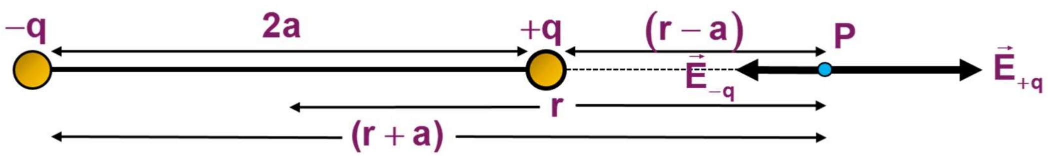
12th Physics All Derivations

ELECTRIC CHARGES AND FIELDS

1. ELECTRIC FIELD AT A POINT ON AXIAL LINE (V Imp)

Consider an electric dipole consisting of two charges +q and – q as shown. We have to find electric field due to this dipole at a point P on axial line at distance r from the centre of this dipole. Clearly, the distance of P from – q is (r + a) and from +q is (r – a).



Electric field at P due to +q is

$$E_{+q} = \frac{q}{4\pi\varepsilon_o}\frac{1}{(r-a)^2}$$

And electric field at P due to – q is

$$E_{-q} = \frac{q}{4\pi\varepsilon_o}\frac{1}{(r+a)^2}$$

Therefore, net field at P is

$$E_{axial} = E_{+q} - E_{-q} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_o}\left[\frac{(r+a)^2-(r-a)^2}{\left(r^2-a^2\right)^2}\right]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_o}\left[\frac{\left(r^2+a^2+2ar\right)-\left(r^2+a^2-2ar\right)}{\left(r^2-a^2\right)^2}\right]$$

$$\Rightarrow \frac{2aq(2r)}{4\pi\varepsilon_o\left(r^2-a^2\right)^2}$$

$$\Rightarrow \boxed{\frac{2pr}{4\pi\varepsilon_o\left(a^2-r^2\right)^2}} \qquad [p = 2aq]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_o}\left[\frac{\left(r^2+a^2+2ar-r^2-a^2+2ar\right)}{\left(r^2-a^2\right)^2}\right]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_o}\frac{4ar}{\left(r^2-a^2\right)^2}$$

For short dipole $r >> a$

$$E = \frac{2rp}{4\pi\varepsilon_o r^4}$$

$$\Rightarrow \boxed{E = \frac{2p}{4\pi\varepsilon_o r^3}}$$

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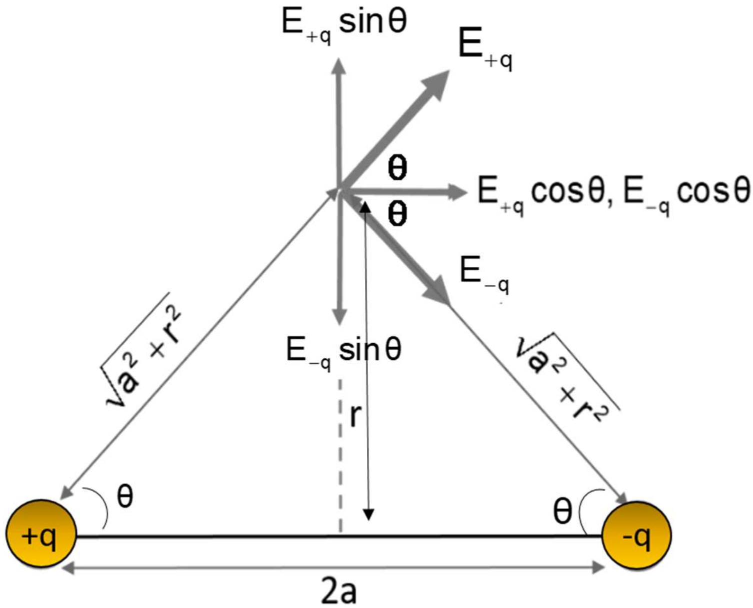
2. ELECTRIC FIELD AT A POINT ON EQUATORIAL LINE (V Imp)

Consider an electric dipole consisting of two charges +q and – q as shown. We have to find electric field due to this dipole at a point P on axial line at distance r from the centre of this dipole. Clearly, the distance of P from – q is (r + a) and from +q is (r – a).

Due to symmetry electric field at P due to both +q and – q will be same which is given by



$$E_{-q} = E_{+q} = \frac{q}{4\pi\varepsilon_o\left(r^2+a^2\right)}$$

The directions of E_{-q} and E_{+q} are as shown in the figure. The components normal to the dipole axis $\left(E_{+q}\sin\theta \text{ and } E_{-q}\sin\theta\right)$ cancel out and $\left(E_{+q}\cos\theta \text{ and } E_{-q}\cos\theta\right)$ will add up

$$E_{eq} = E_{+q}\cos\theta + E_{-q}\cos\theta$$

$$\Rightarrow E_{eq} = 2E_{+q}\cos\theta$$

$$\Rightarrow E_{eq} = \frac{2q}{4\pi\varepsilon_o\left(r^2+a^2\right)}\cos\theta$$

$$\Rightarrow E_{eq} = \frac{2q}{4\pi\varepsilon_o\left(r^2+a^2\right)} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$\Rightarrow E_{eq} = \frac{2aq}{4\pi\varepsilon_o\left(r^2+a^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \boxed{\frac{p}{4\pi\varepsilon_o\left(r^2+a^2\right)^{\frac{3}{2}}}}$$

For short dipole $r >> a$

$$\Rightarrow \boxed{E = \frac{p}{4\pi\varepsilon_o r^3}}$$

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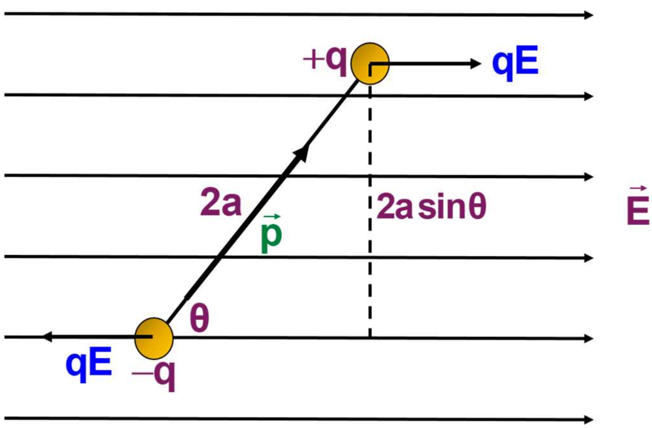
3. TORQUE ACTING ON AN ELECTRIC DIPOLE IN EXTERNAL ELECTRIC FIELD (V imp)

Consider an electric dipole consisting pf charges – q and + q and of length 2a placed in a uniform electric field making an angle θ with electric field.



Force on charge – q = $-qE$ acting opposite to the field

Force on charge +q = qE acting along the field

Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.

$$\tau = \text{Force} \times \text{perpendicular distance between the two forces}$$
$$\tau = qE \times AN$$
$$\Rightarrow \tau = qE \times 2a\sin\theta$$
$$\Rightarrow \tau = (q \times 2a)E\sin\theta$$
$$\Rightarrow \tau = pE\sin\theta$$
$$\Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

SPECIAL CASES

1. If $\theta = 0^\circ$, $\sin 0^\circ = 0, \therefore \tau = 0$, this condition is called stable equilibrium. When the dipole is displaced from this orientation it always comes back to the same configuration.
2. $\theta = 180^\circ, \sin 180^\circ = 0, \therefore \tau = 0$

   this condition is called unstable equilibrium because once displaced the dipole never comes back to this orientation instead it aligns itself parallel to the field.
3. $\theta = 90^\circ, \sin 90^\circ = 1, \therefore \tau = pE\,(\text{maximum})$

Please note: - In a non-uniform electric field $F_{net} \neq 0, \tau \neq 0$, therefore dipole executes both translation and oscillation.

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4. DERIVATION OF COULOMB'S LAW FROM GAUSS LAW (IMP)

Consider an isolated positive point charge q at O. Imagine a sphere of radius r with centre O. the magnitude of electric field intensity **E** at every point on the surface is the same and it is directed radially outwards.

Therefore, the direction of vector **ds** representing a small area element on the surface of sphere is along E only i.e. $\theta = 0^\circ$.

According to Gauss law

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_o}$$

$$\Rightarrow \oint E ds \cos 0^\circ = \frac{q}{\varepsilon_o}$$

$$\Rightarrow E \oint ds \cos 0^\circ = \frac{q}{\varepsilon_o}$$

$$\Rightarrow E\left(4\pi r^2\right) = \frac{q}{\varepsilon_o}$$

$$\Rightarrow E = \frac{q}{4\pi r^2}$$

This is the electric field intensity at any point P distant r from an isolated point charge q at the centre of the sphere. If another point charge q_o were placed at point P, then force on q_o would be

$$F = q_o \times E$$

$$\Rightarrow F = \frac{qq_o}{4\pi\varepsilon_o r^2}$$

Which is Coulomb's law.

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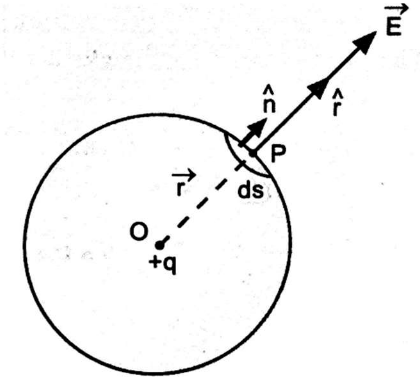
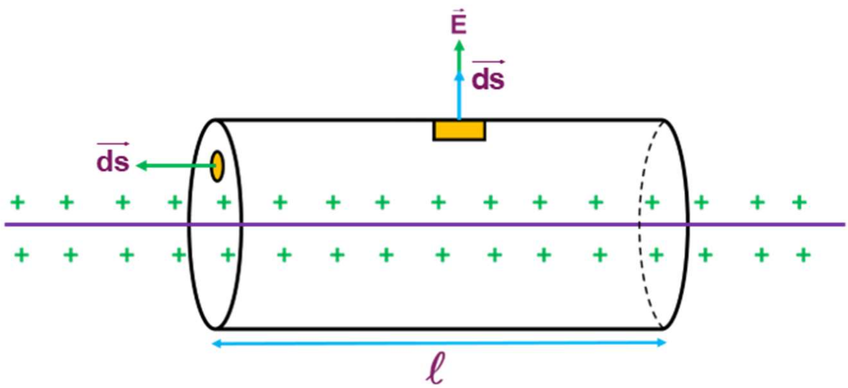
5. ELECTRIC FIELD DUE TO A STRAIGHT LONG CHARGED CONDUCTOR (V IMP)



Consider a straight charged conductor of length *l* as shown. Consider a cylindrical Gaussian surface of radius r around this conductor. Let ds be small area on this surface. As conductor is positively charged, electric field due it is outwards. Therefore, electric field and area vector are in same direction. Applying gauss theorem, we have



$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_o}$$

$$\Rightarrow \oint Eds\cos 0^\circ = \frac{q}{\varepsilon_o}$$

$$\Rightarrow \oint Eds = \frac{q}{\varepsilon_o}$$

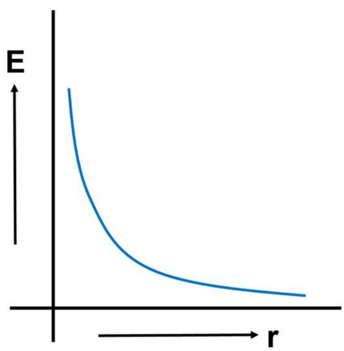
$$\Rightarrow E(2\pi rl) = \frac{q}{\varepsilon_o}$$

$$\Rightarrow E = \frac{q}{2\pi\varepsilon_o rl}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}}$$

$\lambda = \frac{q}{l}$, λ is called linear charge density

Clearly, $E \propto \frac{1}{r}$. Therefore, the variation of E with r is shown graphically in the figure shown below:



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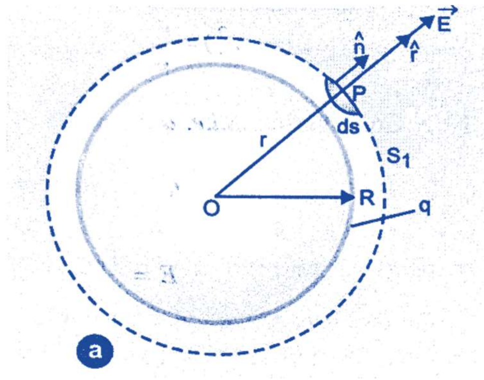
6. ELECTRIC FIELD DUE TO SPHERICAL SHELL (IMP)

When point P lies outside the spherical shell

Suppose that we have to calculate electric field at the point P at a distance *r* (*r* > *R*) from its centre. Draw the Gaussian surface through point P so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius *r* and centre O.

Let $\vec{E}$ be the electric field at point P. Then, the electric flux through area element $\vec{ds}$ is given by,



$$d\varphi_E = \vec{E}.\vec{ds}$$

Since $\vec{ds}$ is also along normal to the surface,

$$d\varphi = E ds \cos 0^\circ = E ds$$

∴ Total electric flux through the Gaussian surface is given by,

$$\varphi = \oint E ds = E \oint ds$$

Now,

$$\oint ds = 4\pi r^2$$

$$\therefore \varphi = E \times 4\pi r^2 \quad ....(i)$$

Since the charge enclosed by the Gaussian surface is *q*, according to Gauss theorem,

$$\varphi = \frac{q}{\varepsilon_o} \quad ....(ii)$$

From equations (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_o r^2} \qquad (\text{for } r > R)$$

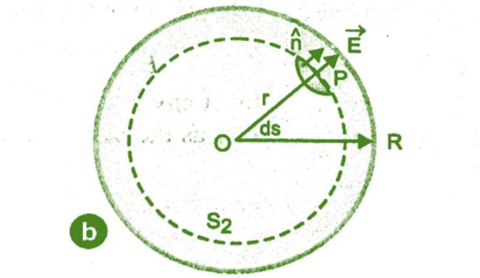
When point P lies inside the spherical shell

In such a case, the Gaussian surface encloses no charge.

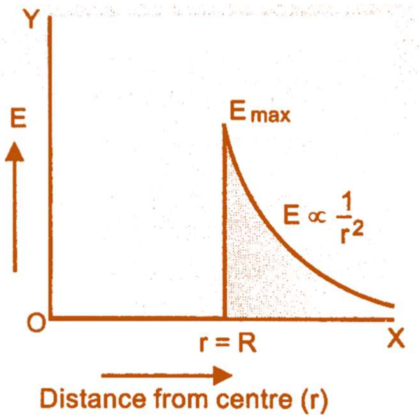
According to Gauss law,

$$E \times 4\pi r^2 = 0$$

$$\Rightarrow E = 0$$



Hence, the field due to a uniformly charged spherical shell is zero at all points inside the shell. The variation of electric field intensity E with distance from the centre of a uniformly charge spherical shell is shown:



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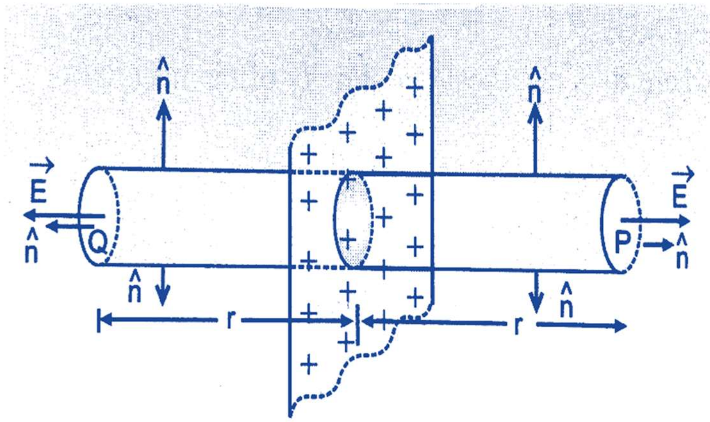
7. ELECTRIC FIELD DUE TO INFINITE PLANE SHEET OF CHARGE (V IMP)

Consider an infinite thin plane sheet of positive charge having a uniform surface charge density σ on both sides of the sheet.

Let P be the point at a distance '*a*' from the sheet at which electric field is required.



Draw a Gaussian cylinder of area of cross-section A through point P. The electric flux crossing through the Gaussian surface is given by,

Φ = E × Area of the circular caps of the cylinder

Since electric lines of force are parallel to the curved surface of the cylinder, the flux due to electric field of the plane sheet of charge passes only through the two circular caps of the cylinder.

