Assignment 1

Q1. The overall initial state is

$$\left(\alpha|01\rangle - \beta|10\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$$

$$= \frac{\alpha}{\sqrt{2}}|0100\rangle + \frac{\alpha}{\sqrt{2}}|0111\rangle - \frac{\beta}{\sqrt{2}}|1000\rangle - \frac{\beta}{\sqrt{2}}|1011\rangle$$

Applying CNOT on 2 and 3 qubit:

$$= \frac{\alpha}{\sqrt{2}}|0110\rangle + \frac{\alpha}{\sqrt{2}}|0101\rangle - \frac{\beta}{\sqrt{2}}|1000\rangle - \frac{\beta}{\sqrt{2}}|1011\rangle$$

Followed by H gate on 2^{nd} qubit.

$$= \frac{\alpha}{2}\left(|0010\rangle - |0110\rangle + |0001\rangle - |0101\rangle\right)$$

$$\frac{-\beta}{2}\left(|1000\rangle + |1100\rangle + |1011\rangle + |1111\rangle\right)$$

Now a CNOT gate on 3 and 4 qubit →

$$= \frac{\alpha}{2}\left(|0011\rangle - |0111\rangle + |0001\rangle - |0101\rangle\right)$$

$$-\frac{\beta}{2}\left(|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right)$$

Now a controlled Z gate on 2 and 4

$$= \frac{\alpha}{2}\left(|0011\rangle + |0111\rangle + |0001\rangle + |0101\rangle\right)$$

$$-\frac{\beta}{2}\left(|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right)$$

Since finally 2^{nd} is $|1\rangle$ and 3^{rd} $|0\rangle$, the final state after measurement is $\alpha|0101\rangle - \beta|1100\rangle$

$\therefore$ $A = \alpha|0\rangle - \beta|1\rangle$ $\quad B = \alpha|1\rangle - \beta|0\rangle$

Q2. Initial joint state = $\alpha|010\rangle + \beta|011\rangle$
Applying bell circuit on $|01\rangle$ (first two qubits)
gives us $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

∴ The state at A is

$$\frac{\alpha}{\sqrt{2}}|010\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$$

Followed by a CNOT on 2^{nd} and 3^{rd} qubit with 3^{rd} qubit as control.
The state after this is

$$\frac{\alpha}{\sqrt{2}}|010\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|001\rangle + \frac{\beta}{\sqrt{2}}|111\rangle$$

Now a hadamard gate on the 3^{rd} qubit -

$$\frac{\alpha}{2}|010\rangle + \frac{\alpha}{2}|011\rangle + \frac{\alpha}{2}|100\rangle + \frac{\alpha}{2}|101\rangle$$

$$+ \frac{\beta}{2}|000\rangle - \frac{\beta}{2}|001\rangle + \frac{\beta}{2}|110\rangle - \frac{\beta}{2}|111\rangle$$

Now a inverse CNOT gate on 1 and 3 with 3 as control.

$$\frac{\alpha}{2}|110\rangle + \frac{\alpha}{2}|011\rangle + \frac{\alpha}{2}|000\rangle + \frac{\alpha}{2}|101\rangle$$

$$+ \frac{\beta}{2}|100\rangle - \frac{\beta}{2}|001\rangle + \frac{\beta}{2}|010\rangle - \frac{\beta}{2}|111\rangle$$

Now a controlled Z on 2^{nd} and 1^{st} with 2^{nd} as control.

$$-\frac{\alpha}{2}|110\rangle + \frac{\alpha}{2}|011\rangle + \frac{\alpha}{2}|000\rangle + \frac{\alpha}{2}|101\rangle$$

$$+ \frac{\beta}{2}|100\rangle - \frac{\beta}{2}|001\rangle + \frac{\beta}{2}|010\rangle + \frac{\beta}{2}|111\rangle$$

The measurement gives 2^{nd} and 3^{rd} as $|0\rangle$ ∴

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$$\therefore \boxed{|\phi\rangle = \alpha|0\rangle + \beta|1\rangle}$$

Q3. For creating GHZ state we input 3 $|0\rangle$ qubits to the circuit: