10+2 PCM NOTES

BY

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(PDF version handwritten notes of Maths, Physics and Chemistry for 10+2 competitive exams like JEE Main, WBJEE, NEST, IISER Entrance Exam, CUCET, AIPMT, JIPMER, EAMCET etc.)

10+2

With best wishes from Joyoshish Saha

M

Set Theory.

* Definition of Set: A set is well defined collection of distinct objects.

eg. Set of all complex numbers, set of all stars in space, set of all states in India.

* Elements of the Set: If x is an element of set A, then $x \in A$.

eg. $A = \{1, 2, 3\}$ $\quad 1 \in A$

* Representation of Set:

1. Tabulation Method or Roster: Elements are enclosed in braces after separating them by commas.

2. Set Builder Method: The stating properties which its elements are to satisfy.

eg: If $A = \{1, 2, 3, 4\}$ then in set builder method,

$A = \{x \in N : x \leq 4\}$ $\quad N \rightarrow$ Set of natural numbers.

* Notations for sets of numbers:

1. $N \rightarrow$ Set of all natural numbers.

2. $I \rightarrow$ Set of all integers.

3. $I_0 \rightarrow$ Set of integers excluding zero.

4. $E \rightarrow$ Set of even integers.

5. $O \rightarrow$ Set of odd integers.

6. $Q \rightarrow$ Set of rational numbers.

$Q = \{xz : x = \frac{p}{q}$, where $p \& q \in I$, $q \neq 0\}$.

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7. $Q_0 \rightarrow$ Set of non-zero rational numbers.

8. $R \rightarrow$ Set of real numbers. (Set of rational + Set of Irrational numbers).

9. $C \rightarrow$ Set of complex numbers.

$$C = \{a+ib : a, b \in R \ \& \ i = \sqrt{-1}\}.$$

10. $N_a \rightarrow$ Set of all natural numbers which are less than or equal to a when a is positive integer.

* Different types of Sets:

1. Null Set (Void Set, ϕ): Set having no element. ϕ is a subset of every set & never written within braces. eg. $\{x : x \in R,\ x^2+1=0\} = \phi$.

2. Singleton or Unit Set: Set having one & only one element. eg. $\{x : x-3=4\}$ is a Unit Set.

3. Subset: A set's every element is also B set's element. $\Rightarrow$ A is called subset of B. $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$.

4. Equal Set: $[A \subseteq B \wedge B \subseteq A] \Leftrightarrow A = B$.

5. Power Set: Set of all the subsets of a given set A is the power set of A (P(A)). $P(A) = \{x : x \in A\}$

6. Super Set: $A \subseteq B \Rightarrow B \supseteq A$, B is called super set of A.

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7. Proper Subset: If every element of A is an element of B & B has at least one element which is not an element of A then A is proper subset of B. $A \subset B$.

8. Finite & Infinite Sets: A set in which the number of elements is finite is finite set, otherwise infinite set.

9. Cardinal number of finite set: $n(A)$.
The number of distinct elements in a finite set.

10. Universal Set: All the sets under consideration are likely to be subsets of a set is called the Universal set & is denoted by U.

11. Union of Sets: Union of two sets A & B is the set of all those elements which are either in A or in B or in both. $A \cup B = \{x : x \in A \vee x \in B\}$.
($\vee \rightarrow$ or)

$$A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n = \bigcup_{i=1}^{n} A_i$$

12. Intersection of Sets: Intersection of two sets A & B is the set of all elements which are common in A & B. $A \cap B = \{x : x \in A \wedge x \in B\}$
($\wedge \rightarrow$ and).

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13. Desjoint Sets: If A & B have no common elements, $A \cap B = \phi$ then they are called desjoint or mutually exclusive.

14. Difference of Sets: If A & B are two given sets, then the set of all those elements of A which do not belong to B is called difference of sets A & B. $\underset{\text{or } A - B}{A \sim B} = \{x : x \in A \wedge x \notin B\}$

$A \sim B \neq B \sim A$ | $A \cap B$, $A - B$, $B - A$ are desjoint.

15. Symmetric difference of two sets:

Symmetric difference of sets A & B is the set $(A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$ & is denoted by $A \Delta B$.

16. Complement Set: Let U be the universal set and A be a set such that $A \subset U$. Then the complement of A with respect to U is denoted by A' or A^c.

$$A^c = U - A = \{x : x \in U \wedge x \notin A\}.$$

* Total no. of Subsets: If a set has n elements then the number of subsets $= 2^n$.

* Number of power sets $= 2^n$.

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Set Theory.

* Laws & Theorems:

1. Idempotent Laws: For any A,

   i) $A \cup A = A$ ii) $A \cap A = A$.

2. Commutative Laws:

   i) $A \cup B = B \cup A$ ii) $A \cap B = B \cap A$.

3. Associative Laws:

   i) $A \cup (B \cup C) = (A \cup B) \cup C$

   ii) $A \cap (B \cap C) = (A \cap B) \cap C$.

4. Identity Laws:

   i) $A \cup \phi = A$ ii) $A \cap \phi = \phi$.

   iii) $A \cup U = U$ iv) $A \cap U = A$.

5. Distributive Laws:

   i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. $P(A) \cap P(B) = P(A \cap B)$

   $P(A) \cup P(B) \subseteq P(A \cup B)$.

7. Demorgan's Laws:

   i) $(A \cup B)^c = A^c \cap B^c$

   ii) $(A \cap B)^c = A^c \cup B^c$

   iii) $A - (B \cup C) = (A - B) \cap (A - C)$

   iv) $A - (B \cap C) = (A - B) \cup (A - C)$.

8. $(A^c)^c = A$, $U^c = \phi$, $A \cap A' = \phi$.

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* Results on Operation of Sets:

1. $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cap B \subseteq A$, $A \cap B \subseteq B$.
2. $A - B = A \cap B^c$
3. $(A - B) \cup B = A \cup B$
4. $(A - B) \cap B = \phi$
5. $A \subseteq B \Leftrightarrow B' \subseteq A'$
6. $A - B = B' - A'$
7. $(A \cup B) \cap (A \cup B') = A$
8. $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$
9. $A - (A - B) = A \cap B$
10. $A - B = B - A \Leftrightarrow A = B$
11. $A \cup B = A \cap B \Leftrightarrow A = B$.

* Cardinal Number of Some Sets:

1. $n(A') = n(U) - n(A)$.
2. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. $n(A \cap B') = n(A) - n(A \cap B)$
4. $n(A \cup B) = n(A) + n(B)$. [A and B disjoint]
5. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
6. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
7. $n(A - B) = n(A) - n(A \cap B)$
8. $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$
9. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
10. If $A_1, A_2, \ldots, A_n$ are disjoint sets.

$$n(A_1 \cup A_2 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + \cdots + n(A_n).$$

* Venn Diagrams (Euler-Venn Diagrams):

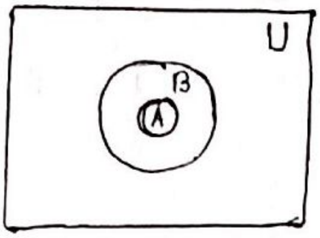
Diagrams drawn to represent sets are called Venn Diagrams. Universal set is represented by points within rectangle & subset A is represented by interior of a circle. Joyoshish Saha

Set Theory.

* Venn Diagrams of Sets :

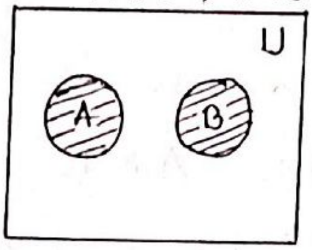
1. Subset.

$A \subseteq B$

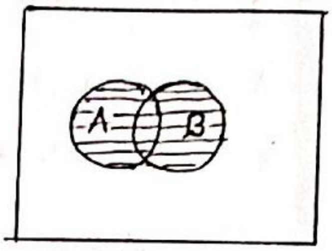


2. Union of Sets:

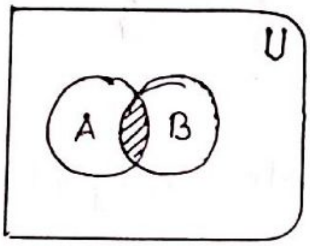
$A \cup B \;,\; A \cap B = \phi$



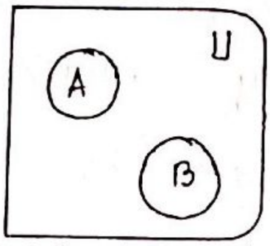
$A \cup B$



3. Intersection of Sets.

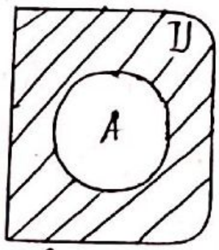


$A \cap B$



$A \cap B \,,\; A \cap B = \phi$

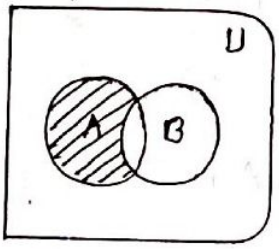
4. Complement Set :



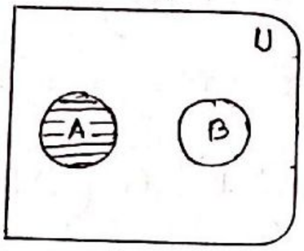
$A^c = U - A$

5. Difference of Sets :

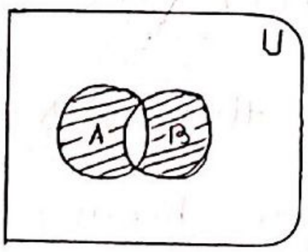
$A - B$



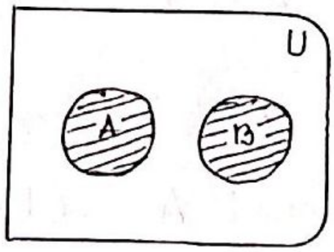
$A - B \;,\; A - B = A \;,\; A \cap B = \phi$



6. Symmetric Difference.



$A \Delta B$



$A \Delta B \,,\; A \cap B = \phi$.

* **Ordered Pair:** If A be a set & $a, b \in A$, then the ordered pair of elements a & b in A denoted by (a, b), where a is called the first co-ordinate & b the second.

(a, b) & (b, a) are different, $(a, b) = (c, d) \Rightarrow a = c,\; b = d$

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* Cartesian Product of Two Sets:

The cartesian product of two sets A & B is the set of all those ordered pairs whose first co-ordinate belongs to A & second co-ordinate belongs to B. This set is denoted by $A \times B$.

$$A \times B = \{(a,b) : a \in A \wedge b \in B\}.$$

- $A \times B \neq B \times A$.
- If A has p elements & B has q elements then $A \times B$ has pq elements.

- Important theorems on Cartesian product:

If A, B, C are three sets,

1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
2. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
3. $A \times (B - C) = (A \times B) - (A \times C)$
4. $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$
5. If $A \subseteq B$, $(A \times C) \subseteq (B \times C)$.
6. If $A \subseteq B$, $(A \times B) \cap (B \times A) = A^2$.
7. If $A \subseteq B$ & $C \subseteq D$, then $A \times C \subseteq B \times D$.

* Relations: A relation (or binary relation) R, from a non-empty set A to another non-empty set B, is a subset of $A \times B$.

$$R \subseteq A \times B.$$

$$R \subseteq \{(a,b) : a \in A \wedge b \in B\}.$$

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If (a,b) be an element of R then we write aRb.

// Any subset of $A \times A$ is a relation A.

// If A has m elements & B has n elements then $A \times B$ has mn elements & total number of different relation from A to B is 2^{mn}.

// If $R = A \times B$, then $\text{dom}.R = A$, Range $R = B$.

* Inverse Relation: If R is a relation from set A to set B, then the inverse relation of R, to be denoted by R^{-1}, is a relation from B to A.

$$R^{-1} = \{(b,a) : (a,b) \in R\}.$$

$\text{dom}(R^{-1}) = \text{range}(R)$ & $\text{range}(R^{-1}) = \text{dom}(R)$.

$(R^{-1})^{-1} = R$

* Identity Relation: The identity relation on a set A is the set of ordered pairs belonging to $A \times A$ is denoted by I_A.

$$I_A = \{(a,a) : a \in A\}.$$

* Universal Relation: If R is the set of $A \times A$, then the relation R in the set A is called the universal relation.

* Void Relation: ϕ is a relation on A & called void relation if $\phi \subset A \times A$

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* Various Types of Relations:

1. Reflexive : If $aRa \ \forall \ a \in A$

   i.e. $(a,a) \in R \ \forall \ a \in A$.

2. Symmetric : If $aRb \Rightarrow bRa \ \forall \ a,b \in A$

   i.e. $(a,b) \in R \Rightarrow (b,a) \in R \ \forall \ a,b \in A$.

3. Anti-symmetric: If aRb & $bRa \Rightarrow a=b \ \forall \ a,b \in A$

4. Transitive : If aRb & $bRc \Rightarrow aRc \ \forall \ a,b,c \in A$.

   i.e. $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R \ \forall \ a,b,c \in A$.

5. Equivalence Relation: Relation R on a set A is said to be an equivalence relation on A when R is reflexive, symmetric & transitive. It is denoted by $\sim$.

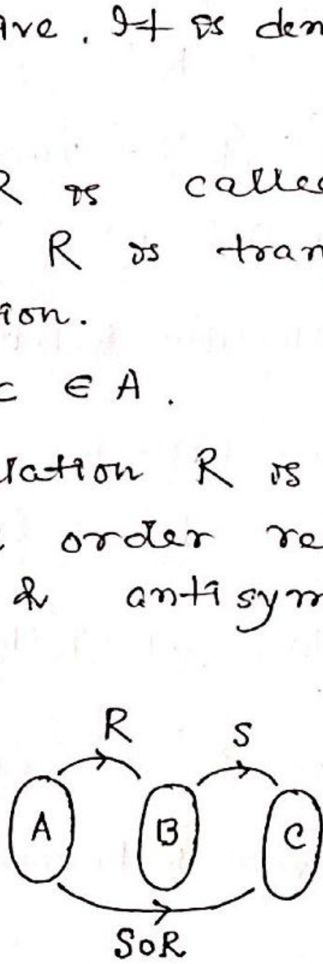
6. Ordered Relation: A relation R is called ordered if R is transitive but not an equivalence relation.

   $aRb, bRc \Rightarrow aRc \ \forall \ a,b,c \in A$.

7. Partial Order Relation: A relation R is called partial order relation if R is reflexive, transitive & antisymmetric.

* Composition of two relations:

If A, B & C are three sets such that $R \subseteq A \times B$ and $S \subseteq B \times C$ then $(SoR)^{-1} = R^{-1} o S^{-1}$.



$aRb, bRc \Rightarrow a \ SoR \ c$

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Set Theory

* Theorems on Binary Relation:

If R is a relation on a set A then

1. R is reflexive $\Rightarrow R^{-1}$ is reflexive
2. R is symmetric $\Rightarrow R^{-1}$ is symmetric.
3. R is transitive $\Rightarrow R^{-1}$ is transitive.

* Congruence: Let m be a positive integer, then the two integers a & b are said to be congruent modulo m if a-b is divisible by m. i.e. $a-b = m\lambda$.

$$a \equiv b \pmod{m}.$$