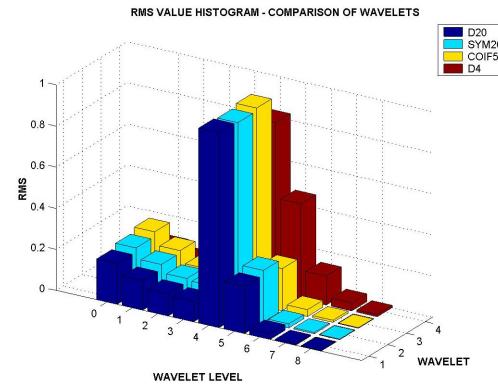




Title: Understanding wavelet analysis and filters for engineering applications



Chethan Parameswariah

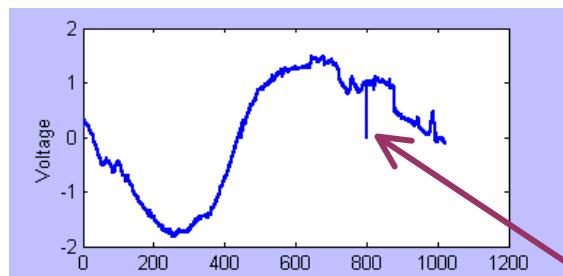


CONTENTS

- Wavelet Basics
- Wavelet filter: pole – zero locations and magnitude – phase characteristics – comparison between different families.
- Methods of identification and classification of inrush and fault current - wavelet for protective relaying technique
- Energy Distribution of wavelet output decomposition
- Frequency Bandwidth characteristics of wavelet output bands
- Wavelet Seismic Event Detection
- Conclusion



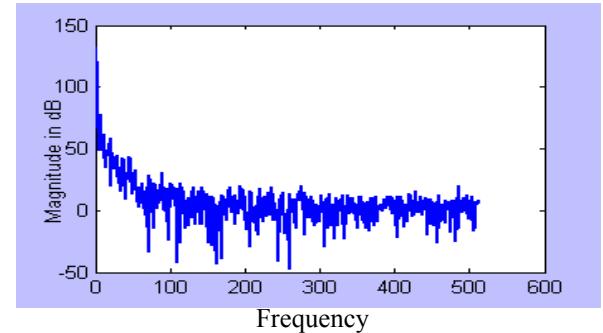
WAVELET BASICS



Time Series – Input Signal

Fourier Transform →

Frequency Domain

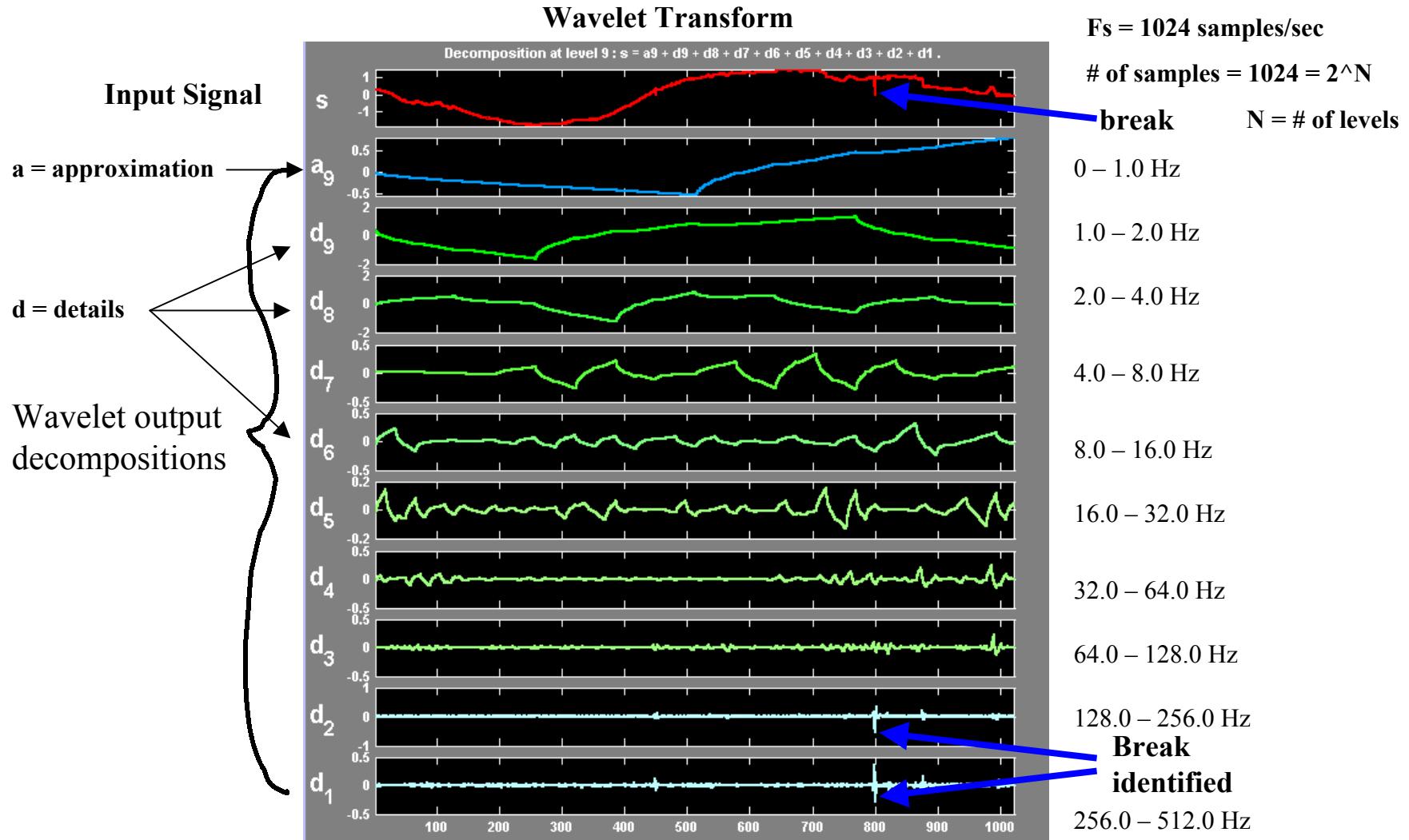


Time information lost

- ✓ Time – Scale Domain.
- ✓ Multiple outputs with different frequency bands called scales (levels).
- ✓ X axis of each output in time units – no time information lost.
- ✓ Y axis of each output in magnitude units.

NEXT PAGE

Other Method :
STFT – Short Time
Fourier Transform





Fourier Transform:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad \text{basis functions – sines and cosines :}$$
$$e^{j\omega t} = \cos \omega \cdot t + j \sin \omega \cdot t$$

Wavelet Transform:

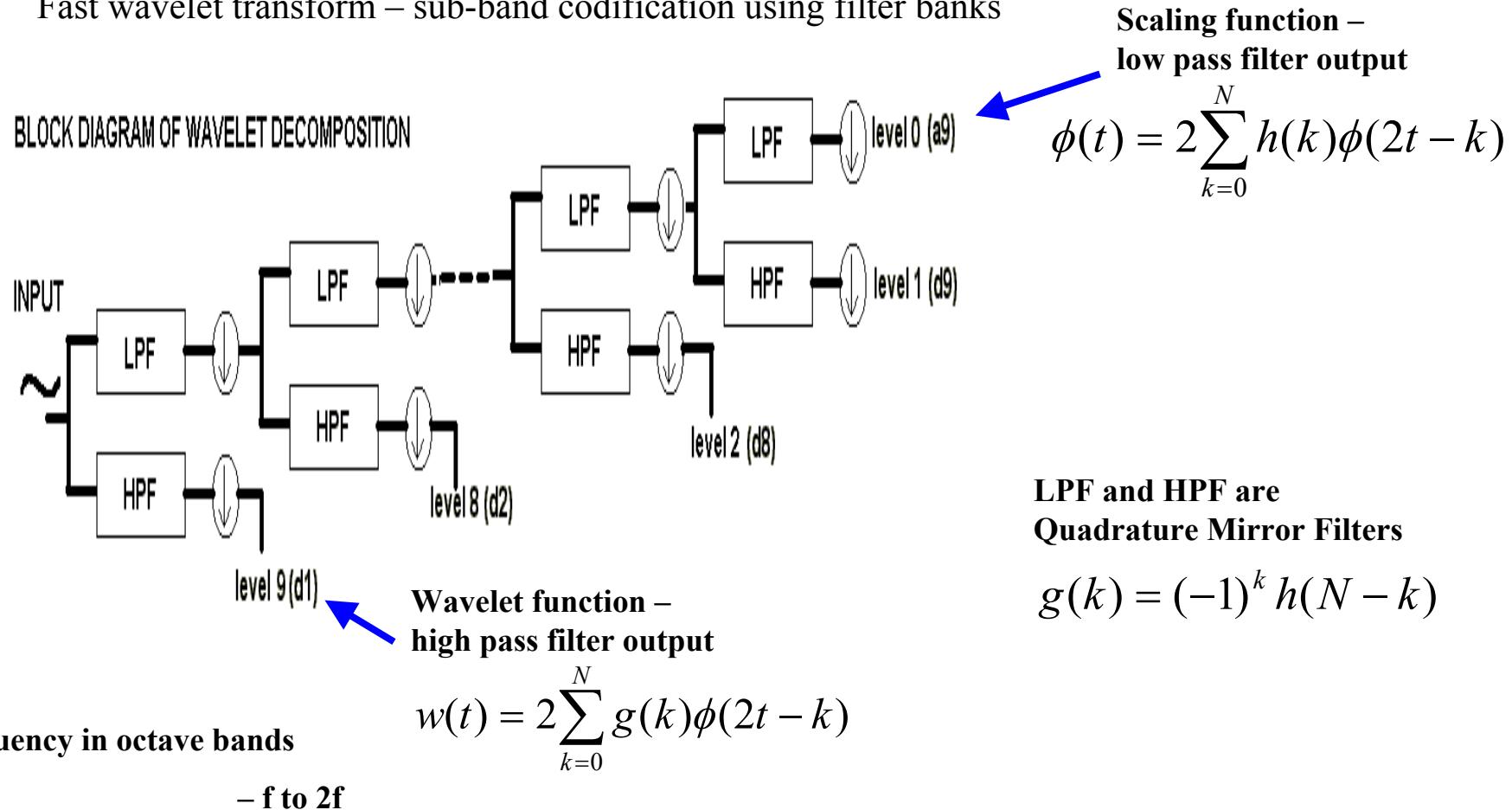
$$CWT(b,a) = \int_{-\infty}^{\infty} f(t) k_{b,a}(t) dt \quad \text{basis functions – compressed and translated functions of mother wavelet:}$$
$$k_{b,a}(t) = \frac{1}{\sqrt{a}} w * \left(\frac{t-b}{a} \right)$$

a is the scale variable of the wavelet replaces the frequency variable in Fourier transform and *b* is the time shift variable.



Thanks to Mallat's multiresolution wavelet analysis

Fast wavelet transform – sub-band codification using filter banks





Ideal : Filters should have perfect box function with sharp edges and no overlap

In practice : it is very important to select wavelet coefficients that lead to good filter characteristics.