$\varphi = E \times 2A$ ......(i)

According to Gauss theorem, we have

$$\varphi = \frac{q}{\varepsilon_o}$$

Here, the charge enclosed by the Gaussian surface,

$q = \sigma A$ where σ is the surface charge density (q/A)

$$\therefore \varphi = \frac{\sigma A}{\varepsilon_o} \quad \text{..........(ii)}$$

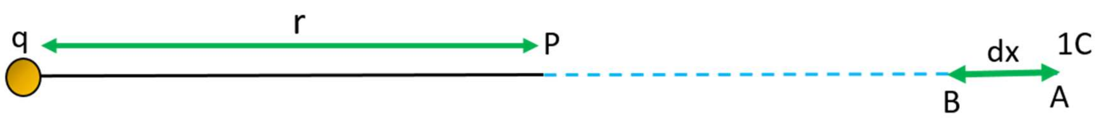
From equations (i) and (ii), we obtain

$$E \times 2A = \frac{\sigma A}{\varepsilon_o} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

ELECTRIC POTENTIAL AND CAPACITANCE

(1) Potential at a point due to a point charge (IMP)

Let a charge of 1 C be placed at a distance x from a charge q. Work done by electrostatic force if we move this charge from A to B towards q through small distance dx



$$dW = Fdx\cos 180^o$$

$$\Rightarrow dW = -\frac{kq(1)}{x^2}dx$$

Work done to move this charge from $x = \infty$ to $x = r$ is

$$W = -\int_{\infty}^{r} \frac{kq}{x^2}dx$$

$$\Rightarrow W - kq\left(\frac{x^{-1}}{-1}\right)_{\infty}^{r}$$

$$\Rightarrow W = kq\left(\frac{1}{x}\right)_{\infty}^{r}$$

$$\Rightarrow W = kq\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$\Rightarrow W = \frac{kq}{r}$$

By definition, this is the potential at P. Thus potential at a distance r due to a charge q is

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}}$$

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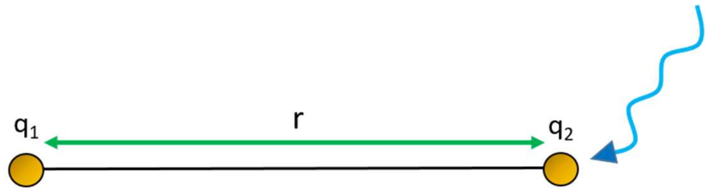
(2) Electric potential energy

Consider a charge kept at A. Let another charge q_2 be brought from infinity to point B at a distance r from it, then work done to bring it at P is



$$W = q_2 V$$

$$\Rightarrow W = q_2\left(\frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}\right)$$

$$\Rightarrow W = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$$

This work is stored in the system of two charges as electric potential energy. Thus

$$\boxed{U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}}$$

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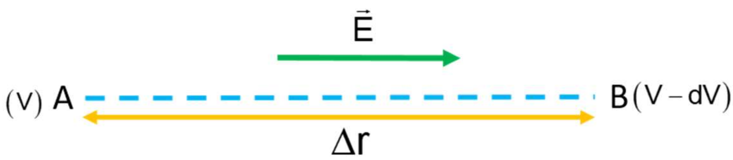
(3) Relation between electric field and electric potential (potential gradient) **(IMP)**

Consider a charge q moving from A to B in the direction of electric field as shown



Small amount of work done is

$$dW = q(V_B - V_A)$$

$$\Rightarrow dW = q(V - dV - V)$$

$$\Rightarrow dW = -qdV$$

$$\Rightarrow dW = Fdr = qEdr$$

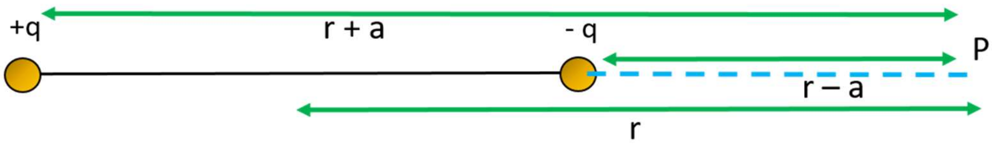
$$\Rightarrow -qdV = qEdr$$

$$\Rightarrow \boxed{E = -\frac{dV}{dr}}$$

Or $dV = -\vec{E}.\vec{dr}$

<u>POTENTIAL DUE TO DIPOLE</u>

(4) At a point on axial line



Potential at P due to +q

$$V_{+q} = \frac{kq}{(r+a)}$$

Potential at P due to –q

$$V_{-q} = \frac{kq}{(r-a)}$$

Therefore, total potential at P is

$$V_{axial} = V_{+q} + V_{-q}$$

$$\Rightarrow V_{axial} = \frac{kq}{(r+a)} + \frac{kq}{(r-a)}$$

$$\Rightarrow V_{axial} = \frac{kq(r-a) - kq(r+a)}{r^2 - a^2}$$

$$\Rightarrow V_{axial} = \frac{kqr - kqa - kqr - kqa}{r^2 - a^2}$$

$$\Rightarrow V_{axial} = \frac{(-2aq)k}{r^2 - a^2}$$

$$\Rightarrow \boxed{V_{axial} = \frac{-kp}{r^2 - a^2}}$$

For short dipole a<<r

$$V_{axial} = -\frac{kp}{r^2}$$

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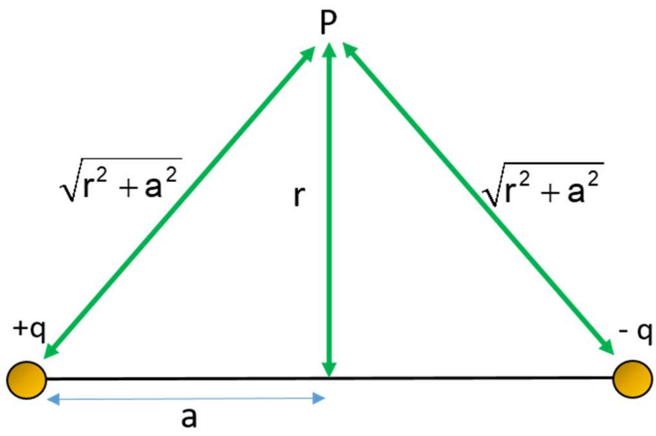
(5) At a point on equatorial line

As shown in the diagram, potential at P due to +q

$$V_{+q} = \frac{kq}{\sqrt{a^2 + r^2}}$$



Potential at P due to –q

$$V_{+q} = \frac{k(-q)}{\sqrt{a^2+r^2}}$$

Therefore, total potential at P is

$$V_{eq} = V_{+q} + V_{-q} = 0$$

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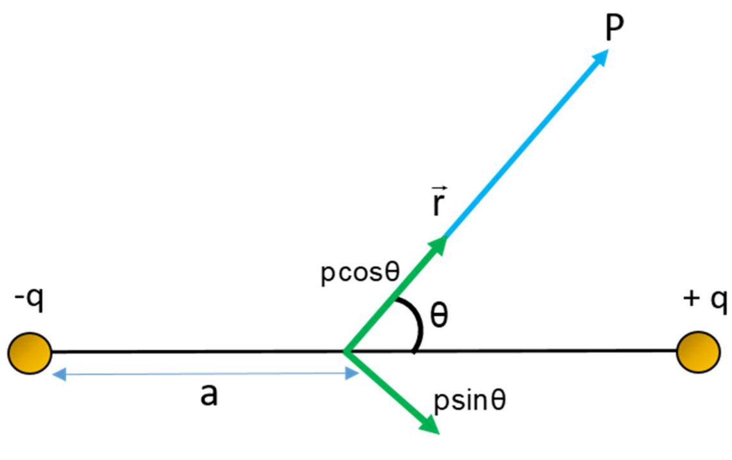
(6) Potential at any arbitrary point

Consider a point P at a distance along a line making an angle θ with the dipole axis. If we resolve $\vec{p}$ into two rectangular components as shown.

Point P lies on the axial line of the dipole with dipole moment $p\cos\theta$ and on equatorial line of the dipole with the dipole moment $p\sin\theta$



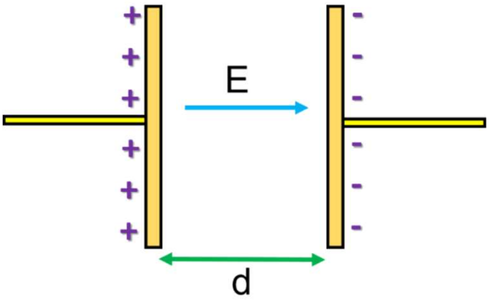
$$V = \frac{kp\cos\theta}{r^2} + 0$$

$$\boxed{V = \frac{kp\cos\theta}{r^2}}$$

CAPACITANCE

(7) Capacitance of a parallel plate capacitor (IMP)

Consider a parallel plate capacitor as shown.



Let

V = potential difference between the plates

Q = charge on the capacitor

E = potential difference between the plates

σ = Surface charge density of the plates

d = distance between the plates

$$\text{As } C = \frac{Q}{V} = \frac{\sigma A}{V} \qquad [\because \sigma = \frac{Q}{A}]$$

$$\because V = Ed$$

$$\therefore C \frac{\sigma A}{Ed}$$

$$\text{field between plates capacitor is } E = \frac{\sigma}{\varepsilon_o}$$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_o} d} \Rightarrow \boxed{C = \frac{\varepsilon_o A}{d}}$$

If there is a medium of dielectric constant k between the plates, then

$$k = \frac{\varepsilon}{\varepsilon_o} \Rightarrow \varepsilon = k\varepsilon_o$$

$$\therefore \boxed{C = \frac{k\varepsilon_o A}{d}}$$

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(8) Energy stored in capacitor **(not in syllabus for session 2022-23)**

Let dW be the small amount of work by the battery to store small charge dq

So, dW = Vdq, where V is the voltage of the battery

$$\because V = \frac{q}{C}$$

$$dW = \frac{q}{C}dq$$

Then, the total work done to store charge Q is

$$\int dW = \int_0^Q \frac{q}{C}dq$$

$$\Rightarrow W = \frac{1}{C}\int_0^Q qdq$$

$$\Rightarrow W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\Rightarrow W = \frac{1}{2C}\left[Q^2 - (0)^2\right]$$

$$\Rightarrow W = \frac{Q^2}{2C}$$

This work is stored in the capacitor in the form of electrostatic energy

$$\because \boxed{U = \frac{Q^2}{2C}}$$

$$\because Q = CV$$

$$\therefore U = \frac{C^2V^2}{2C}$$

$$\therefore \boxed{U = \frac{1}{2}CV^2}$$

$$\text{or } C = \frac{Q}{V} \;\therefore U = \frac{Q^2}{2\frac{Q}{V}}$$

$$\boxed{U = \frac{1}{2}QV}$$

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(9) Energy density (u)

$$u = \frac{\text{Energy stored}}{\text{volume}}$$

$$= \frac{\frac{1}{2}CV^2}{Ad}$$

$$= \frac{1}{2}\frac{\varepsilon_o A}{d}\frac{E^2 d^2}{(Ad)}$$

$$\therefore \boxed{u = \frac{1}{2}\varepsilon_o E^2}$$

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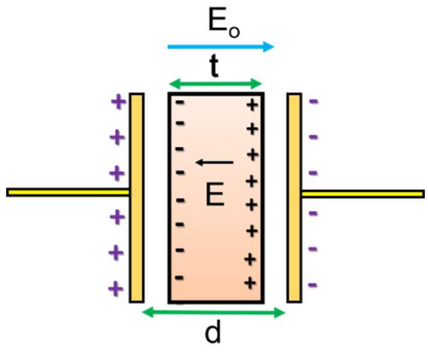
(10) Capacitance of parallel plate capacitor with dielectric slab between the plates (V IMP)

Consider a slab of thickness t inserted between the plates as shown



Potential difference between the plates is given by

$$V = E_o(d-t) + Et$$

$$\Rightarrow V = E_o(d-t) + \frac{E_o}{k}t$$

$$\Rightarrow V = E_o\left[d - t + \frac{t}{k}\right]$$

Let new capacitance be C'

$$C' = \frac{Q}{V}$$

$$\Rightarrow C' = \frac{Q}{E_o\left[d - t + \frac{t}{k}\right]}$$

$$\Rightarrow \boxed{C' = \frac{\varepsilon_o A}{d - t\left(1 - \frac{1}{k}\right)}}$$

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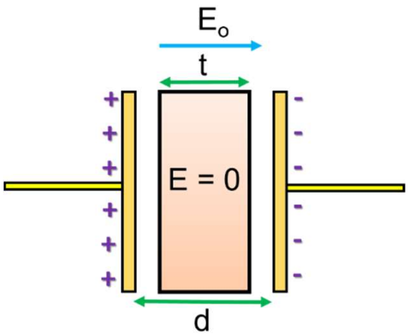
(11) Capacitance of a parallel plate capacitor with conducting slab between the plates (V IMP)

Consider a conducting slab placed between the plates of a parallel plate capacitor as shown

Since, electric field inside the conducting slab is zero, potential difference between the plates is given by



$$V = E_o(d-t) + Et$$

$$\Rightarrow V = E_o(d-t) + (0)t$$

$$\Rightarrow V = E_o(d-t)$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_o}(d-t)$$

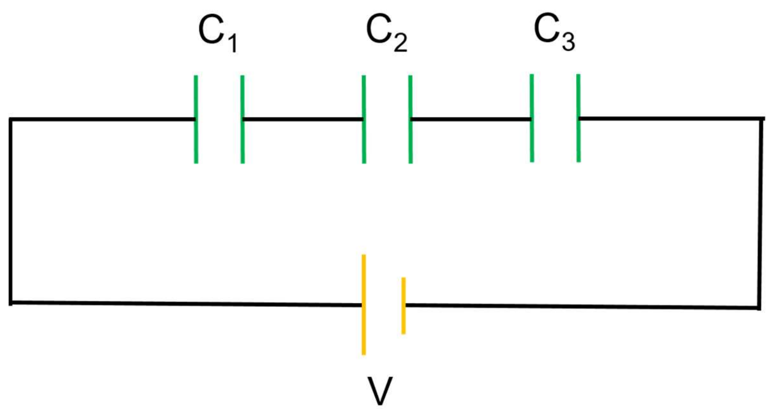
$$\Rightarrow V = \frac{\sigma}{\varepsilon_o}(d-t)$$

$$\therefore C' = \frac{Q}{V} = \frac{\cancel{\phi}}{\frac{\cancel{\phi}}{A\varepsilon_o}(d-t)} \Rightarrow \boxed{C' = \frac{\varepsilon_o A}{d-t}}$$

COMBINATION OF CAPACITORS

(12) Series combination

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series as shown. Let potential difference across them be V_1, V_2 and V_3 and charge stored by each is Q.



If V is applied voltage, then

$$V = V_1 + V_2 + V_3$$

$$\because V = \frac{Q}{C}$$

$$\therefore V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

If equivalent capacitance is C_{eq}

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

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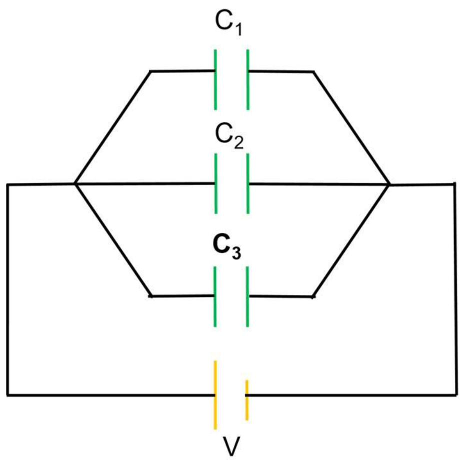
(13) Parallel combination

Figure shown three capacitors connected in parallel, let charge stored by each is Q_1, Q_2 and Q_3 and potential difference across each is V. If charge supplied by battery be Q, then



$$Q = Q_1 + Q_2 + Q_3$$

$$\because Q = C_{eq}V,\ C_{eq} = \text{equivalent capacitance}$$

$$Q = C_1V + C_2V + C_3V$$

$$\Rightarrow C_{eq}\cancel{V} = \cancel{V}\left(C_1 + C_2 + C_3\right)$$

$$\Rightarrow \boxed{C_{eq} = C_1 + C_2 + C_3}$$

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(14) Common potential

If two capacitors of capacitances C_1 and C_2 are charged to potential V_1 and V_2 and are connected together, then, the charge flows from the capacitor at higher potential to the other at lower potential till the potential of both become equal, this equal potential is called common potential.

