Voyage Into the Unknown

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Question 1: Why does re-planning only occur when blocks are discovered on the current path? Why not whenever knowledge of the environment is updated?

Answer: Because A* finds the optimal path, it shouldn't matter if there are any obstacles that are not in the path as we will continue on our current path. It would be a waste of time to re-plan every time there's an update in the environment as planning takes at best $O(n)$ each time and if there are at most n blocked nodes, we can see that this situation would quickly rise to exponential time.

Question 2: Will the agent ever get stuck in a solvable maze? Why or why not?

Answer: Because we use the A* algorithm, we will never get stuck in a solvable maze. This is because A* will explore every node that's reachable until there are no more nodes to explore, in which case the maze has to be unsolvable. The agent *does* get stuck in a maze when its neighbors are arranged in a way that it is blocked from progressing even if it back tracks *(an unsolvable maze)*. Let's take an example of one:

```
[0, 0, 1, 1, 1, 1, 1, 1, 1, 1]
[1, 1, 1, 1, 1, 1, 1, 1, 1, 0]
[0, 0, 0, 0, 0, 1, 1, 0, 1, 1]
[1, 1, 0, 0, 0, 1, 1, 1, 0, 0]
[1, 0, 1, 1, 1, 1, 0, 1, 1, 1]
[1, 1, 0, 1, 1, 0, 0, 0, 1, 0]
[1, 1, 1, 0, 0, 1, 1, 0, 1, 1]
[1, 1, 0, 1, 1, 0, 1, 1, 1, 0]
[0, 1, 0, 1, 1, 0, 1, 1, 1, 1]
[1, 0, 1, 0, 1, 1, 0, 1, 1, 0]
```

We clearly see that the above maze is not solvable because once the agent takes a step to the right, there is no additional step to take other than back to the start and once it back tracks there is no way to move forward. Now lets see an example of a solvable maze:

```
[0, 1, 0, 0, 0, 0, 0, 0, 0, 1]
[0, 0, 0, 0, 0, 0, 0, 0, 1, 0]
[0, 0, 1, 0, 0, 1, 1, 0, 0, 1]
[1, 0, 1, 0, 0, 0, 0, 0, 0, 1]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 1, 0, 0, 0]
[1, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[1, 0, 0, 0, 0, 0, 0, 0, 1, 0]
[0, 0, 0, 0, 0, 1, 0, 0, 0, 0]
```

Once we run repeated A* we find the following path:

```
[2, 1, 0, 0, 0, 0, 0, 0, 0, 1]
[2, 2, 0, 0, 0, 0, 0, 0, 1, 0]
[0, 2, 1, 0, 0, 1, 1, 0, 0, 1]
[1, 2, 1, 0, 0, 0, 0, 0, 0, 1]
[0, 2, 2, 2, 2, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 2, 2, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 2, 1, 0, 0, 0]
[1, 0, 0, 0, 0, 2, 2, 2, 0, 0]
[1, 0, 0, 0, 0, 0, 0, 2, 1, 0]
[0, 0, 0, 0, 0, 1, 0, 2, 2, 2]
```

Now although the agent had to recalculate **4** times due to obstacles in its original path, our agent is always going to move backwards and find a different path each time. We do this by first checking if the current path is blocked and returning the section before the blocked node.

```
if complete_grid.gridworld[curr[0]][curr[1]] == 1:
    # update our knowledge of blocked nodes
    discovered_grid.update_grid_obstacle(curr)
    # remove the path starting with the blocked node
    path = path[:index]
    return path
```

Then we go back and create a new path (avoiding the visited neighbor nodes) using repeated A*.

```
last_node = new_path.pop()
last_unblock = last_node.curr_block
# append the rest to the final path
final_path.extend(new_path)
# check if the path made it to the goal node
if last_unblock == (dim-1, dim-1):
    final_path.append(last_node)
    break
# create a new path from the last unblocked node
new_path = path_planner(last_unblock, discovered_grid, dim, euclidian)
```

By using this smart method of backtracking in the case of a blockage in the path, we can avoid getting stuck in a solvable maze as we can always move back node by node to check if there are any unvisited paths that were missed in the first go around.

Question 3: Once the agent reaches the target, consider re-solving the now discovered gridworld for the shortest path (eliminating any backtracking that may have occurred). Will this be an optimal path in the complete gridworld? Argue for, or give a counter example.

Answer: While the A* algorithm will always give us the shortest path; repeated A* might not give us the most optimal path. For example let us take the following maze:

```
[0,0,0,0,0,0]
[0,1,1,1,1,0]
[0,1,1,0,0,0]
[0,1,0,0,1,0]
[0,1,0,1,1,0]
[0,0,0,1,1,0]
```

Depending on how we setup our queue and heuristic functions, our agent has two options:

