Replies to referee 1

[Authors] We thank the referee for the provided feedback. Please find below the point-by-point replies to the raised questions and comments. A list of references is provided at the end of this document

The authors present a comparison between local (LPC) and global (GPC) current-free boundary conditions applied to an axisymmetric, hybrid simulation of a Hall effect thruster. They apply these boundary conditions to three separate system configurations: one with an external cathode (EC) inside the magnetic field separatrix (MS), one an EC outside the MS, and one with an internally mounted cathode, using four separate domain sizes, i.e., 12 different cases. There are several issues noted for this manuscript to be considered for PSST in its current form:

Major Points:

1) The authors should address how the simulations are validated against experimental data. It is indeed an interesting study from the perspective of Hall thruster simulations, but the paper reads more like a “sensitivity study,” without detailed discussions of the physics. So, what is the novelty in the physics that the authors discovered or investigated with this new computational model? There are some mentions of experimental measurements which agree with the cathode location study, but there are no direct numerical comparisons to any experimental results (e.g., thruster performance, oscillation amplitude or frequency, etc.).

Thank you for your comments. The referee is right: we do not present a direct comparison of our numerical results with experimental ones, and this is an important backstep that limits the validation of our simulation results. This is due to the fact that we lack experimental data to perform a quantitative comparison against our simulation results. In our research group, we are currently doing our best to obtain data from real prototypes to advance on this aspect. As the referee may know, this is not straightforward, and we hope the referee can understand this situation.

While full validation of our simulation results awaits experimental data, this study offers a valuable contribution to modeling the magnetized near plume of a Hall effect thruster (HET). A central focus is the formulation of boundary conditions at the downstream plume boundary (P), consistent with a globally current-free plasma and a monotonically decreasing electric potential at infinity ϕ_∞. The commonly used local plume condition (LPC) imposes a zero current at every point on P, which is overly restrictive and cannot predict the final plume potential. Our main contribution is the global plume condition (GPC), which enforces a global current-free constraint $I_P = 0$ and allows for estimation of ϕ_∞. The GPC proves more robust against plume truncation, particularly when the cathode lies outside the magnetic separatrix (cathode location C2 in the manuscript), a typical HET configuration. In contrast, the LPC yields plume currents that vary with domain size, undermining result reliability. Thus, the GPC offers a solid foundation for future validation against plume diagnostics once experimental data becomes available.

As discussed in the manuscript, several results show good qualitative agreement with previous studies. First, the LPC model perturbes the electron transport at the lateral plume boundary leading to enhanced electron collection, an effect also reported in Ref. [1], while the GPC avoids this issue. Second, unlike the LPC, the GPC can estimate the potential at infinity, yielding total potential fall values in the off-simulation region consistent with those from a kinetic paraxial model of plasma expansion in a magnetic nozzle MN (Ref. [2]). Third, our results confirm that, downstream of the magnetic separatrix, ions detach inward from magnetic field lines, in agreement with the two-fluid, 2D model for a MN in Ref. [3]. Finally, as noted by the referee, trends with cathode location in key parameters—such as coupling voltage, thrust, thrust efficiency, plume divergence, electric potential, electron temperature, and plasma density—align well with prior experimental and numerical studies, as detailed in Section IV.

2) The authors do not discuss the details of the hybrid code, HYPHEN. It can be understood that the authors did not want to make the paper any longer (already 16 pages), but it is critical to explain the basics of the code. The major comments include the following. (i) The E-module is perhaps the most important. Please add some detailed explanation for readers who are not familiar with hybrid models in Hall thrusters, e.g., what kinds of equations are solved and how are the equations solved? In particular, the quasineutral drift diffusion model solves an elliptic PDE for the potential, so how one can “force” the current condition via the electron equations is of great interest and importance (e.g., eq (1) perhaps determines the balance between the electric field and pressure gradient). (ii) The sheath (S) module might also play an important role in the investigation. Please elaborate on “The local sheath potential fall is also determined” and “retains non-Maxwellian features of the electron velocity distribution function (VDF)”. (iii) Please include particle statistics in the report or demonstrate particle convergence. How many macroparticles per cell were used in this study? Are the presented results instantaneous or time-averaged? (iv) Can you include any comments about the computational cost of the model?

