{-4}$$

Problem 15 :-

Laser $\rightarrow P_s = 0.2\ mW$

$\eta = 50\%$

$P_c = ??$

$\because\ \eta = \frac{P_c}{P_s} = 50\% \Rightarrow P_c = \frac{1}{2} P_s = 0.1\ mW$

Problem 16 :-

$P_{in} = 1.5\ mW \longrightarrow$

$\longrightarrow P_{out_{min}} = 2\ \mu W$

$\alpha_{dB} = 0.5\ dB/km$

$L = ??$

$$\alpha_{dB/km} L = 10 \log \frac{P_{in}}{P_{out}}$$

$$L_{max} = \frac{10}{\alpha_{dB/km}} \log \frac{P_{in}}{P_{out\ min}}$$

$$l_{max} = \frac{10}{0.5} \log \frac{1.5 \times 10^{-3}}{2 \times 10^{-6}}$$

$$l_{max} = 57.5\ km$$