German University in Cairo - GUC
Faculty of Media Engineering and Technology - MET
Department of Digital Media Engineering and Technology
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Monday, November 22nd, 2021

DMET 502 Computer Graphics

Winter Semester 2021/2022

Midterm Exam Model Answers (Version I)

Barcode

Major: Pick one

☐ DMET
☐ CSEN

Instructions: **Read Carefully Before Proceeding.**

1- Non-programmable calculators are allowed.
2- This is a **closed book exam**.
3- Write your solutions in the space provided.
4- The exam consists of **(4) questions.**
5- This exam booklet contains **(10) pages** including this page. The last two pages are a formula sheet. **Keep them attached**.
6- Total time allowed for this exam is **(120) minutes.**
7- When you are told that time is up, stop working on the test.

Good Luck!

Don't write anything below ;-)

Question	**1**	**2**	**3**	**4**	**∑**
Possible Marks	18	23	9	20	70
Final Marks					

Question 1 [2D Graphics]:

a) [6 marks] Listed is the algorithm we use to assign outcodes for Cohen-Sutherland clipping algorithm. The order of the bits (from left to right) in the outcode represents top followed by bottom followed by right and then left.

Algorithm

Input: x, y, x_{min}, x_{max}, y_{min}, y_{max}
Output: *outcode*

```
1: outcode = 0000
2: if (y > y_max) then
3:     outcode = outcode OR 1000
4: else if (y < y_min) then
5:     outcode = outcode OR 0100
6: end if
7: if (x > x_max) then
8:     outcode = outcode OR 0010
9: else if (x < x_min) then
10:    outcode = outcode OR 0001
11: end if
12: return outcode
```

end

What would be changed if the order of the bits becomes bottom followed by right followed by top and then left?

Answers to Question 1 a):

3: outcode = outcode OR 0010 [1.5 mark]
5: outcode = outcode OR 1000 [1.5 mark]
8: outcode = outcode OR 0100 [1.5 mark]
10: outcode = outcode OR 0001 [1.5 mark]

b) [12 marks] The listed is the digital differential analyzer (DDA) algorithm used to draw lines when the absolute magnitude of slope (i.e., $|m|$) is less than or equal to 1.

Without swapping coordinates or reflecting about the line y=x, modify it to draw lines when $|m|$>1. Keep x as the loop variable.

Algorithm

Input: x_0, y_0, x_1, y_1

1: $m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$

2: $y = y_0$

3: **for** $(x = x_0 \text{ to } x_1)$ **do**

4: Plot $[x, \lfloor y + 0.5 \rfloor]^T$

5: $y = y + m$

6: **end for**

end

Answers to Question 1 b):

It can be written that

$$x_i = \frac{1}{m}(y_i - B)$$

Consequently, x_{i+1} is calculated as by incrementing the value of y by 1; so, we can write

$$x_{i+1} = \frac{1}{m}(y_{i+1} - B) = \frac{1}{m}(y_i + 1 - B) = \underbrace{\frac{1}{m}(y_i - B)}_{x_i} + \frac{1}{m} = x_i + \frac{1}{m}$$

2: $y = y_0$

3: $for\ \left(x = x_0\ to\ x_1\ step\ \frac{1}{m}\right)$

4: $Plot\ [\lfloor x + 0.5 \rfloor, y]^T$

5: $y = y + 1$

[The idea: 4 marks; lines from 2 thru 5: 8 marks]

Question 2 [2D Transformations]:

a) [10 marks] The equation of a 2D line segment is expressed in $x'y'$-coordinates as

$$y' = -3x' + 11\sqrt{2}.$$

If the x'- and y'-axes are obtained as the x- and y-axes are rotated through an angle of 45º about the origin, express the same linear equation in the old xy-coordinate system.