Since total charge before and after remains same, therefore

$$C_1V + C_2V = C_1V + C_2V$$

$$\Rightarrow \boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

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(15) Loss of energy on sharing of charges

When charge is shared between the capacitors, energy is lost in the form of heat

Total energy before sharing

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

total energy after sharing

$$U_f = \frac{1}{2}(C_1 + C_2)V^2$$

$\therefore$ Heat loss, $\Delta U = U_i - U_f$

$$\Delta U = \frac{1}{2}\left\{C_1V_1^2 + C_2V_2^2 - (C_1 + C_2)V^2\right\}$$

$$\Delta U = \frac{1}{2}\left\{C_1V_1^2 + C_2V_2^2 - (C_1 + C_2)\left[\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2}\right]\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}\left\{\frac{C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2}\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}\left\{\frac{\cancel{C_1^2V_1^2} + C_1C_2V_1^2 + C_1C_2V_2^2 + \cancel{C_2^2V_2^2} - \cancel{C_1V_1^2} - \cancel{C_2^2V_2^2} - 2C_1C_2V_1V_2}{C_1 + C_2}\right\}$$

$$\Rightarrow \Delta U = \frac{1}{2}C_1C_2\left\{\frac{V_1^2 + V_2^2 - 2V_1V_2}{C_1 + C_2}\right\}$$

$$\Rightarrow \boxed{\Delta U = \frac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}}$$

CURRENT ELECTRICITY

(1) Drift velocity (V IMP)

We may define **drift velocity** as the average velocity with which electrons get drifted towards the positive terminal of the battery under the influence of an external electric field.

Let the initial velocities of electrons (in the absence of battery) be $u_1, u_2, u_3 \ldots\ldots\ldots\ldots u_n$, then, $\frac{u_1 + u_2 + u_3 \ldots\ldots\ldots\ldots + u_n}{n} = 0$.

When the battery is applied, acceleration of each electrons is $a = \frac{eE}{m}$. When electrons move in a conductor, they keep colliding with the heavy ions present in it and come to a momentary rest. Time gap between two successive collisions is called relaxation time (τ).

Thus, if $v_1, v_2 \ldots\ldots\ldots v_n$ be the final velocities of electrons then, by definition, drift velocity is

$$v_d = \frac{v_1 + v_2 \ldots\ldots\ldots + v_n}{n}.$$

Since, $v_1 = u_1 + a\tau_1,\ v_2 = u_2 + a\tau_2,\ v_3 = u_3 + a\tau_3 \ldots\ldots\ldots v_n = u_n + a\tau_n$. Therefore v_d becomes

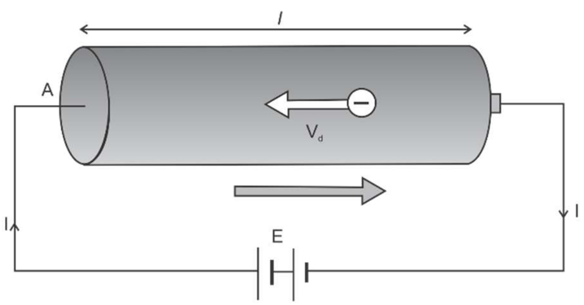
$$v_d = \frac{(u_1 + a\tau_1) + (u_2 + a\tau_2) + (u_3 + a\tau_3) \ldots\ldots\ldots + (u_n + a\tau_n)}{n}$$

$$\Rightarrow v_d = \left(\frac{u_1 + u_2 \ldots\ldots\ldots + u_n}{n}\right) + a\left(\frac{\tau_1 + \tau_2 \ldots\ldots\ldots\ldots + \tau_n}{n}\right)$$

Or $\boxed{v_d = \frac{eE}{m}\tau}$, where τ is average relaxation time.

---

(2) Relation between current and drift velocity (V IMP)



Consider a conductor of length ℓ and area of cross section A connected to battery of potential difference V. Then, volume of the conductor is $A\ell$. If number density of electrons in the conductor (number of electrons per unit volume) is n, then total number of electrons in conductor is $A\ell n$. Hence, total charge is, $q = A\ell ne$. Therefore, current in the conductor is given by $I = \frac{q}{t} \Rightarrow I = \frac{A\ell ne}{\left(\frac{\ell}{v_d}\right)}$.

or $\boxed{I = Anev_d}$.

---

(3) Proof of Ohm's law and formula for resistance (and resistivity): (IMP)

$\because I = Anev_d$ and $v_d = \frac{eE}{m}\tau$

$$\therefore I = Ane\left(\frac{eE}{m}\tau\right)$$

$$\Rightarrow I = \frac{Ane^2E}{m}\tau$$

$$\Rightarrow I = \frac{Ane^2}{m}\left(\frac{V}{\ell}\right)\tau$$

$$\Rightarrow V = \frac{m\ell}{Ane^2\tau}I$$

If physical conditions are constant $\frac{m\ell}{Ane^2\tau}$ is constant. Therefore, $V \propto I$.

Comparing (i) and (ii), we get $\boxed{R = \frac{m\ell}{Ane^2\tau}}$

Microscopic or vector form of ohm's law.

$$\because J = \frac{I}{A}$$

$$\therefore J = \frac{Anev_d}{A} \Rightarrow J = ne\left(\frac{eE}{m}\tau\right)$$

$$\Rightarrow J = \frac{ne^2\tau}{m}E$$

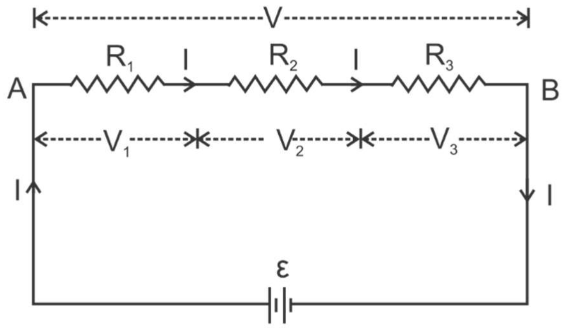
or $\boxed{\vec{J} = \sigma\vec{E}}$

---

(4) Combination of resistors (NOT IN SYLLABUS FOR SESSION 2022-23)

Series Combination

Consider two resistors R_1 and R_2 in series. The charge which leaves R_1 must be entering R_2 .



Since current measures the rate of flow of charge, this means that the same current I flows through R_1 and R_2 . By Ohm's law:

Potential difference across $R_1 = V_1 = IR_1$, and

Potential difference across $R_2 = V_2 = IR_2$.

Potential difference across $R_3 = V_3 = IR_3$

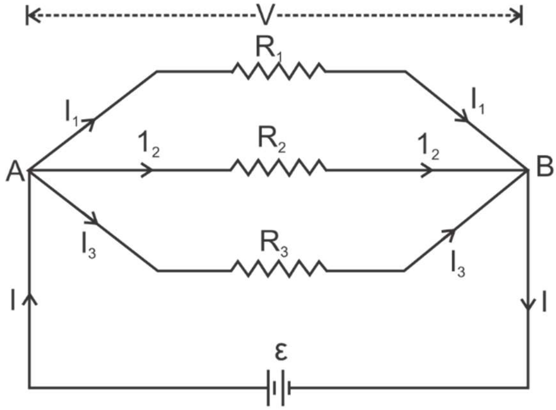
The potential difference V across the combination is $V_1 + V_2 + V_3$. Hence, $V = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$ This is as if the combination had an equivalent resistance R_{eq} , which by Ohm's law is

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

This obviously can be extended to a series combination of any number n of resistors $R_1, R_2, \ldots\ldots\ldots, R_n$. The equivalent resistance R_{eq} is

$$\boxed{R_{eq} = R_1 + R_2 + R_3 \ldots\ldots\ldots\ldots\ldots + R_n}$$

Parallel combination.



The currents I, I_1, I_2 and I_3 shown in the figure are the rates of flow of charge at the points indicated. Hence,

$I = I_1 + I_2 + I_3$

The potential difference between A and B is given by the Ohm's law applied to R_1

$V = I_1R_1$

Also, Ohm's law applied to R_2 and R3 gives

$V = I_2R_2$, $V_3 = IR_3$

$\therefore I = I_1 + I_2 + I_3$

$$\Rightarrow \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Or $\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

If n resistors are connected in parallel, then,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots\ldots\ldots\ldots\ldots + \frac{1}{R_n}}$$

---

(5) Relation between internal resistance, terminal potential difference and emf (IMP)

Let ε be emf of the cell, V be the terminal potential difference, r be the internal resistance, R be external resistance and I be the current flowing in the circuit then, potential drop across internal resistance is Ir. Therefore, potential drop across external resistance is,

$\boxed{V = \varepsilon - IR}$

$\Rightarrow Ir = \varepsilon - V$

$\Rightarrow r = \dfrac{\varepsilon - V}{I}$

$\Rightarrow r = \dfrac{\varepsilon - V}{\frac{V}{R}}$

$\Rightarrow r = \left(\dfrac{\varepsilon - V}{V}\right) \times R$

Or $\boxed{r = \left(\dfrac{\varepsilon}{V} - 1\right) \times R}$

Charging. During charging of a cell, current flows in reverse direction with the help of external agency, so the terminal potential difference becomes $\boxed{V = \varepsilon + IR}$

---

(6) Combination of cells (V IMP)

Like resistors, cells can also be connected in series and parallel combination.

Series combination. Consider two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series.

If V_1 and V_2 be the terminal potential differences of the two cells, then V = $V_1 + V_2$

$\Rightarrow V = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$

$\Rightarrow V = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$

Comparing this with $V = \varepsilon_{eq} - Ir_{eq}$ we get

$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$

This result can be extended to series combination of n cells as

$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 .... \varepsilon_n}$

Parallel combination

If two cells are connected in parallel, terminal potential difference across them is same but current is different, $\therefore$ total current

$$I = I_1 + I_2$$

$$\Rightarrow I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_1 - V}{r_2}$$

$$\Rightarrow I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\Rightarrow V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$

$$\Rightarrow V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

Comparing this with $V = \varepsilon_{eq} - I r_{eq}$ we get

$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}$$

This result can be extended to parallel combination of n cells as

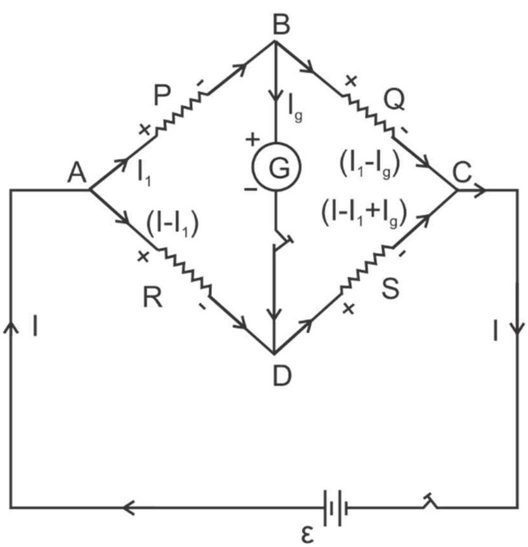
$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} \ldots\ldots\ldots\ldots + \frac{\varepsilon_n}{r_n}}$$

---

(7) Wheatstone bridge (V IMP)

Wheatstone bridge is a circuit which is used to measure accurately an unknown resistance.

Principle. It states that when the bridge is balanced (i.e. when $I_g = 0$), the product of resistances of opposite arms is equal.



. Applying Kirchhoff's second law to loop ABDA, we get

$$I_1P + I_gG - (I - I_1)R = 0$$

SInce $I_g = 0$

$$\therefore I_1P - (I - I_1)P = 0$$

$$\Rightarrow I_1P = (I - I_1)R \qquad \text{..........(i)}$$

Applying second law in loop BCDB, we get

$$(I_1 - I_g)Q - (I - I_1 + I_g)S - I_gG = 0$$

$$\because I_g = 0$$

$$\therefore I_1Q - (I - I_1)S = 0$$

$$\Rightarrow I_1Q = (I - I_1)S \qquad \text{..........(ii)}$$

From (i) and (ii) we get

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

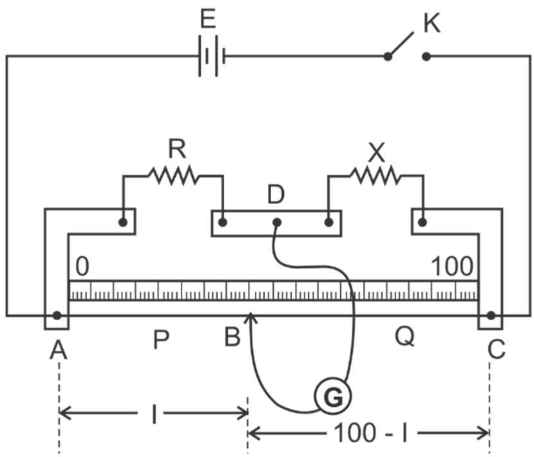
Or $\boxed{PS = QR}$

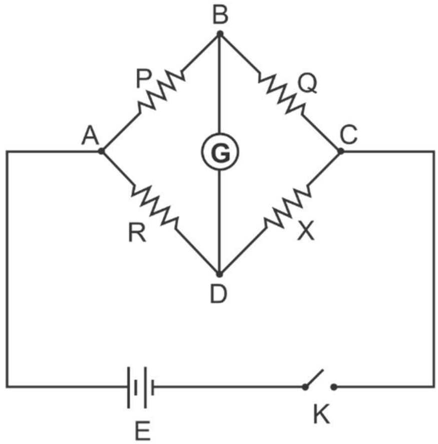
---

(8) Finding unknown resistance using slide wire bridge (NOT IN SYLLABUS FOR SESSION 2022-23)

It a practical form of a Wheatstone bridge which is used to find an unknown resistance. Its operation is based on the principle of wheat stone bridge.

As shown in the figure introduce a suitable value of R and close key K. Move the jockey on the wire AC to obtain the null point (i.e. zero reading of the galvanometer). Let point B be the null point on the wire AC**.** Let length AB be ℓ, therefore length BC is $100 - \ell$. As the bridge is balanced, therefore, by Wheatstone bridge principle, we have





$$\frac{P}{Q} = \frac{R}{S}$$

If r be the resistance per cm length of the wire, then

P = resistance of length ℓ of the wire = ℓr

Q = resistance of length $100-\ell$ of the wire = $(100-\ell)r$

$$\therefore \frac{\ell r}{(100-\ell)r} = \frac{R}{S}$$

Or $\boxed{S = \left(\frac{100-\ell}{\ell}\right) \times R}$

Knowing ℓ and R, S can be determined.

---

(9) Proof of working principle of potentiometer (NOT IN SYLLABUS FOR SESSION 2022-23)

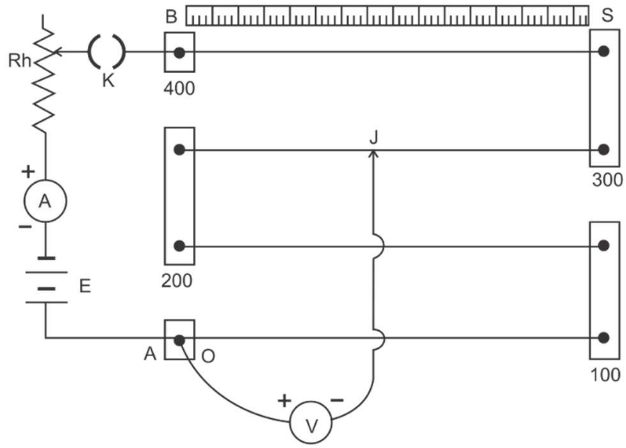
Principle. The working of potentiometer is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross section and a constant current is flowing through it.