Wavelet Research community –

presented several wavelet families

- Daubechies, Symlets, Coiflets, Haar

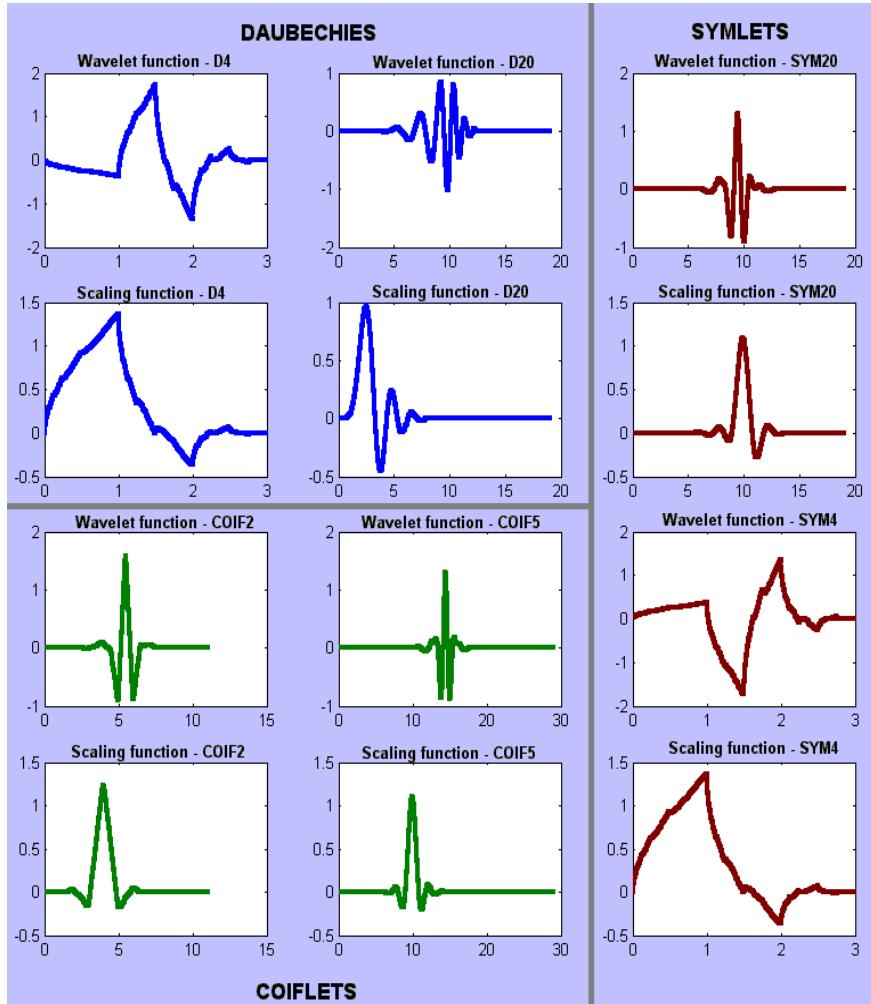
**Each with different shape and lengths
of mother wavelet leading to different
wavelet filters with different properties.**

Choosing the right wavelet for a specific application has been an open question due to lack of sufficient understanding.

“ It is a misunderstanding that any wavelet is suitable for any signal and any applications. Choosing or designing the right wavelet is crucial for a successful WT application” [18]



Some Wavelet families and properties



Type	Filter	Symmetry	Orthogonality
Haar	FIR	Symmetric	Orthogonal
Daubechies	FIR	Asymmetric	Orthogonal
Symlets	FIR	Near Symmetric	Orthogonal
Coiflets	FIR	Near Symmetric	Orthogonal

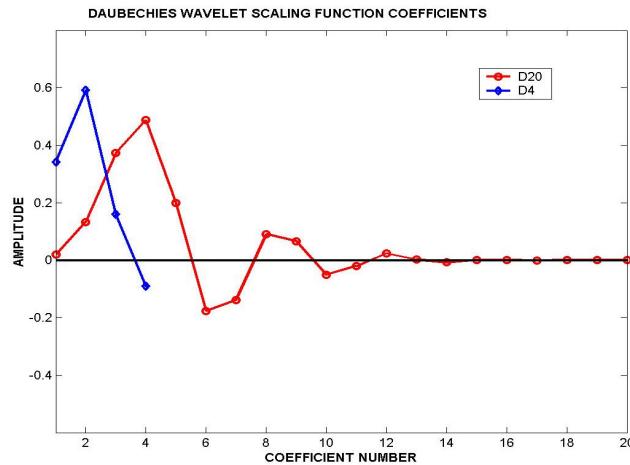


WAVELET FILTER'S CHARACTERISTICS.

So, discrete wavelet transform is equivalent to filtering it by a bank of filters of non-overlapping bandwidths which differ by an octave.

- coefficients of these filter banks determined by the mother wavelet design
- important to know the behavior of these filters with these wavelet coefficients

How are they different from the other filters ?



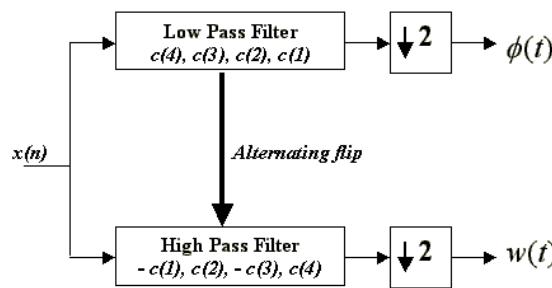
Daubechies D4 scaling coefficients

$$c(1) = 0.3415, c(2) = 0.5915, \\ c(3) = 0.1585, c(4) = -0.0915$$

Daubechies D4 low pass filter coefficients

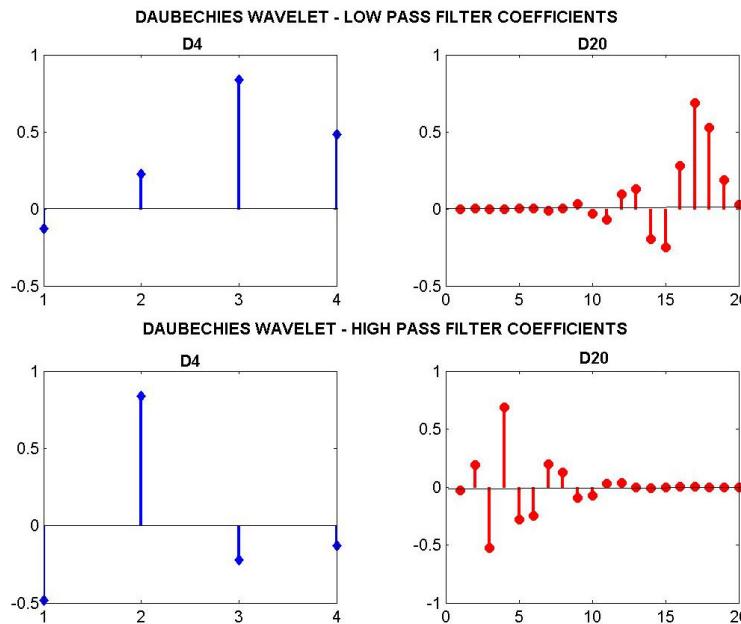
$$\text{reverse}(\sqrt{2} * (c(1), c(2), \dots, c(n)))$$

$$c(1) = 0.4830, c(2) = 0.8365, \\ c(3) = 0.2241, c(4) = -0.1294$$



Block Diagram of wavelet filter bank

High Pass Filter Coefficients are obtained by reversing the low pass filter coefficients and then negating the odd coefficients.



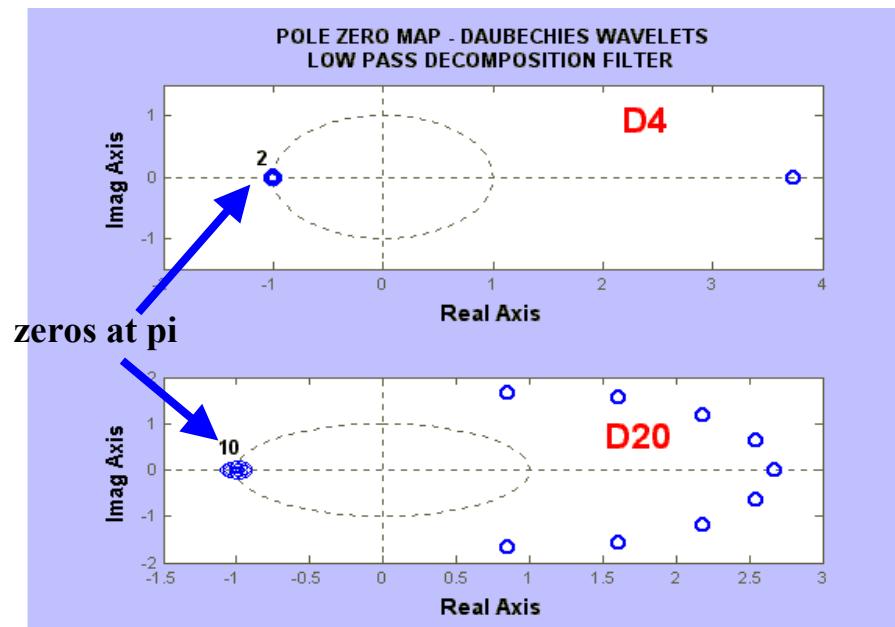
Filter coefficients

- Daubechies D4 and D20

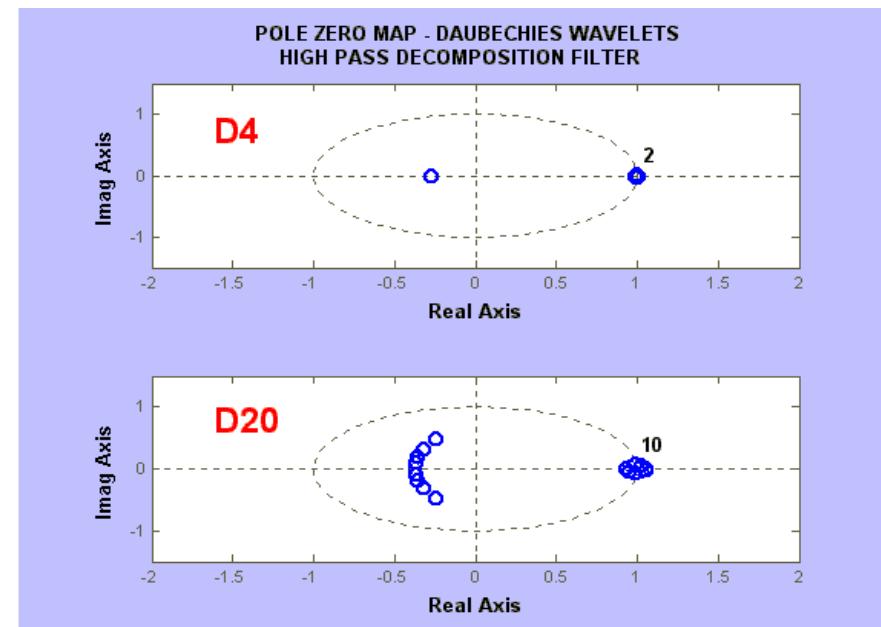


- zeros at π ($z = -1$ in the figure) for the low pass decomposition filter are at the heart of wavelet theory.
- For the filter to behave well, they must have an extra property not built into earlier designs
 - sufficient number of “zeros at π ” [22].
- Daubechies D₄ wavelet with 4 coefficients has 2 zeros at π and Daubechies D₂₀ with 20 coefficients has 10 zeros at π .

Low Pass filter: pole – zero map



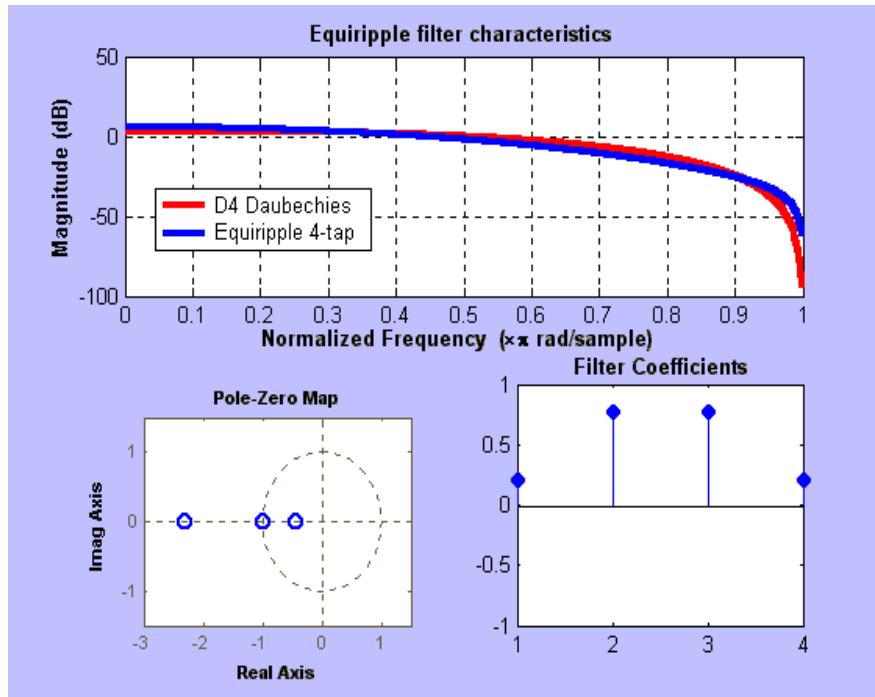
High Pass filter: pole – zero map





Comparing to equiripple filters:

- equiripple filters have smallest ripple in both passband and stopband
- equiripple filter with 4 coefficients has one less zero at pi than Daubechies D4
- equiripple filters not optimal for iteration – decimation by 2 operators mix up the frequency bands

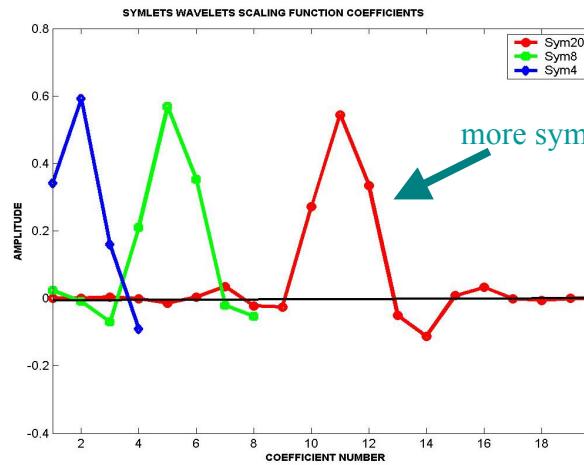


For every p th zero at π for wavelet low pass filter:

- there exists p vanishing moments in the high pass filter for orthogonal wavelets.
- The decay towards low frequencies of high pass filter governed by p .
- smoothness (s) of the wavelet function is less than the order of the polynomial p and corresponds to the decay at high frequencies of low pass filter

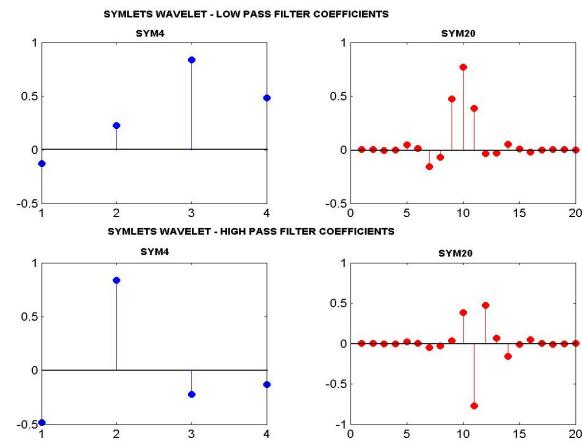


Scaling function

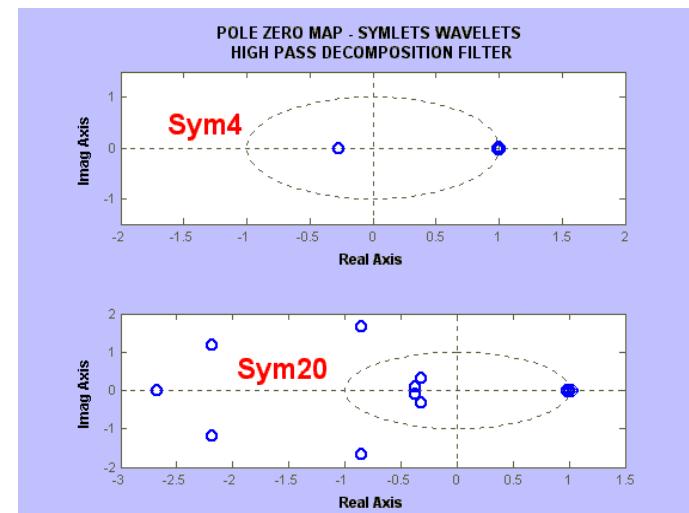
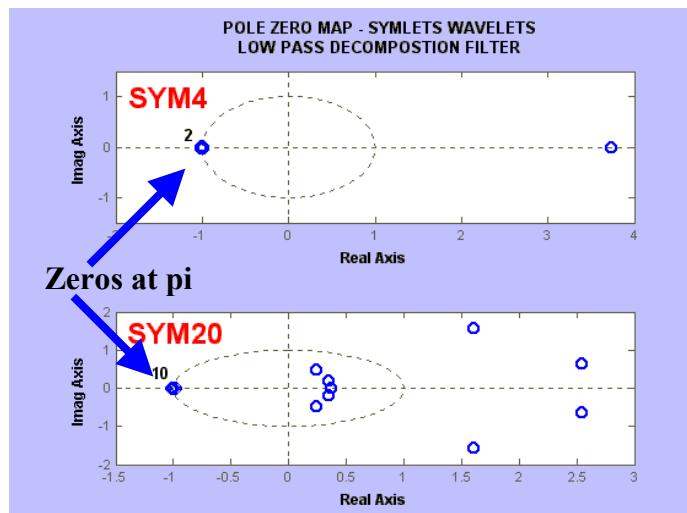


Symlets wavelets

Coefficients

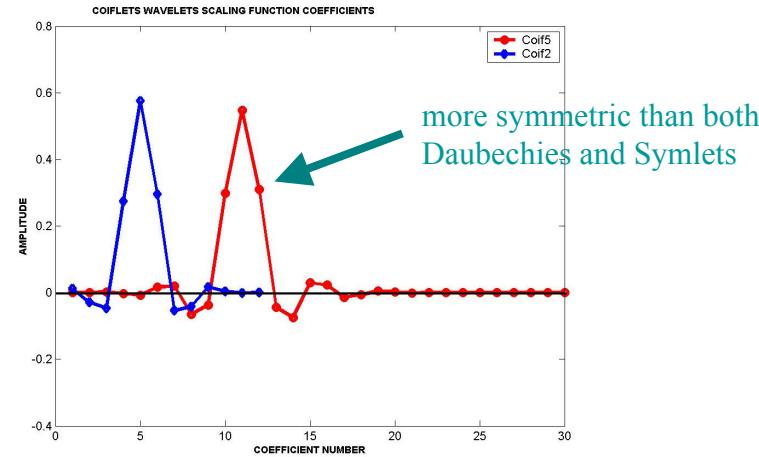


pole – zero maps



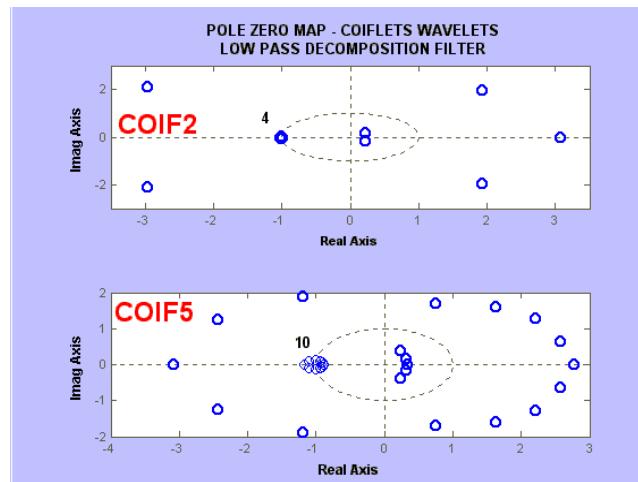
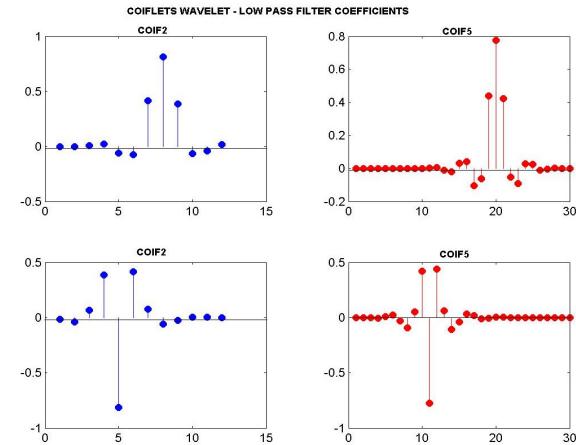


Scaling function

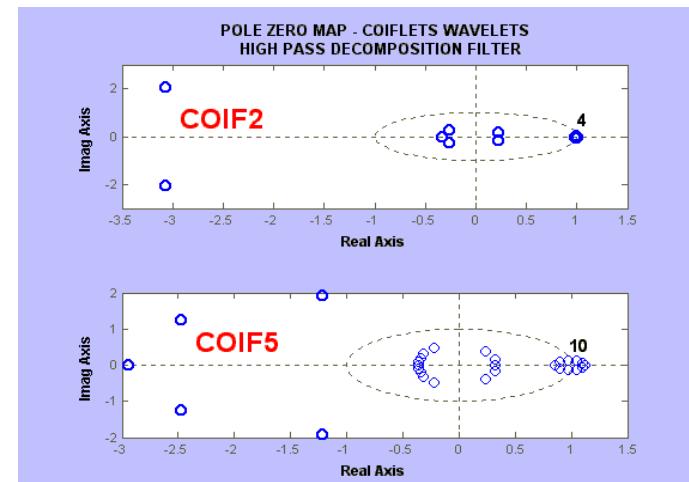


Coiflets wavelets

Coefficients

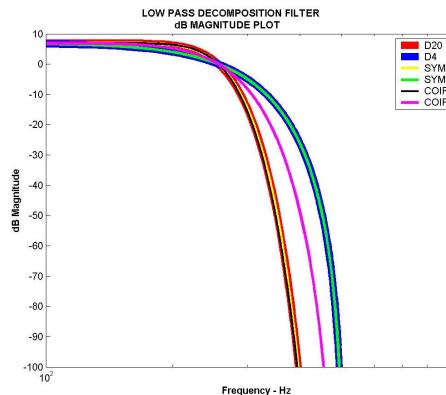
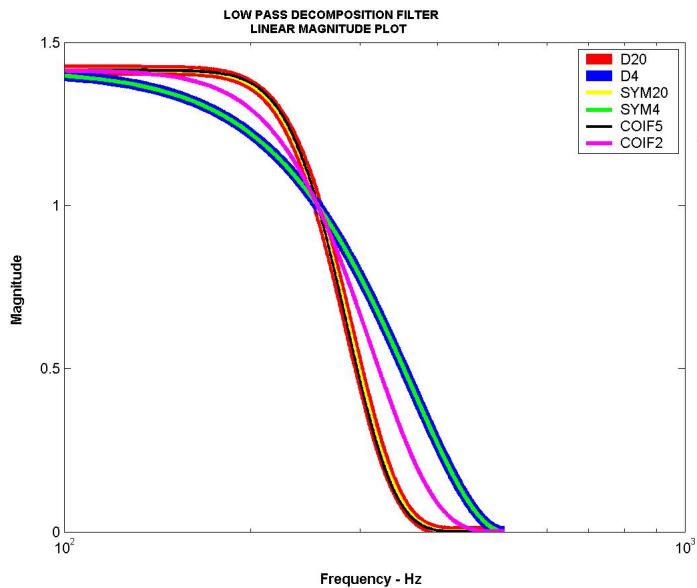


