

Replies to referee 2

[Authors] We thank the referee for the provided feedback. Please find below the point-by-point replies to the raised questions and comments. A list of references is provided at the end of this document

In this paper, the authors present a two-dimensional radial-axial hybrid particle-in-cell (PIC) simulation of a Hall thruster. The hybrid model is a well-established approach for simulating Hall thruster discharges, and the presented simulation results appear credible. While the simulation outcomes are of interest, the authors' discussion regarding the newly introduced plume boundary condition remains unclear. My comments primarily concern the impact of the proposed boundary condition model, which the authors claim to be a significant contribution.

We thank the referee for the valuable comments. We would like to emphasize the main reasons why we consider the newly introduced Global Plume Condition (GPC) a significant contribution to the numerical modelling of Hall effect thruster (HET) plumes. As discussed in the manuscript, the GPC yields a more physically grounded solution for the electron current in the plume region, which is essential for meaningful comparison with experimental plasma diagnostics, most of which are performed in the near plume.

In particular, our results for cathode location C2, which corresponds to a laterally-mounted cathode positioned off-axis and outside the magnetic separatrix (a configuration commonly encountered in HETs), demonstrate that the widely used Local Plume Condition (LPC) fails to provide reliable current solutions in the near plume. In this configuration, the LPC produces artificial features such as stagnation points and nonphysical current loops, and the solution becomes more strongly dependent on the size of the simulated domain. This behavior reveals the flaws of the LPC for such configurations.

The GPC overcomes this limitation by enforcing a net zero-current condition over the entire plume boundary, thereby limiting the effect of the plume size on the current solution in the near plume. Crucially, by enabling reliable current maps in reduced, computationally efficient simulation domains, the GPC establishes a solid foundation for future validation of hybrid models against experimental measurements, particularly in plume diagnostics where accurate representation of electron transport is essential. Additionally, the GPC enables the estimation of key physical quantities, such as the final potential reached in the far plume, ϕ_{∞} , which are not accessible under the LPC framework.

For these reasons, we believe the introduction of the GPC model represents a substantial step forward in enabling accurate and scalable numerical predictions of HET plume behavior and in paving the way for future validation of simulations against experimental plume diagnostics.

Major Comment:

1) The effect of the new boundary condition model—referred to as the Global Plume Condition (GPC)—is not demonstrated quantitatively. The authors state that the GPC minimizes the truncation effects of the plume boundary. Although differences between the results using the LPC and the GPC are illustrated in Figures 2 and 3, it is not convincingly shown that the results obtained using the GPC are more accurate. The manuscript claims that the GPC yields solutions “much closer” to those obtained with a larger simulation domain and provides a “significantly more robust solution,” yet these assertions lack quantitative discussion. While Table II presents a comparison of thruster performance parameters (e.g., discharge current), the data do not clearly support the claimed advantages of the GPC. A quantitative assessment—such as the convergence behavior of key physical parameters with respect to simulation domain size—would strengthen the argument and clarify the benefit of the proposed boundary model.

We agree with the reviewer that the advantages of the GPC model over the LPC were not sufficiently quantified in the original version manuscript. In the revised version of the manuscript, we have expanded the quantitative discussion in Section III. In particular:

- a) We have evaluated the relative displacement of the plasma stagnation point, defined as the location where the longitudinal plasma current density vanishes ($\tilde{j} = 0$), as a function of the plume domain size, taking the result for the largest domain (size P4) as reference. This analysis reveals that the LPC leads to larger displacements of the stagnation point with plume size than the GPC, highlighting the stronger sensitivity of the LPC to domain truncation and further confirming the robustness of the GPC solution.
- b) Figure 4 has been improved to highlight the differences in the electric potential ϕ_p and the electron temperature T_{eP} at the lateral part of the plume boundary P induced by the plume condition for plume size P3, and these differences are now better quantified in the manuscript. In particular, the LPC yields a larger electron flow at the lateral P boundary. As a consequence, ϕ_p in case LP3C1 is up to approximately 30% higher than in case GP3C1, with differences on the order of the local electron temperature, and thus relevant for near-plume diagnostics.
- c) Additionally, these differences in ϕ_p have been found to affect the dynamics of slow ions generated in the plume through charge exchange collisions (CEX). The current density of singly charged CEX ions is now shown in Figure 4(c), and the distinct behavior of this ion population under both the GPC and LPC models is discussed and quantified in the revised manuscript.