We thank the reviewer for requesting clarification regarding the details of the hybrid code HYPHEN. While a full description of the code would considerably increase the length of the manuscript and would be essentially a copy of texts of previous papers, we agree that the reader should be properly directed to relevant references where the model equations and their numerical implementation are detailed. In the revised version of the manuscript, we have updated the text in Section II.A to include specific references and guidance for the interested reader. Below we address each subpoint in detail:

(i) The full set of equations solved by the E-module is detailed in Section III.A of Ref. [4] and in Section 2.1 of Ref. [5]. The time and spatial discretization of these equations is described in the Appendix B of Ref. [5]. These references have been included in the manuscript.

(ii) We have refined the description of the sheath model in Section II.A of the manuscript. In particular, we now refer the reader to the Appendix of Ref. [4], where the sheath model implemented in HYPHEN is described in detail. The most relevant aspect of the sheath model for the discussion and objectives of the present study is that it determines the local sheath potential drop from the quasineutral plasma to the wall, and provides the appropriate boundary conditions for both the wall-collected electron current density and the energy flux at each (quasineutral) MFAM boundary face. This is now explicitly mentioned in the text. For the sake of both completeness and conciseness, we now refer the reader to Section III.A of Ref. [4], where the implementation details for both types of walls are provided.

(iii) We have now included in Section II.B details about the number of particles per cell used for the simulations. We also clarify that all results shown in the study are time-averaged over a sufficiently large number of breathing mode cycles.

(iv) The computational times have been included in Section II.B.

3) The anomalous scattering frequency is assumed to be a function of axial position only. Are the same values used in the same magnetic field lines or radial direction? Past studies (cf, Hall2De) have shown that an anomalous scattering frequency is also needed for the cathode-to-plume coupling. It would seem to me that in a paper investigating the cathode boundary condition these details would be important and should at the very least be mentioned. [Page 3, Paragraph 4]

Thank you for pointing out this aspect: we use the same value of alpha along each magnetic line, and we have clarified this point in Section II.A in the revised version of the manuscript.

We agree with the referee on the fact that anomalous transport mechanisms can play a role in the cathode-plume coupling. In our study, we do apply an anomalous scattering frequency in the plume region. However, (1) this parameter is uncertain and unfortunately, we lack experimental data to evaluate it more accurately; and (2) in our study we analyze only the effect of plume size, plume condition and cathode position while keeping constant the anomalous collision frequency. We now acknowledge this aspect in Section II.A and we cite Ref. [1], which presents a more elaborated study with Hall2De on these matters.

4) Figure 2 is interesting, but what is the meaning of showing all four cases? There are only 2 paragraphs explaining the results. It seems like P1 is different when the domain is truncated but P2-P4 looks pretty similar, i.e., the near-field plume is not affected, and the far-field plume shows a circulation of electrons at far field. The paper could be significantly shortened by modifying this discussion. [Page 3, Section B, Paragraph 2]

We agree with you on this point. We have now better quantified the differences on the electric current solution downstream the cathode magnetic line (CML) by reporting the location of the electric current stagnation point. Since the solution is similar for P4 cases, we have omitted the results for size P4 on Figure 2 so that the paper is shortened, as suggested by the referee.

5) It is hard to see “The main differences among cases are found in the outwards flow of electrons, especially in the lateral part of the plume.” Also, I am confused by this sentence: “the surface-averaged values over P of B and the effective Hall parameter, for plume sizes P1 to P4, decrease from 7.4 to 1.8 G, and from 212.2 to 68.1 G, respectively.” What is the significance of this statement? This just comes from the smaller domain? [p. 6]

The first sentence has been rephrased and clarified in the revised version of the manuscript. Our aim with the second sentence is to highlight the fact that the electron population in the plume is well-magnetized even for the largest domain considered. Along with the effective

Hall parameter, ranging from 212.2 to 68.1 for plume sizes P1 to P4, we report the average B value for completeness.