Answers to Question 2 a):

Example 3.23:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned} x' &= x\cos(\theta) + y\sin(\theta) \\ &= x\cos(45) + y\sin(45) = \tfrac{x+y}{\sqrt{2}}. \end{aligned}$$

$$\begin{aligned} y' &= y\cos(\theta) - x\sin(\theta) \\ &= y\cos(45) - x\sin(45) = \tfrac{y-x}{\sqrt{2}}. \end{aligned}$$

[Eq: 2 marks; value: 2 marks + Eq: 2 marks; value: 2 marks]

Substituting the values of x' and y' in the given linear equation, we get [2 marks]

$$\begin{aligned} y' &= -3x' + 11\sqrt{2} \\ \tfrac{y-x}{\sqrt{2}} &= \tfrac{-3(x+y)}{\sqrt{2}} + 11\sqrt{2} \end{aligned}$$

$$y = 5.5 - 0.5x$$

b) [13 marks] A 2D object is to be sheared by an amount ε along an axis inclined at an angle θ with respect to the x-axis. Show the steps of transformations. **Use inhomogeneous coordinates**.

Answers to Question 2 b):

Example 3.15:

1. Translate the object such that the point of intersection between the axis of shearing and the y-axis $[0, y0]^T$ is moved to the origin (e., $[0, -y0]^T$). [Step: 1 mark; vector: 1 mark]

2. Rotate the object through an angle $-\theta$ such that the axis coincides with the x-axis. [Step: 1 mark; angle with sign: 1 mark]
3. Shear the object along the x-axis. [1 mark]
4. Rotate back through an angle θ. [1 mark]
5. Translate back using the same translation vector but along the opposite direction (i.e., $[0, y0]^T$). [1 mark]

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathrm{R}}(\theta)} \underbrace{\begin{bmatrix} 1 & \varepsilon \\ 0 & 1 \end{bmatrix}}_{\dot{\mathrm{Sh}}_x(\varepsilon)} \underbrace{\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}}_{\dot{\mathrm{R}}(-\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 - y_0 \end{bmatrix}}_{\dot{\mathbf{p}}_1 - \dot{\mathbf{p}}_0} + \underbrace{\begin{bmatrix} 0 \\ y_0 \end{bmatrix}}_{\dot{\mathbf{p}}_0}$$

[each matrix or vector:1 mark; order: 1 mark]

OR

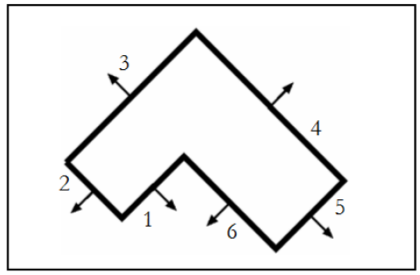
1. Translate the object such that the point of intersection between the axis of shearing and the y-axis $[0, y0]^T$ is moved to the origin (e., $[0, -y0]^T$). [Step: 1 mark; vector: 1 mark]
2. Rotate the object through an angle $90-\theta$ such that the axis coincides with the y-axis. [Step: 1 mark; angle with sign: 1 mark]
3. Shear the object along the y-axis. [1 mark]
4. Rotate back through an angle θ-90. [1 mark]
5. Translate back using the same translation vector but along the opposite direction (i.e., $[0, y0]^T$). [1 mark]

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}}_{\dot{\mathbf{p}}_2} = \underbrace{\begin{bmatrix} \sin(\theta) & \cos(\theta) \\ -\cos(\theta) & \sin(\theta) \end{bmatrix}}_{\dot{\mathrm{R}}(\theta-90)} \underbrace{\begin{bmatrix} 1 & 0 \\ -\varepsilon & 1 \end{bmatrix}}_{\dot{\mathrm{Sh}}_y(-\varepsilon)} \underbrace{\begin{bmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{bmatrix}}_{\dot{\mathrm{R}}(90-\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 - y_0 \end{bmatrix}}_{\dot{\mathbf{p}}_1 - \dot{\mathbf{p}}_0} + \underbrace{\begin{bmatrix} 0 \\ y_0 \end{bmatrix}}_{\dot{\mathbf{p}}_0}$$

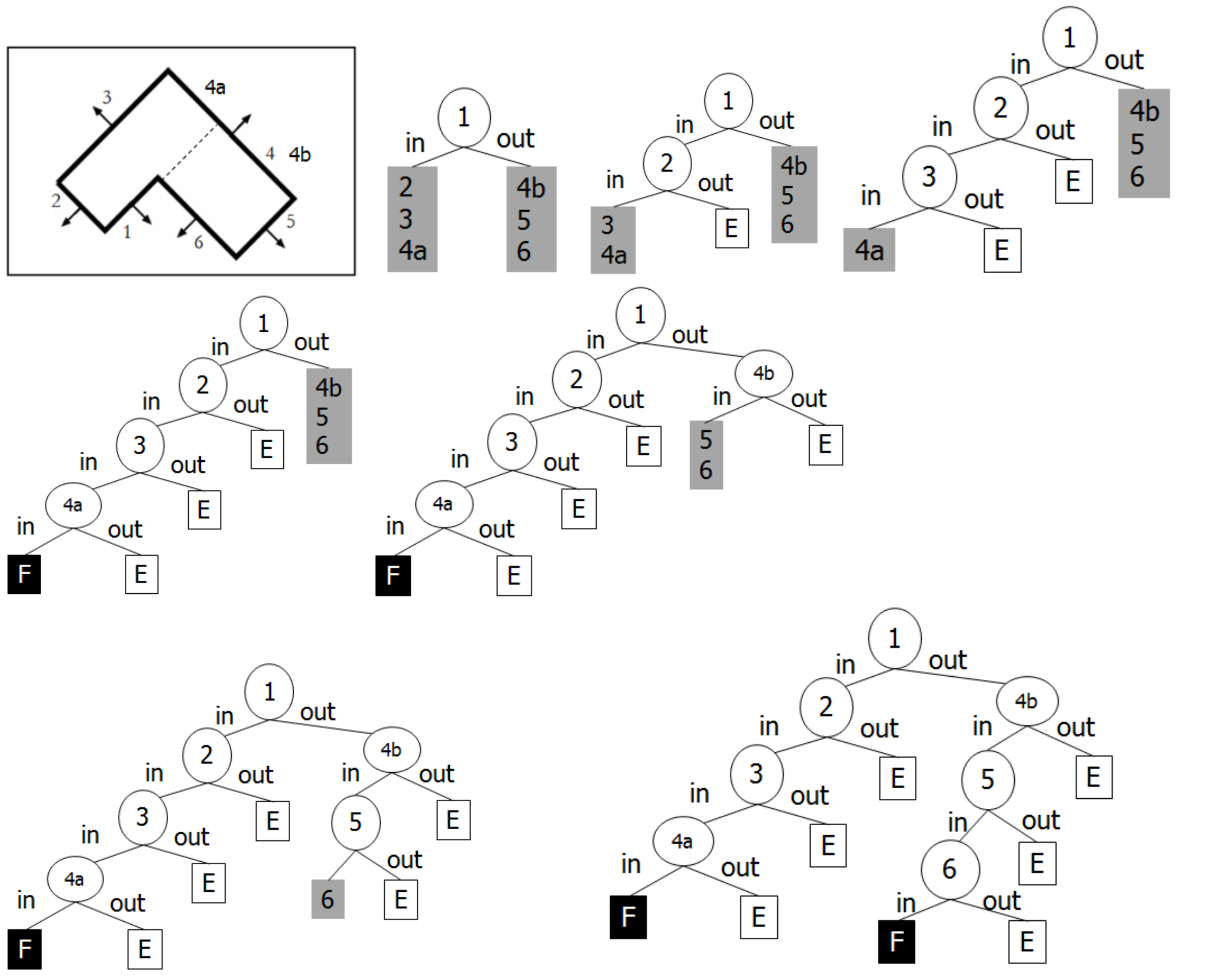
[each matrix or vector:1 mark; order: 1 mark]

Question 3 [Solids]:

[9 marks] The top view of a 3D model is shown. The arrows shown indicate the outside directions of the faces. Considering only the top view, represent this model using a BSP tree. Start with face "1." **Show <u>ALL</u> the steps of constructing this tree**. Missing steps will result in deduction.