pole – zero
maps





Magnitude Characteristics

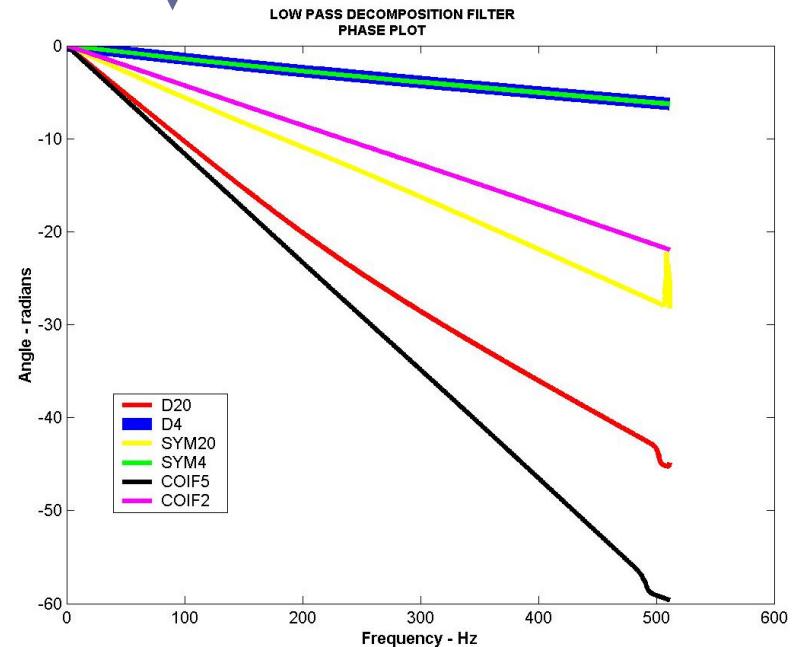


Magnitude
Characteristics
in dB

LOW PASS WAVELET FILTERS

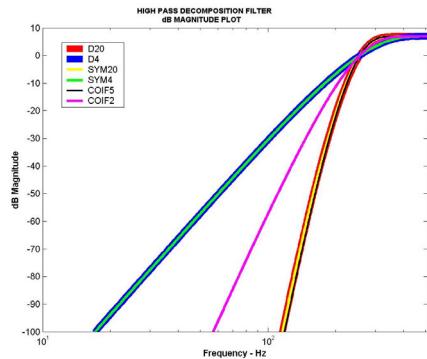
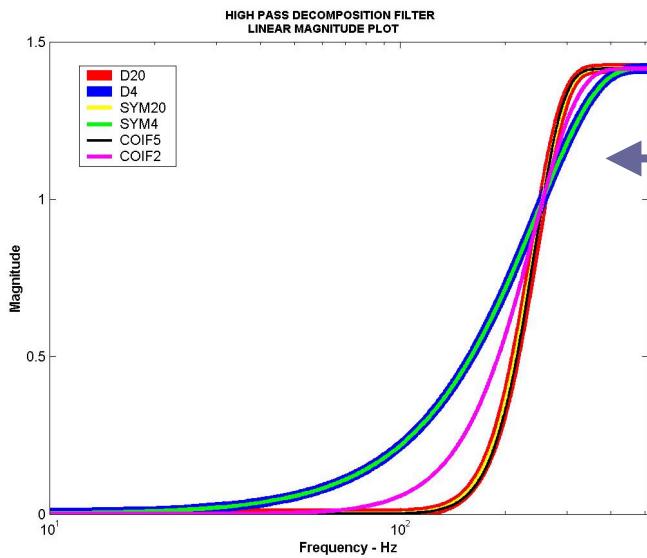
Some filters have exactly same magnitude characteristics but different phase characteristics

Phase Characteristics





Magnitude Characteristics

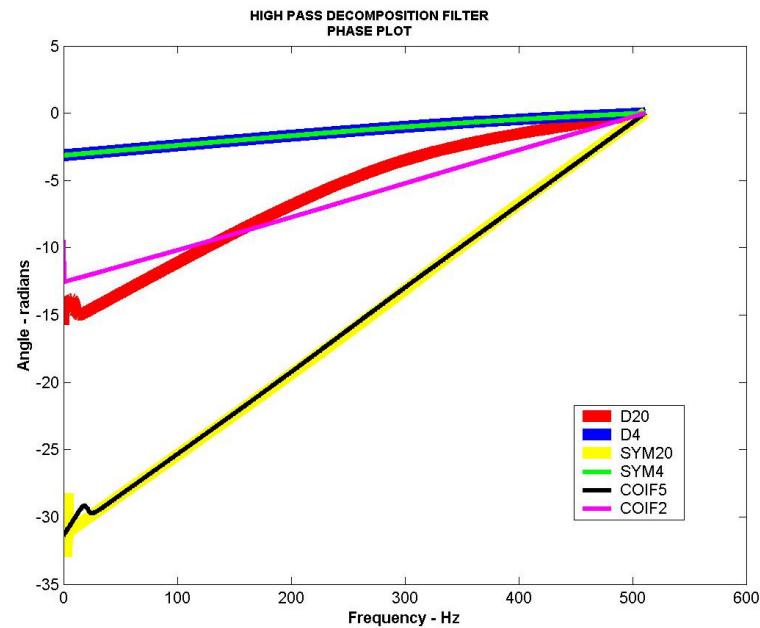


Magnitude
Characteristics
in dB

HIGH PASS WAVELET FILTERS

Some filters have exactly same magnitude characteristics but different phase characteristics

Phase Characteristics





Observations:

For use of wavelets in signal processing,

- ❖ knowledge of the magnitude and phase response of filters is important.
- ❖ information obtained - one of the properties that might be useful in choosing the right wavelet.
- ❖ Two wavelet filters - same magnitude characteristics but different phase characteristics.

For applications with importance of time,

- ❖ the wavelet filter with lesser slope is chosen since it has a smaller time delay.

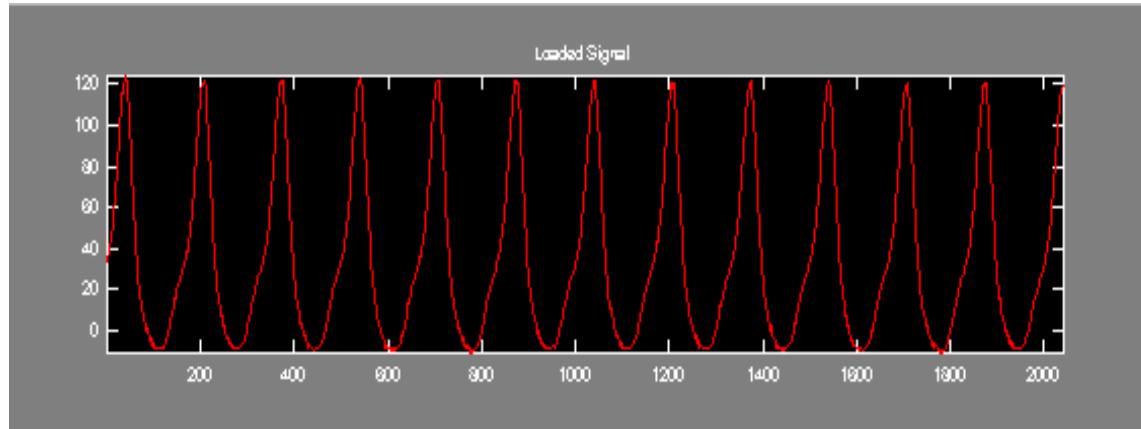
So lets see an application of wavelets to fast detection of fault current in a transformer !!!



METHODS OF INRUSH AND FAULT CURRENT IDENTIFICATION/PROTECTION USING WAVELETS

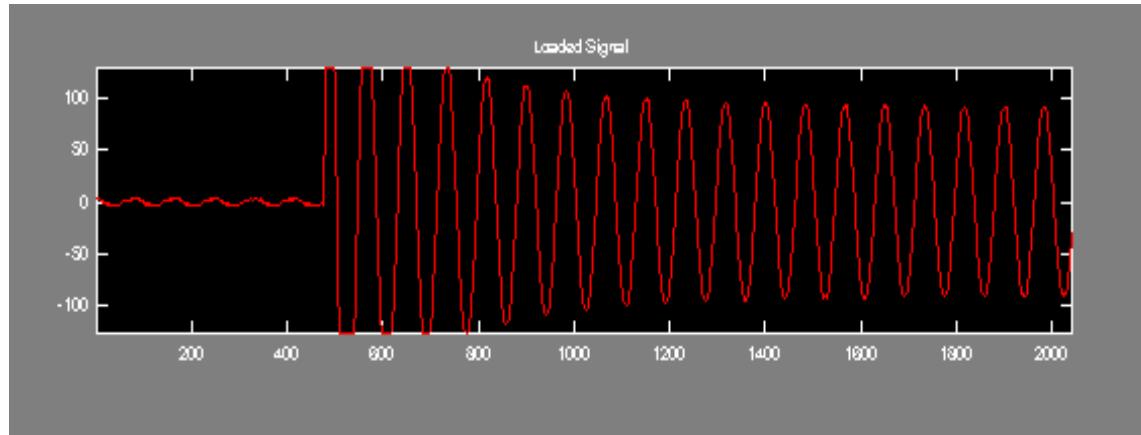
- Inrush current
 - transient current which occurs when transformer turned on
 - Amplitude much higher than rated full load current
 - causes false tripping of differential relays used for protection.
- Fault current
 - short circuit of output or high current at output.
 - transient at the time of occurrence

Inrush Current :



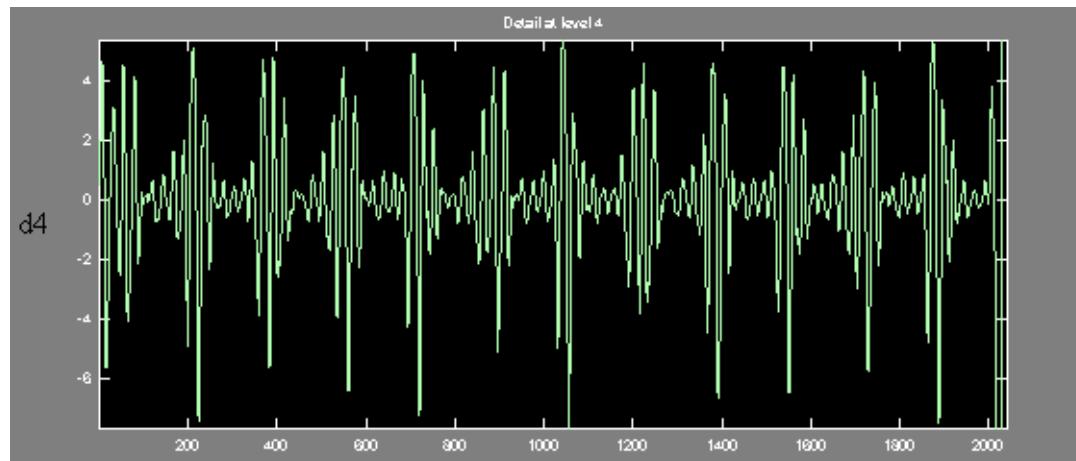
- inrush current has intermittent peaks with valleys in between them

Fault Current :



- fault current on the other hand is perfectly symmetric with both positive and negative peaks

Applying Daubechies D4 and D20 wavelet transform to inrush current:

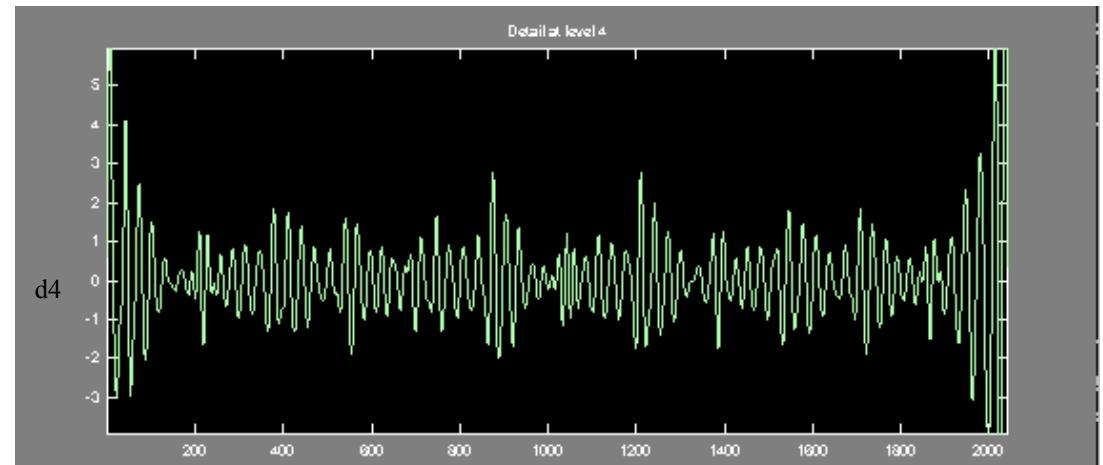


D4 – detail d4

- large amplitude peaks corresponding to the peaks of the inrush current and smaller amplitude signal almost closer to a value of zero corresponding to the valleys (null period) in between the peaks

D20 – detail d4

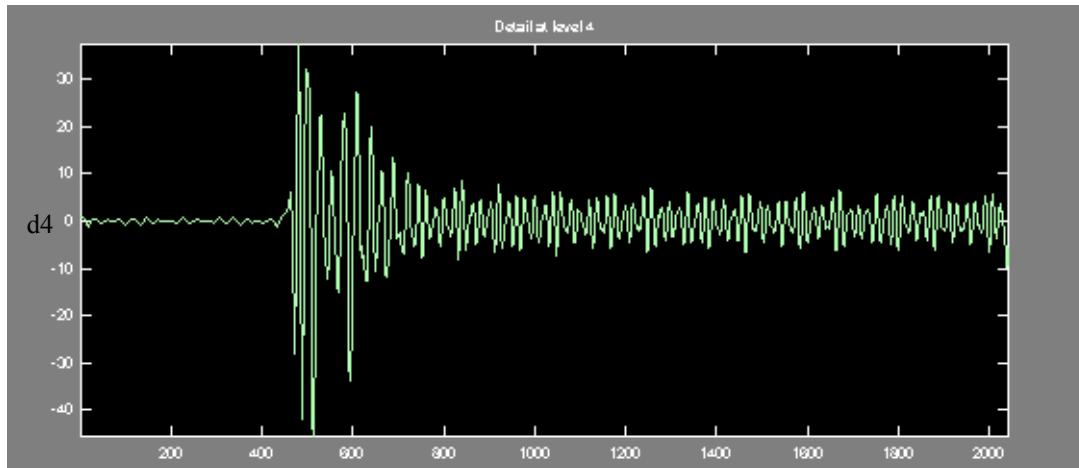
- Higher number of coefficients smoothens the output decomposition so peaks and valleys indistinguishable.