6) Figure 4 seems to be a very busy figure but is least effective in addressing the scientific messages. It is perhaps more effective to just show the comparison of the 2D figures of C1, C2, and C3 with GP3 and LP3, which are shown in Figs. 6 and 7. To make the paper more concise, the authors may make a figure comparing LP3/GP3C1, C2, and C3 as one figure. Comparing LP3/GP3 vs LP4/GP4 seems less scientifically interesting. If the authors feel otherwise, then the recommendation would be to add more scientific discussions about what "physics" is captured and is important to be captured with the larger domain. The scientific messages of Fig 4 can be done without it and what the authors want to talk about in Fig 5 can be better captured.

Figure 4 has been improved in the revised version of the manuscript to better address the corresponding scientific messages. First, we have simplified it by omitting the ion and electron current profiles and energy profiles without losing the associated scientific messages in the text. Second, we highlight the differences in the electric potential ϕ_P and the electron temperature T_{eP} at the lateral part of the plume boundary P induced by the plume condition. These differences are now better quantified in the manuscript. In particular, the LPC yields a larger electron flow at the lateral P boundary. As a consequence, ϕ_P in case LP3C1 is up to approximately 30% higher than in case GP3C1, with differences on the order of the local electron temperature, and thus relevant for near-plume diagnostics. Finally, these differences in ϕ_P have been found to affect the dynamics of slow ions generated in the plume through charge exchange collisions (CEX). The current density of singly charged CEX ions is now shown in Figure 4(c), and the distinct behavior of this ion population under both the GPC and LPC models is discussed and quantified in the revised manuscript.

Figures 6 and 7 have been revised to improve conciseness and focus. In particular, for cathode C3, now Figure 7 only shows results for plume size P3. On the other hand, for cathode C2, the maps of electron current density for P4 have been omitted. However, the electric current density maps for plume sizes P3 and P4 are kept in Figure 6, since the comparison between the GPC and the LPC for these two plume sizes reveals a key finding: that the current solution provided by the LPC is inherently dependent on the simulation domain size under this cathode configuration, and is therefore unreliable. This result strongly supports the need for the GPC in such scenarios. Accordingly, Section IV has been revised to center the discussion on the most relevant physical aspects induced by cathode positioning when using the GPC model.

7) Figure 5 seems interesting, but it should be clearly mentioned that all results assume the same anomalous mobility profile. The cathode location may significantly change the near-field electron transport profile.

Thank you for your comment. In connection with our reply to point 3) above, we now explicitly state in Section II.A that all simulation results presented in our study assume the same anomalous mobility profile.

8) The calculation of the thrust is non-trivial. Please elaborate on whether the thrust was found to be constant beyond a certain control volume and on how the control volume / surface to evaluate the thrust is chosen.

The thrust is computed as the surface integral over the downstream plume boundary P of the total axial momentum flux of the plasma, considering all plasma species (i.e., ions, electrons and neutrals):

$$F = \int_P \vec{1}_z \cdot \sum_s (m_s n_s \vec{u}_s \vec{u}_s + \bar{\bar{p}}_s) \cdot \vec{1}_n dA,$$

where m_s, n_s, $\vec{u}_s$ and $\bar{\bar{p}}_s$ are the elementary mass, the particle density, the macroscopic velocity and the pressure tensor of each plasma species s, and $\vec{1}_n$ is the outward normal unit vector to the downstream plume boundary P.

The plasma momentum conservation equation shows that the thrust can also be approximated by the magnetic thrust

$$F \approx F_m = \int_V \vec{1}_z \cdot (\vec{j} \wedge \vec{B}) dV = \int_V - j_\theta B_r dV,$$

being the integral extended to the volume of the simulation domain. Further details on thrust contributions can be found in Ref. [5].

As we comment in the manuscript, the net increase in thrust found in our simulations with increasing plume domain is due to the magnetic force on the plasma. Net increase in F from P2 to P4 is below 1%. We therefore assume that the residual magnetic thrust from P to infinity is negligible.

