Boolean Algebra

$x + 0 = x$ $\qquad$ $x * 0 = 0$

$x + 1 = 1$ $\qquad$ $x * 1 = x$

$x + x = x$ $\qquad$ $x * x = x$

$x + x' = 1$ $\qquad$ $x * x' = 0$

$(x')' = x$

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* Distributivity:

$(x+y) \cdot (z+w) = xz + xw + yz + yw$

$x + yz = (x+y)(x+z)$

* DeMorgan's Law:

$(x+y)' = x' \cdot y'$

$(x' \cdot y)' = x + y'$

$(x' + y)' = x \cdot y'$

Ex1: "Common Factor" (1)

$f = x + x.y$ ⟵ 2 logic Gates

$= x.(1+y)$

$= x.1$

$f = x$ ⟵ 0 logic Gates

Ex2: "Distributivity" (2)

□ + □'○

$x + x'y$

$y + y'z$

$f = x + x'y$

$= (x+x').(x+y)$

$= 1.(x+y)$

$f = x + y$

Ex3: "Common Factor" + "Distributivity" (3)

$f = xy + xy'z$

$= x(y + y'z)$

$= x(y+y')(y+z)$

$= x.1.(y+z)$

$f = x(y+z)$

$F = ABC + ABC' + A'B$

$= AB(C + C') + A'B$

$= AB * 1 + A'B$

$= AB + A'B$

$= BA + BA'$

$= B(A + A')$

$= B * 1$

$= B$

Distributivity

$x + x' = 1$

$x * 1 = x$

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Distributivity

$x + x' = 1$

$x * 1 = x$

Note:

$F = xy + xy'$

$= x(y + y')$

$= x$

$F = xy + xz$

$= x(y + z)$

$P = (A+B)'(A'+B')$ — DeMorgan's law

$= (A'B')(A'+B')$ — Distributivity

$= A'A'B' + A'B'B'$ — $x \cdot x = x$

$= A'B' + A'B'$ — Distr.

$= A'B'(1+1)$ — $1+1=1$

$= A'B'$

P = $(A + B' + AB')(AB + A'C + BC)$

$= (A + AB' + B') \cdot (AB + A'C + BC)$

$= (A(1 + B') + B') \cdot (AB + A'C + BC)$

$= (A * 1 + B') \cdot (AB + A'C + BC)$

$= (A + B')(AB + A'C + BC)$

$= AAB + AA'C + ABC + ABB' + A'B'C + BB'C$

$= AAB + ABC + A'B'C$

$= AB + ABC + A'B'C$

$= AB(1 + C) + A'B'C$

$= AB * 1 + A'B'C$

$\boxed{F = A.B + A'.B'.C}$

6 Logic Gates

$f = P'XY + PX'Y + PXY' + PXY$

$= P'xy + Px'y + Px(y' + y)$

$= P'xy + Px'y + Px * 1$

$= P'xy + Px'y + Px$

$= P'xy + P(x'y + x)$

$= P'xy + P(x' + x)(y + x)$ ($x' + x = 1$)

$= P'xy + P(y + x)$

$= P'xy + Py + Px$

$= yP'x + yP + Px$

$= y(P'x + P) + Px$

$= y(P' + P)(x + P) + Px$ ($P' + P = 1$)

$$\boxed{f = y \cdot (x + P) + P \cdot x}$$ 4 Logic Gates

$f = yx + yP + Px$ 5 logic Gates

Exercise 11-2

Given the following Boolean expression, simplify it to a minimum number of literals using the Boolean algebra. Please mention the applied rules.

$$((A+B)(B'+C'+D'))+B'C'(A+B'+C)+A'C+D$$

Hint: The circuit of the simplified expression consists of zero gates.