Proof. Let A be the area of cross section, ρ be the resistivity of the material pf the wire, V be potential difference across length ℓ whose resistance is R. Let I be the current flowing through the wire, then by Ohm's law

$V = IR$

$\text{As } R = \rho \frac{\ell}{A}$

$\text{we have } V = I\rho \frac{\ell}{A}$



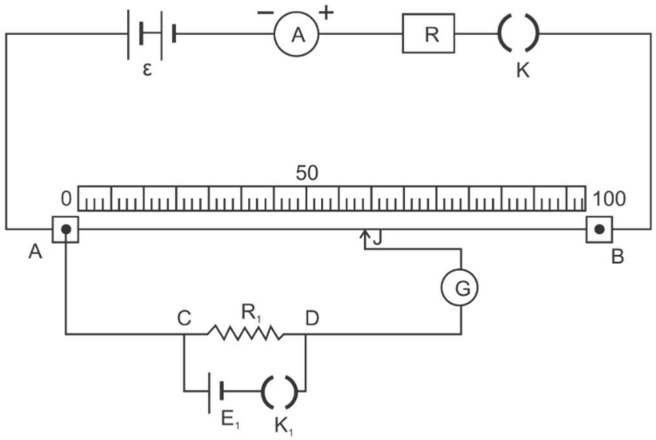
$\text{or } V = \left(\frac{I\rho}{A}\right)\ell$

$\text{or } V \propto \ell$

$\frac{V}{\ell}$ is called potential gradient of the wire i.e. fall in potential per unit length of the wire.

---

(10) Determining a potential difference using potentiometer **(NOT IN SYLLABUS FOR SESSION 2022-23)**



Close key K and adjust the value of R so that fall of potential across the potentiometer wire is greater than the potential difference to be measured. Close key K_1. Adjust the position of jockey on potentiometer wire where is pressed, the galvanometer shows no deflection. Let that position be J. Let length AJ be ℓ. If k is the potential gradient of potentiometer wire, then potential difference across R_1 i.e.

$V = k\ell$

If r is the resistance of potentiometer wire of length L, then current through potentiometer wire is , $I = \frac{\varepsilon}{R+r}$

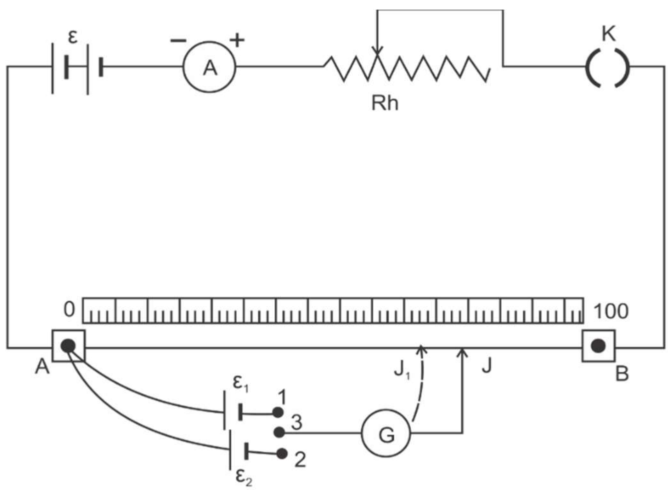
Potential drop across potentiometer wire = Ir = $\left(\frac{\varepsilon}{R+r}\right)r$

Potential gradient of potentiometer wire $k = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L}$

$$\therefore \boxed{V = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L} \times \ell}$$

---

(11) Comparing emfs of two cells using potentiometer **(NOT IN SYLLABUS FOR SESSION 2022-23)**



Two cells whose emfs are to compared are connected as shown in the figure. First connect terminal 1 with terminal 3 such that cell with emf ε_1 comes in the circuit. If ℓ_1 is the balancing length in this case, we can write

$\varepsilon_1 = k\ell_1$ ....(i)

Now disconnect 1 and 3 and connect 2 and 3. Now cell with emf ε_2 comes in the circuit. If ℓ_2 is the balancing length in this case, then

$\varepsilon_2 = k\ell_2$ ....(ii)

From (i) and (ii) we get $\boxed{\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}}$

---

(12) Determining internal resistance of a cell (NOT IN SYLLABUS FOR SESSION 2022-23)

Close key K and note the balancing length. Let it be ℓ_1. Now, emf of the cell, ε = potential difference across length ℓ_1 of the potentiometer wire

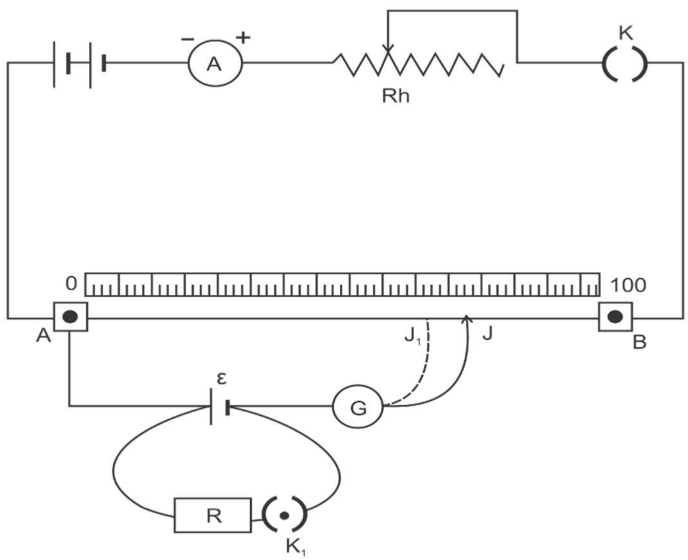
Or $\varepsilon = k\ell_1$

Now close key K_1 so that the resistance R is introduced in the circuit. Again, find the position of null point. Let balancing length in this case be ℓ_2. Then, potential difference between two terminals of the cell, V = potential difference across length ℓ_2 of the potentiometer wire

i.e. $V = k\ell_2$

$\frac{\varepsilon}{V} = \frac{\ell_1}{\ell_2}$

$\therefore \frac{\varepsilon}{V} = \frac{\ell_1}{\ell_2}$



$\because r = \left(\frac{\varepsilon}{V} - 1\right) \times R$ v

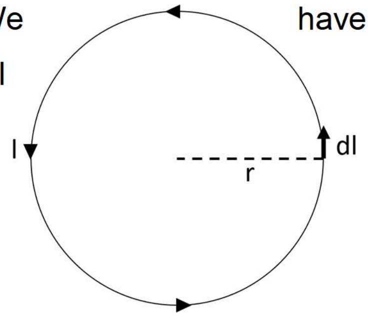
$\therefore \boxed{r = \left(\frac{\ell_1}{\ell_2} - 1\right) \times R}$

Knowing the values of ℓ_1, ℓ_2 and R, internal resistance of the cell can be determined.

MAGNETIC EFFECTS OF CURRENT

(1) Magnetic field at the centre of a circular loop carrying current (V IMP)

Consider a circular current carrying loop carrying current I. We have to find magnetic field at the centre of this loop. Consider a small current element dl on the circumference of this loop.



Clearly angle between dl and r is 90°. Applying Biot Savart's law, we get

$$dB = \frac{\mu_o}{4\pi}\left(\frac{Idl\sin 90^o}{r^2}\right)$$

$$\Rightarrow dB = \frac{\mu_o}{4\pi}\frac{Idl}{r^2}$$

Integrating both sides we get

$$\int dB = \int \frac{\mu_o}{4\pi}\frac{Idl}{r^2}$$

$$\Rightarrow B = \frac{\mu_o}{4\pi}\frac{I}{r^2}\int dl$$

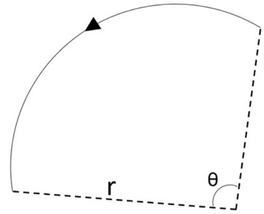
$$\Rightarrow B = \frac{\mu_o}{4\pi}\frac{I}{r^2} \times 2\pi r$$

$$\Rightarrow \boxed{B = \frac{\mu_o I}{2r}}$$

---

(2) Magnetic field due to arc

As complete circle is also an arc which subtends an angle 2π at the centre so by applying the unitary method, we can find the magnetic field at the centre of arc as follows:

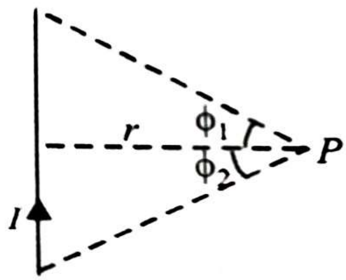


Angle	Magnetic field
2π	$\frac{\mu_o I}{2r}$
1 radian	$\left(\frac{\mu_o I}{2r}\right)\times\frac{1}{2\pi}=\frac{\mu_o I}{4\pi r}$
Any angle θ	$\boxed{B=\frac{\mu_o I}{4\pi r}\times\theta}$

---

(3) Magnetic field due to a straight conductor

Magnetic field at point P at a perpendicular distance r from from a straight cinductor carrying current I is



$$B=\frac{\mu_o I}{4\pi r}(\sin\varphi_1+\sin\varphi_2)$$

Special cases

When length of wire is infinite (or very long) and distance r is very small then

✓ If P lies near one end , then $\varphi_1=90^\circ$ and $\varphi_2=0^\circ$

$$\text{so, } B=\frac{\mu_o I}{4\pi r}(\sin 90^\circ+\sin 0^\circ)$$

$$\Rightarrow \boxed{B=\frac{\mu_o I}{4\pi r}}$$

✓ If P lies near centre, then $\varphi_1 = 90^\circ$ and $\varphi_2 = 90^\circ$

$$\text{so, } B = \frac{\mu_o I}{4\pi r}\left(\sin 90^\circ + \sin 90^\circ\right)$$

$$\Rightarrow \boxed{B = \frac{\mu_o I}{2\pi r}}$$

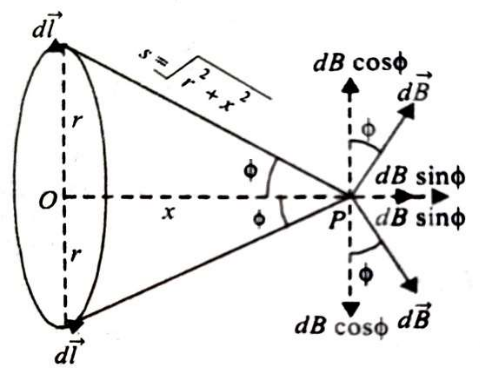
---

(4) Magnetic field on the axis of a circular loop (M IMP)

Small magnetic field due to current element Idl of circular loop of radius r at a point P at distance x from its centre is

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin 90^\circ}{s^2} = \frac{\mu_o}{4\pi}\frac{Idl}{\left(r^2 + x^2\right)}$$

Component $dB\cos\varphi$ due to current element at point P is cancelled by equal and opposite



component $dB\cos\varphi$ of another diagonally opposite current element, whereas the sine components $dB\sin\varphi$ add up to give net magnetic field along the axis. So net magnetic field at point P due to entire loop is

$$\int dB\sin\varphi = \int_0^{2\pi r} \frac{\mu_o}{4\pi}\frac{Idl}{\left(r^2+x^2\right)}.\frac{r}{\left(r^2+x^2\right)^{1/2}}$$

$$\Rightarrow B = \frac{\mu_o Ir}{4\pi\left(r^2+x^2\right)^{\frac{3}{2}}}\int_0^{2\pi r} dl$$

$$\Rightarrow B = \frac{\mu_o Ir}{4\pi\left(r^2+x^2\right)^{\frac{3}{2}}}.2\pi r$$

$$\Rightarrow \boxed{B = \frac{\mu_o Ir^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}}.$$

Which is directed along the axis (a) towards the loop if current in it is in clockwise direction (b) away from the loop if current in it is in anticlockwise direction.

Special points

If point P is far away from the centre of the loop i.e. x >> r then magnetic field at point P is

$B = \frac{\mu_o Ir^2}{2x} = \frac{\mu_o I\pi r^2}{2\pi x^3}$ or $B = \frac{\mu_o IA}{2\pi x^3}$ where A is the area of the circular loop.

If circular loop has N turns then magnetic field strength at its centre is $B = \frac{\mu_o NI}{2r}$ and at any point on the axis of circular loop is $B = \frac{\mu_o NIr^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}$

---

(5) Ampere's circuital law (V IMP)

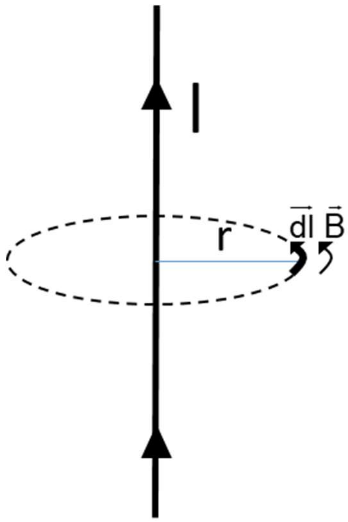
It states that the line integral of magnetic field intensity over a closed loop is μ_o times the total current threading the loop.

$$\int \vec{B}.\vec{dl} = \mu_o I$$

Proof:

Consider a straight conductor carrying current as shown in the figure. Consider a circular Amperian loop of radius r around the conductor. As $\vec{B}$ and $\vec{dl}$ are in same direction so angle between them is 0. Therefore

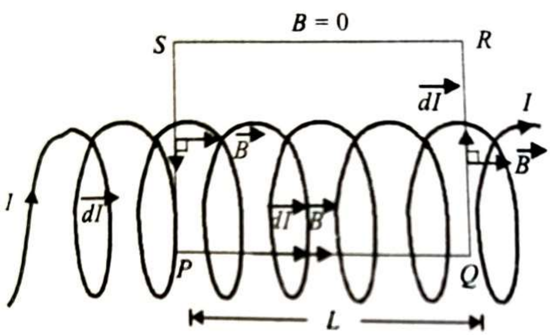
$$\int \vec{B}.\vec{dl}$$
$$= \int B dl \cos 0^\circ$$
$$= \int B dl$$
$$= B \int dl$$
$$= \frac{\mu_o I}{2\pi r} \times 2\pi r$$
$$= \mu_o I$$
$$\therefore \int \vec{B}.\vec{dl} = \mu_o I$$



Applications of ampere's circuital law

(6) Magnetic field intensity at the centre of a long solenoid (V IMP)

Let a solenoid consists of n no. of turns per unit length and carry current I. Then magnetic field lines inside the solenoid are parallel to its axis whereas outside the solenoid the magnetic field is zero. Line integral of magnetic field over a closed loop PQRS shown in the figure is



$$\int \vec{B}.\vec{dl} = \int_P^Q \vec{B}.\vec{dl} + \int_Q^R \vec{B}.\vec{dl} + \int_R^S \vec{B}.\vec{dl} + \int_S^P \vec{B}.\vec{dl}$$

$$= \int_P^Q B.dl\cos 0^\circ + \int_Q^R B.dl\cos 90^\circ + 0 + \int_S^P B.dl\cos 90^\circ$$
$$= B\int_P^Q dl + 0 + 0 + 0 = BL$$

But by Ampere's circuital law

$$\int \vec{B}.\vec{dl} = \mu_o \times \text{total current threading loop PQRS}$$
$$= \mu_o \times \text{number of turns in solenoid PQRS} \times I$$
$$= \mu_o nLI$$

Therefore

$$BL = \mu_o nLI$$
$$\Rightarrow \boxed{B = \mu_o nI}$$

Note: at the ends of the solenoid the magnetic field is $\boxed{B = \frac{1}{2}\mu_o nI}$

---

(7) Force acting on a charged particle moving in a magnetic field

If a charge q is moving with velocity v in a magnetic field of intensity B such that the angle between velocity vector and magnetic field vector is θ, then a force F acts on the particle such that

i) $F \propto q$
ii) $F \propto v$
iii) $F \propto B$
iv) $F \propto \sin\theta$

Combining all these, we get

$$F \propto qvB\sin\theta$$
$$\Rightarrow \boxed{F = qvB\sin\theta}$$

As the value of constant in this relation is 1 in SI units.

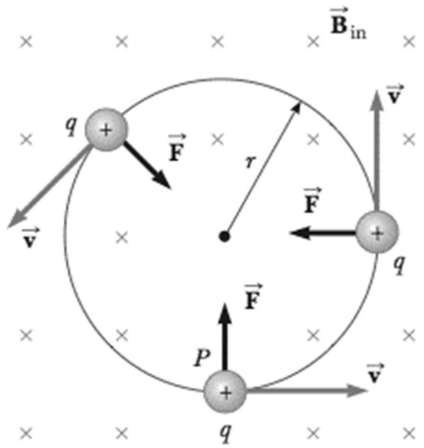
In vector form

$\vec{F} = q(\vec{v} \times \vec{B})$, thus F is perpendicular to the plane containing v and B.