9) Table II is interesting (especially, ϕ_∞) but more details are needed. (i) Why do we not see the convergence of results? For instance, LP2C1 = 13.22 V; LP3C2 = 14.15 V; LP4C1 = 10.97 V. (ii) The small thrust with smaller domain is attributed to magnetic force. (iii) Please provide more clarification and physics discussions. The authors mention that the results are “converging”, but the main interesting physics is what is the correct value of ϕ_∞? For instance, if the cathode potential is 0 V, the far-field potential should be Vcc, which is a positive value with respect to the cathode potential? (v) Finally, while it is appreciated that the authors provide all values thoroughly, many of the values seem to be less interesting. For instance, all discharge power, current, efficiencies are not that sensitive to the far-field boundaries.

(i) We thank the reviewer for pointing out this important aspect, which highlights a difficulty associated with the use of a Magnetic Field Aligned Mesh (MFAM) for solving the electron fluid equations. We have clarified this point in Section III of the revised version of the manuscript, as detailed below.

The variations in the coupling voltage V_{cc} reported in Table II for C1 cases are not due to fundamental changes in the plasma solution with plume size, but rather stem from numerical differences in the construction of the MFAM. The MFAM is generated from the applied magnetic field. The cells of the mesh are defined by the intersection of the magnetic field streamlines and equipotential lines, which are obtained through the integration of the magnetic field. For each plume size, the MFAM is built to balance accuracy and computational cost, ensuring a controlled cell skewness and resolution. Since the applied magnetic field is highly non-uniform in the near-plume region, the resulting distribution of magnetic field lines is also highly non-uniform (refer to Fig. 1(a) below). Thus, the optimal selection of magnetic isolines that define the MFAM cells becomes a nontrivial task, which typically results in variations in the cell layout across different plume sizes. Consequently, the cathode magnetic line (CML), defined within each MFAM by the field line passing through the cathode injection center, may differ slightly between plume sizes. This leads to small differences in the intersection point between the CML and the thruster channel midline used to estimate V_{cc}, as illustrated in Fig. 1(b). These geometric differences explain the variations in V_{cc} among the C1 cases (maximum of about 1% of V_d for plume sizes P2-P4), and do not reflect significant physical changes in the cathode-beam coupling process. In fact, compared to C1 cases, variations in V_{cc} among C3 cases reported in Tab. IV are lower (maximum of about 0.3% of V_d). This is because the CML is the symmetry axis for C3 cases (cathode at the axis), and the magnetic line that connects it to the thruster channel midline is the magnetic separatrix (MS), which is the same for every MFAM regardless of the plume size (refer to Fig. 1). For C1 cases, mesh-related variations in V_{cc} have been quantified and explicitly stated in the manuscript.

On the other hand, mesh-related variations in ϕ_∞ are lower, since this quantity is related to far field plume dynamics, and they have also been quantified and explicitly stated in the manuscript. Furthermore, it has been verified that these variations do not affect the qualitative trend found for ϕ_∞ with the plume size.

Finally, the calculation of V_{cc} has been improved for C1 cases through a finer interpolation of the electric potential solution to the crossing point between the CML and the thruster channel midline.