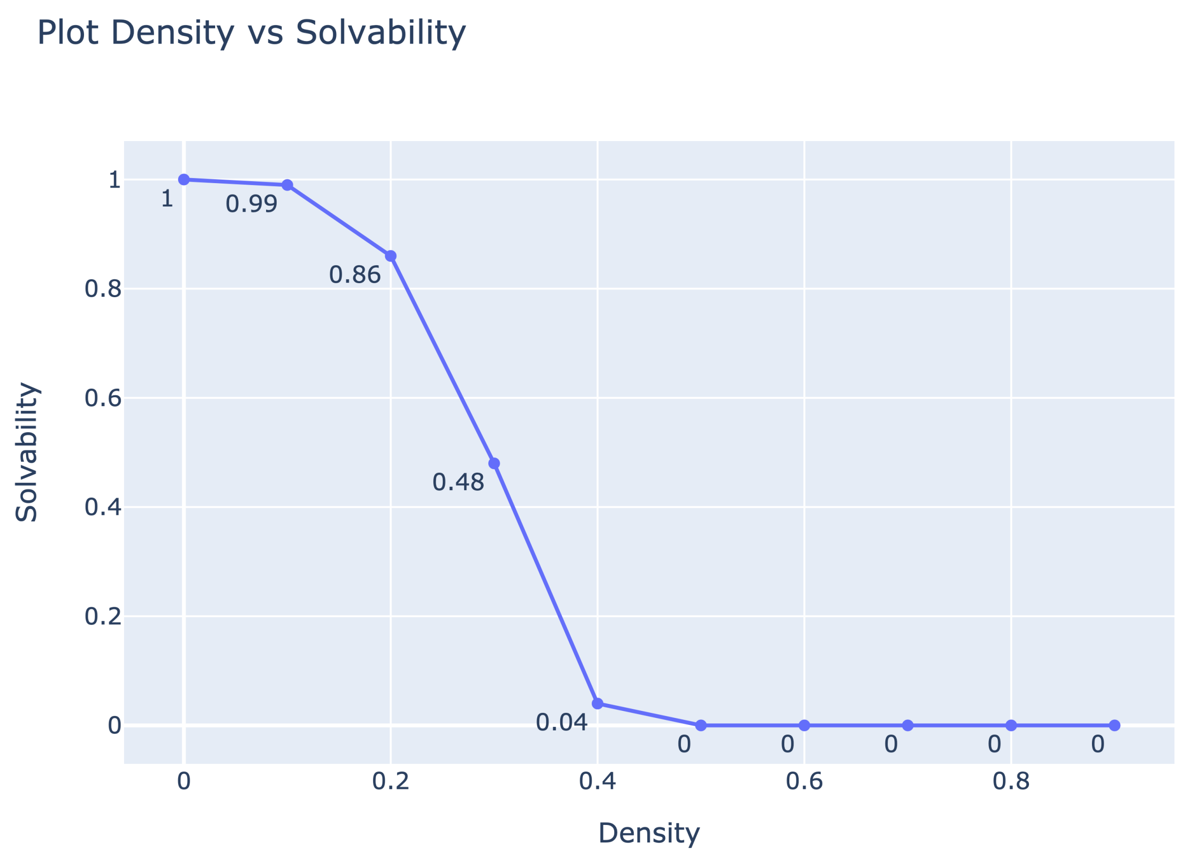
1. If the agent takes a step to the right first then it is on its way to find the shortest path. In this case our discovered gridworld once the agent moves through it will contain the shortest path and A* will follow this path to the finish node.
2. If the agent takes a step down first then it will have to follow the longer path. In this case our discovered gridworld will not contain the shortest path of the complete gridworld and therefore will not be able to traverse it once we re run A8 on the discovered world.

Therefore as you can see, if we were to re-solve our discovered gridworld once a path was found; though it is certainly possible that the shortest (most optimal) path is found it is by no means a guarantee.

This means that if we were to re-solve the discovered gridworld after finding a path, the new path is guaranteed to be the shortest due to the way our heuristic/priority is measured. To back up my argument, let me explain how new nodes are added to the priority queue:

Question 4: A gridworld is solvable if it has a clear path from start to goal nodes. How does solvability depend on p? Given $dim = 101$, how does solvability depend on p? For a range of p values, estimate the probability that a maze will be solvable by generating multiple environments and checking them for solvability. Plot density vs solvability, and try to identify as accurately as you can the threshold p_0 where for $p < p_0$, most mazes are solvable, but $p > p_0$, most mazes are not solvable. Is A* the best search algorithm to use here, to test for solvability? Note for this problem you may assume that the entire gridworld is known, and hence only needs to be searched once each.

Answer: We can assume that once density of blocks increases, there will be a steep decline in solvability and the number of paths to the finish node will be severely reduced. To test our hypothesis we calculated solvability after running 100 trials of the algorithm at p being each tenth of a decimal between 0.1 to 0.9. After running our tests, we got this resulting plot.



As we can see, there's a clear and steep decline of the solvability of the gridworld once we increase p_0 to 0.4. Therefore our hypothesis has been tested and true; solvability does indeed fall steeply once you reach a threshold probability of $p_0=0.4$ when solvability decreases to 0.05 according to our testing.

We do believe that A* is the best search algorithm as it is guaranteed to find a path in any solvable gridworld due to it's backtracking, path planning, and heuristics methodologies. Therefore if A* did not succeed it means that the gridworld was not a solvable one which is what we are trying to measure.

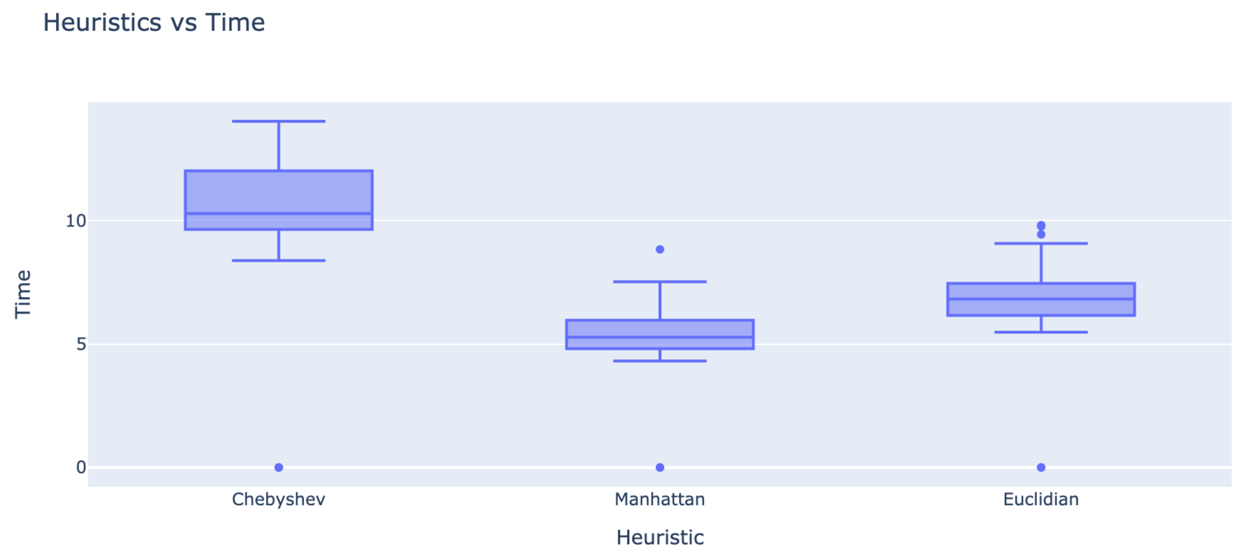
Question 5: Among environments that are solvable, is one heuristic uniformly better than the other for running A*? How can they be compared? Plot the relevant data and justify your conclusions. Again, you may take each gridworld as known, and thus only search once.

