Advanced Empirical Finance: Topics and Data Science

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Multifactor asset pricing models

Multifactor asset pricing models

- FF (1992, 1993) have shown that parts cross-sectional variation in expected returns can be captured using the following factors:

1. the return on the market portfolio in excess of the risk-free rate of return
2. a zero net investment (spread) portfolio long in small firm stocks and short in large firm stocks (SMB)
3. a spread portfolio long in high book-to-market stocks and short in low book-to-market stocks (HML)

- Basic CAPM risk-return trade-off: high risk comes along with a high expected return
- Conditional versions: risk and risk premium could vary with macro economy variables such as the default spread
- The Arbitrage Pricing Theory (APT) was introduced by Ross (1976) as an alternative to the Capital Asset Pricing Model

Market anomalies

Betting against beta

- **Last block**: high beta stocks deliver negative CAPM α
- Frazzini and Pedersen (2014): α declining in β among many asset classes

Size effect

- Banz (1981), Basu (1983), Fama and French (1992): small stocks outperform

Value effect

- Higher average returns on high book-to-price ratios stocks than growth stocks

Momentum

- short and long-term reversals, intermediate-term momentum (Carhart, 1997)
- In contrast: “time series momentum” (Moskowitz, 2012)

Liquidity

- Effect of liquidity on expected returns (Pastor and Stambaugh, 2003)

Given these anomalies: Are markets efficient?

- The CAPM is not sufficient
- “an alternative risk-based model would capture all anomalous patterns in asset prices. Markets are in general efficient and the risk-return trade-off applies. The price is right up to transaction cost bounds”
- “asset prices are subject to behavioral biases. Asset returns are too volatile to be explained by changing fundamental values”
- Cochrane (2011): There is a discount rate and equivalent distorted probability that can rationalize any (arbitrage-free) data

$$1 = E_t\left(mR_{t+1}\right)$$

- “The market went up, risk aversion must have declined” is as vacuous as “the market went up, sentiment must have increased.” Any model only gets its bite by restricting discount rates or distorted expectations, ideally tying them to other data

Arbitrage Pricing Theory

- APT assumes that markets are competitive and frictionless and that the return-generating process for asset returns being considered is

$$R_i = a_i + b_i' f + \varepsilon_i$$

  where R_i is the return for asset i, a_i is the intercept of the factor model, b_i is a $(K \times 1)$ vector of factor sensitivities (loadings) and f is a $(K \times 1)$ vector of common factor realizations
- **Note:** f is not necessarily a tradeable risk factor
- For the system of N assets we have

$$R = a + Bf + \varepsilon$$

  where B is a $(N \times K)$ matrix of factor loadings and $\mathrm{Cov}(\varepsilon) = \Sigma$
- Estimation is straightforward if f are traded portfolios

APT (Ross, 1976)

- The absence of arbitrage implies

$$E(R) = \mu = \iota\lambda_0 + B\lambda_k$$

  where λ_0 is the zero-beta parameter (usually the risk-free rate) and λ_k is the $(K \times 1)$ vector of risk premia. Then, the estimation rests on the regression

$$Z_t = a + BZ_{K,t} + \varepsilon_t$$

  where Z_i are excess returns
- All the exact factor pricing models allow one to estimate the expected return on a given asset
- One needs measures of the factor sensitivity matrix B, the riskfree rate or the zero-beta expected return, and the factor risk premia λ_k
- In the case where the factors are the excess returns on traded portfolios, the risk premia can be estimated directly from the sample means of the excess returns on the portfolios

Open question: How to choose the factors?

Considerations

- Models may overfit the data because of data-snooping bias; in this case, they will not be able to predict asset returns in the future (you can always add another factor to increase in-sample goodness of fit)
- Models may capture empirical regularities that are due to market inefficiency or investor irrationality; in this case, they may continue to fit the data but they will imply Sharpe ratios for factor portfolios that are too high to be consistent with a reasonable underlying model of market equilibrium