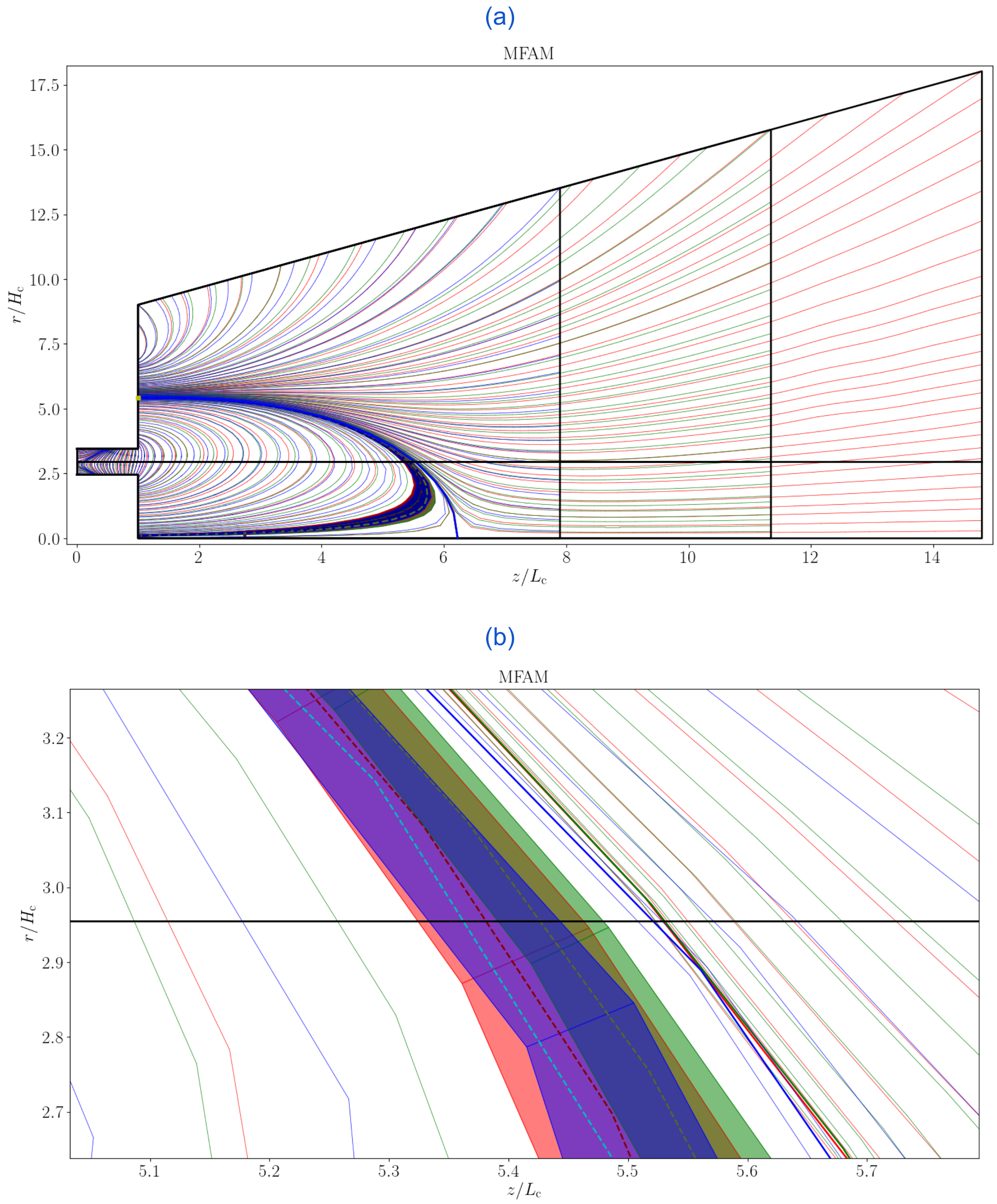

Fig. 1. (a) MFAM streamlines comparison for P2 (blue), P3 (green) and P4 (red). (b) Detail of the CML (dashed lines) and corresponding magnetic tube between two consecutive MFAM magnetic streamlines. The horizontal thick black line corresponds to the thruster channel midline. The MS is the same for the three MFAMs and it is depicted with a thicker blue line in (a).

(ii) As commented in our reply to point 8), the net increase in the thrust with the plume size found in our simulations is due to the (small) magnetic thrust over the still magnetized plasma (mainly electrons) in the near plume, and can be seen from the plasma momentum conservation equation.

(iii) We do not use the word “converging” for V_{cc} nor for ϕ_{∞}. On the one hand, V_{cc} is a value in the very near field plume, that results from the complex cathode-beam coupling process. On the other hand, ϕ_{∞} is related to the plasma expansion in the far plume, i.e., from the downstream plume boundary P to infinity. In this region, it is assumed that the expanding plasma beam is current-free, and that the electric potential decreases monotonically to a value ϕ_{∞} at infinity (with respect to the cathode). Indeed our results show that $\phi_{\infty} < V_{cc}$ for all cases. As we indicate in the manuscript, our results for plume sizes P2-P4 suggest that simulating a plume size larger than P4 would be necessary for a precise estimation of ϕ_{∞}, which requires the use of optimal MFAMs to limit numerical errors or the use of cylindrical meshes for the electron model instead, and is left for future work.