Answer: Acording to our findings the manhattan distance is uniformly better than the others for running A*. To find the best heuristic we first fixed $p=0.1$ and $dimension=500$ and we ran the following loop 100 times recording the time each loop and each heuristic. To be fair to all three heuristics we made sure to use the same grid for each one every time we ran the loop.

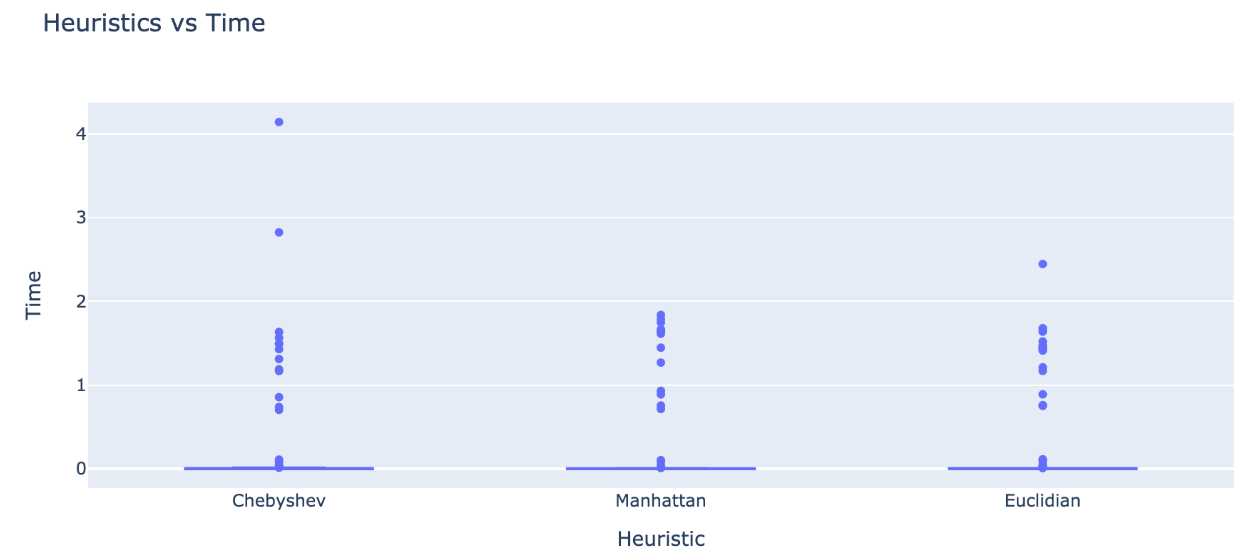
```
# create the gridworld
    complete_grid = Gridworld(dim, prob, False)
    final_path = None

    # times: chebyshev, manhattan, euclidian
    times = []

    for i,h in enumerate([chebyshev, manhattan, euclidian]):
```

```
        starting_time = time()
        path_planner((0,0), final_path, complete_grid, dim, h)
        times.append(time() - starting_time)

    print(times)
```

Then we saved all of the times and graphed the data using a bar plot.



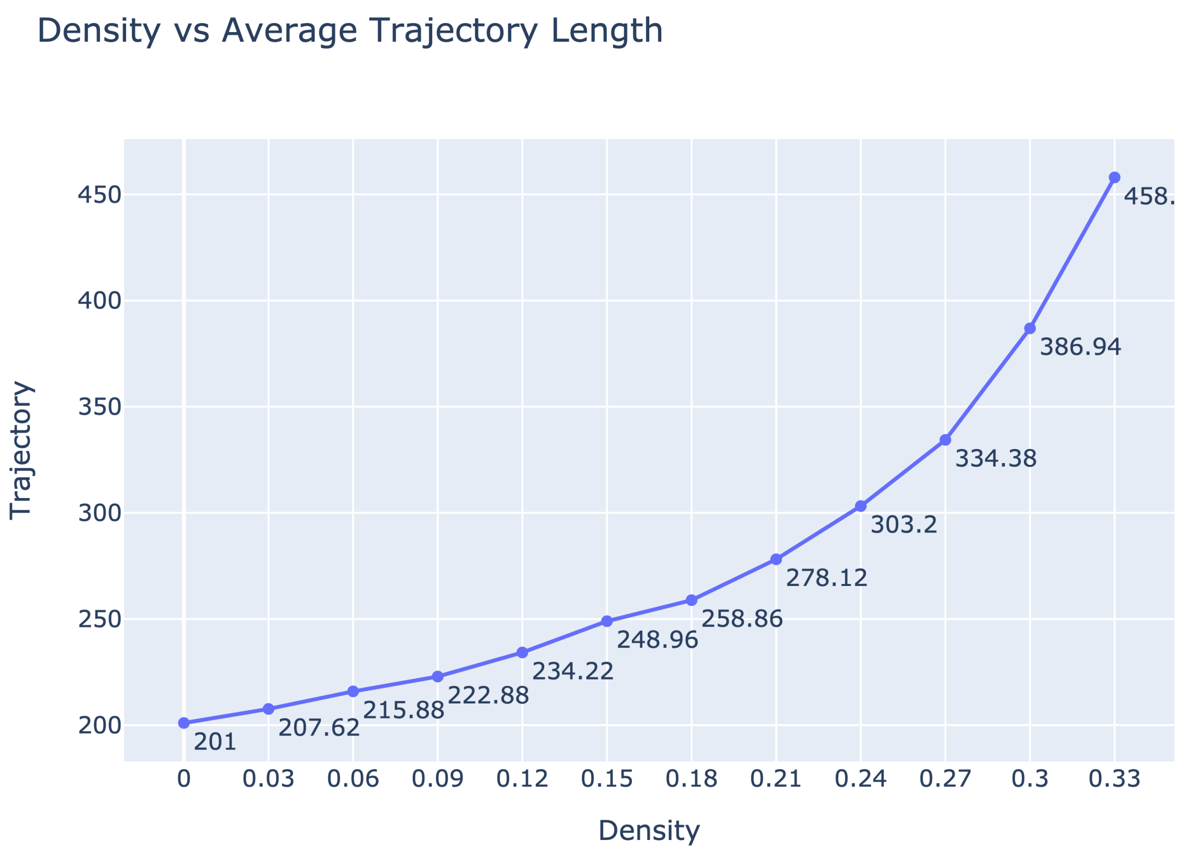
As you can see the Manhattan distance had the lowest run time out of all the other heuristics. To make sure that this data is supported we re-ran our loop with $p=0.4$ this time which was the threshold for solvability as we saw before. The bar plot is shown below.