Showed only relevant wavelet output



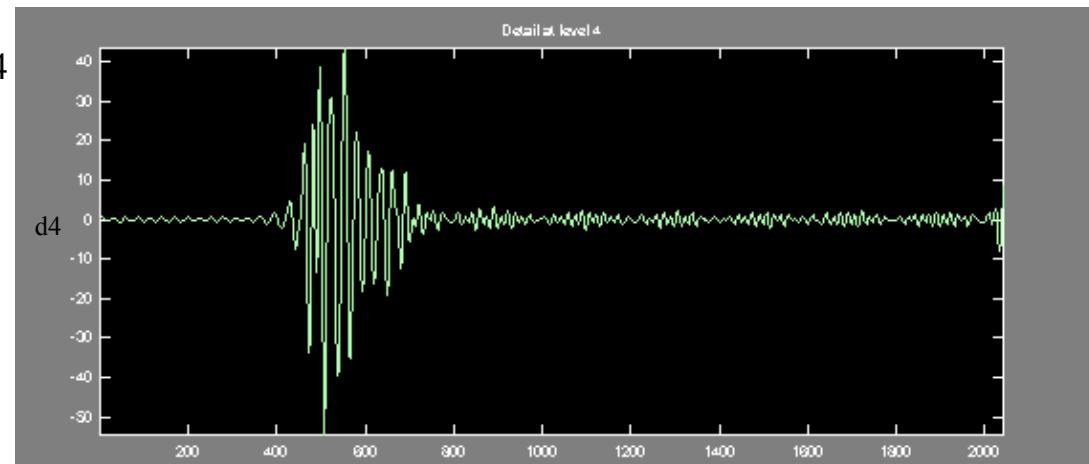
Applying Daubechies D4 and D20 wavelet transform to fault current:



D4 – detail d4

do not show any distinctive behavior except for a few large spikes at the state of transition from the normal current to the fault current.

D20 – detail d4



Shown only relevant wavelet output



D4 Daubechies wavelet :

- ✓ clearly distinguishes the inrush current from fault current
- ✓ better compared to Daubechies D20 for this application
- ✓ uses the detection principle based on the null period that exists in the inrush current

better than the existing second harmonic component method used [28], [29].

Protective relaying techniques:

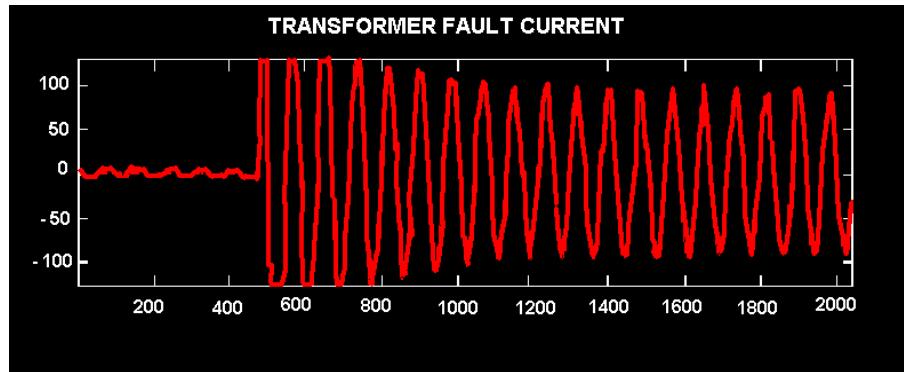
- **Identification of the fault current in the power system and shutting down the power**
- **detection and classification - reliably conclude, in a very short time (1 – 2 cycles), whether and which type of fault occurs under a variety of time-changing operating conditions [51]**
- **important characteristics of a good relaying system is its short time response.**

Wavelets have very interesting characteristics that can provide answers to the time response of the relay algorithm

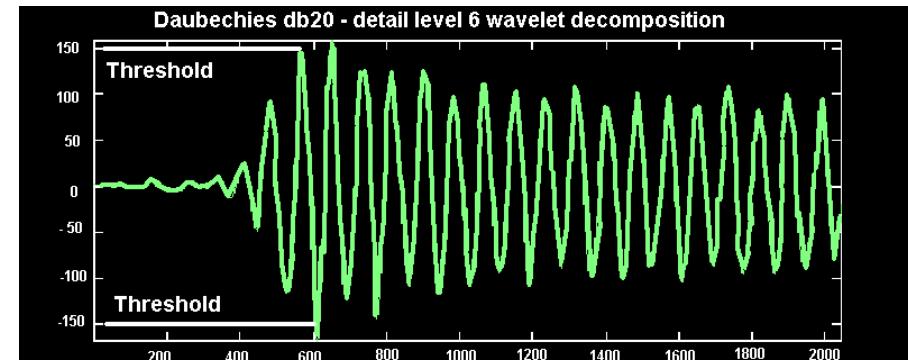
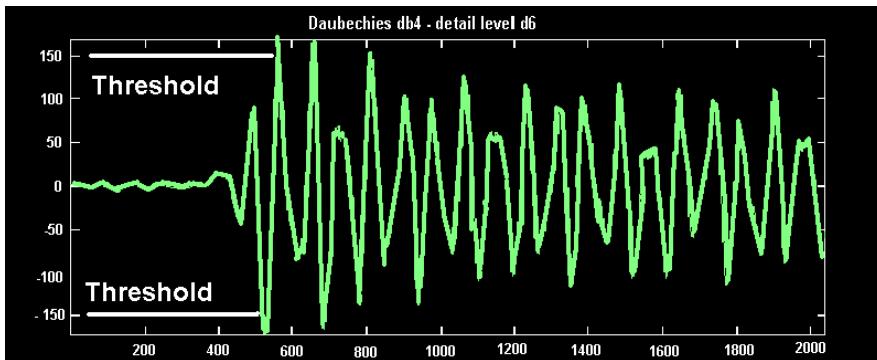


Fast detection of fault current and response:

Daubechies D4



Daubechies D20



- Daubechies D4 – with fewer coefficients has smaller phase slope, hence smaller delay and faster response.
- Promising tool for further study towards applying them to the area of power protection systems.



ENERGY DISTRIBUTION OF WAVELET DECOMPOSITIONS

Previous fault current example:

- Can we identify the frequency composition of the fault signal using wavelets ?

Daubechies D4:

- the output amplitude is maximum in the detail d6 (level 5) after fault
- detail d5 (level 6) having approximately half the maximum amplitude

Daubechies D20: *Frequency composition of signal easily identified.*

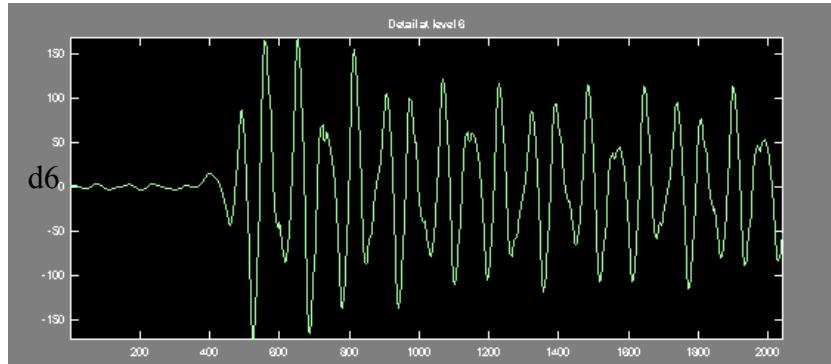
- the output amplitude is maximum in the detail d6 (level 5) after fault
- detail d5 (level 6) having very small ($1/5^{\text{th}}$ the maximum) amplitude

Energy of the output wavelet decomposition using D4 is more spread out between two frequency bands compared to output wavelet decomposition using D20

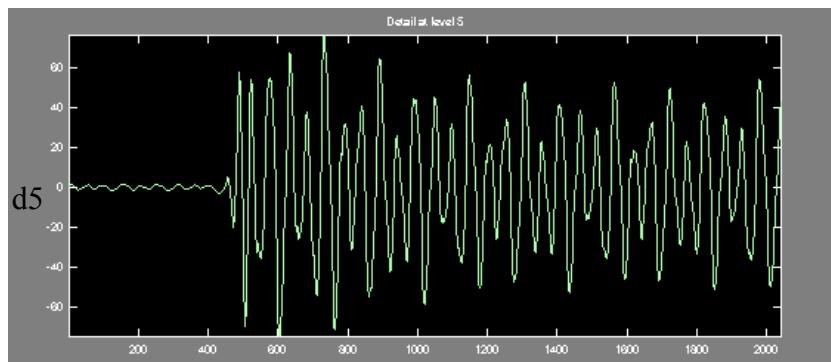


Wavelet decomposition output of transformer fault current signal

Daubechies D4

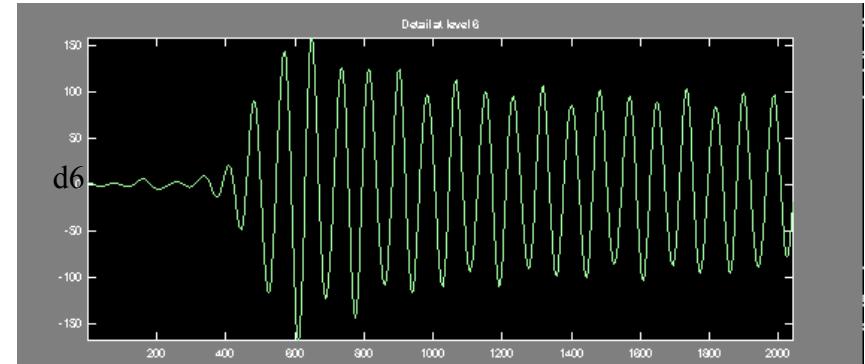


Frequency band: 40 – 80 Hz

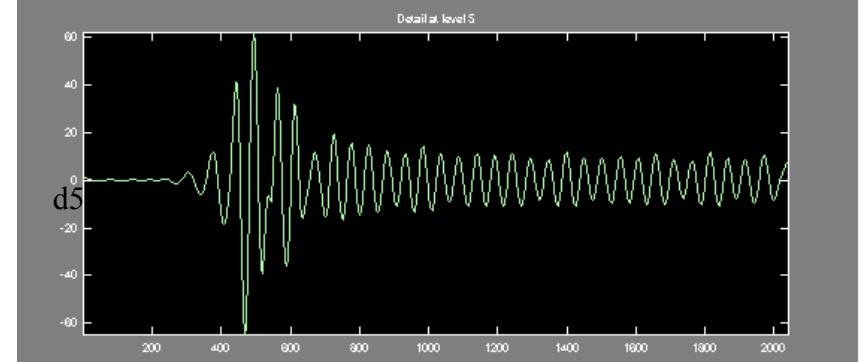


Frequency band: 80 – 160 Hz

Daubechies D20



Frequency band: 40 – 80 Hz



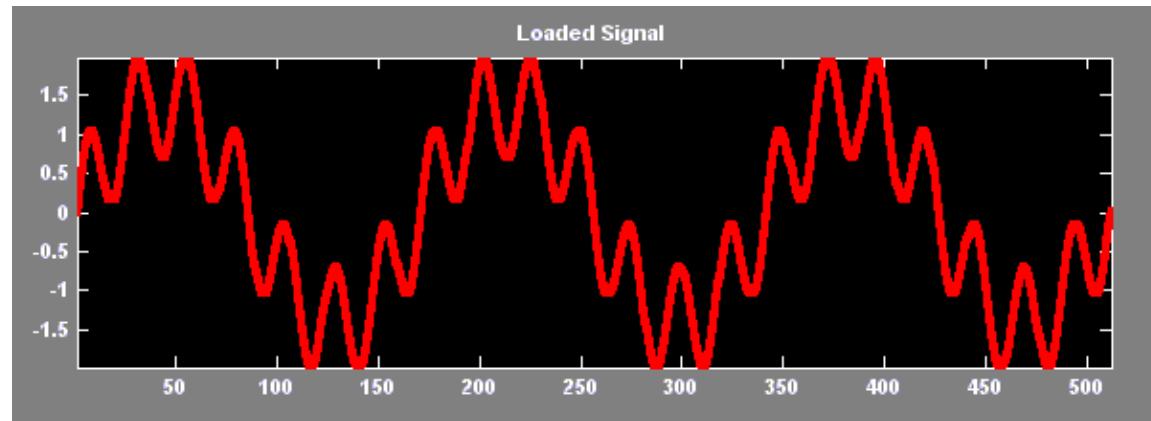
Frequency band: 80 – 160 Hz



To further study:

Consider a simulated sinusoidal signal with 60 Hz and 420 Hz

$$f(t) = \sqrt{2} \sin(2\pi 60t) + 0.5 \times \sqrt{2} \sin(2\pi 420t)$$



Three cycles of the signal at 60 Hz with 512 data points and sampled at a 10,240 Hz

$$rms(f(t)) = \sqrt{1.25} = 1.118$$

$$\text{Energy of the signal} = (rms(f(t)))^2 = 1.25$$



- Perform wavelet transform on the signal
- Calculate the rms and energy values for each output frequency bands.