(iv) Thank you. We have omitted some of the performance values in the tables, and are now just reported in the text for simplicity. Partial efficiencies are kept in Tables to facilitate the comparison between C1, C2 and C3 cases, as discussed in Section IV.

10) It is perhaps important to mention the difference between ϕ_P vs ϕ_∞. The sheath theory in Eq 4 should only be applicable for ion-attracting electron-repelling sheath. Do the authors apply Eq 4 for ion-repelling ion-attracting sheath as well?

We agree with the referee. The difference between ϕ_P and ϕ_{∞} has been clarified in the revised version of the manuscript (see Section II.C).

Eq. (4) is valid for an electron-repelling sheath, that is what we find for all the cases and all locations on the downstream boundary. This has been clarified in Section II.C.

11) Section IV mainly discusses the observations in Tables III and IV. In the text, the decrease and increase of certain values are shown, but it seems to lack a concrete scientific discussion. If there is no scientific discussion, one could consider moving this to the appendix or significantly shortening the discussions. Specifically, the increase in ϕ_∞ in Table III (when increasing the domain size) is counterintuitive.

We thank the reviewer for this valuable comment. Section IV has been fully rewritten in the revised manuscript to improve both clarity and conciseness, and to better emphasize the key scientific insights derived from the results. Superfluous descriptions have been removed, and the discussion now focuses on two main points. First, the analysis of the current solution for C2 (and C3 cases). For case C2 the key finding is that the current solution provided by the LPC inherently depends on the plume size, and it is therefore unreliable. Therefore, the remainder of the discussion focuses on the GPC cases, emphasizing the impact of cathode location on cathode-beam coupling, main performance figures and the plasma solution in the plume. The trends found on the main plasma parameters show good agreement with previous numerical and experimental studies.

The values of ϕ_∞ are found to monotonically decrease with plume size for all cathode locations. Larger variations with plume size are observed as the cathode radial position increases, which suggests that simulating larger plumes is particularly important to obtain a precise estimation of ϕ_∞ when the cathode is located outside the MS.

Minor points:

12) The authors mention “zero current condition at the plume downstream”. It would be better to briefly explain that current-free plasma at the far field plume or the assumption that the accelerated ion beam is neutralized at the far field plume.

We thank the reviewer for the suggestion. We have rewritten the abstract to clarify the goals and findings of the study.

13) The explanation of the electron energy flux should be made more carefully (cf. eq (2)). If one considers a half-Maxwellian, it can be considered that c = 2. 5/2 comes from the 3/2 p + p in the energy equation (but without any q). So, mathematically where do 9/2 and 13/2 come from?

Previous simulations of an electrodeless plasma thruster in Ref. [5] investigated the effect of c using the LPC. Values of c = 5/2 (qe = 0), 9/2 (qe = 2neTeue) and 13/2 (qe = 4neTeue) were simulated. The results revealed a significant impact of c on the electron temperature, with c = 9/2 providing the best agreement for two plume domain sizes. Since in our present study we have a MN-like topology downstream the CML, similar to that of an EPT, we set c = 9/2 when the LPC is applied. As mentioned in the manuscript, $c = -e\varepsilon_{neP}/(j_{neP}T_{eP}) = 4.5$ set for the LPC [refer to Eq. (2)], tries to reproduce the values $-e\varepsilon_{neP}/(j_{neP}T_{eP}) \simeq$4.0-9.2 obtained when the GPC is applied.

14) More discussion is needed on the modeling details. (i) In the introduction, there is no mention about whether ions = kinetic and electrons = fluid. Also, state-of-the-art models use mainly drift diffusion approximation but it is not specified that the authors use QDD until later. The use of QDD is important in how the local vs global current-free conditions are applied. (ii) The model is described as an axisymmetric r-z simulation but the drift-diffusion model will only work in Cartesian (i.e., Coriolis/centrifugal forces will not be captured)? Please elaborate.

(i) We thank the reviewer for this observation. We agree that the hybrid nature of the model should be clarified early in the manuscript. Accordingly, we have now added a sentence in the Introduction specifying that in hybrid models the ions and neutrals are treated kinetically using the particle-in-cell (PIC) method, while electrons are modeled as a fluid.