From this plot we see that our initial observation holds true even when we increase our blocking probability to 0.4. Though there are significantly more unsolvable grids.

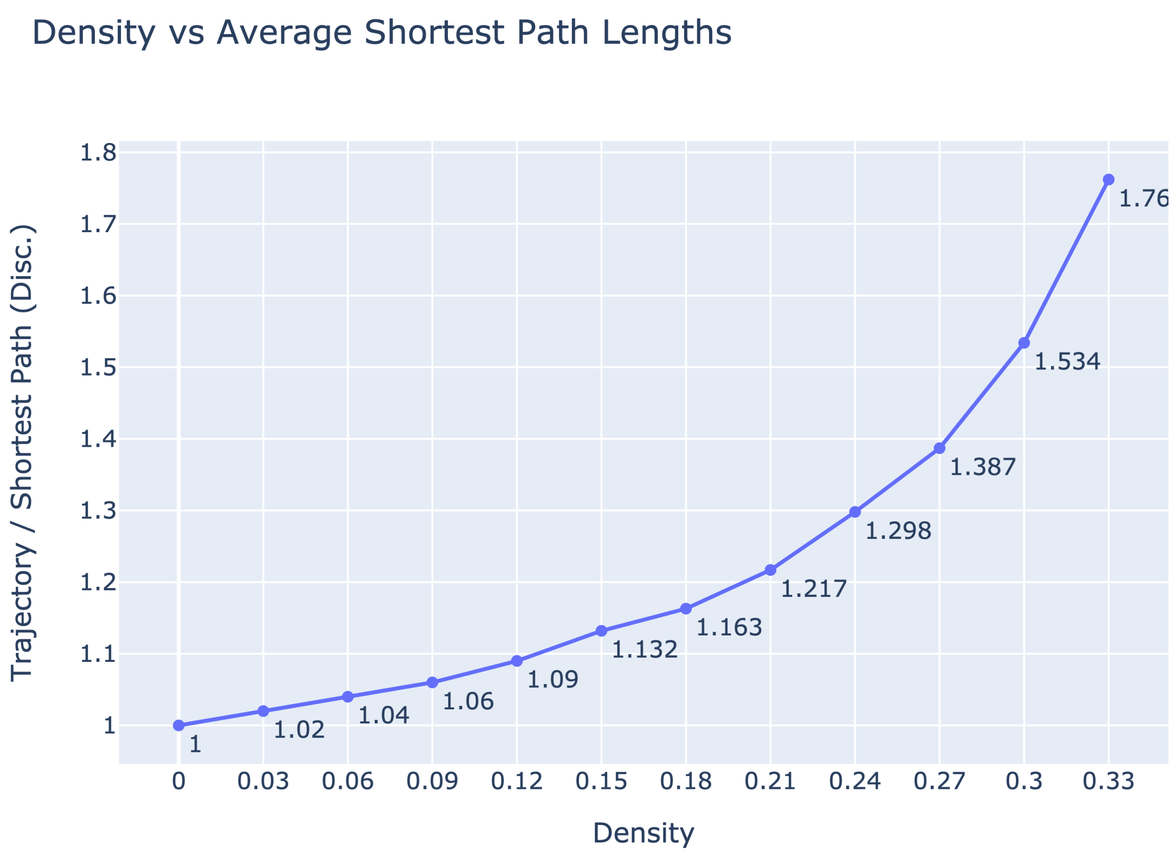
Question 6: Taking $dim = 101$, for a range of density p values from 0 to min $(p_0; 0.33)$, and the heuristic chosen as best in Q5, repeatedly generate gridworlds and solve them using Repeated Forward A*. Use as the field of view each immediately adjacent cell in the compass directions. Discuss your results. Are they as you expected? Explain.

Answer: To calculate density average trajectory length first split our tests into 100 trials. In each trial we we ran the repeated A* algorithm 12 times between $p=0$ and $p=0.33$ at intervals of 0.03 to see how the trajectory would increase when density is increased. Then we took the average of the hundred trials for each p-value ending up with 12 total data points for trajectory.



