Avg case ≡ Best case

Worst case → sorted. // pivot doesn't move.

Lecture 5 Quick Sort

i p j j j j j j r

ex | 8 | 1 | 6 | 4 | 0 | 3 | 9 | 5 | // initial pointers

↳ pivot

Partition (A,p,r)

pivot = A[r]

i = p-1

for (j = p to r-1) do

if (A[j] ≤ pivot then

i++;

exchange A[i] and A[j]

end

end

Exchange A[i+1] and A[r]

return i+1   // index of pivot.

1 4 0 3 6 8 9 5 = $\overbrace{1\ 4\ 0\ 3}^{<5}$ 5 $\overbrace{8\ 9\ 6}^{>5}$

do again on both sides. till sorted array.

Quick Sort

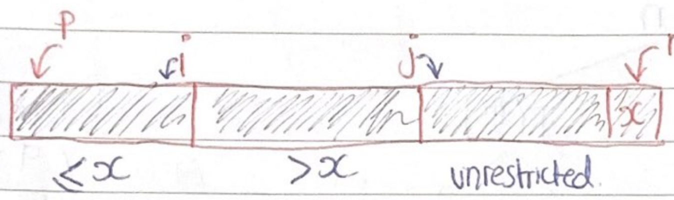
if(p<r) then

q = partition (A,p,r)

QuickSort (A,p,q-1)

QuickSort (A,q+1, r)

end



- initialization
- 2 cases (maintenance) → (a) A[j] > x
  (b) A[j] ≤ x swaps A[i] and A[j]
- termination

Quick Sort Complexity

↳ different recurrence based on cases.

| ≤5 | 5 | >5 |

| ≤4 | 4 |

Split not equal

Avg

$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$ ↳ best

↓

$O(n \log n)$

$T(n) = T(n-1) + \Theta(n)$

worst

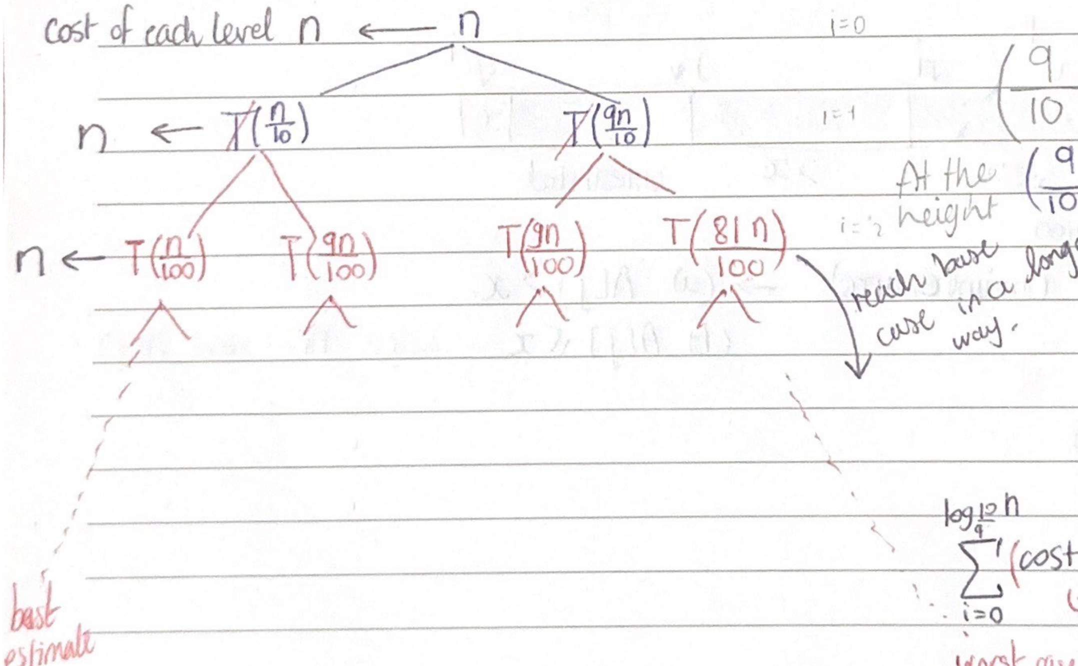
↓

$O(n^2)$

e.g. $T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + O(n)$

↳ not applicable for Master Theorem

→ Recursion Tree. $T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + O(n)$



$\left(\frac{9}{10}\right)^i n$

At the height $\left(\frac{9}{10}\right)^h n = 1$

$\left(\frac{9}{10}\right)^h = \frac{1}{n}$

$\left(\frac{10}{9}\right)^h = n$

$\therefore \boxed{h = \log_{\frac{10}{9}} n}$

$\sum_{i=0}^{\log_{\frac{10}{9}} n} (\text{cost of each level})$

↳ $\boxed{n}$

worst case estimate ↳ $O(n \log(n))$

Big(Oh)

∇ not a balanced tree.

↳ leaves not @ same height

ex (4) (PA4) $T(n) = T(n/2) + 5^{\lfloor \log_5 n \rfloor}$ $T(1) = \Theta(1)$

✓ in the form of Master theorem

→ which case?

Cost of leaves $n^{\log_b a} = n^{\log_2 1} = n^0 = c$

cost of root $= 5^{\lfloor \log_5 n \rfloor}$

n power of 5

$f(n) = 5^{\log_5 n}$

$f(n) = n^{\log_5 5} = n$

n not power of 5

$f(n) = 5^{\lfloor \log_5 n \rfloor}$

$= 5^{\log_5 n - (\log_5(n) \% 1)}$

$= n \cdot 5^{-\boxed{\log_5(n) \% 1}^{d}}$ decimal part

$0 < d < 1$

$\frac{1}{5} < 5^{-d} < 1$

e

$\therefore f(n) = en$

$en \geq n^{0+\varepsilon}$

regularity condition

$n \geq n^{0+\varepsilon}$ ✓

$5^{\lfloor \log_5 (\frac{n}{2}) \rfloor} \leq c\, 5^{\lfloor \log_5 (n) \rfloor}$

m , $2m$

$5^{\lfloor \log_5 m \rfloor} \leq c \cdot 5^{\lfloor \log_5 (2m) \rfloor}$

m power of 5

m not power of 5

$5^{\log_5 m} \leq c \cdot 5^{\lfloor \cancel{\log_5(2)} + \log_5(m) \rfloor}$

floor gets rid of fraction part

$\cancel{5^{\log_5 m}} \leq c \cdot \cancel{5^{\log_5 m}}$

$1 \leq c$ contradiction as $0 < c < 1$

$\therefore$ can't be done by master theorem → Try it by tree.

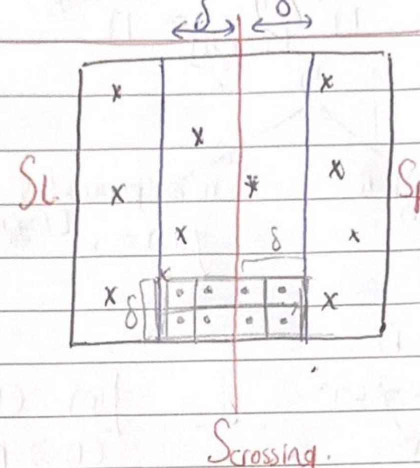
Midterm

* Master Theorem
* Recursion Tree
* Design for Divide & Conqu
* Correctness proof → Harder than Quiz 1

Design Question

Closest pair problem

↳ similar to largest subRange



Brute Force → Naive Way

$\boxed{n^2}$

find better complexity

$\delta = \min(S_L, S_R)$

$T(n) = 2T(\frac{n}{2}) + \Theta(n \log(n))$

↳ $\Theta(n \log^2(n))$