In addition, an important asset of the GPC is the determination of the potential at infinity, ϕ_∞ , which cannot be obtained by the LPC. This is a key parameter characterizing the current-free expansion of the plasma plume into free space (or into a large vacuum chamber with low background pressure). In particular, the surface-averaged values of $e\phi_{\infty P}/T_e$ obtained in our study agree very well with those reported in Ref. [1], as we comment in the manuscript, for all cathode configurations. Furthermore, as detailed in the manuscript, an added value of the GPC is that it yields the average energy deposited at P by exiting

electron, which range from $4.0T_{eP}$ to $9.2T_{eP}$ for case GP3C1. In contrast, the LPC cannot provide this energy and instead sets it (rather arbitrarily) to cT_{eP} , with $c \sim 2.5-4.5$.

Finally, as discussed at the beginning of this document, the LPC is not reliable for cathode location C2, and yields an electron current solution that inherently depends on the plume size. This is a central aspect in favor of the GPC, since the C2 configuration is commonly encountered in many HETs. Section IV has been refined in the revised version of the manuscript to improve clarity, deliver a more direct and focused message, and better emphasize the benefits of the GPC over the LPC.

Minor Comments:

2) Is it possible to implement a nonlinear sheath model at the anode boundary alongside the new nonlinear plume boundary condition? In conventional electron fluid models for Hall thrusters, the anode sheath is often modeled using a nonlinear approach, whereas the plume boundary typically employs a linearized model—likely due to an issue of numerical stability. In the current study, the anode boundary is modeled with a Dirichlet condition on potential, whereas the plume employs a nonlinear condition. It would be beneficial for the authors to discuss whether a nonlinear sheath model could also be applied to the anode boundary in combination with the nonlinear plume model.

We thank the referee for the comment. Indeed, HYPHEN applies a nonlinear sheath model at the anode, which is a conducting boundary wall at known potential V_d . The nonlinear sheath model is linearized and solved iteratively if required together with the electron momentum and continuity equations, as detailed in Section III.A of Ref. [2].

We have revised the description of the sheath model in Section II.A of the manuscript to improve clarity. In particular, we now refer the reader to the Appendix of Ref. [2], where the sheath model implemented in HYPHEN is described in detail. We clarify that the model is restricted to electron-repelling sheaths and is applied both at the dielectric walls of the thruster and at the conducting anode wall at known potential V_d . Besides, for the sake of both completeness and conciseness, we now refer the reader to Section III.A of Ref. [2], where the implementation details for both types of walls are provided.

3) In Figure 3, the authors discuss differences in electron flow near the lateral plume boundary. While the directional vectors highlight these differences, the electron current density in that region is relatively small (on the order of 10^{-5} to 10^{-3}), suggesting that the impact may be limited. Could you provide a more quantitative assessment of this difference? For instance, does the altered electron flow significantly influence the trajectories of high-divergence fast ions or charge-exchange (CEX) ions?

Thank you for pointing out this aspect. In Section 3, when discussing the values of the ion beam current I_{iP} listed in Table II, we quantify the fraction $I_{eP} (=I_{iP})$ that is collected at the lateral boundary for the cases LP3C1 and GP3C1 (whose 2D solutions for the in-plane

electron current density are shown in Figure 3). These fractions are 14% for LP3C1 and 10% for GP3C1, illustrating the physical relevance of the difference in this lateral electron flow.

Furthermore, we refer here to our reply to point 1) above regarding differences on the electric potential (and electron temperature) at the lateral plume boundary and the effects on the slow CEX ions. We thank the referee for raising the point about the CEX ions, which we believe has contributed to enriching the contents of the manuscript.

4) Figure 4 may be difficult for readers to interpret. It is recommended that the authors highlight selected lines or regions in the figure to better emphasize the key differences being discussed.

Thank you for the comment. Figure 4 has been upgraded according to your recommendations, as we indicate in our reply to point 1) above.

REFERENCES

- [1] Ahedo, E., Correyero, S., Navarro, J., and Merino, M., "Macroscopic and parametric study of a kinetic plasma expansion in a paraxial magnetic nozzle", *Plasma Sources Science and Technology*, Vol. 29, No. 4, 2020, pp. 045017.
- [2] Perales-Díaz, J., Domínguez-Vázquez, A., Fajardo, P., Ahedo, E., Faraji, F., Reza, M., and Andreussi, T., "Hybrid plasma simulations of a magnetically shielded Hall thruster", *Journal of Applied Physics*, Vol. 131, No. 10, 2022, pp. 103302.