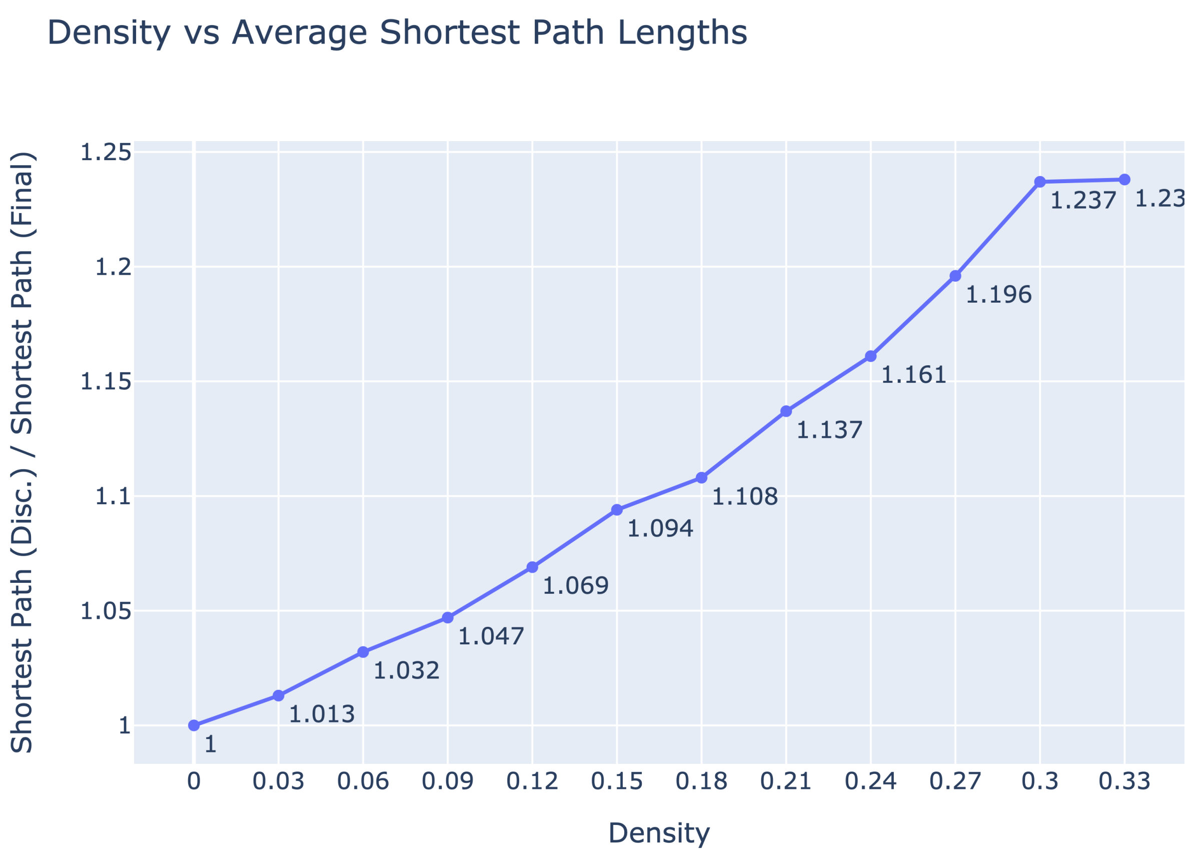
As you can see we achieved the expected result which is the positive correlation between density and average trajectory length. Obviously our algorithm had to take more steps when there are more blockages in its path.

For the next graph we followed the same experiment procedure but in addition to calculating trajectory we also calculated the length of the shortest path in the discovered gridworld by running A* on the discovered grid after running repeated A*. Then after taking the average we divided the averages of the trajectories by the averages of the shortest path.