The rms value of the individual wavelet level L is given by:

$$rms(levelL) = \sqrt{\frac{1}{512} \times sum(A(L,:)^2)}$$

where $A(L,:)$ is the coefficients of the wavelet decomposition level L signal

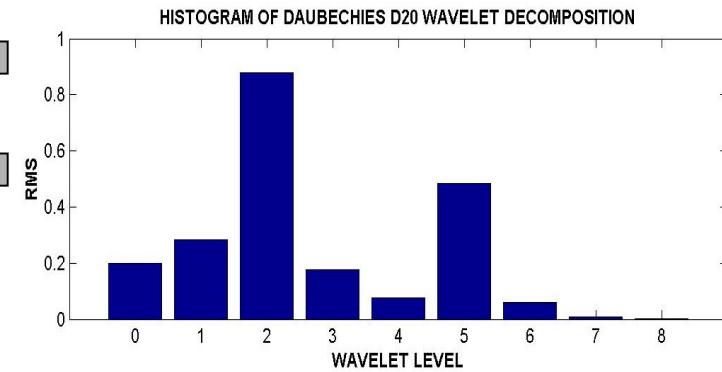
Total energy of the signal is given by sum of energies of individual output frequency bands:

$$\sum_{n=1}^N |f(n)|^2 = \sum_{n=1}^N |a_j(n)|^2 + \sum_{j=1}^l \sum_{n=1}^N |d_j(n)|^2$$



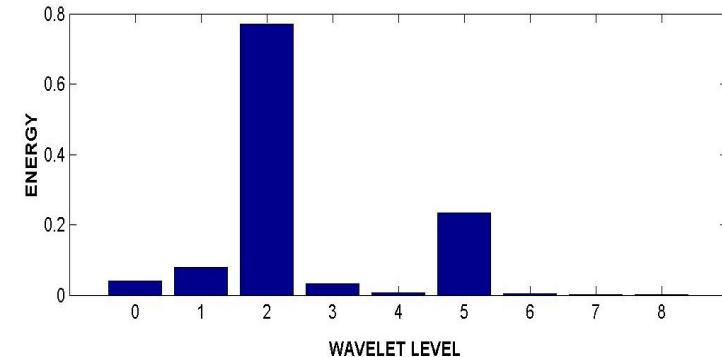
Daubechies D20 wavelet decomposition of the simulated signal

Wavelet Level	Frequency band	Center frequency	Rms value	Energy
0 (a8)	0-20 Hz	10 Hz	0.1981	0.0392
1 (d8)	20 Hz – 40 Hz	30 Hz	0.2827	0.0799
2(d7)	40 Hz – 80 Hz	60 Hz	0.8779	0.7706
3(d6)	80 Hz – 160 Hz	120 Hz	0.1777	0.0316
4(d5)	160 Hz – 320 Hz	240 Hz	0.0775	0.0060
5(d4)	320 Hz – 640 Hz	480 Hz	0.4831	0.2334
6(d3)	640 Hz – 1280 Hz	960 Hz	0.0596	0.0035
7(d2)	1280 Hz – 2560 Hz	1920 Hz	0.0074	0.0001
8(d1)	2560 Hz – 5120 Hz	3840 Hz	0.0021	0.0000



$$\sqrt{\sum (rmsvalue)^2} = \sqrt{(0.1981)^2 + (0.2827)^2 + (0.8779)^2 + (0.1777)^2 + (0.0775)^2 + (0.4831)^2 + (0.0596)^2 + (0.0074)^2 + (0.0021)^2}$$

$$rms(f(t)) = \sqrt{1.16} = 1.08 \quad Energy = 1.16$$





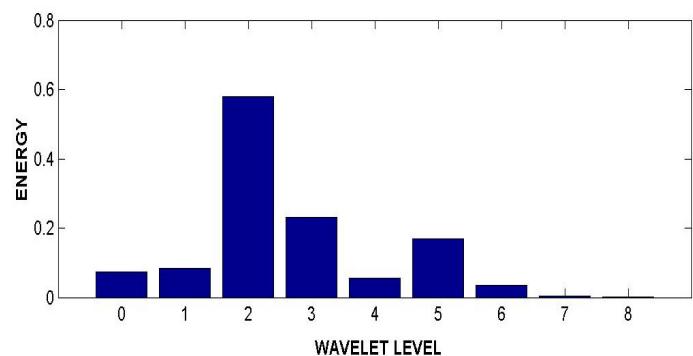
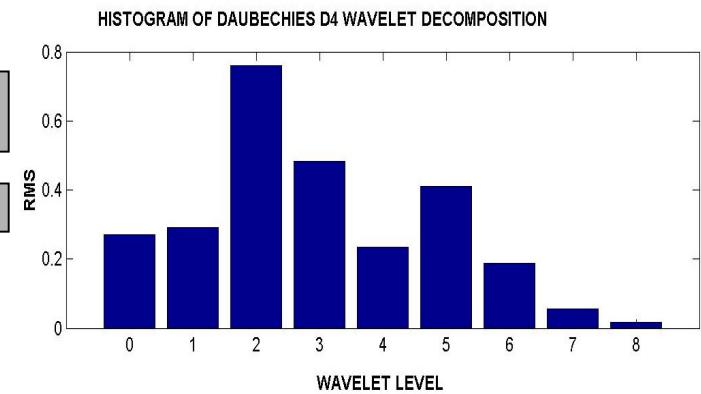
Daubechies D4 wavelet decomposition of the simulated signal

Wavelet Level	Frequency band	Center frequency	Rms value	Energy
0 (a8)	0-20 Hz	10 Hz	0.2704	0.0731
1 (d8)	20 Hz – 40 Hz	30 Hz	0.2909	0.0846
2(d7)	40 Hz – 80 Hz	60 Hz	0.7603	0.5780
3(d6)	80 Hz – 160 Hz	120 Hz	0.4817	0.2320
4(d5)	160 Hz – 320 Hz	240 Hz	0.2342	0.0548
5(d4)	320 Hz – 640 Hz	480 Hz	0.4111	0.1690
6(d3)	640 Hz – 1280 Hz	960 Hz	0.1890	0.0357
7(d2)	1280 Hz – 2560 Hz	1920 Hz	0.0562	0.0032
8(d1)	2560 Hz – 5120 Hz	3840 Hz	0.0156	0.0002

$$\sqrt{\sum (rmsvalue)^2} = \sqrt{(0.2704)^2 + (0.2909)^2 + (0.7603)^2 + (0.4817)^2 + (0.2342)^2 + (0.4111)^2 + (0.1890)^2 + (0.0562)^2 + (0.0156)^2}$$

$$rms(f(t)) = \sqrt{1.23} = 1.11$$

$$Energy = 1.23$$





Observations

Daubechies D20:

- energy is concentrated in wavelet level 2 (d7: 40 Hz to 80 Hz) and wavelet level 5 (d4: 320 Hz to 640 Hz).

Daubechies D4:

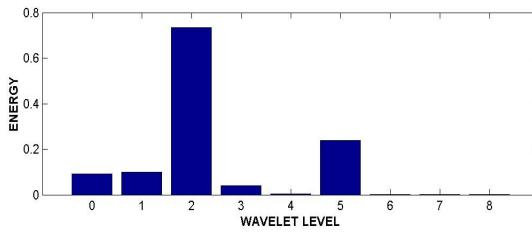
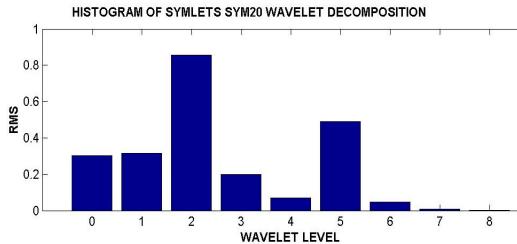
- energy is still concentrated in wavelet level 2 (d7: 40 Hz to 80 Hz) and wavelet level 5 (d4: 320 Hz to 640 Hz).
- the energy is also considerably high in level 3 (d6: 80 Hz to 160 Hz)
- and increased energy levels in adjacent bands compared to Daubechies D20.

Energy distribution varies within wavelet levels for different wavelet families

So what is the frequency bandwidth characteristics of these wavelet levels?

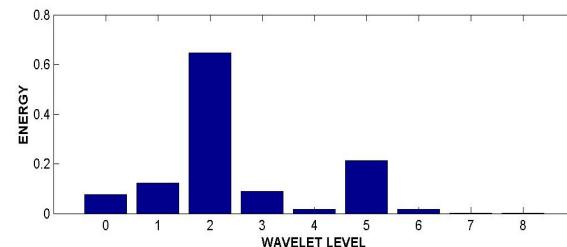
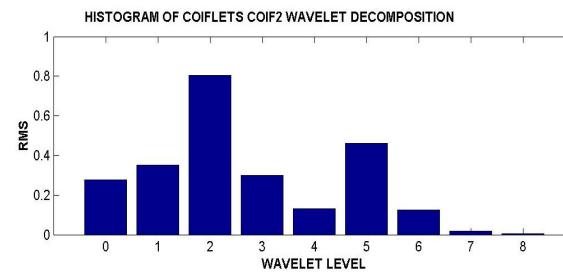


Energy distribution in output wavelet decomposition of simulated signal

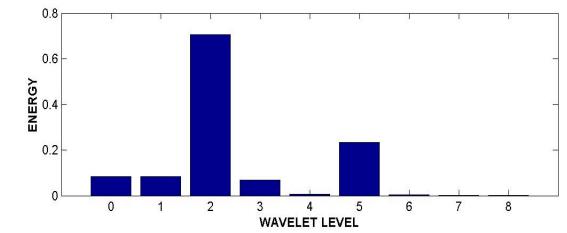
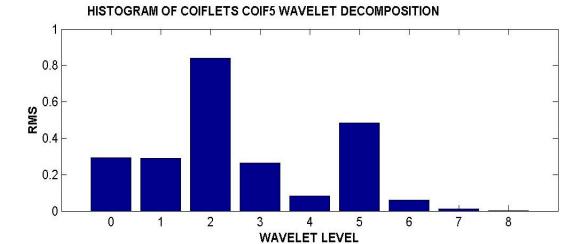