Answers to Question 3:



[Splitting face 4: 2 marks; each step 1 mark]

Question 4 [3D Transformations]:

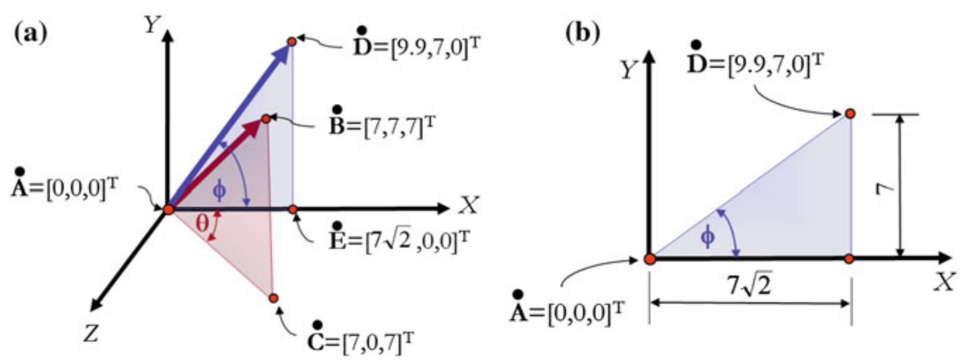
[20 marks] Derive a transformation matrix that shears an object using amounts 0.2 and 0.5 relative to an axis passing through the points $[0, 0, 0]^T$ and $[7, 7, 7]^T$. **Show <u>ALL</u> the steps**. Missing steps will result in deduction.

[Added as a separate sheet:] You may use any of (or all) the following factors:

- $sh_{xy} = 0.2$ and $sh_{xz} = 0.5$
- $sh_{yx} = -0.4232$ and $sh_{yz} = 0.3330$
- $sh_{zx} = -0.5330$ and $sh_{zy} = -0.0768$

Answers to Question 4:

Example 5.5 and 5.16:



Angles: The angle θ is the inclination angle of the line **AC** with respect to the x-axis where the line **AC** is the projection of **AB** onto the zx-plane. ➔ θ should be 45° [2 marks]

When you rotate the line **AB**, the origin **A** will not change. However, point **B** will be rotated to **D**. Apply the rotation matrix about the y-axis

$$\mathbf{D} = \mathrm{R}_y(45)\mathbf{B} = \begin{bmatrix} \cos(45) & 0 & \sin(45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(45) & 0 & \cos(45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 7 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 7\sqrt{2} \\ 7 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 9.8995 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$ [2 marks]

The angle φ is the inclination angle of the line **AD** with respect to the x-axis.

$$\phi = \tan^{-1}\left(\frac{7}{7\sqrt{2}}\right) = 35.26°.$$

[2 marks]

1. Rotate the line **AB** about the y-axis through an angle θ=45 so that we get the line **AD** on the xy-plane. [axis: 1 mark; angle sign: 1mark]
2. Rotate the line **AD** about the z-axis through an angle $-\varphi$=−35.26 to coincide with the x-axis. [axis: 1 mark; angle sign: 1mark]
3. Shear relative to the x-axis using amounts 0.2 and 0.5. [1 mark]
4. Rotate through an angle of 35.26° about the z-axis. [1 mark]
5. Rotate through an angle of −45° about the y-axis. [1 mark]

$$\mathrm{M}_1 = \mathrm{R}_y(45) = \begin{bmatrix} \cos(45) & 0 & \sin(45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(45) & 0 & \cos(45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [matrix: 1 mark]

Answers to Question 4 (cont.):