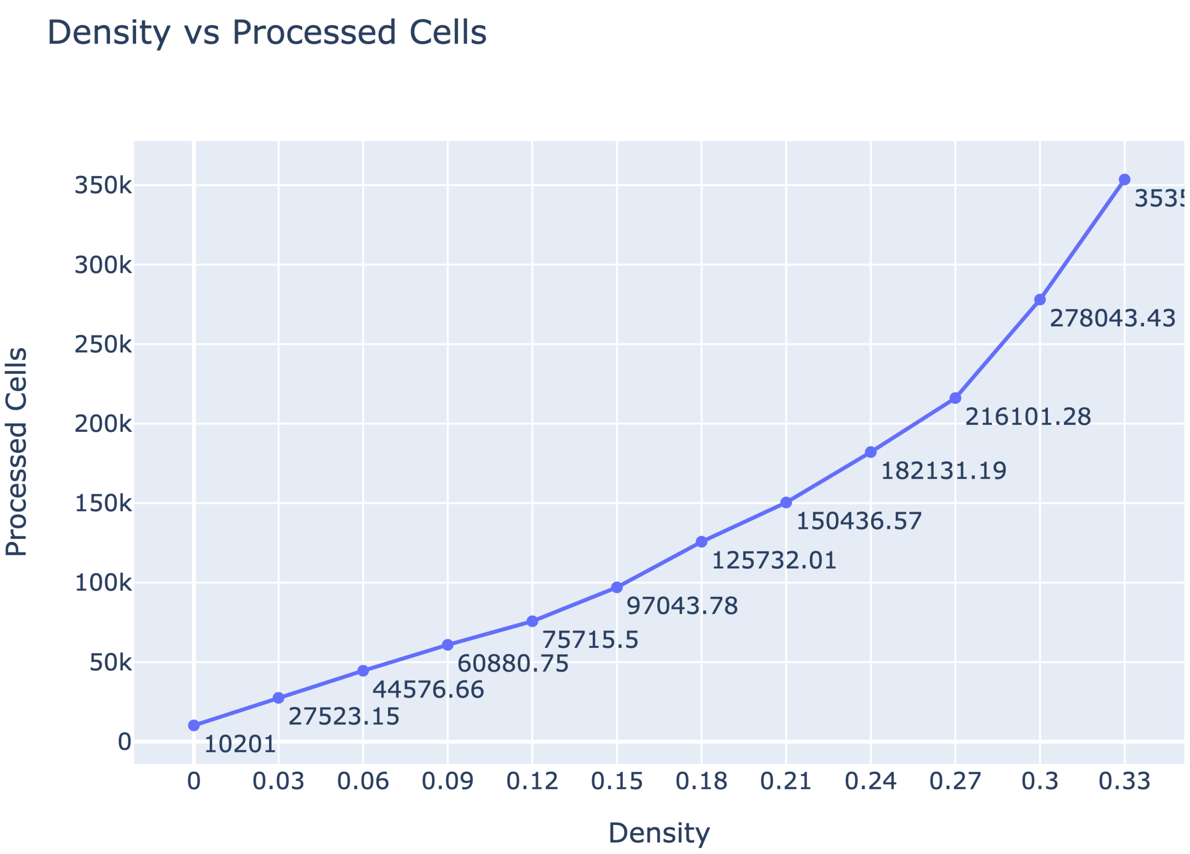


As you can see, initially while density was $p=0$ the trajectory (number of total steps taken by repeated A*) and the shortest path is the same. However as the density of obstacles increases, repeated A* will have to backtrack way more times which means that the fraction will obviously increase as the numerator increases quicker than the denominator; this is because the shortest path will never backtrack as we run A* on the already discovered gridworld which will return the most optimal path.

Next, we look at running the shortest path on the discovered gridworld divided by the shortest path on the complete gridworld. One can expect that similar to the previous part, running shortest path on the complete grid will be much more efficient than the discovered as repeated A* might not have discovered a more efficient path initially due to blockers.



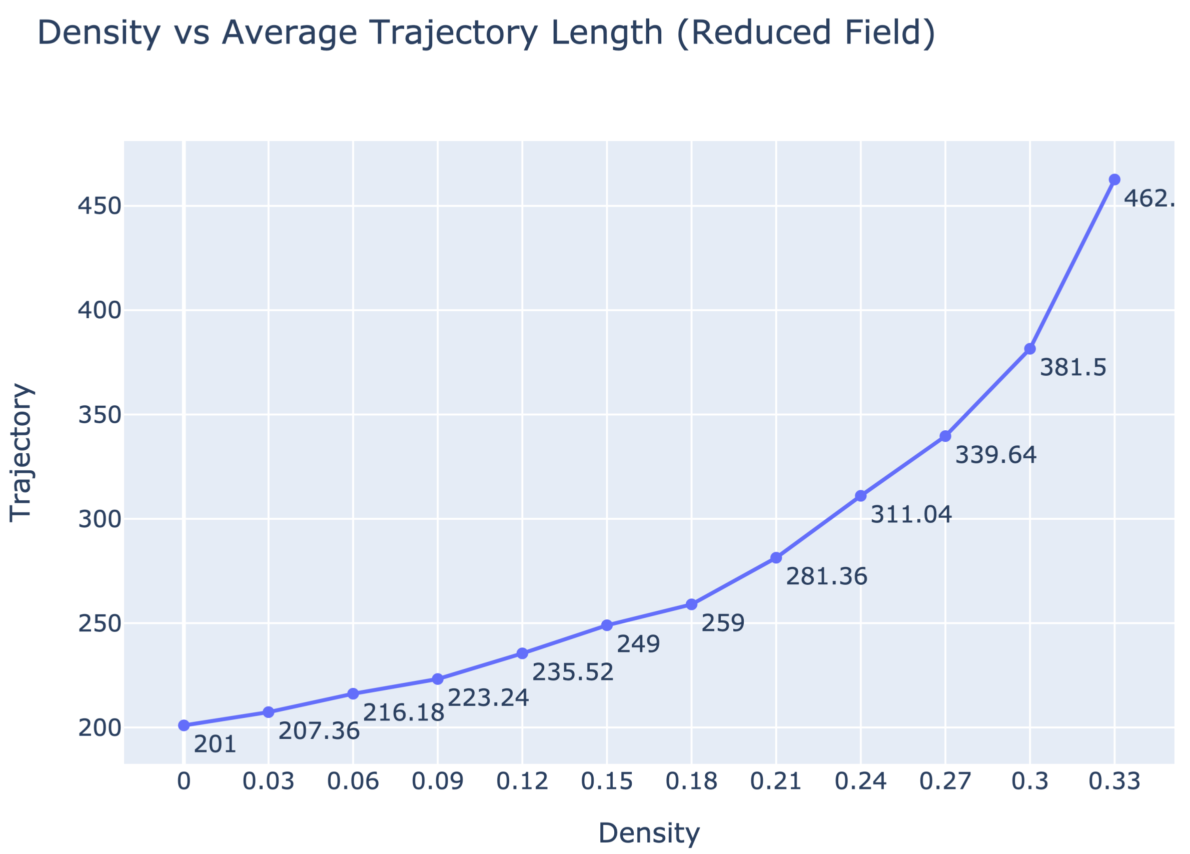
We see that this is true, however unlike the last part, our new graph increases linearly and plateaus at the end when density is $p=0.33$. This most likely means that at a certain density, the discovered grid will start to act like the complete grid as more of it will be discovered by the agent due to the increased blockages in its path. To see if this is true let's turn to our last graph.



And again our hypothesis is proven true as there is a clear exponential increase in the number of cells that are processed(discovered) once density is increased. This is because each time there is a block, our agent looks at every neighbor and assesses other options.