**SYMLETS
SYM20**

**COIFLETS
COIF2**

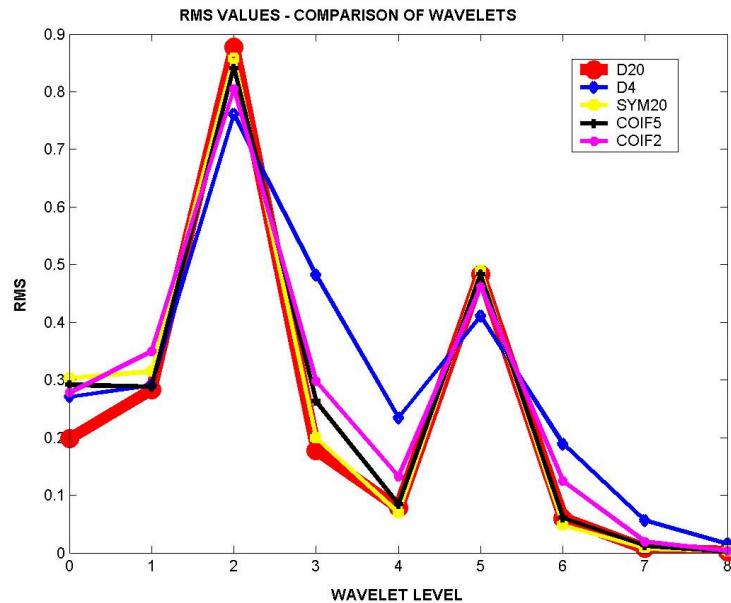


**COIFLETS
COIF5**

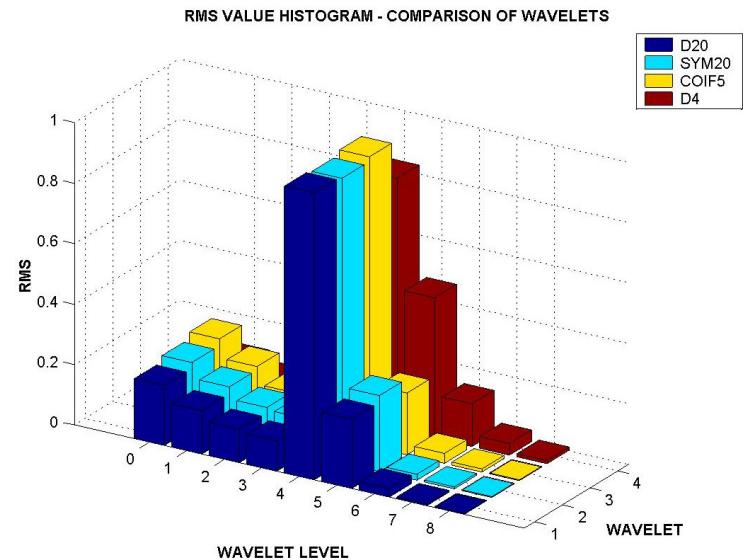




COMPARISON



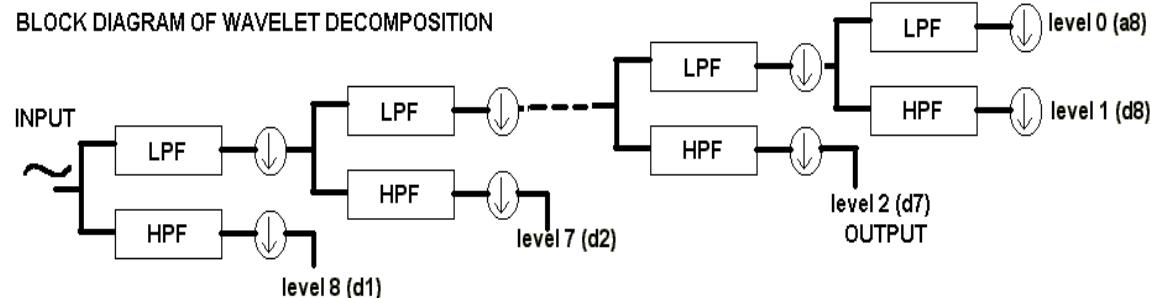
Signal with single frequency
at the center of the whole
frequency range





FREQUENCY BANDWIDTH CHARACTERISTICS

- Select say, wavelet level 2 (d7: 40 Hz to 80 Hz), sweep the sinusoidal signal's frequency from 20 Hz to 100 Hz
- calculate the rms value of the wavelet level at different frequency points along the sweep
- For output level 2 (d7), input signal passes through few low pass filters and one high pass filter with down-sampling in between each filter.

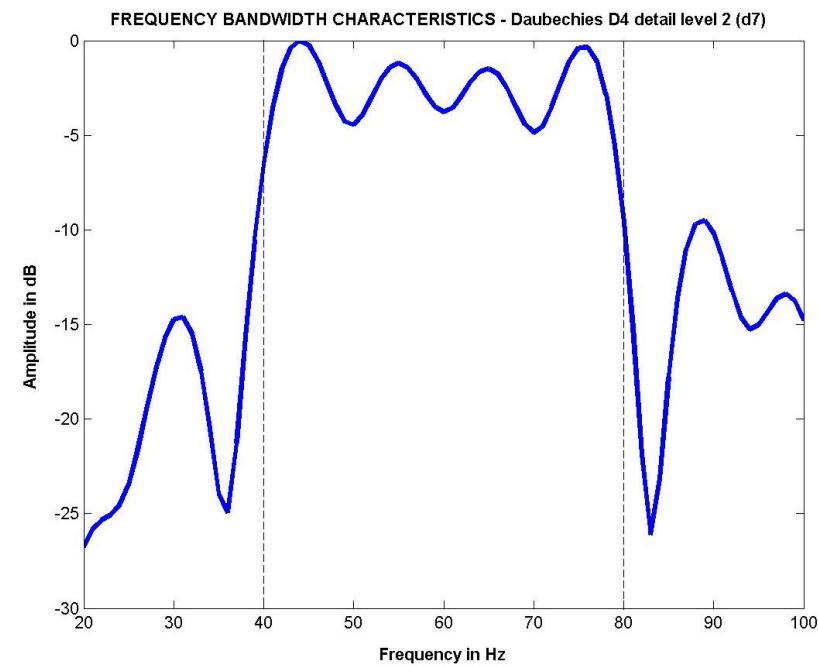


512 data points which provides us with 9 wavelet levels

FREQUENCY BANDWIDTH CHARACTERISTICS

DAUBECHIES D4 wavelet level d7

Freq Range: 40 – 80 Hz



Freq in Hz	RMS value	decibel value (dB)
20	0.2489	-26.7633
21	0.2609	-25.8221
22	0.2671	-25.3493
23	0.2709	-25.0644
24	0.2779	-24.5555
25	0.2939	-23.4407
26	0.3214	-21.6459
27	0.3583	-19.4763
28	0.3979	-17.3782
29	0.4324	-15.7179
30	0.4540	-14.7393
31	0.4570	-14.6103
:	:	:
:	:	:

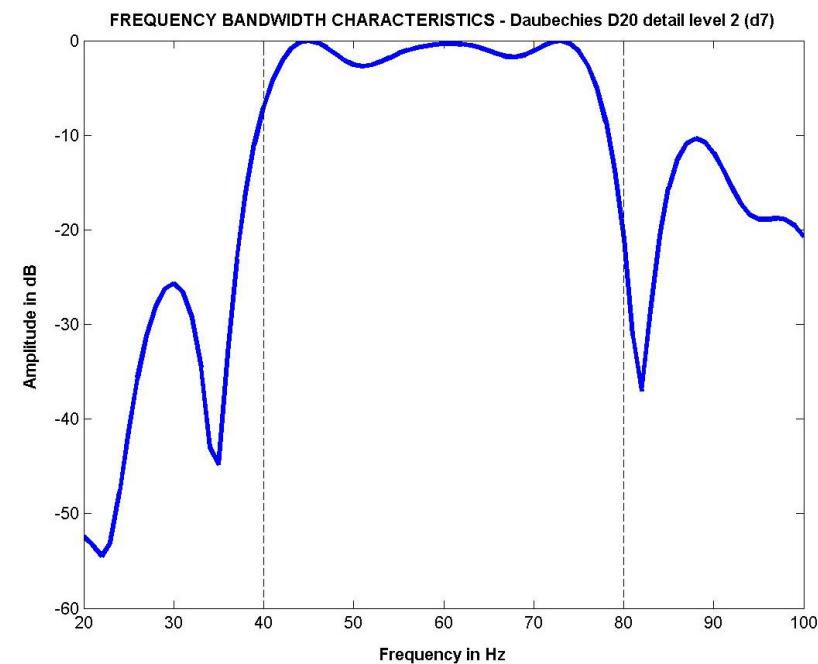
Freq in Hz	RMS value	decibel value (dB)
:	:	:
:	:	:
90	0.5698	-10.1972
91	0.5324	-11.5540
92	0.4903	-13.2042
93	0.4568	-14.6163
94	0.4422	-15.2676
95	0.4473	-15.0380
96	0.4638	-14.3151
97	0.4798	-13.6351
98	0.4859	-13.3815
99	0.4772	-13.7455
100	0.4533	-14.7734



FREQUENCY BANDWIDTH CHARACTERISTICS

DAUBECHIES D20 wavelet level d7

Freq Range: 40 – 80 Hz



Freq in Hz	RMS value	decibel value (dB)
20	0.0659	-52.4409
21	0.0630	-53.3332
22	0.0594	-54.5011
23	0.0638	-53.0709
24	0.0833	-47.7443
25	0.1152	-41.2637
26	0.1529	-35.5995
27	0.1904	-31.2079
28	0.2225	-28.0998
29	0.2441	-26.2465
30	0.2510	-25.6867
31	0.2400	-26.5807
:	:	:
:	:	:

Freq in Hz	RMS value	decibel value (dB)
:	:	:
:	:	:
90	0.5004	-11.8857
91	0.4602	-13.5594
92	0.4182	-15.4772
93	0.3831	-17.2284
94	0.3612	-18.4082
95	0.3528	-18.8767
96	0.3529	-18.8686
97	0.3543	-18.7896
98	0.3511	-18.9724
99	0.3403	-19.5953
100	0.3222	-20.6934



Observations:

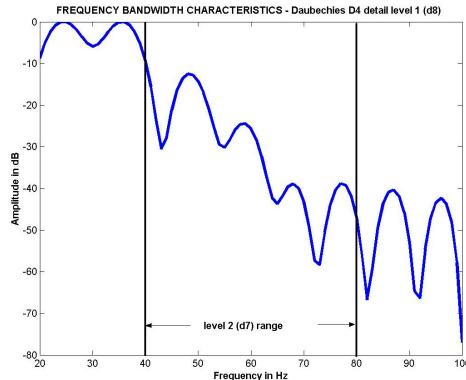
- The passband and stop band regions are not flat.
- Daubechies D20 wavelet decomposition is *more flat* in the passband.
- The lower frequency side lobe is better in the Daubechies D20.
- The higher frequency side lobes are same in both.

*Daubechies D20 with higher number of coefficients is better wavelet
for quantifying the signal frequencies in the passband.*



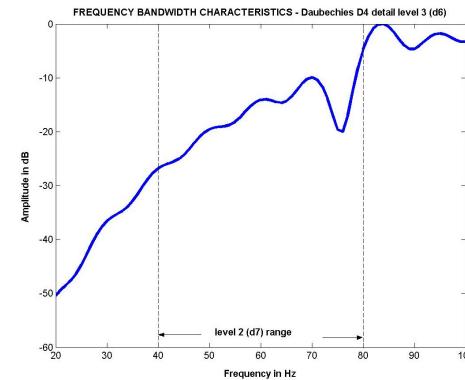
Energy Leakage into adjacent bands.