If a charge q enters perpendicularly into a magnetic field, then its path will be circular as force always acts in a direction perpendicular to the direction of motion of motion of the charge. Centripetal force required for circular motion is provided by the magnetic force acting on the particle. Thus

$$\frac{mv^{\cancel{2}}}{r} = q\cancel{v}B$$

$$\frac{mv}{r} = qB$$



1. **Radius of the path (r)**

$$r = \frac{mv}{Bq}$$

2. **Velocity (v)**

$$v = \frac{Bqr}{m}$$

3. **Time period (T)**

$$T = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{Bq\cancel{r}}{m}} = \frac{2\pi m}{Bq}$$

4. **Frequency**

$$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

5. **Angular frequency**

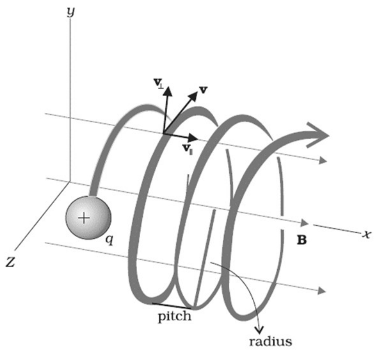
$$\omega = 2\pi\nu = 2\pi \times \frac{Bq}{2\pi m} = \frac{Bq}{m}$$

6. **Kinetic energy**

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Bqr}{m}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}\cancel{m}\frac{B^2q^2r^2}{m^{\cancel{2}}} = \frac{1}{2}\frac{B^2q^rr^2}{m}$$

If charge particle enters at an angle with the direction of magnetic field then split its velocity into rectangular components $v\cos\theta$ along the field and $v\sin\theta$ perpendicular the field as shown. Due to these two components, the motion of the charge is helical.



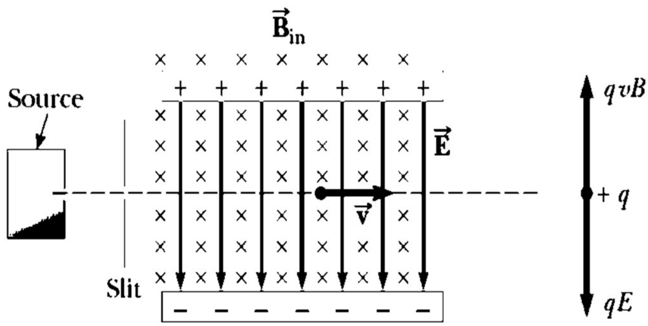
Distance between two turns of the helix is called **pitch(d)** which is given by

$$d = v\cos\theta \times \text{time period} = v\cos\theta \times \frac{2\pi m}{Bq}$$

---

(8) Velocity selector or velocity filter

Consider a situation as shown in the figure in a charge is moving perpendicularly to both electric and magnetic fields such the force the force acting on charge due to both the fields is equal and opposite i.e.



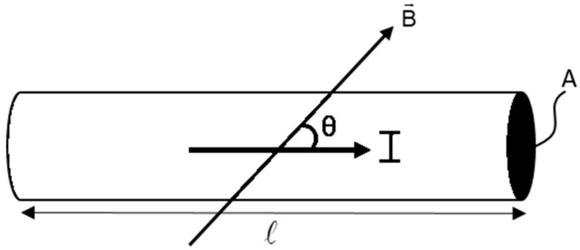
qE = qvB

$$\therefore v = \frac{E}{B}$$

This result is used in velocity selectors or velocity filters in which we have to select a particle with a particular value of velocity.

---

(9) Force acting on a current carrying conductor placed in a magnetic field (V IMP)



Consider a conductor of length ℓ and area of cross section A carrying current I placed in a magnetic field at an angle θ as shown. If number density of electrons in the conductor is n then total number of electrons in the conductor is $A\ell n$.

As force acting on one electron is $f = ev_d B\sin\theta$ where v_d is the drift velocity of electrons.

So the total force acting on the conductor is

$$A\ell nf = A\ell n\left(ev_d B\sin\theta\right)$$
$$= \left(Anev_d\right)\ell B\sin\theta$$
$$\Rightarrow \boxed{F = I\ell B\sin\theta}$$

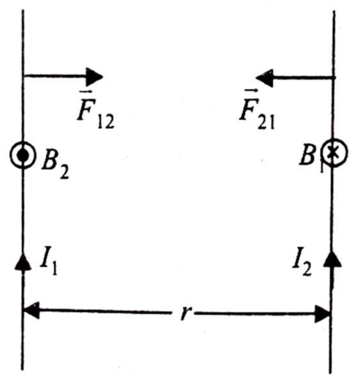
Direction of this force can be determined by Fleming's left hand rule.

---

(10)Force between two parallel straight conductors carrying current (M IMP)

When the currents are in same direction

When two current carrying conductors are placed parallel to each other, each conductor produces a magnetic field around itself. So, one conductor is placed in the magnetic field produced by the other. Using Fleming's left hand rule it can be easily shown that the forces on them are such that they attract each other. Force acting on 1st conductor is given as



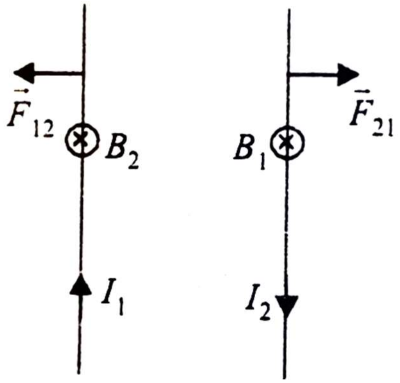
$$F_1 = I_1\ell B_2\sin 90^\circ$$
$$F_1 = I_1\ell\frac{\mu_o I_2}{2\pi r}$$
$$\Rightarrow \boxed{\frac{F_1}{\ell} = \frac{\mu_o I_1 I_2}{2\pi r}}$$

Now force acting on conductor 2 is given by

$$F_2 = I_2 \ell B_1 \sin 90^\circ$$

$$F_2 = I_2 \ell \frac{\mu_o I_1}{2\pi r}$$

$$\Rightarrow \boxed{\frac{F_2}{\ell} = \frac{\mu_o I_1 I_2}{2\pi r}}$$



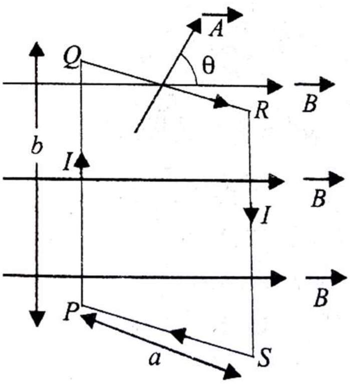
Therefore $F_1 = F_2$

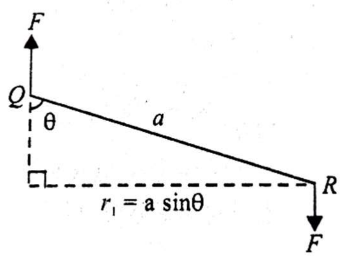
Note: When the current is in opposite direction the conductors will repel each other the magnitude of force will be same as derived above.

---

(11)Torque acting on a current carrying conductor placed in a magnetic field (M IMP)

When a rectangular loop PQRS of sides 'a' and 'b' carrying current I is placed in uniform magentic field B, such that area vector A makes an angle θ with direction of magnetic field, then forces on the arms QR and SP of loop are equal, opposite and collinear, thereby perfectly cancel each other, whereas forces on arms PQ and RS of loop are equal and opposite but not collinear, so they give rise to torque on loop.





Force on side PQ or RS of loop is $F = IbB\sin 90^\circ = IbB$

Perpendicular distance between two non collinear forces $r_\perp = a\sin\theta$

So, torque on the loop is

$$\tau = F_{\perp} = IbBa\sin\theta = I(ab)B\sin\theta$$

or $\boxed{\tau = IAB\sin\theta}$

If loop has N turns then $\boxed{\tau = NIAB\sin\theta}$.

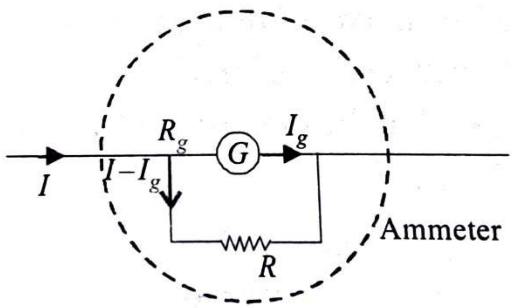
In vector form $\tau = \vec{M}\times\vec{B}$ where M = NIA is called magnetic dipole moment of current loop abd is directed in direction of area vector.

- ✓ If the plane of the loop is normal to the direction of magnetic field i.e. θ = 0° between $\vec{B}$ and $\vec{A}$ then the loop does not experience any torque i.e. $\tau_{min} = 0$
- ✓ If the plane of the loop is parallel to the direction of magnetic field i.e. θ = 90° between $\vec{B}$ and $\vec{A}$ then the loop experience maximum torque $\tau_{max} = NIAB$

---

(12)Conversion of galvanometer into ammeter (M IMP)

A galvanometer can be converted into ammeter by connecting a low shunt resistance in parallel with it, so that most of the current by passes through the shunt resistance, enabling the galvanometer to measure much larger currents.



Thus if a galvanometer of resistance R_g which gives full scale deflection at I_g is to be used to convert into an ammeter capable of measuring a maximum current I , we connect a shunt resistance R in parallel with it which is obtained as

$$V_R = V_G$$

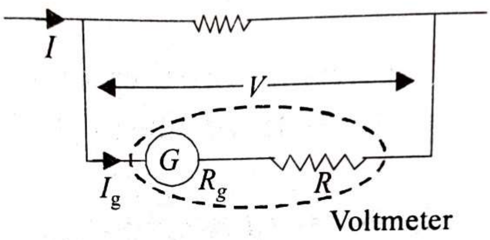
$$\Rightarrow (I - I_g)R = I_g R_g$$

$$\Rightarrow \boxed{R = \frac{I_g R_g}{I - I_g}}$$

---

(13)Conversion of galvanometer into voltmeter (V IMP)

A galvanometer can be converted into voltmeter by connecting high resistance in series with it, so that most of the voltage applied drops across it, enabling the galvanometer to measure much larger voltages.



Thus is the galvanometer of resistance R_g which gives full deflection at current I_g, is to be converted into voltmeter capable of measuring maximum voltage up to V volts, then a high resistance R is connected in series with it which is given by

$$V = I_g R_g + I_g R \quad \text{or} \quad V - I_g R_g = I_g R$$

$$\text{or} \quad \boxed{R = \frac{V}{I_g} - R_g}$$

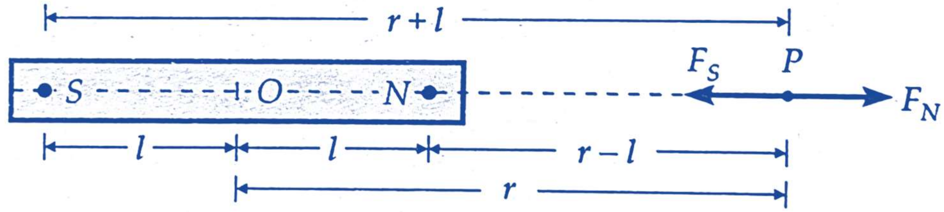
MAGNETISM AND MATTER

In syllabus of 2022-23 its mentioned “qualitative aspects” which means you don’t need to prepare the derivations of this chapter for board exam. But your teacher can give these in half yearly or pre board exams

(1) Magnetic field of a bar magnet at an axial point

Let NS be a bar magnet of length 2l and of pole strength q_m. Suppose the magnetic field is to be determined at a point P which lies on the axis of the magnet at a distance r from its centre, as shown.

Imagine a unit north pole placed at point P. Then from Coulomb's law of magnetic forces, the force exerted by N- pole of strength q_m on unit north pole will be



$$F_N = \frac{\mu_o}{4\pi}\frac{q_m}{(r-\ell)^2}, \quad \text{along } \overrightarrow{NP}$$

Similarly, the force exerted by S pole on unit north pole is

$$F_S = \frac{\mu_o}{4\pi}\frac{q_m}{(r+\ell)^2}, \quad \text{along } \overrightarrow{PS}$$

Therefore, the strength of the magnetic field $\vec{B}$ at point P is

$B_{axial} = F_N - F_S \ (\because F_N > F_S)$

$$= \frac{\mu_o q_m}{4\pi}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

$$= \frac{\mu_o q_m}{4\pi}\cdot\frac{4r\ell}{(r^2-\ell^2)^2}$$

But $q_m 2\ell = M$ (magnetic dipole moment)

So, $\boxed{B_{axial} = \frac{\mu_o}{4\pi}\cdot\frac{2Mr}{(r^2-\ell^2)^2}}$

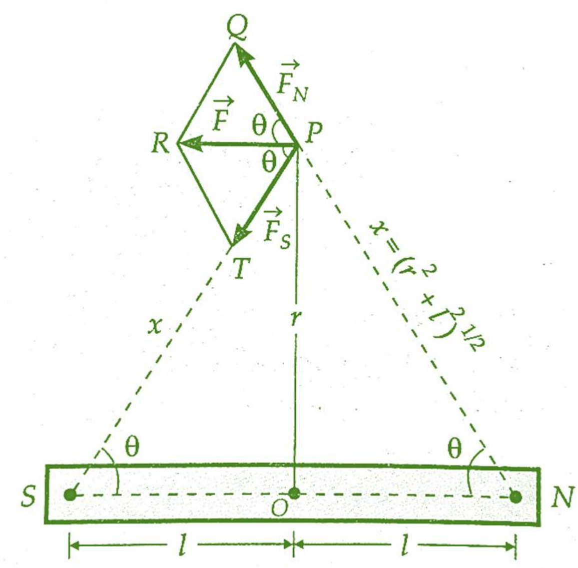
For short bar magnet $\ell << r$, therefore , we have

$$B_{axial} = \frac{\mu_o}{4\pi}\frac{2M}{r^3}$$

---

(2) Magnetic field of a bar magnet at an equatorial point

Consider a bar magnet NS of length 2ℓ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P lying on the equatorial line of the magnet NS at a distance r from its centre as shown.



Imagine a unit north pole placed at point P. Then from Coulomb's law of magnetism, the force exerted by north pole of the magnet on unit north pole is

$$F_N = \frac{\mu_o}{4\pi}.\frac{q_m}{x^2} \text{ along NP}$$

Similarly, the force exerted by the S pole of the magnet on unit north pole is

$$F_S = \frac{\mu_o}{4\pi}.\frac{q_m}{x^2} \text{ along PS}$$

As the magnitude of FS and FN are equal, so their vertical components get cancelled while the horizontal components add up along PR.

Hence, magnetic field at the equatorial point is

$$B_{eq} = F_N \cos\theta + F_S \cos\theta$$

$$\Rightarrow B_{eq} = 2F_N \cos\theta$$

$$\Rightarrow B_{eq} = 2\frac{\mu_o}{4\pi}\frac{q_m}{x^2}\frac{\ell}{x}$$

$$\Rightarrow \boxed{B_q = \frac{\mu_o}{4\pi}\frac{M}{\left(r^2+\ell^2\right)^{\frac{3}{2}}}}$$

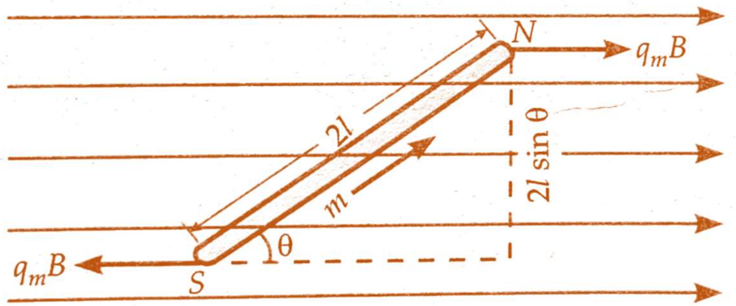
For short magnet $\ell << r$, so we have

$$B_{eq} = \frac{\mu_o}{4\pi}\frac{M}{r^3} \text{ along PR}$$

---

(3) Torque on a magnetic dipole in a magnetic field

Consider a bar magnet NS of length 2ℓ placed in a uniform magnetic field B. Let q_m be the pole strength of each pole. Let the magnetic axis of the bar magnet make an angle θ with the field B as shown



Force on north pole $= q_m B$ along B

Force on south pole = $= q_m B$ opposite to B

The forces on the two poles are equal and opposite. They form a couple. So, torque is given by

$\tau = \text{Force} \times \text{perpendicular distance}$

$\Rightarrow \tau = q_m B \times 2\ell$

$\Rightarrow \tau = \left(q_m 2\ell\right)B$

$\boxed{\tau = \vec{M} \times \vec{B}}$

Special cases

1. When angle between $\vec{M}$ and $\vec{B}$ is 0°, $\sin 0^\circ = 0$, therefore $\tau = 0$, this is the condition of stable equilibrium.
2. When angle between $\vec{M}$ and $\vec{B}$ is 180°, sin180° = 0, therefore $\tau = 0$, this is the condition of unstable equilibrium.
3. If angle between $\vec{M}$ and $\vec{B}$ is 90°, $\sin 90^\circ = 1 \therefore \tau_{max} = MB$

---

(4) Potential energy of a magnetic dipole in a magnetic field

The torque which acts on magnetic dipole in external magnetic field tends to align the dipole in the direction of magnetic field. If the dipole is rotated against the action of this torque, work has to be done, this work is stored in the dipole in the form of potential energy.