Example 5.5 and 5.16:

$$M_2 = R_z(-35.26) = \begin{bmatrix} \cos(-35.26) & -\sin(-35.26) & 0 & 0 \\ \sin(-35.26) & \cos(-35.26) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [matrix: 1 mark]

$$M_3 = Sh_x(0.2, 0.5) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [matrix: 1 mark]

$$M_4 = R_z(35.26) = \begin{bmatrix} \cos(35.26) & -\sin(35.26) & 0 & 0 \\ \sin(35.26) & \cos(35.26) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [matrix: 1 mark]

$$M_5 = R_y(-45) = \begin{bmatrix} \cos(-45) & 0 & \sin(-45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-45) & 0 & \cos(-45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [matrix: 1 mark]

Thus, the overall transformation is expressed as

$$M = M_5 M_4 M_3 M_2 M_1$$

$$= \begin{bmatrix} 0.7487 & -0.2513 & -0.2513 & 0 \\ 0.0943 & 1.0943 & 0.0943 & 0 \\ 0.1570 & 0.1570 & 1.1570 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ [Order: 1 mark; final matrix 1 mark]

There are more solutions (12 in total). Among them

1. $R_z(-45)\ R_x(35.26)\ Sh_y(-0.4232, 0.3330)\ R_x(-35.26)\ R_z(45)$
2. $R_x(-45)\ R_y(35.26)\ Sh_z(-0.5330, -0.0768)\ R_y(-35.26)\ R_x(45)$

Formula Sheet

Input: x_0, y_0, x_1, y_1

$steep = |y_1 - y_0| > |x_1 - x_0|$
if $(steep = TRUE)$ **then**
  swap (x_0, y_0)
  swap (x_1, y_1)
end if

if $(x_0 > x_1)$ **then**
  swap (x_0, x_1)
  swap (y_0, y_1)
end if

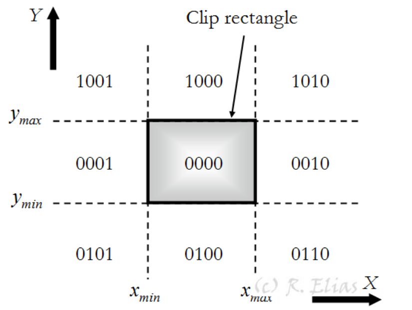
if $(y_0 > y_1)$ **then**
  $\delta y = -1$
else
  $\delta y = 1$
end if

$\Delta x = x_1 - x_0$
$\Delta y = |y_1 - y_0|$
$y = y_0$
$error = 0$

for $(x = x_0$ to $x_1)$ **do**
  if $(steep = TRUE)$ **then**
    Plot $[y, x]^T$
  else
    Plot $[x, y]^T$
  end if
  $error = error + \Delta y$
  if $(2 \times error \geq \Delta x)$ **then**
    $y = y + \delta y$
    $error = error - \Delta x$
  end if
end for

end

1. Determine outcode for each endpoint.
2. Dealing with the two outcodes:
    a. Bitwise-OR the bits. If this results in 0000, trivially accept.
    b. Otherwise, bitwise-AND the bits. If this results in a value other than 0000, trivially reject.
    c. Otherwise, segment the line. The outpoint is replaced by the intersection point. Go to Step 2.
3. If trivially accepted, draw the line.



$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1} + \underbrace{\begin{bmatrix}t_x\\t_y\end{bmatrix}}_{\mathbf{t}}.$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\\1\end{bmatrix}}_{\mathbf{P}_2} = \underbrace{\begin{bmatrix}1&0&t_x\\0&1&t_y\\0&0&1\end{bmatrix}}_{\mathrm{T}([t_x,t_y]^T)} \underbrace{\begin{bmatrix}x_1\\y_1\\1\end{bmatrix}}_{\mathbf{P}_1}$$

$$\begin{aligned}\sin(\alpha+\theta) &= \sin(\alpha)\cos(\theta) + \cos(\alpha)\sin(\theta)\\ \cos(\alpha+\theta) &= \cos(\alpha)\cos(\theta) - \sin(\alpha)\sin(\theta)\end{aligned}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}\cos(\theta)&-\sin(\theta)\\\sin(\theta)&\cos(\theta)\end{bmatrix}}_{\dot{\mathrm{R}}(\theta)} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}s_x&0\\0&s_y\end{bmatrix}}_{\dot{\mathrm{S}}(s_x,s_y)} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}1&0\\0&-1\end{bmatrix}}_{\dot{\mathrm{Ref}}_x} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}-1&0\\0&1\end{bmatrix}}_{\dot{\mathrm{Ref}}_y} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}1&sh_x\\0&1\end{bmatrix}}_{\dot{\mathrm{Sh}}_x(sh_x)} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix}x_2\\y_2\end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix}1&0\\sh_y&1\end{bmatrix}}_{\dot{\mathrm{Sh}}_y(sh_y)} \underbrace{\begin{bmatrix}x_1\\y_1\end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}-t_x\\-t_y\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\\1\end{bmatrix} = \begin{bmatrix}1&0&-t_x\\0&1&-t_y\\0&0&1\end{bmatrix} \begin{bmatrix}x\\y\\1\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\cos(\theta)&\sin(\theta)\\-\sin(\theta)&\cos(\theta)\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\frac{1}{s_x}&0\\0&\frac{1}{s_y}\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}1&0\\0&-1\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}-1&0\\0&1\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix}$$

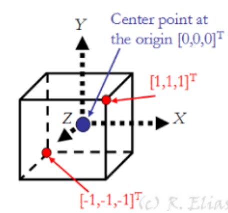
Vertex #	x	y	z

Edge #	Start vertex	End vertex

Edge	Vertices		Faces		Left traverse		Right traverse	
Name	Start	End	Left	Right	Pred	Succ	Pred	Succ

Vertex	edge

Face	edge

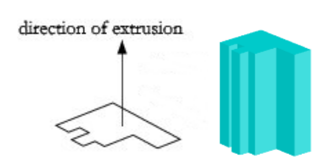


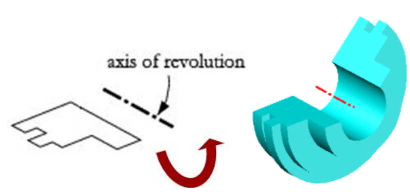
translate(scale(Block, < 1, 1.5, 1.5 >), < 1, 2, 3 >)

Face	Vertices
A	$[x_1, y_1, z_1]^T$, $[x_2, y_2, z_2]^T$, $[x_3, y_3, z_3]^T$
B	$[x_2, y_2, z_2]^T$, $[x_4, y_4, z_4]^T$, $[x_3, y_3, z_3]^T$
⋮	⋮

Vertex	Coordinates
1	$[x_1, y_1, z_1]^T$
2	$[x_2, y_2, z_2]^T$
⋮	⋮

Face	Vertices
A	1, 2, 3
B	2, 4, 3
⋮	⋮





$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1} + \underbrace{\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}}_{\mathbf{t}}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}}_{\mathbf{P}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathtt{T}([t_x,t_y,t_z]^T)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}}_{\mathbf{P}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}_x(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}}_{\dot{\mathtt{R}}_y(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{R}}_z(\theta)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}}_{\dot{\mathtt{S}}(s_x,s_y,s_z)} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{xy}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{yz}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Ref}}_{zx}} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ sh_{xy} & 1 & 0 \\ sh_{xz} & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_x(sh_{xy},sh_{xz})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & sh_{yx} & 0 \\ 0 & 1 & 0 \\ 0 & sh_{yz} & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_y(sh_{yx},sh_{yz})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$

$$\underbrace{\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\dot{\mathbf{P}}_2} = \underbrace{\begin{bmatrix} 1 & 0 & sh_{zx} \\ 0 & 1 & sh_{zy} \\ 0 & 0 & 1 \end{bmatrix}}_{\dot{\mathtt{Sh}}_z(sh_{zx},sh_{zy})} \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}}_{\dot{\mathbf{P}}_1}$$