Factor selection: Principal Components

- The framework above assumes that the identity of the factors is known
- Procedures: Statistical or theoretical
- Well-known statistical procedure: Principal components
- Principal components is a technique to reduce the number of variables being studied without losing too much information in the covariance matrix
- The principal components serve as the factors. The first principal component is the (normalized) linear combination of asset returns with maximum variance
- Let the $(N \times 1)$ vector x_1^* be the solution to

$$\max_{x_1} x_1' \hat{\Omega} x_1 \quad s.t. \quad x_1' x_1 = 1$$

  where $\hat{\Omega}$ is the sample variance-covariance matrix of the returns
- x_1^* is the eigenvector associated with the largest eigenvalue of $\hat{\Omega}$
- For principal component x_2^*, solve the same optimization problem with the additional constraint that $x_2' x_1^* = 0$
- Instead, partial least squares perform dimension reduction by directly exploiting covariation of predictors with the forecast target
- `R` implementation in package `pls`

Factor selection: Theoretical approach

- Theoretical approach: specify macroeconomic and financial market variables that are thought to capture the systematic risks of the economy or specify characteristics of firms that are likely to explain differential sensitivity to the systematic risks and then form portfolios of stocks based on the characteristics
- For more information, consider Chapter 5 of “Econometrics of Financial Markets”

A Bayesian mindset would not compromise on a single model!

- Instead, assess the probability that a candidate model generates the observed dynamics of asset returns
- Integrate over factor models weighted by their posterior probabilities
- Approach follows directly from the Bayes rule: integrated factor model summarizes the uncertainties about the joint dynamics of stock returns

“Integrating Factor Models”, Avramov et al. (2022)

- Flexible to consider models with time-varying parameters
- Prior beliefs about the entire parameter space are economically interpretable and weighted against model mispricing and the inclusion of macro predictors
- Posterior probability is weighted against deviations from the unconditional CAPM
- Penalize model complexity, i.e., an incremental factor, beyond the market, or macro predictor is retained only if it considerably improves model pricing abilities

Integrating Factor Models

- Excess returns: multivariate asset pricing regression

$$r_{t+1} = \alpha(z_t) + \beta(z_t) f_{t+1} + u_{r,t+1}, u_{r,t+1} \sim N\left(0, \Sigma_{RR}\right)$$

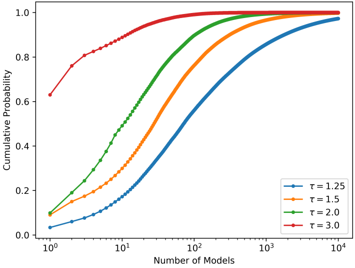
- Factors: multivariate predictive regression

$$f_{t+1} = \alpha_f + a_F z_t + u_{f,t+1}, u_{f,t+1} \sim N\left(0, \Sigma_{FF}\right)$$

- r_{t+1}: N-vector of excess returns
- f_{t+1}: K-vector of factors
- z_t: M-vector of macro predictors
- $\alpha(z_t) = \alpha_0 + \alpha_1 z_t$, fixed and time-varying mispricing
- $\beta(z_t) = \beta_0 + \beta_1 \left(I_K \otimes z_t\right)$, fixed and time-varying factor loadings
- a_F captures time-varying risk premia

Results

- Model space: 14 asset pricing factors and 13 macro predictors (more than 50 million models)



	$\tau = 1.25$	$\tau = 1.5$	$\tau = 2$	$\tau = 3$
Conditional Model Probability	1.000	1.000	1.000	1.000
Mispricing Probability	0.641	0.686	0.579	0.057
Average T_0	4,693	2,112	880	330
Average Shrinkage $\frac{T_0}{T_0+T}$	0.897	0.799	0.630	0.398

The reproducibility crisis, p-hacking, and non-standard errors

What can go wrong when doing empirical research?

Reproducibility

Discuss (see FT article)

- Research should be reproducible - enforcement becomes more critical
- What does reproducible mean? Would you say your reports are reproducible?

Minimum requirements for reproducibility

1. A set of files with the raw data and code. It is possible to create the tables and any data-derived charts/graphics/visualizations by running the code
2. Details about the system being used to run the analysis: operating system, patches, random number seeds (`set.seed(3010)`), and specific versions of all software/packages/libraries are listed
3. Open/transparent. All the data and materials are available (as opposed to “available upon request”) – e.g., posted on GitHub
4. Code is written in a way that can be readily understood

(Obvious) partial solution

- Literate programming. Files are provided which import the data, produce all the results, insert the results into the text of the report, and format the report

Collect ideas: Is empirical research in finance reproducible?

- What are limiting factors?
- What problems arise with non-reproducible research (in academia and finance)?

P-hacking: What is odd with this figure?