Small amount of work done dW done in rotating the dipole through small angle $d\theta$ is $dW = \tau d\theta = MB\sin\theta d\theta$

Total work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta$$

$$\Rightarrow W = MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow W = -MB\left[\cos\theta_2 - \cos\theta_1\right]$$

This work is stored in the dipole in the form of potential energy

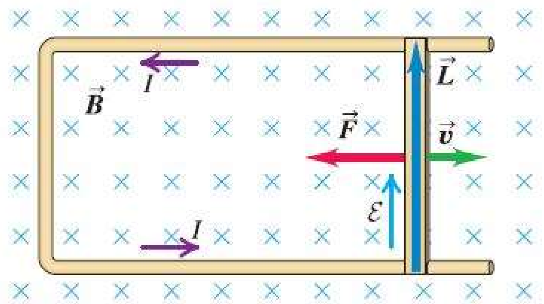
Thus, $\boxed{U = -MB\left[\cos\theta_2 - \cos\theta_1\right]}$

ELECTROMAGNETIC INDUCTION

(1) MOTIONAL EMF

- **In case of a conductor in translational motion in a magnetic field (V imp)**

The figure shows a rectangular conducting loop PQRS in the plane of the paper. The conductor is free to move. Let the conductor QR be moved towards the right with a constant velocity v. The area enclosed by the loop PQRS increases.



Therefore, the amount of magnetic flux linked with the loop increases. An e.m.f. is induced in the loop.

If the length QR = l and the distance through which is it pulled is *x*, then emf induced between ends Q and R is , $\varepsilon = -\dfrac{d\phi}{dt}$

As $\phi = BA\cos 0^\circ = BA$

$$\therefore \varepsilon = -\frac{d}{dt}(BA) = -B\frac{d}{dt}(A) = -B\frac{d}{dt}(\ell x)$$

$$\therefore \varepsilon = -B\ell\frac{dx}{dt}$$

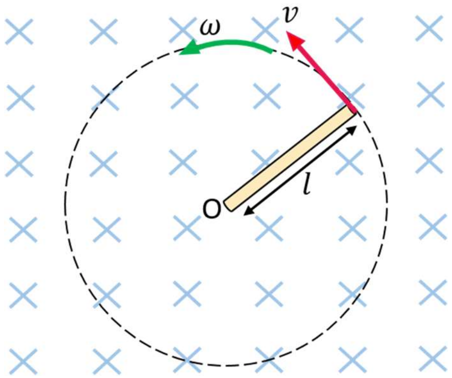
or $\boxed{\varepsilon = -B\ell v}$

This is called motional emf as it is produced due to motion of a conductor in a magnetic field.

- **In case of a conductor in rotational motion in a magnetic field (imp)**

Average linear velocity of the rod is $v_{avg} = \frac{0+\omega l}{2} = \frac{\omega l}{2}$

therefore, magnitude of emf induced between the ends of the rod is



$$\varepsilon = Bl\left(\frac{\omega l}{2}\right) = \boxed{\frac{1}{2}Bl^2\omega}$$

---

(2) Energy considerations **(v imp)**

Force on the movable arm

$$F = I\ell B \sin 90^\circ = \left(\frac{B\ell v}{R}\right)\ell B = \frac{B^2\ell^2 v}{R}$$

Power delivered by external force $= Fv = \left(\frac{B^2\ell^2 v}{R}\right)v = \frac{B^2\ell^2 v^2}{R}$

Power dissipated as heat loss = $I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2 v^2}{R}$

Clearly, mechanical power delivered = electrical power dissipated, which proves the law of conservation of energy.

---

(3) Self-inductance of solenoid **(M IMP)**

Let us consider a solenoid of N turns with length ℓ and area of cross section A. It carries a current I. If B is the magnetic field at any point inside the solenoid, then

Magnetic flux per turn = B $\times$ area of each turn

But, $B = \dfrac{\mu_0 N I}{\ell}$

Therefore, magnetic flux per turn $= \dfrac{\mu_0 N I A}{\ell}$

Hence, the total magnetic flux (φ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\phi = \frac{\mu_0 N I A}{\ell} \times N$$

$$\text{i.e. } \phi = \frac{\mu_0 N^2 I A}{\ell} \qquad \text{....... (i)}$$

If L is the coefficient of self-induction of the solenoid, then

$$\phi = LI \qquad \text{........ (ii)}$$

From equations (i) and (ii)

$$LI = \frac{\mu_0 N^2 I A}{\ell}$$

$$\therefore \qquad \boxed{L = \frac{\mu_0 N^2 A}{\ell}}$$

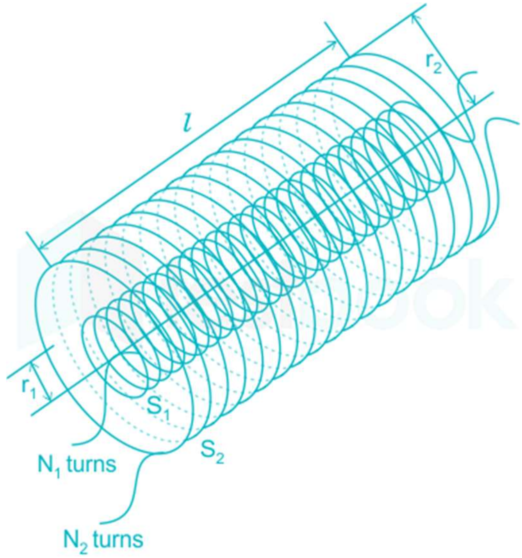
If the core is filled with a magnetic material of permeability μ, then

$$L = \frac{\mu N^2 A}{\ell}$$

---

(4) Mutual inductance of two solenoids (M IMP)

S_1 and S_2 are two long solenoids each of length l. The solenoid S_2 is wound closely over the solenoid S_1. N_1 and N_2 are the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1. The magnetic field B_1 produced at any point inside the solenoid S_1 due to the current I_1 is



$$B_1 = \frac{\mu_o N_1 I_1}{\ell} \quad \text{....... (1)}$$

The magnetic flux linked with each turn of S_2 is equal to B_1A.Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1 A N_2$$

Substituting for B_1 from equation (1)

$$\phi_2 = \left(\frac{\mu_o N_1 I_1}{\ell}\right) A N_2$$

$$\phi_2 = \left(\frac{\mu_o N_1 N_2 I_1}{\ell}\right) A \quad \text{..... (2)}$$

$$\text{But } \phi_2 = M I_1 \quad \text{..... (3)}$$

Where M is the coefficient of mutual induction between S_1 and S_2.

From equations (2) and (3)

$$M I_1 = \left(\frac{\mu_o N_1 N_2 I_1}{\ell}\right) A$$

$$\therefore M = \frac{\mu_o N_1 N_2 A}{\ell}$$

If the core is filled with a magnetic material of permeability

$$\boxed{M = \frac{\mu N_1 N_2 A}{\ell}}$$

Thus, the coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity. The unit of coefficient of mutual induction is henry.

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

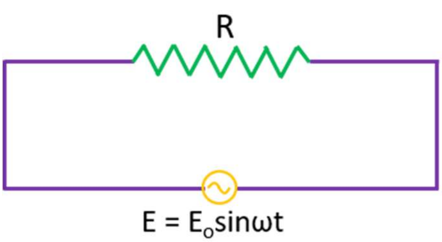
The coefficient of mutual induction between a pair of coils depends on the following factors

1) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.
2) Proximity of the coils

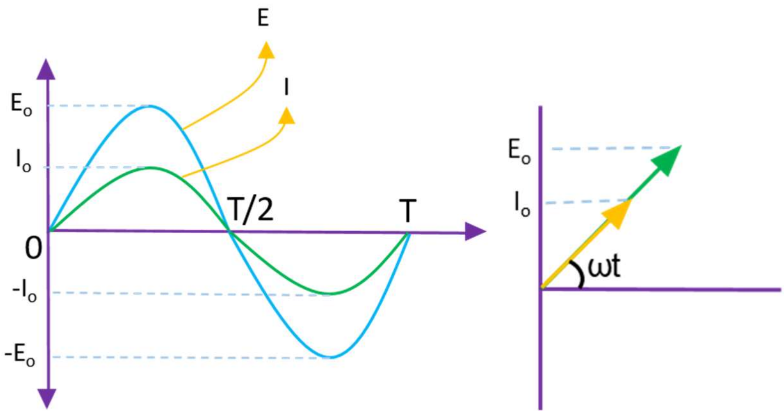
ALTERNATING CURRENT

(1) AC circuit containing resistor only (IMP)

Consider a resistor of resistance R connected to an alternating emf source as shown.



Let the applied emf be $E = E_o \sin\omega t$.



Dividing both sides by R, we get

$$\frac{E}{R}=\frac{E_o}{R}\sin\omega t$$

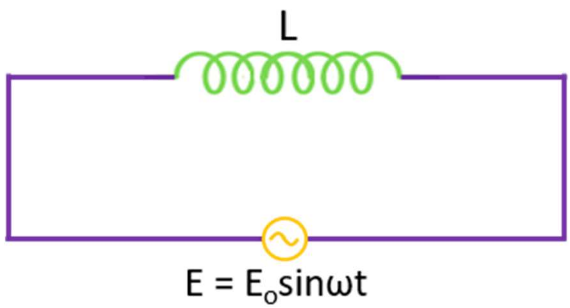
$$\frac{E}{R}=\frac{E_o}{R}\sin\omega t$$

$$\Rightarrow \boxed{I=I_o\sin\omega t}$$

Therefore, current and voltage are in same phase.

---

(2) AC circuit containing inductor only (V IMP)



Consider an inductor of inductance L connected to an AC source as shown

Let the applied emf be $E=E_o\sin\omega t$.

Since $E=L\frac{dI}{dt}$

Therefore $dI=\frac{E}{L}dt$

$$dI = \frac{E}{L}dt$$

$$\Rightarrow dI = \frac{E_o \sin \omega t}{L}dt$$

$$\Rightarrow \int dI = \int \frac{E_o}{L} \sin \omega t dt$$

$$\Rightarrow I = \frac{E_o}{L} \int \sin \omega t dt$$

$$\Rightarrow I = \frac{E_o}{\omega L}[-\cos \omega t]$$

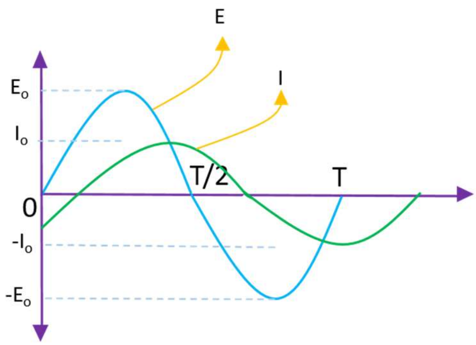
$$\Rightarrow I = -\frac{E_o}{\omega L}\left[\sin\left(\frac{\pi}{2} - \theta\right)\right]$$

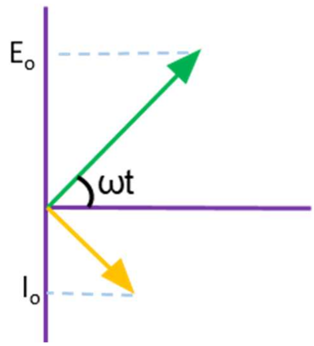
$$\Rightarrow I = \frac{E_o}{\omega L}\left[\sin\left(\theta - \frac{\pi}{2}\right)\right]$$

$$\Rightarrow \boxed{I = I_o\left[\sin\left(\theta - \frac{\pi}{2}\right)\right]} \qquad \text{where } I_o = \frac{E_o}{\omega L}$$

Thus, there is a phase difference of $\frac{\pi}{2}$ between current and voltage in a purely inductive circuit.

Phasors





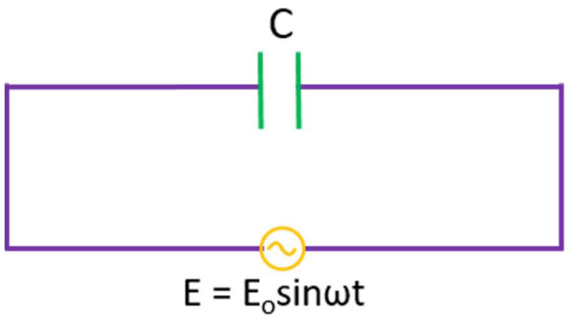
---

(3) AC circuit containing capacitor only (V IMP)

Consider an inductor of inductance L connected to an AC source as shown

Let the applied emf be $E = E_o \sin \omega t$.

Since Q = CE



$$Q = CE_o \sin \omega t$$

$$Q = CE_o \sin \omega t$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}[CE_o \sin \omega t]$$

$$\Rightarrow I = \omega CE_o \cos \omega t$$

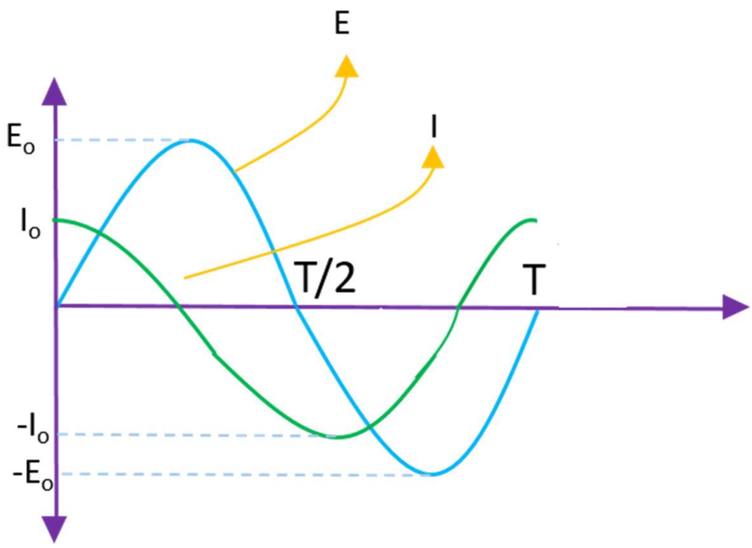
$$\Rightarrow I = \frac{E_o}{\frac{1}{\omega C}} \cos \omega t$$

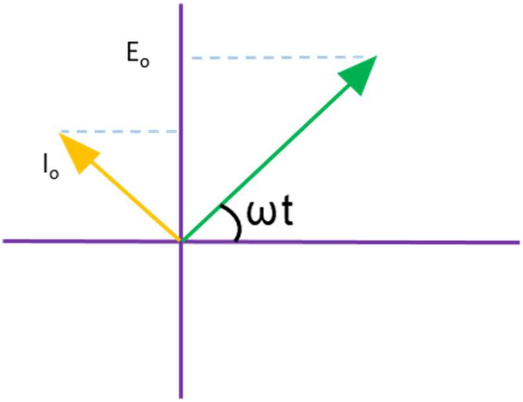
$$\Rightarrow \boxed{I = I_o \sin(\omega t + \frac{\pi}{2})}$$

Where where $\frac{E_o}{\frac{1}{\omega C}} = I_o$

Thus current leads the voltage by a phase of $\frac{\pi}{2}$ in a purely capacitive circuit.