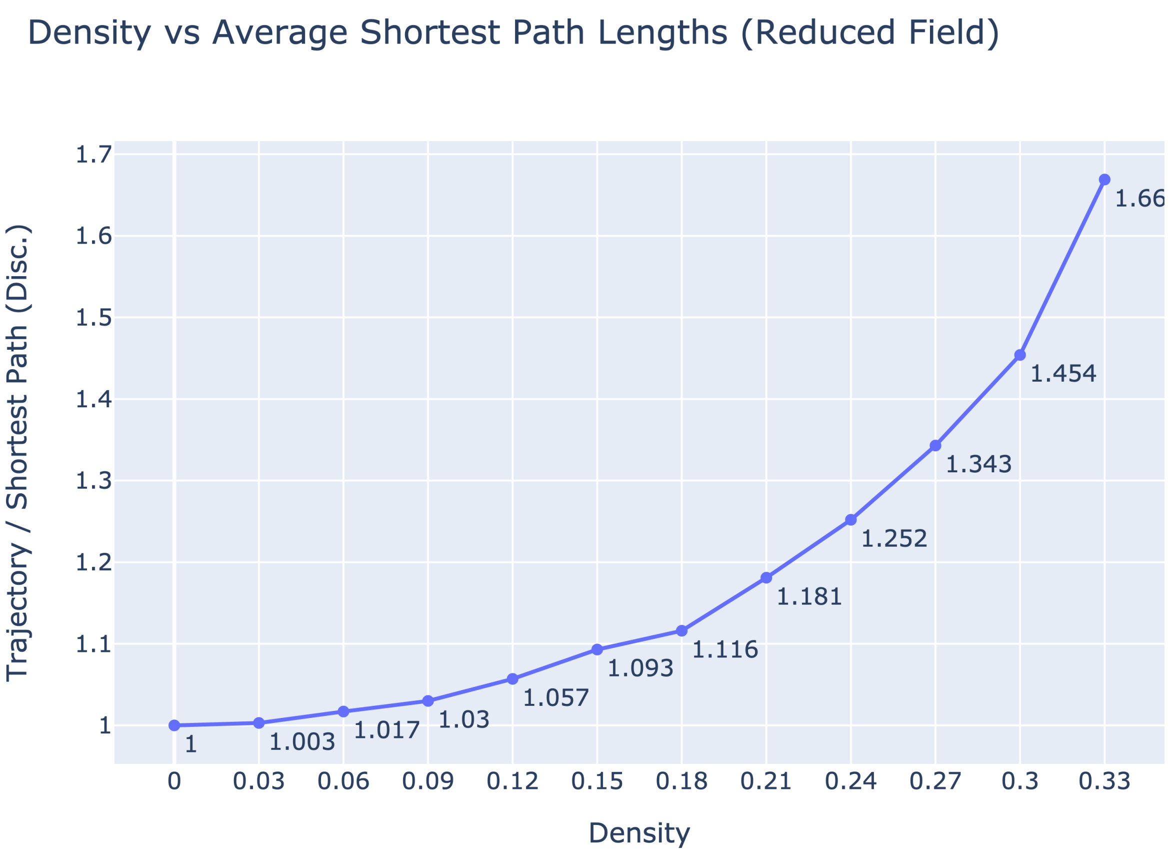
Question 7: Generate and analyze the same data as in Q6, except using only the cell in the direction of attempted motion as the eld of view. In other words, the agent may attempt to move in a given direction, and only discovers obstacles by bumping into them. How does the reduced field of view impact the performance of the algorithm?

Answer: Because less of the gridworld is updated on each move of the agent, you would think that trajectory length might be a bit longer in the reduced field of view as you might need to back track more.



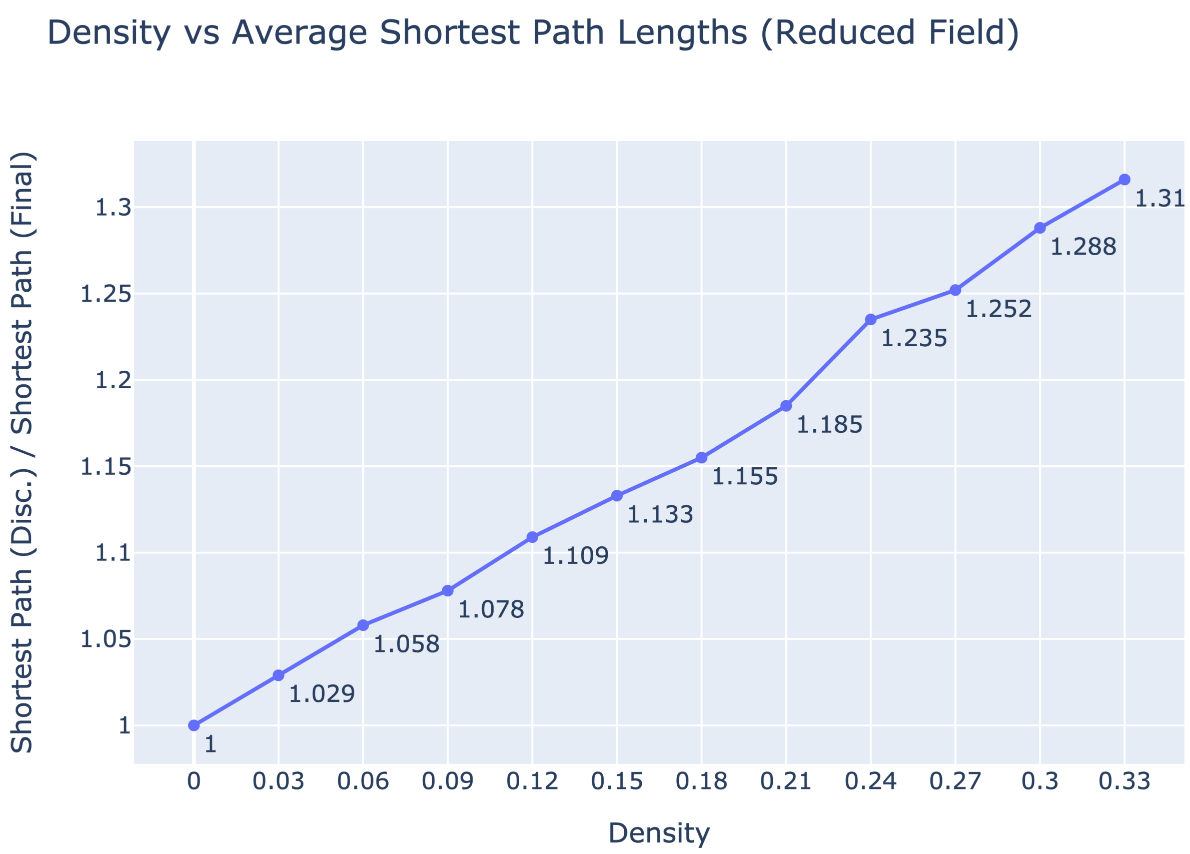
While this might be true to an extent at higher densities of blockers, we really did not see that much of an increase in the trajectory length overall in our A* implementation. This might mean that while doing repeated A* it really doesn't make much of a difference in the end overall trajectory to update neighbors while traversing the gridworld. However it takes a longer amount of time to execute the algorithm and find the trajectory.