- Publication bias: reported *t*-stats of asset pricing *factors*

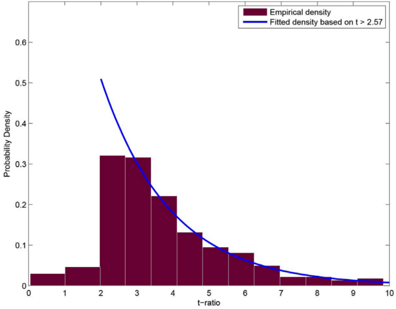
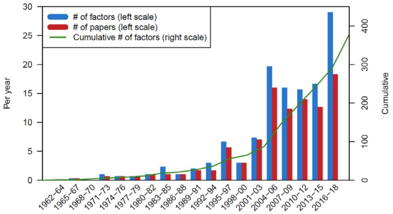


Figure 1. Distribution of reported *t*-statistics from factor studies, 1963 to 2012. Evidence from Harvey, Liu, and Zhu (2016).

- Number of studies reporting t-statistics in the range of 2.0 to 2.57 is almost the same as the number reporting in the range of 2.57 to 3.14 (Harvey, 2017)
- Notice very few papers with negative results (t-statistic less than 2.00)

P-hacking and the factor zoo

Figure 1. Out of control factor production - through January 2019



* Journals published through December 2018. Data collection in January 2019.

- Cochrane (2011): “Now we have a zoo of new factors”
- Do we really believe in 300+ risk factors?
- Machine Learning aim: Tame the factor zoo
- See this site for an updated sheet with published factors and some descriptions

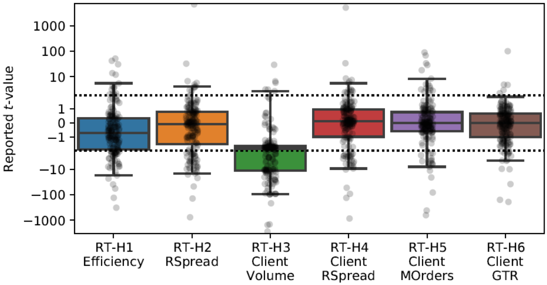
Related: Non-standard errors

- Large research project, led by Albert J. Menkveld (Paper available here)
- Uncertainty from the **evidence generating process** (EGP)
- EGP variation across researchers adds uncertainty: non-standard errors
- 164 teams test six hypotheses on the same sample

Residence of research-team members

Related: Non-standard errors

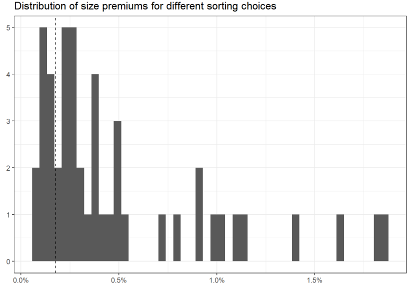
- Non-standard errors are sizeable, on par with standard errors
- Their size (i) co-varies only weakly with team merits, reproducibility, or peer rating, (ii) declines significantly after peer feedback, and (iii) is heavily underestimated by participants



Recap: The size effect

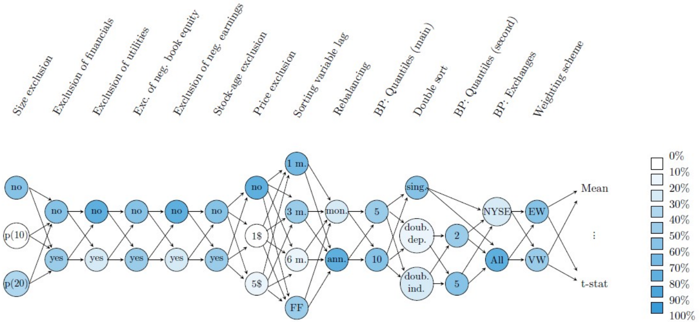
1. Portfolio sorts based on size (small-minus-big)
2. Risk-adjusted excess returns (α) of the long-short portfolio

- What can go wrong? Discuss the choices of the researcher along the way



- See also Weiss et al (2022), Hasler (2022), Soebhag et al (2022)

Non-standard errors in portfolio sorts



- sample construction: Exclusions based on market equity, industry, firm-months with negative book equity or negative earnings, stock-age or stock price
- portfolio construction: lag of the sorting variables, portfolio rebalancing frequency, the number of portfolios, listing exchanges for breakpoints, weighting scheme
- color saturation indicates how often the 109 papers analyzed by Hou et al. (2020) implemented each choice.