Phasors





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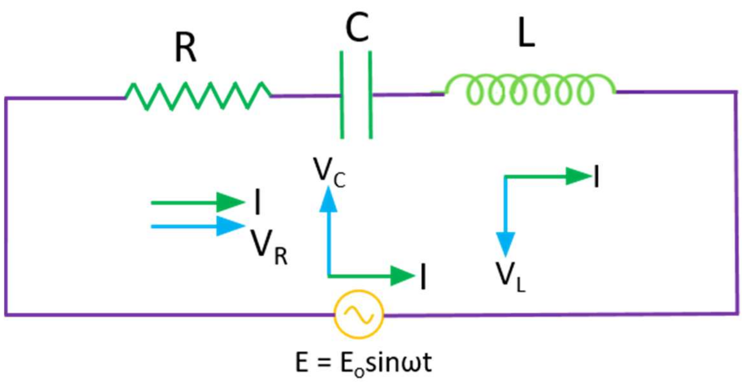
(4) Impedance in series LCR circuit (M IMP)

Consider a resistor of resistance R, inductor of inductance L and capacitor of capacitance C connected in series to an alternating EMF source as shown:



Voltage across all the components is shown in the diagram below

$$V = \sqrt{(V_L - V_C)^2 + V_R^2}$$

$$\because V_L = IX_L,\ V_R = IR,\ V_C = IX_C$$

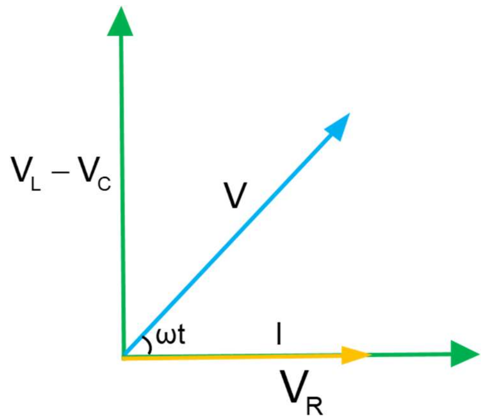
$$\therefore V = \sqrt{(IX_L - IX_C)^2 + (IR)^2}$$

$$\Rightarrow V = \sqrt{I^2\left\{(X_L - X_C)^2 + R^2\right\}}$$

$$\Rightarrow V = I\sqrt{\left\{(X_L - X_C)^2 + R^2\right\}}$$

$$\Rightarrow \frac{V}{I} = \sqrt{\left\{(X_L - X_C)^2 + R^2\right\}}$$

$$\Rightarrow Z = \sqrt{(X_L - X_C)^2 + R^2}$$



Where Z is called the impedance of the circuit.

---

(5) Resonating frequency in series LCR circuit (IMP)

Resonance occurs when inductive reactance becomes equal to capacitive reactance

$$X_L = X_C$$

$$\Rightarrow 2\pi v_r L = \frac{1}{2\pi v_r C}$$

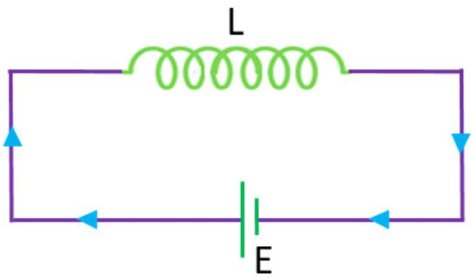
$$\Rightarrow (2\pi v_r)^2 = \frac{1}{LC}$$

$$\Rightarrow 2\pi v_r = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \boxed{v_r = \frac{1}{2\pi\sqrt{LC}}}$$

---

(6) Energy stored in an inductor (M IMP)



Consider an inductor of inductance L connected to a voltage source E as shown in figure above. Let current at any instant be I.

As we know that instantaneous power is given by

$$P = EI$$

$$\text{As } E = L\frac{dI}{dt}$$

$$\text{so, } P = LI\frac{dI}{dt}$$

$$\because P = \frac{dW}{dt}$$

$$\therefore \frac{dW}{dt} = LI\frac{dI}{dt}$$

$$\Rightarrow dW = LIdI$$

So, total work done by source to build a max. current I_o in the circuit is

$$\Rightarrow W = \int_0^{I_o} LIdI$$

$$\Rightarrow W = L\left[\frac{I^2}{2}\right]_0^{I_o}$$

$$\Rightarrow W = L\left[\frac{I_0^2}{2} - 0\right]$$

$$\Rightarrow W = \frac{1}{2}LI_o^2$$

This work is stored in the circuit as magnetic potential energy. So,

$$\boxed{U_B = \frac{1}{2}LI_o^2}$$

(7) Power in series LCR circuit (M IMP)

Let a voltage $E = E_o \sin\omega t$ be applied to a series LCR circuit and current flowing through it is $I_o \sin(\omega t - \varphi)$, so instantaneous power supplied to the source is

$$P = EI = E_o \sin\omega t \times I_o \sin(\omega t - \varphi)$$
$$= \frac{E_o I_o}{2}\left[\cos\varphi - \cos(2\omega t + \varphi)\right]$$

The average power over a cycle is given by the average of the two terms in RHS of above equation. It is only the second term which is time dependent. Its average is zero (the positive half of the cosine cancels the negative half).

Therefore

$$P = \frac{E_o I_o}{2}\cos\varphi = \frac{E_o}{\sqrt{2}}\frac{I_o}{\sqrt{2}}\cos\varphi$$
$$\Rightarrow \boxed{P = E_{rms} I_{rms} \cos\varphi}$$

For purely inductive or purely capacitive circuit

$\varphi = 90^\circ \Rightarrow \cos\varphi = 0 \therefore P = 0$

ANSWERS

RAY OPTICS

Derive a relation between critical angle and refractive index of a medium.

Consider a light ray travelling from denser medium (b) to a rarer medium a.

According to Snell's law,

$$ {}^{a}\mu_{a} = \frac{\sin i}{\sin r} $$

Where, b represents denser medium to rarer medium.

$$ \text{At } i = i_c,\ r = 90^\circ $$

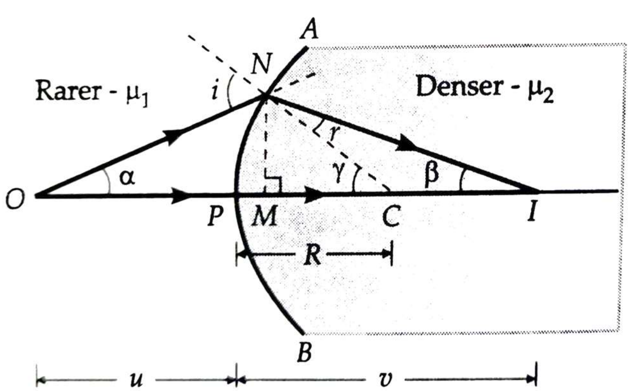
$$ {}^{b}\mu_{a} = \frac{\sin i_c}{\sin 90^\circ} = \sin i_c $$

$$ \text{But } {}^{b}\mu_{a} = \frac{1}{{}^{a}\mu_{b}} $$

Therefore

$$ \boxed{ {}^{a}\mu_{b} = \frac{1}{\sin i_c} } $$

Derive the relation between the distance of object, distance of image and radius of curvature of convex spherical surface, when refraction takes place from rarer to denser medium and image formed is real.



Consider an object placed at O and its real image is formed at I as shown.

In ΔNOC, i is an exterior angle, therefore,

$i = \alpha + \gamma$

Similarly, from ΔNIC, we have

$\gamma = r + \beta$

$\Rightarrow r = \gamma - \beta$

Suppose, all the rays are paraxial. Then the angles i, r, α, β and γ will be small.

$$\therefore \alpha \approx \tan\alpha = \frac{NM}{OM} \approx \frac{NM}{OP}$$

$$\beta \approx \tan\beta = \frac{NM}{MI} \approx \frac{NM}{PI}$$

$$\gamma \approx \tan\gamma = \frac{NM}{MC} \approx \frac{NM}{PC}$$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and r are small, therefore

$$\frac{i}{r} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \mu_1 i = \mu_2 r$$

$$\Rightarrow \mu_1(\alpha + \gamma) = \mu_2(\gamma - \beta)$$

$$\Rightarrow \mu_1\left(\frac{NM}{OP} + \frac{NM}{PC}\right) = \mu_2\left(\frac{NM}{PC} - \frac{NM}{PI}\right)$$

$$\Rightarrow \mu_1\left(\frac{1}{OP} + \frac{1}{PC}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right)$$

$$\Rightarrow \frac{\mu_1}{OP} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using Cartesian sign convention,

Object distance $\quad OP = -u$

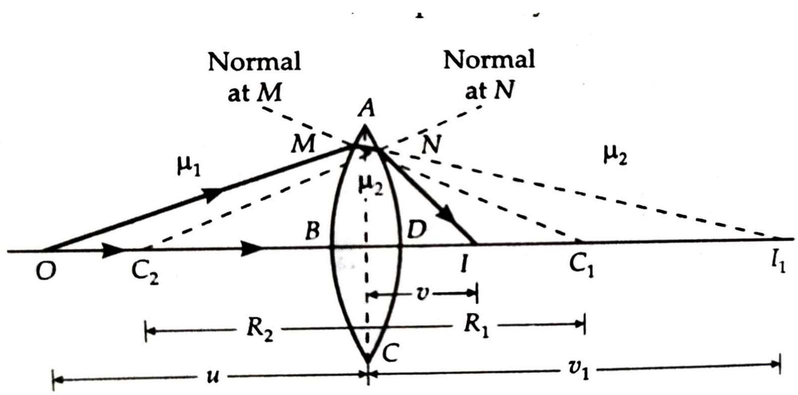
Image distance $\quad PI = +v$

Radius of curvature PC = +R

$$\therefore \boxed{\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}}$$

Derive the expression for lens maker's formula.

Consider an object placed at O whose final image is formed at I as shown. Let the image formed by first surface is at I_1. This image will act as on abject for the second surface.



For refraction at first surface, we have

$$\therefore \frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \qquad \text{(i)}$$

for refraction at second surface we have

$$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{R_2} \qquad \text{(ii)}$$

adding (i) and (ii) we get

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

If object is placed at infinity ($u=\infty$), the image is formed at focus, i.e. v = f. Therefore,

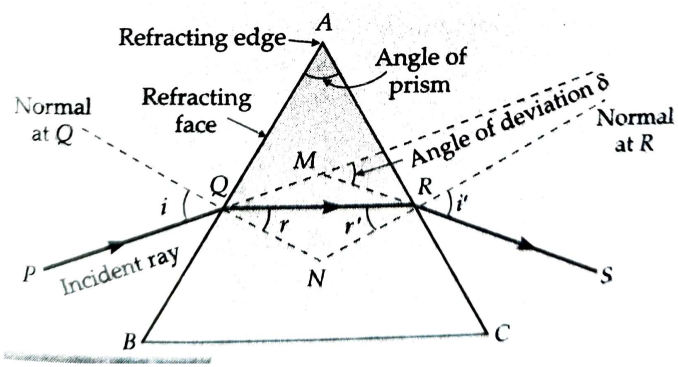
$$\frac{1}{f}=\left[\frac{\mu_2-\mu_1}{\mu_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{f} = \left[\frac{\mu_2}{\mu_1} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \boxed{\frac{1}{f} = \left({}^1\mu_2 - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

This result is lens maker's formula.

Derive a relation between angle of deviation, angle of prism and refractive index of prism.



Consider a ray PQ incident of one face of a prism as shown. The path of ray inside the prism and refracted ray is also shown.

From quadrilateral AQNR

$$A + \angle QNR = 180^\circ$$
From the triangle QNR
$$r + r' + \angle QNR = 180^\circ$$
$$\therefore A = r' + r$$

Now, from the triangle MQR, the deviation produced by the prism

$$\delta = \angle MQR + \angle MRQ = (i - r) + (i' - r')$$
$$\text{or} \quad \delta = (i + i') - (r + r')$$
$$\text{or} \quad \delta = i + i' - A$$
$$\text{or} \quad i + i' = A + \delta$$

For refraction at face AB, we have

$$\mu = \frac{\sin i}{\sin r} = \frac{i}{r} \Rightarrow i = \mu r$$

For refraction at face AC, we have

$$\mu = \frac{\sin i'}{\sin r'} = \frac{i'}{r'} \Rightarrow i' = \mu r'$$

Hence deviation produced by the prism is

$$\delta = i + i' - A = \mu r + \mu r' - A$$
$$\Rightarrow \delta = \mu(r + r') - A = \mu A - A$$
$$\Rightarrow \boxed{\delta = (\mu - 1)A}$$

Derive prism formula

Or

Derive a relation for refractive index of a prism in terms of angle of minimum deviation.

When a prism is in the position of minimum deviation, a ray of light passes symmetrically (parallel to base) through the prism, so that

$$i = i',\ r = r',\ \delta = \delta_m$$

As

$$A + \delta = i + i'$$
$$\therefore A + \delta_m = i + i' \quad \text{or} \quad i = \frac{A + \delta_m}{2}$$
$$\text{Also} \quad A = r + r' = r + r = 2r$$
$$\therefore \qquad r = \frac{A}{2}$$

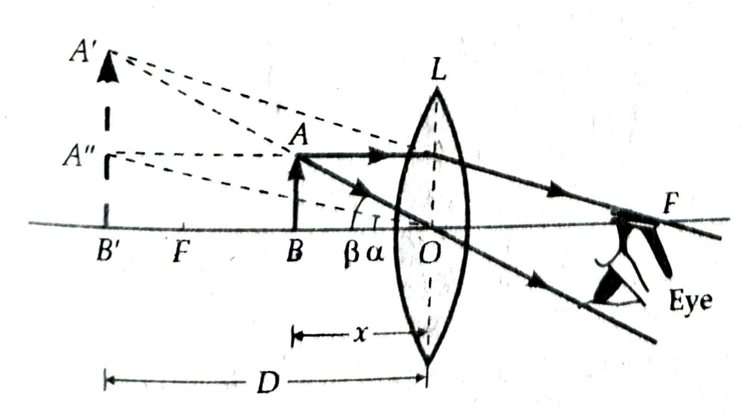
From Snell's law, the refractive index of the material of the prism will be

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \boxed{\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$

Derive an expression for magnifying power of a simple microscope when final image in formed at

a. Least distance of distinct vision.
b. Infinity

When final image is formed at least distance of distinct vision



The image A'B' of an object AB is formed at least distance of distinct vision 'D' as shown. Let $\angle A'OB' = \beta$. Imagine the object AB to be placed to position $A''B'$ at distance D from the lens. Let $\angle A''OB' = \alpha$. Then, magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad \text{[since } \alpha \text{ and } \beta \text{ are small]}$$

$$= \frac{AB/OB}{A''B'/OB'} = \frac{AB/OB}{AB/OB'} \qquad [\because A''B' = AB]$$

$$= \frac{OB'}{OB} = \frac{-D}{-x}$$

$$\text{or } m = \frac{D}{x}$$

Let f be the focal length of the lens. As the image is formed at least distance of distinct vision from the lens, so

$$v = -D$$

Using thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we get

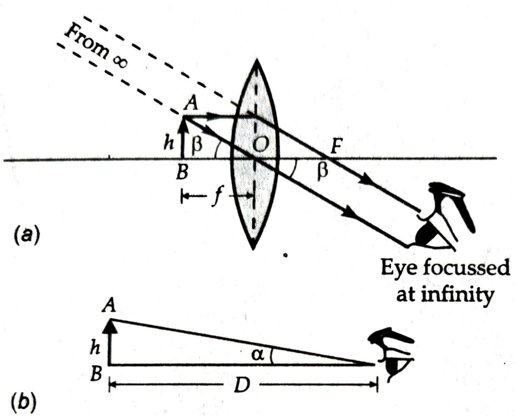
$$\frac{1}{-D}-\frac{1}{-x}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{D}+\frac{1}{x}$$

$$\Rightarrow \frac{D}{x}=1+\frac{D}{f}$$

$$\Rightarrow \boxed{m=1+\frac{D}{f}}$$

When final image is formed at infinity



From fig (a)

$$\tan\beta=\frac{h}{f}$$

From fig (b)

$$\tan\alpha=\frac{h}{D}$$

$$\therefore m=\frac{h/f}{h/D}$$

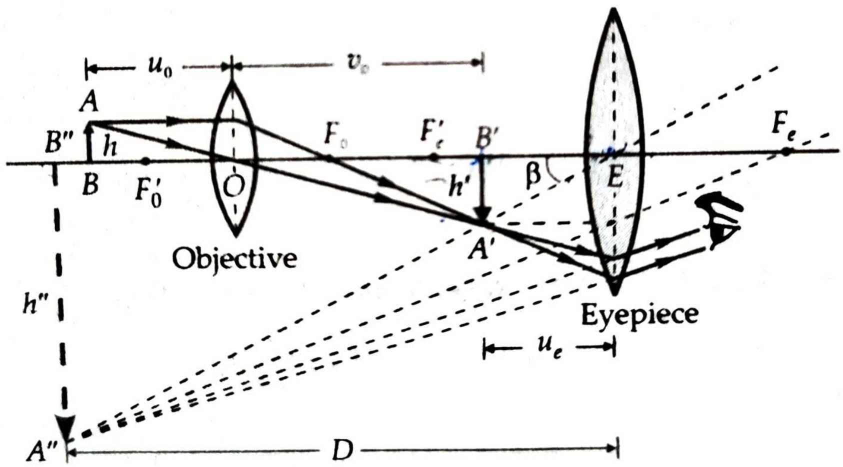
$$\Rightarrow \text{or } \boxed{m=\frac{D}{f}}$$