Next we looked at the average length of the trajectory divided by the average length of the shortest path on the discovered grid. Because trajectories did not increase as much you would expect the shortest path to not have been too impacted as well as both rely on the density in a similar way.



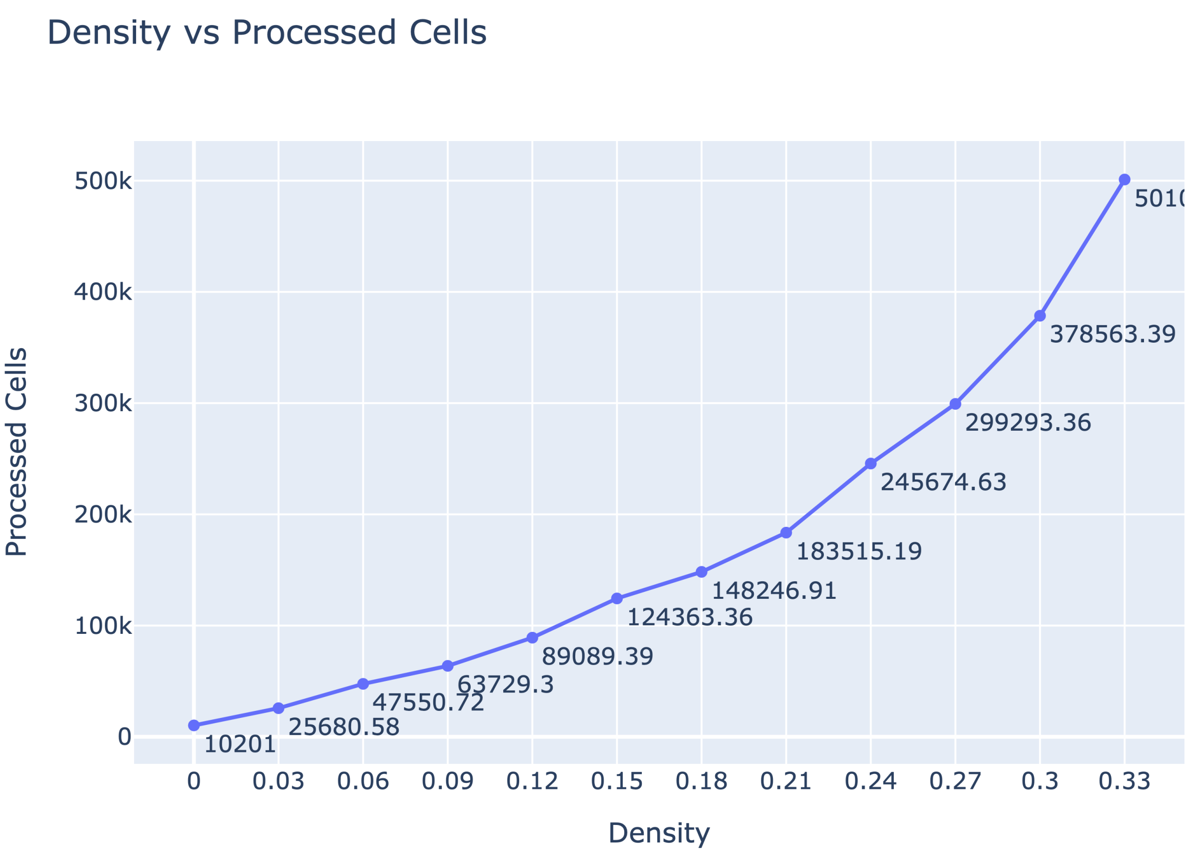
We can again see that the ratio really did not change as much again. However at higher densities the ratio between reduced and full field of vision decreasse by about a tenth. Which means that the shortest path increased in higher densities. This is because we have less of the gridworld discovered when we run A* therefore we are less likely to find an optimal path on the gridworld which leads to an increased distance in the shortest path

Our previous conclusion directly impacts our next hypothesis. If the shortest path on the discovered grid world increased this means that our ratio between discovered and complete shortest paths increases. This is because the shortest path on the final grid will likely stay the same.



We can see from our plot that our hypothesis is supported. Even at higher densities the ratio keeps rising unlike our full field of vision as the shortest path in the reduced continues to rise.

Finally if we explore less while we're moving through the path we're surly going to have to process more cells as the only way to discover obstacles is to now encounter them and to add them to the fringe.



Here we can clearly see that the number of nodes required to be processed is significantly greater if we don't explore neighbors at each step in the path. This is because the agent would now have to move more and bump into more nodes to figure out what the landscape looks like.

Question 9: A* can frequently be sped up by the use of inadmissible heuristics - for instance weighted heuristics or combinations of heuristics. These can cut down on runtime potentially at the cost of path length. Can this be applied here? What is the efect of weighted heuristics on runtime and overall trajectory? Try to reduce the runtime as much as possible without too much cost to trajectory length.

Answer: