10+2 PCM NOTES

BY

JOYOSHISH SAHA

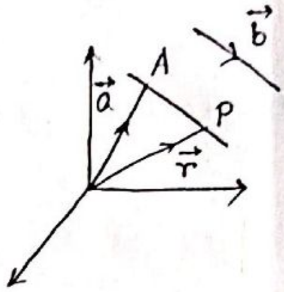
(PDF version handwritten notes of Maths, Physics and Chemistry for 10+2 competitive exams like JEE Main, WBJEE, NEST, IISER Entrance Exam, CUCET, AIPMT, JIPMER, EAMCET etc.)

10+2

3D Geometry.

* Straight Line:

1. Equation of a straight line passing through a given point & parallel to a given vector:



- Vector form: $\vec{r} = \vec{a} + t\vec{b}$

  [$t \rightarrow$ parameter].

- Cartesian form:

$A \equiv (x_1, y_1, z_1)$

$\vec{b} = l\hat{i} + m\hat{j} + n\hat{k}$

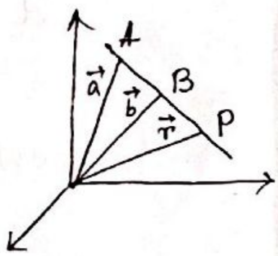
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = t$$

- Non-parametric Vector Equ^n:

$$(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}.$$

2. Equation of a straight line passing through two given points;



- Vector form: $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$

- Cartesian form:

$A \equiv (x_1, y_1, z_1)$

$B \equiv (x_2, y_2, z_2)$.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

- Non-parametric Vector Equ^n:

$$(\vec{r} - \vec{a}) \times (\vec{b} - \vec{a}) = \vec{0}$$

* Angle Between Two Lines:

- Vector form: $\vec{r} = \vec{a} + t\vec{b}$; $\vec{r} = \vec{c} + t\vec{d}$

$$\theta = \cos^{-1}\left|\frac{\vec{b} \cdot \vec{d}}{|\vec{b}|\,|\vec{d}|}\right|.$$

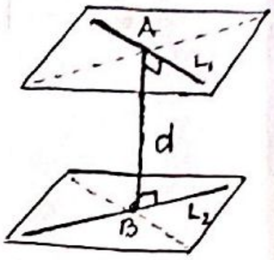
Joyoshish Saha

- Cartesian form:

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \; ; \; \frac{x-x_1}{a_2} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_2}.$$

$$\theta = \cos^{-1}\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\;\sqrt{a_2^2+b_2^2+c_2^2}}\right|.$$

* Skew Lines: Two lines which are neither intersecting nor parallel are called skew lines.



* Shortest distance between two lines:

1. Shortest distance between two intersecting lines is zero.

2. Parallel Lines: $\vec{r} = \vec{a_1} + t\vec{b}$

$\vec{r} = \vec{a_2} + t'\vec{b}$

$$d = \frac{|\vec{b} \times (\vec{a_2} - \vec{a_1})|}{|\vec{b}|}$$

3. Skew Lines: $\vec{r} = \vec{a_1} + t\vec{b_1}$

$\vec{r} = \vec{a_2} + t'\vec{b_2}$

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

- Cartesian form: $l_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

$l_2 \Rightarrow \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}.$

$$d = \left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

Joyoshish Saha

* Condition for two lines to intersect:

$\vec{r} = \vec{a_1} + t\vec{b_1}$

$\vec{r} = \vec{a_2} + t'\vec{b_2}$.

$|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})| = 0$.

Cartesian –

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

* Collinearity of Three Points:

Three points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear iff

$$\lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c} = 0 \;\&\; \lambda_1 + \lambda_2 + \lambda_3 = 0.$$

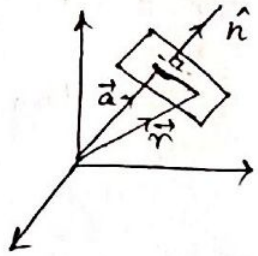
[$\lambda_1, \lambda_2, \lambda_3$ not all zeroes].

* Plane:

1. Equation of a plane passing through a given point & perpendicular to a vector:

- Vector form: $(\vec{r} - \vec{a}) \cdot \hat{n} = 0$

$\Rightarrow \vec{r} \cdot \hat{n} = \vec{a} \cdot \hat{n}$



- Cartesian form:

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\hat{n} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

If passing through origin, $\vec{r} \cdot \vec{n} = 0$.

2. Equation of the plane when distance from the origin and unit normal is given:

- Vector form: $\vec{r} \cdot \hat{n} = p$. $\left[\begin{array}{l} \hat{n} \to \text{unit normal} \\ p \to \text{distance} \end{array}\right]$.

- Cartesian form: $\hat{n} = l\hat{i} + m\hat{j} + z\hat{k}$

$$lx + my + nz = p.$$

3. Equ^n of the plane passing through a given point & parallel to two vectors:

- Vector form: $\vec{r} = \vec{a} + t\,\vec{b} + u\,\vec{c}$

[$\vec{a}$ – given point
parallel to $\vec{b}$ & $\vec{c}$].

- Cartesian form:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{b} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$

$\vec{c} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$

- Non parametric form: $[(\vec{r} - \vec{a})\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow [\vec{r}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}].$$

4. Equation of the plane passing through two given points & parallel to a given vector:

- Vector form:

$\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t\vec{v}$ [$\vec{a}, \vec{b}$ – position vectors of points, parallel to $\vec{v}$].

- Non - parametric:

$$[(\vec{r} - \vec{a})\ (\vec{b} - \vec{a})\ \vec{v}] = 0.$$

Joyoshish Saha

- Cartesian form:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0.$$

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$
$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$
$\vec{v} = l\hat{i} + m\hat{j} + n\hat{k}.$

5. Equation of the plane passing through three given non-collinear points:

- Vector form: $\vec{r} = \vec{a} + s(\vec{b}-\vec{a}) + t(\vec{c}-\vec{a})$
  [$\vec{a}, \vec{b}, \vec{c}$ — position vectors of points].

- Non-parametric: $[(\vec{r}-\vec{a})\ (\vec{b}-\vec{a})\ (\vec{c}-\vec{a})] = 0.$

- Cartesian form:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$$

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$
$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$
$\vec{c} = x_3\hat{i} + y_3\hat{j} + z_3\hat{k}.$

6. Equation of a plane passing through the line of intersection of two given planes:

- Vector form: Equ^n of plane passing through the line of intersection of planes $\vec{r}\cdot\vec{n}_1 = q_1$ & $\vec{r}\cdot\vec{n}_2 = q_2$ is

$$(\vec{r}\cdot\vec{n}_1 - q_1) + \lambda(\vec{r}\cdot\vec{n}_2 - q_2) = 0.$$

- Cartesian form: $a_1x + b_1y + c_1z + d_1 = 0$
  $a_2x + b_2y + c_2z + d_2 = 0.$

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0.$$

Joyoshish Saha

7. Equ^n of the plane which contains two given lines :

- Vector form: lines - $\vec{r} = \vec{a}_1 + t\vec{u}$

$$\vec{r} = \vec{a}_2 + s\vec{v}$$

$$[(\vec{r} - \vec{a}_1)\ \vec{u}\ \vec{v}] = 0 \quad \& \quad [(\vec{r} - \vec{a}_2)\ \vec{u}\ \vec{v}] = 0$$

- Cartesian form: $\vec{a}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$$\vec{a}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}.$$

$$\vec{u} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$$

$$\vec{v} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}.$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$$

* The distance between a point & a plane:

If (x_1, y_1, z_1) be a point & $ax + by + cz + d = 0$ be the equation of the plane, then

$$d = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|.$$

- Distance between two parallel planes

~~lines~~ $ax_2 + by_2 + cz_2 + d_1 = 0$ &

$ax_2 + by_2 + cz_1 + d_2 = 0.$ is

$$\left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right|.$$

* Intercept form of the equ^n of a plane:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0.$$

$a, b, c \rightarrow X, Y, Z$ axes intercept

* Coplanarity of two lines:

Lines. $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$

$\vec{r} = \vec{a_2} + \lambda' \vec{b_2}$

Condition – $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0.$

i.e. $$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

* Angle between two planes:

$\vec{r} \cdot \vec{n_1} = q_1$

$\vec{r} \cdot \vec{n_2} = q_2$

$$\theta = \cos^{-1}\left[\frac{\vec{n_1} \cdot \vec{n_2}}{n_1 n_2}\right]$$

* Angle between a line and a plane:

$\vec{r} = \vec{a} + t\vec{b}$

$\vec{r} \cdot \vec{n} = q.$

$$\theta = \sin^{-1}\left[\frac{\vec{b} \cdot \vec{n}}{b n}\right].$$

* Equation of bisectors of the angle between two planes: $ax + by + cz + d = 0$

$a_1 x + b_1 y + c_1 z + d_1 = 0.$ $[d, d_1 > 0]$.

$$\frac{ax + by + cz + d}{\sqrt{a^2 + b^2 + c^2}} = \pm \frac{a_1 x + b_1 y + c_1 z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$$

$aa_1 + bb_1 + cc_1$	acute angle bisector	obtuse angle bisector
>0	$-$	$+$
<0	$+$	$-$

Joyoshish Saha

* Image of a point with respect to a plane mirror: Image of $A(x_1, y_1, z_1)$ wrt $ax+by+cz+d=0$ be $B(x_2, y_2, z_2)$

$$\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{z_2-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

* Feet of perpendicular:

$$\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{z_2-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}.$$

* Reflection of a plane on another plane: The reflection of the plane $ax+by+cz+d=0$ on the plane $a_1x+b_1y+c_1z+d_1=0$ is.

$$2(aa_1+bb_1+cc_1)(a_1x+b_1y+c_1z+d_1) = (a_1^2+b_1^2+c_1^2)(ax+by+cz+d)$$

* An equation of second degree (homogeneous) represents two planes passing through origin.

Condition - $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$

* If θ is the acute angle between two planes $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ then

$$\theta = \tan^{-1}\left\{\frac{2\sqrt{f^2+g^2+h^2-bc-(ca-ab)}}{a+b+c}\right\}.$$

If planes are perpendicular, then

$$a+b+c=0.$$

Joyoshish Saha