$F = (A+B)(B'+C'+D') + B'C'(A+B'+C) + A'C + D$ Dist

$= AB' + AC' + AD' + BB' + BC' + BD' + AB'C' + BB'C' + B'C'C + A'C + D$

$= AB' + AC' + AD' + BC' + BD' + AB'C' + B'C' + A'C + D$

$= AB' + AB'C' + C'B + C'B' + AC' + AD' + BD' + A'C + D$

$= AB'(1+C') + C'(B+B') + AC' + AD' + BD' + A'C + D$

$= AB' + C' + AC' + AD' + BD' + A'C + D$

$= AB' + C' + C'A + AD' + BD' + A'C + D$

$= AB' + C'(1+A) + AD' + BD' + A'C + D$

$= AB' + C' + AD' + BD' + A'C + D$

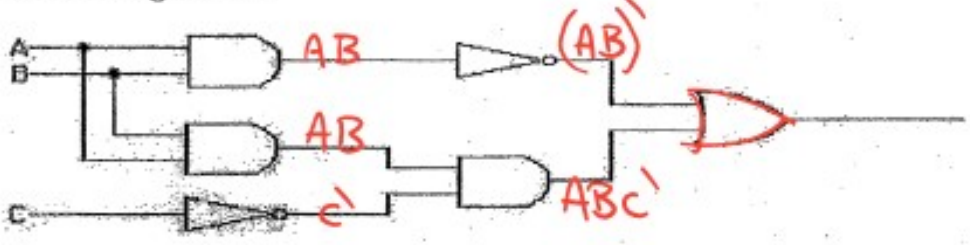
$= AB' + BD' + D + DA + C' + CA'$

$= AB' + BD' + (D+D')(D+A) + (C'+C)(C+A')$

$= AB' + BD' + D + A + A' + C' = 1$

Exercise 11-3

Given the following circuit



a) Determine the boolean expression of the circuit

b) Determine the truth tabel of the circuit.

a) $F = (AB)' + ABC' = A' + B' + ABC'$

$1 + \square$

b)

A	B	C		F
0	0	0		1
0	0	1		1
0	1	0		1
0	1	1		1
1	0	0		1
1	0	1		1
1	1	0		1
1	1	1		0

A=0
B=0
C=0

A=0
B=0
C=1

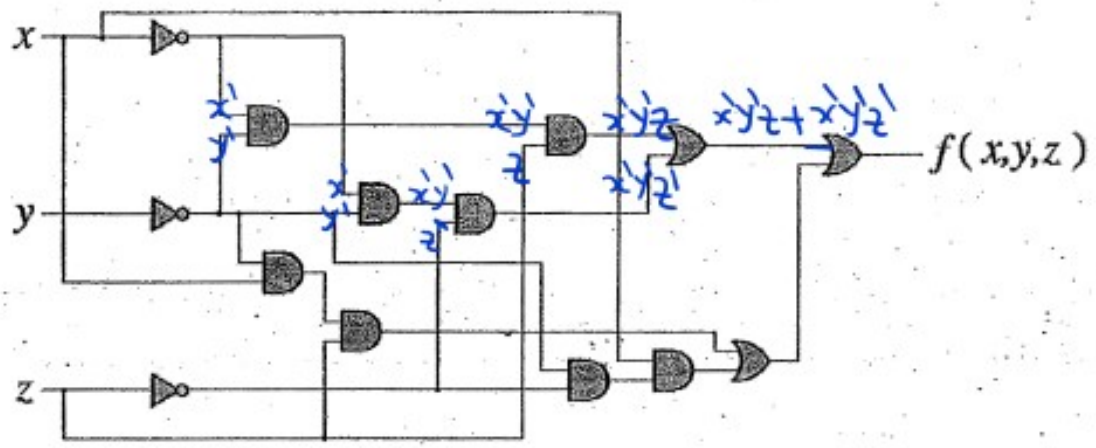
$F = A' + B' + ABC'$

$\downarrow \quad \downarrow$

A=1 $0 + 0 + (1 \cdot 1 \cdot 1) = 1$

B=1

C=0


x
y
z
x'
y'
x'y'
z
x'y'z
x'y'z'
xy'z+x'y'z'
f(x,y,z)

Exercise 7 (3+3+4+2=12 Marks)

Given the following the following truth table, where A, B are the input variables and X, Y and Z are the output variables.

A	B	X	Y	Z
0	0	0	1	0
0	1	0	1	1
1	0	1	0	0
1	1	1	0	1

N 0 1 2 3

R 2 3 4 5

R = N + 2

a) Use the sum-of-products algorithm to find the Boolean expressions that describe the output of the truth table.

b) What is the functionality of the circuit?

Exercise 7 (5+4+5=14 Marks)

Given the following truth table, where A, B are the input variables and X, Y, and Z are the output variables.

A	B	X	Y	Z
0	0	0	0	0
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

N 0 1 2 3

R 0 1 3 7

$R = 2^N - 1$

a) Use the sum-of-products algorithm to find the Boolean expressions that describe the output of the truth table.

b) What is the functionality of the circuit?