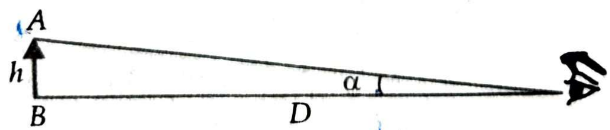
Derive an expression for magnifying power of a compound microscope when final image in formed at

a. **Least distance of distinct vision.**

b. **Infinity**

When final image is formed at least distance of distinct vision





The object AB is placed at u_o slightly larger than the focal length f_o of the objective O. The object forms a real, inverted and magnified image A'B' on the other side of the lens. This image acts as an object for the eyepiece which essentially acts like a simple microscope. The eyepiece E forms a virtual and magnified final image A''B'' of the object.

$$\text{Magnifying power, } m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \frac{h'}{h}.\frac{D}{u_e} = m_o m_e$$

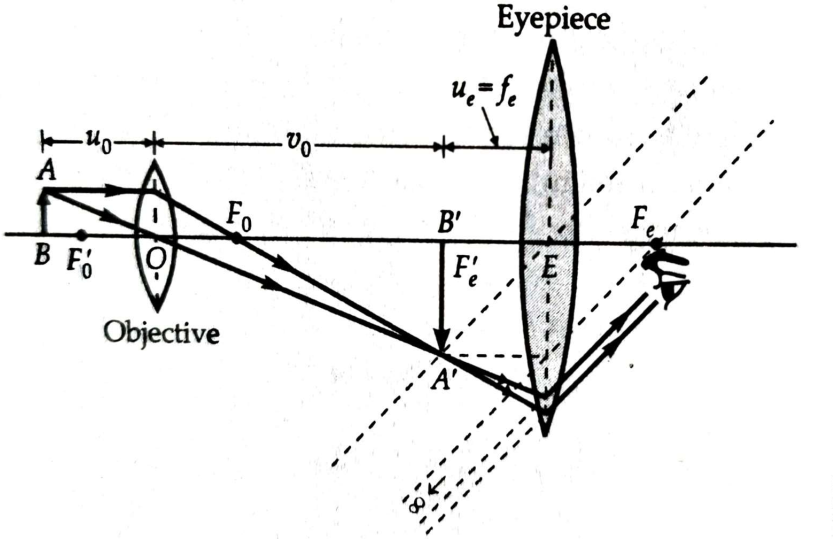
$$\text{Here, } m_o = \frac{h'}{h} = \frac{v_o}{u_o}$$

As the eyepiece acts as a simple microscope, so

$$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e}$$

$$\therefore m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

When final image is formed at infinity



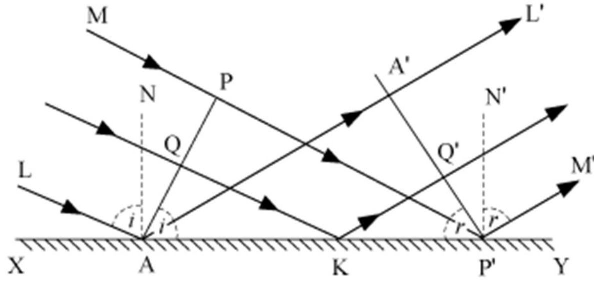
$$m_e = \frac{D}{f_e}$$

$$\therefore \boxed{m = -\frac{L}{f_o} \times \frac{D}{f_e}}$$

WAVE OPTICS

Derive law of reflection using Huygens principles (or wave theory of light).

Consider a beam of light LM, whose wave front AP reaches A'P' in time t, hence for any point Q on the AP wave front must also reach A'P' in time t.



Let speed of light in the medium be c, then

PP' = ct and AA' = ct

In $\Delta AP'P$ and $\Delta AA'P'$

$PP' = AA'$ [Proved above]

$AP' = AP'$ [common]

$\angle APP' = \angle AA'P'$ [both 90°]

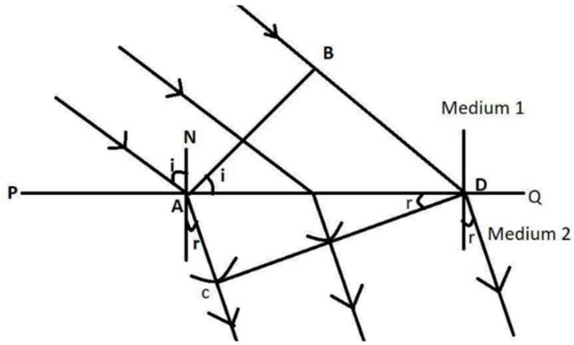
$\therefore \Delta AP'P \cong \Delta AA'P'$ [by RHS congruence rule]

$\therefore \angle i = \angle r$ [by cpct]

Derive Snell's law using Huygens principles (or wave theory of light).

Let speed of light in medium 1 be c_1 and speed of light in medium 2 be c_2.

Let time taken by light to travel from AP to A'P' be t, then



$PP' = c_1 t,\ AA' = c_2 t$

In ΔABD

$$\sin i = \frac{BD}{AD}$$

In ΔADC

$$\sin r = \frac{AC}{AD}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{BD/AD}{AC/AD} = \frac{BD}{AC} = \frac{c_1 t}{c_2 t} = \frac{c_1}{c_2}$$

Also $^1\mu_2 = \dfrac{c_1}{c_2}$

$$\therefore {}^1\mu_2 = \frac{\sin i}{\sin r}$$

Which is Snell's law.

Derive an expression for amplitude of resultant wave when two waves superimpose on each other.

Suppose the displacement of two light waves from two coherent sources at point P on the observation screen at any time t are given by

$$y_1 = a_1 \sin\omega t$$
$$y_2 = a_2 \sin(\omega t + \varphi)$$

Where a_1 and a_2 are the amplitudes of two light waves, φ is the constant phase difference between the two waves. By the superposition principle, the resultant displacement at point P is

$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \varphi)$$
$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\varphi + a_2 \cos\omega t \sin\varphi$$
or
$$y = (a_1 + a_2 \cos\varphi)\sin\omega t + a_2 \sin\varphi \cos\omega t$$

Put

$$(a_1 + a_2\cos\varphi) = A\cos\theta \qquad \text{....(i)}$$
and $$a_2 \sin\varphi = A\sin\theta \qquad \text{.....(ii)}$$

Then

$$y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$$
or $$y = A\sin(\omega t + \theta)$$

Which is the equation of resultant wave.

Squaring and adding (i) and (ii), we get

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\varphi)^2 + a_2^2\sin^2\varphi$$
$$A^2(\cos^2\theta + \sin^2\theta) = a_1^2 + a_2^2(\cos^2\varphi + \sin^2\varphi) + 2a_1a_2\cos\varphi$$
$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$$

Derive an expression for intensity at any point on the observation screen in young's double slit experiment.

Since intensity of a wave $\propto$ (amplitude)^2

Let $I = kA^2$, $I_1 = ka_1^2$, $I_2 = ka_2^2$ where k is proportionality constant.

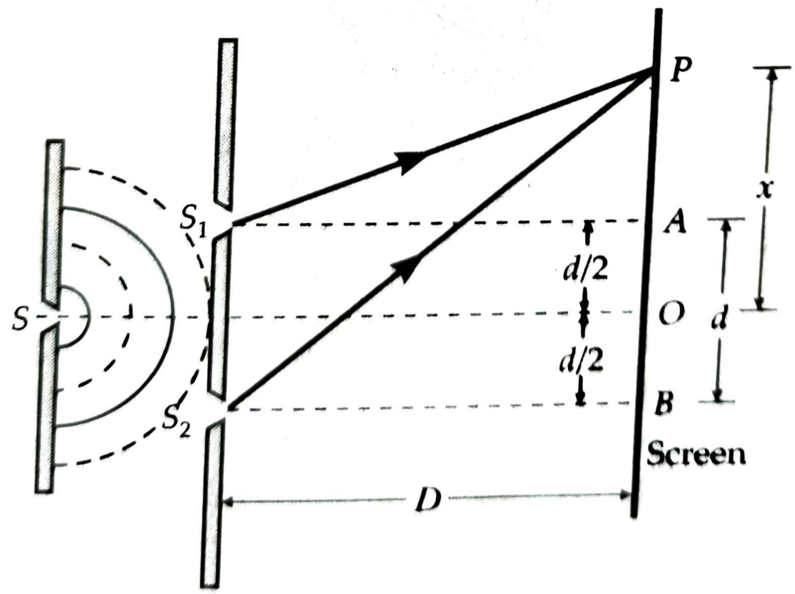
The above equation can be written as

$$kA^2 = ka_1^2 + ka_2^2 + 2\sqrt{k}a_1\sqrt{k}a_2 \cos\varphi$$

or

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi}$$

Derive an expression for fringe width in Young's double slit experiment.



From the diagram of experimental setup of Young's double slit experiment, consider a point P on the screen at distance x from centre O. The nature of the interference at the point P depends on path difference,

$$p = S_2P - S_1P$$

From right angled ΔS_2BP and ΔS_1AP

$$S_2P^2 - S_1P^2 = \left[S_2B^2 + PB^2\right] - \left[S_1A^2 + PA^2\right]$$

$$= \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

Or

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

or $$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

In practice, the point P lies very close to O, therefore

$S_1P \simeq S_2P \simeq D$.Hence

$$p = S_2P - S_1P = \frac{2xd}{2D}$$

or $$p = \frac{xd}{D}$$

Position of bright fringes

$$p = \frac{x_n d}{D} = n\lambda$$

or $$x_n = \frac{n\lambda D}{d}$$

Where x_n is the distance of nth bright band from centre of screen.

Width of dark fringe = distance between two consecutive bright fringes

$$\beta_{dark} = x_n - x_{n-1}$$

$$= \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{D\lambda}{d}$$

Position of dark fringes

$$p = \frac{x_n' d}{D} = (2n-1)\frac{\lambda}{2}$$

or $$x_n' = (2n-1)\frac{D\lambda}{2d}$$

Where x_n' is the distance of nth dark fringe from centre of screen.

Width of bright fringe = separation between two consecutive dark fringes

$$= x_n' - x_{n-1}'$$

$$= (2n-1)\frac{D\lambda}{2d} - [2(n-1)-1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

DUAL NATURE OF MATTER AND RADIATION

Deduce an expression for the de Broglie wavelength of a particle of mass m moving with velocity v. Hence derive de Broglie wavelength of an electron accelerated through a potential difference of V volts.

Considering photon as an electromagnetic wave of frequency v, its energy from Planck's quantum theory is given by

$$E = hv$$

Where h is Planck's constant. Considering photon as a particle of mass m, the energy associated with it is given by Einstein's mass energy relationship as

$$E = mc^2$$

From equations (i) and (ii), we get

$$hv = mc^2$$
$$\Rightarrow \frac{hc}{\lambda} = mc^2$$
$$\Rightarrow \lambda = \frac{h}{mc} = \frac{h}{p}$$

According the de Broglie hypothesis, the above equation must be true for material particles like electrons, protons, neutrons etc. Hence a particle of mass m moving with velocity v must be associated with a matter wave of wavelength λ given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\because p = \sqrt{2m(KE)}$$
$$\text{and } KE = qV$$

$$\therefore \lambda = \frac{h}{\sqrt{2m(qV)}}$$

As, $h = 6.62 \times 10^{-34}$ Js

$m = 9.11 \times 10^{-31}$ kg

$q = 1.6 \times 10^{-19}$ C

We get

$$\lambda = \frac{12.27}{\sqrt{V}} \mathring{A}B$$

ATOMS

Using Bohr's postulates, derive an expression for the velocity of an electron revolving in an orbit. Also show that the velocity of electron in innermost orbit of H atom is 1/137 of times the speed of light.

Consider an electron of mass m and charge e revolving with velocity v around a nucleus having atomic number z. Then the centripetal force required by the electron is provided by electrostatic force of attraction between nucleus and electron according to equation

$$\frac{mv^2}{r} = \frac{k(Ze)e}{r^2}$$

$$\Rightarrow mv^2 = \frac{kZe^2}{r} \qquad ....(i)$$

According to Bohr's quantum condition for angular momentum

$$mvr = \frac{nh}{2\pi} \qquad ....(ii)$$

Expression for velocity

From (i) $r = \frac{kZe^2}{mv^2}$ ....(iii)

From (ii) $r = \frac{nh}{2\pi mv}$ ....(iv)

Therefore

$$\frac{nh}{2\pi mv} = \frac{kZe^2}{mv^2}$$

$$\Rightarrow \boxed{v = \frac{2\pi kZe^2}{nh}}$$

Putting

$\pi = 3.14, k = 9\times 10^9 Nm^2C^{-2}, Z = 1, e = 1.6\times 10^{-19} C, n = 1$ and $h = 6.62\times 10^{-34}$ Js ,

we get

$$v = \left(\frac{1}{137}\right)c$$

Using Bohr's postulates, derive an expression for the radii of the permitted orbits in the hydrogen atom.

Putting the value of v obtained above in equation (iv), we get

$$r = \frac{nh}{2\pi m}\cdot\frac{nh}{2\pi kZe^2}$$

$$\Rightarrow \boxed{r = \frac{n^2h^2}{4\pi^2 mkZe^2}}$$

Using Bohr's postulates, derive an expression for the total energy of an electron revolving in an orbit.

Kinetic energy of electron in nth orbit

$$K.E = \frac{1}{2}mv^2 = \frac{kZe^2}{2r} \quad \text{[Using equation (i)]}$$

Potential energy of electron in nth orbit is

$$P.E = k\frac{q_1q_2}{r} = k\frac{(Ze)(-e)}{r} = -k\frac{Ze^2}{r}$$

Total energy T.E = P.E + K.E

$$T.E = k\frac{Ze^2}{2r} - k\frac{Ze^2}{r} = -k\frac{Ze^2}{2r}$$

Putting the value of r, we get

$$T.E = -\frac{kZe^2}{2} \cdot \frac{4\pi^2 mkZe^2}{n^2h^2}$$

$$\Rightarrow \boxed{T.E = -\frac{2\pi^2 mk^2Z^2e^4}{n^2h^2}}$$

On the basis of Bohr's theory, derive an expression for the wavelength of emitted photon when an electron comes back from a higher state n_2 to a lower state n_1.

From Bohr's theory, the energy of an electron in the nth orbit of hydrogen atom is given by

$$E_n = -\frac{2\pi^2 mk^2Z^2e^4}{h^2} \cdot \frac{1}{n^2}$$

According to Bohr's condition, whenever an electron makes a transition from a higher energy level n_2 to lower energy level n_1, the difference of energy appears in the form of a photon is given by

$$hv = E_{n_2} - E_{n_1}$$

$$\Rightarrow hv = \frac{2\pi^2 mk^2e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{c}{\lambda} = \frac{2\pi^2 mk^2e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{1}{\lambda} = \frac{2\pi^2 mk^2e^4}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where $R = \frac{2\pi^2 mk^2e^4}{ch^3}$, is the Rydberg constant

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NUCLEI

Derive an expression for the density of nucleus. Hence show that the density is independent of mass number.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of nucleus = mA

Volume of nucleus

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi\left(R_o A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi R_o A$$

Therefore, nuclear density

$$\rho = \frac{\text{Mass of nuclues}}{\text{Volume of nucleus}}$$

$$\rho = \frac{mA}{\frac{4}{3}\pi R_o^3 A} = \frac{3m}{4\pi R_o^3}$$

Clearly, density of nucleus is independent of mass number A or the size of the nucleus.

Taking $m = 1.67 \times 10^{-27}\,kg$, $R_o = 1.2 \times 10^{-15}\,m$, we get

$\rho = 2.30 \times 10^{17}\,kgm^{-3}$ which is very large as compared to the density of ordinary matter.