On the other hand, the current free condition at P is not affected by the use of the QDD approximation, and it is required by both QDD and more general electron fluid models that include inertial terms. In fact, even fully kinetic (non-quasineutral) models such as full PIC, must ensure that I_P= 0 at steady state, so that the cathode current equals the anode current and no leakage current exists. Therefore, we do not consider it necessary to modify the

Introduction to explicitly mention the use of the QDD approximation in relation to the boundary condition at P.

(ii) Drift-diffusion fluid models neglect electron inertia, so that centrifugal forces cannot be captured. This is true regardless of the chosen coordinate system (e.g. Cartesian or cylindrical). Regarding the Coriolis force it can only be present when a non-inertial reference frame is used to describe the dynamics, which is not the case for our model.

15) This sentence is appreciated: “Eq. (3) is an implicit equation for ϕ_∞ which is solved iteratively [by linearizing Eq. (4)].” However, this is why the equations for the electron fluid model are recommended. It would be much clearer that ϕ is solved at the “n-th” timestep, but ion current density comes from n-1 th timestep. Then, the interior cells can use the drift-diffusion formulation to solve for ϕ at n-th time step but the boundary condition requires the evaluation of j_neP at the n-th timestep, which may (or may not) need the implicitness for ϕ. The model for ϕ_∞ is very interesting (i.e., novel), but it is recommended to be more specific about the meaning of this, i.e., the global potential value, ϕ_∞, is chosen such that the global current is maintained to be zero.

Thank you for your comments. The referee is right: ϕ_∞ is obtained from Eq. (3) and, therefore, the resulting value of ϕ_∞ satisfies $I_P = 0$. This is now clarified in the revised version of the manuscript.

On the other hand, regarding the model equations and their numerical treatment, we refer to our reply to point 2) above.

16) “[T]urbulent cross-field transport due to high-frequency electron instabilities” should more precisely be “turbulent cross-field transport due to high-frequency plasma waves driven by kinetic and fluid instabilities.”

Thank you. We have corrected the sentence in the revised version of the manuscript.

17) Figure 5 appears to have some stray red dots.

Thank you. These dots are not in the original figure and only appear in the submitted proof document generated by the PSST website. We shall revise the submission package to avoid this effect.

18) What was the justification of choosing GP3 as the reference case for comparison between cathode placement results? Wouldn’t it be better to use the larger domain case? Could this be elaborated upon in the text?

For cathode location C1 and C3, going from plume size P3 to P4 yields only incremental variations in plasma properties in the near plume, and thus the effect on discharge performance is minimal, as discussed in the manuscript. Therefore, we have decided to (1) show results for P3 in Figures 3 and 4 (results for P4 could be shown instead without changing the scientific message), and (2), as suggested by the referee in previous points, omit results for P4 in Figures 2 and 7 for the sake of simplicity and conciseness.

On the other hand, for cathode location C2 the impact of plume size is higher than for C1 and C3, and results for P3 and P4 are shown in Figure 6 to illustrate the fact that the LPC solution is inherently dependent on the plume size, and thus unreliable.

19) Could the authors elaborate on the selection of locations for estimating Vcc (i.e., plume boundary at channel mid-radius vs crossing point between the channel midline and the CML vs crossing point between the channel midline and the MS).

We agree that the definition of Vcc is not unique. For C1 cases, V_{cc} is estimated as the electric potential value at the crossing point between the channel midline and the CML, following the approach proposed in Ref. [8]. In addition to C3 cases, we now also estimate V_{cc} for C2 cases as the electric potential at the crossing point between the channel midline and the MS. This modification does not alter the trends observed in V_{cc} with plume size or cathode location, but enables a more consistent comparison with C1 and C3 cases. Moreover, this choice is physically motivated: in C2 cases, the component of the cathode-emitted electron flow that enters the thruster channel travels along the MS toward the magnetic singular point. Therefore, the MS serves as a meaningful reference magnetic surface for characterizing the coupling between cathode electrons and the ion beam.

20) There seems to be consistently a red “radial” line near the anode (e.g., Fig 2). Does this mean that there is a strip of very large current ? This isn’t the focus of the paper but seems strange. Is this a meshing artifact (see Fig 1b)? [Page 5, Figure 2]

We thank the referee for pointing this out. The red “radial” line near the anode corresponds to a region of high current density caused by the magnetic topology in the near-anode region, which replicates that of the PPS5000 prototype developed by Safran. We have verified that this feature is not a meshing artifact but instead has a physical origin. However, as correctly noted by the referee, this study does not focus on the plasma solution inside the thruster channel. Therefore, we have modified plasma maps to show only the solution in the plume region, in order to better focus the discussion.

21) Figures 5(e) and 5(f) are never discussed or references in text.

Thank you. We have eliminated them in the revised version of the manuscript for simplicity and conciseness, as suggested by the referee.

22) What is meant by “electron momentum flux is transferred to ion momentum flux”? Are the authors trying to imply collisional momentum transfer or referring to the decrease in electron contribution to the net thrust with increasing domain size? Is this just meant to be a description of the magnitude of the ion vs electron current? [Page 7, paragraph 4]

We thank the referee for raising this point. In the expansion of an unmagnetized plasma plume, the transfer of electron momentum flux to ion momentum flux occurs due to the ambipolar electric field, and does not imply collisional momentum exchange. This mechanism leads to a redistribution of momentum between species, but not to a net increase in the total thrust. In our simulations, as the plume size increases, the ion contribution to thrust increases while the electron contribution decreases, consistent with this ambipolar effect. However, a small net increase in total thrust with plume size is observed.

This increase is due to the magnetic force acting on the plasma in the plume volume [refer to the magnetic thrust discussed in our reply to point 8)].

23) The net zero-current condition implies that the cathode current equals the anode current at steady state. This means that there is no leakage current. If this is the case, it would be [4]interesting to explicitly mention that.

Thank you for pointing this out. In our simulations, the cathode current equals the anode current for both the LPC and the GPC, since the net current through the plume boundary is $I_P = 0$ for all cases. We have clarified this point in Section III.

REFERENCES

[1] Lopez Ortega, A. and Mikellides, I. G., “The importance of the cathode plume and its interactions with the ion beam in numerical simulations of Hall thrusters”, Physics of Plasmas, Vol. 23, No. 4, 2016, pp. 043515.

[2] Ahedo, E., Correyero, S., Navarro, J. and Merino, M., “Macroscopic and parametric study of a kinetic plasma expansion in a paraxial magnetic nozzle,” Plasma Sources Science and Technology, Vol. 29, No. 4, 2020, pp. 045017.

[3] Merino, M. and Ahedo, E., “Plasma detachment in a propulsive magnetic nozzle via ion demagnetization,” Plasma Sources Science and Technology, Vol. 23, No. 3, 2014, pp. 032001.

[4] Perales-Díaz, J., Domínguez-Vázquez, A., Fajardo, P., Ahedo, E., Faraji, F., Reza, M. and Andreussi, T., “Hybrid plasma simulations of a magnetically shielded Hall thruster”, Journal of Applied Physics, Vol. 131, No. 10, 2022, pp. 103302.

[5] Zhou, J., Domínguez-Vázquez, A., Fajardo, P. and Ahedo, E., “Magnetized fluid electron model within a two-dimensional hybrid simulation code for electrodeless plasma thrusters”, Plasma Sources Science and Technology, Vol. 31, No. 4, 2022, pp. 045021.

[6] Cichocki, F., Domínguez-Vázquez, A., Merino, M. and Ahedo, E., “Hybrid 3D model for the interaction of plasma thruster plumes with nearby objects”, Plasma Sources Science and Technology, Vol. 26, No. 12, 2017, pp. 125008.

[7] Domínguez-Vázquez, A., Cichocki, F., Merino, M., Fajardo, P. and Ahedo, E., “Axisymmetric plasma plume characterization with 2D and 3D particle codes”, Plasma Sources Science and Technology, Vol. 27, No. 10, 2018, pp. 104009.

[8] Jorns, B. A. and Byrne, M. P., “Model for the dependence of cathode voltage in a Hall thruster on facility pressure”, Plasma Sources Science and Technology, Vol. 30, No. 1, 2021, pp. 015012.