D4



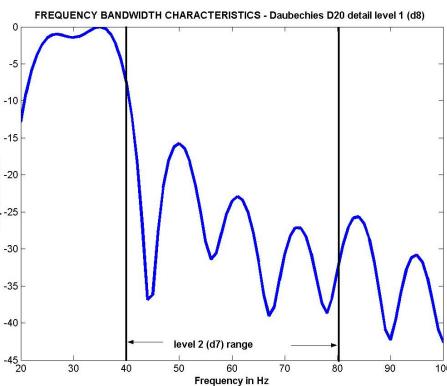
Level d6

D4



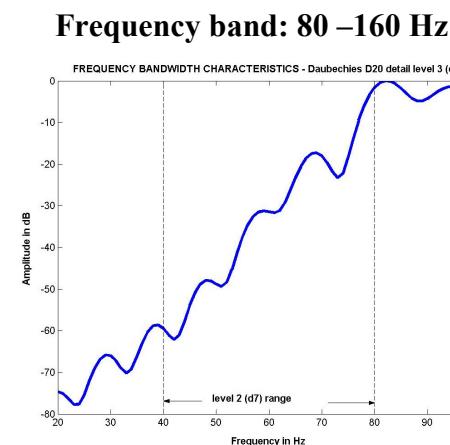
Level d8

D20



Frequency band: 20 - 40 Hz

D20



Frequency band: 80 –160 Hz

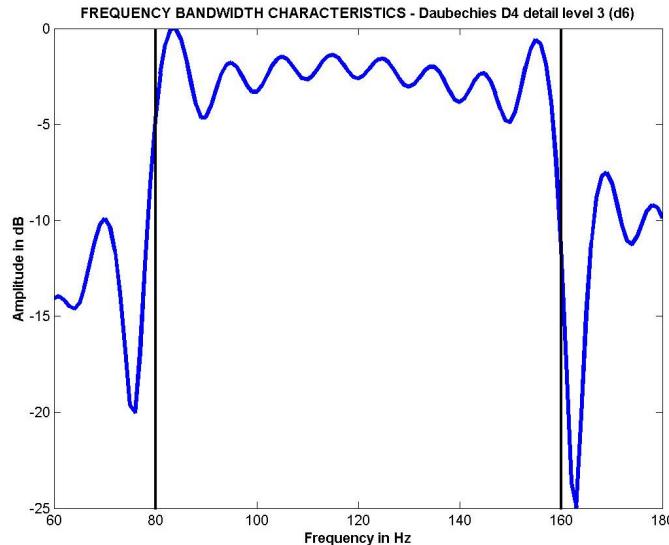


FREQUENCY BANDWIDTH CHARACTERISTICS

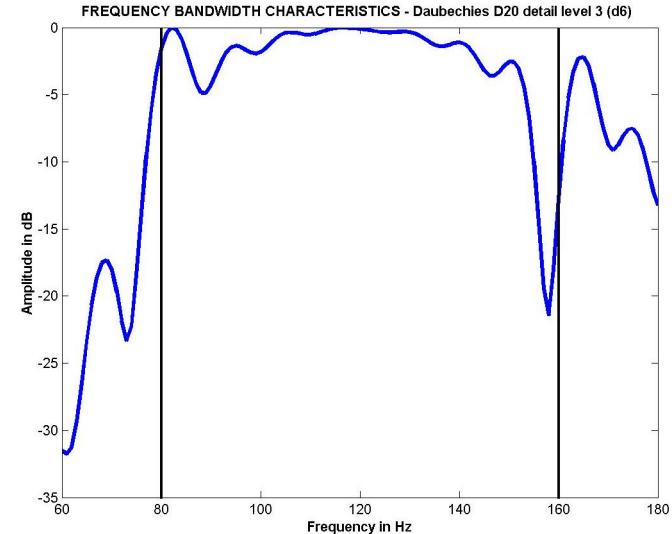
wavelet level d6

Freq Range: 80 – 160 Hz

Daubechies D4



Daubechies D20

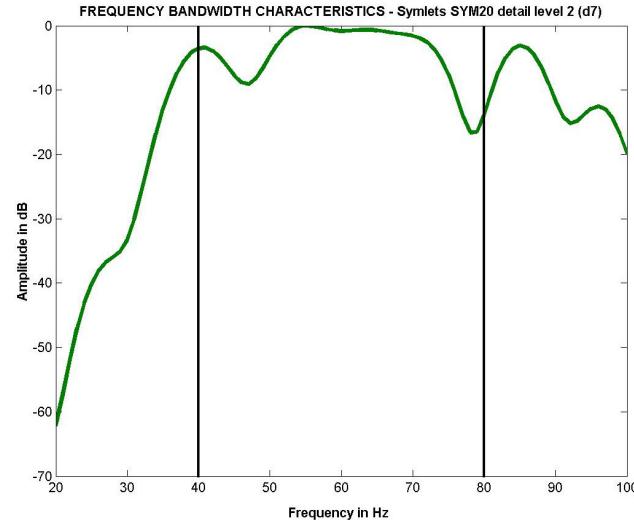




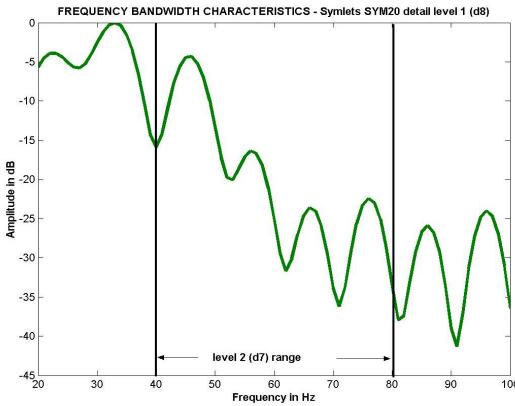
FREQUENCY BANDWIDTH CHARACTERISTICS

Symlets wavelet level d7

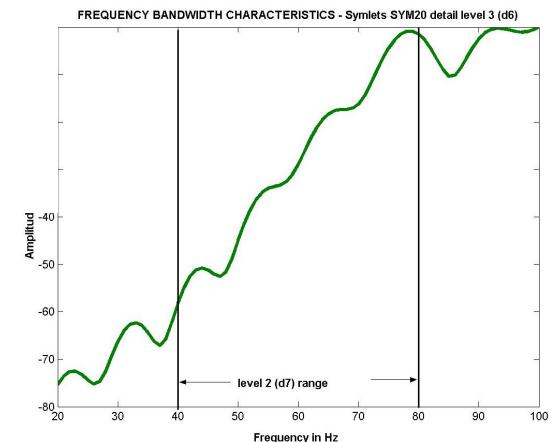
Freq Range: 40 - 80 Hz



wavelet d6



wavelet d8

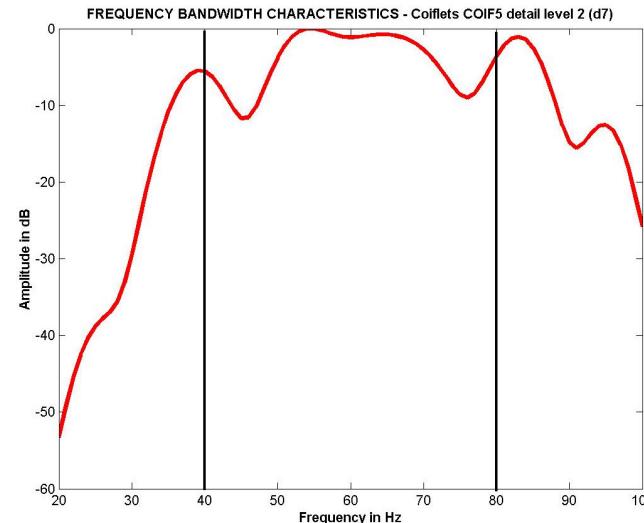




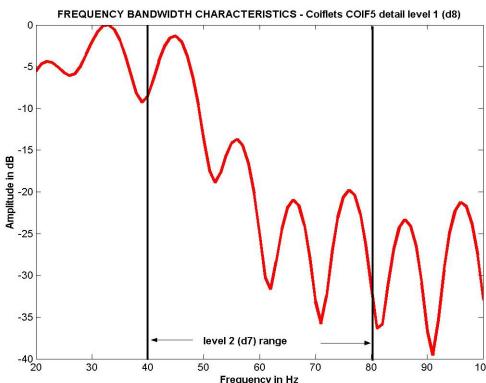
FREQUENCY BANDWIDTH CHARACTERISTICS

Coiflets wavelet level d7

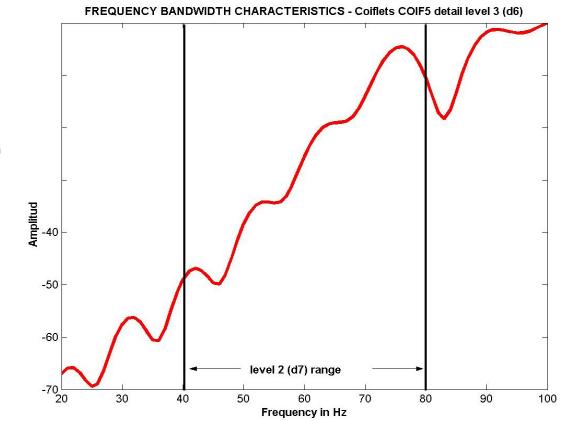
Freq Range: 40 - 80 Hz



wavelet d6



wavelet d8





So from frequency bandwidth characteristics:

- wavelets do not exhibit the ideal non-overlapping bandwidths
- Daubechies D4, the bandwidth characteristics are poorer – lot of leakage
- Daubechies D20 wavelet is better than D4 – narrower transition regions
- ripples in both the pass band and the stop band
- D20, Symlets and Coiflets – flat response in the middle of the band

Careful selection of the number of data points and the sampling frequency to avoid the frequency of interest from falling on the edges of the band



Selection of sampling frequency

Wavelet level	Frequency bands for sampling frequency 1024 Hz	Frequency bands for sampling frequency 10240 Hz
d1	512 Hz to 256 Hz	5120 Hz to 2560 Hz
d2	256 Hz to 128 Hz	2560 Hz to 1280 Hz
d3	128 Hz to 64 Hz	1280 Hz to 640 Hz
d4	64 Hz to 32 Hz	640 Hz to 320 Hz
d5	32 Hz to 16 Hz	160 Hz to 80 Hz
d6	16 Hz to 8 Hz	80 Hz to 40 Hz
d7	8 Hz to 4 Hz	40 Hz to 20 Hz
d8	4 Hz to 2 Hz	20 Hz to 10 Hz
a8	DC to 2 Hz	DC to 10 Hz

- 1024 Hz sampling frequency – 60 Hz at the end region of level d4
- 10240 Hz sampling frequency – 60 Hz in the middle of the level d6

For qualitative and quantitative signal analysis – sampling frequency and number of data points important.



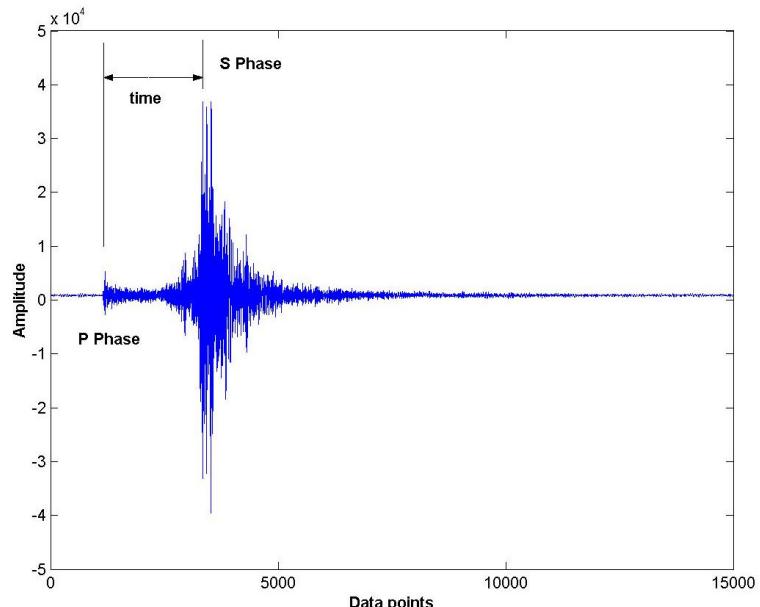
WAVELET SEISMIC EVENT DETECTION

Current Methods:

- STA/LTA (Short Time Average/Long Time Average) phase picker method
- new event detector by Murdock and Hutt the Seismic Research Observatories (1983)

Wavelet Method:

- properties of wavelet transforms - promising potential for use
- idea of using wavelets has been floated around and has created interest



Typical seismic earthquake signal



BACKGROUND

Seismic signal : $s(t) = n(t) + e(t)$ i.e., noise + event

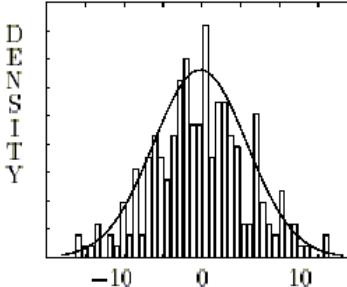
seismic signal can be represented
in its wavelet bases as:

$$s(t) = \sum_{j,n} d_{j,n}(s) \psi_{j,n}(t) \text{ for } j > 0 \text{ and } n \in I$$

where $d_{j,n}(s) = d_{j,n}(n) + d_{j,n}(e)$

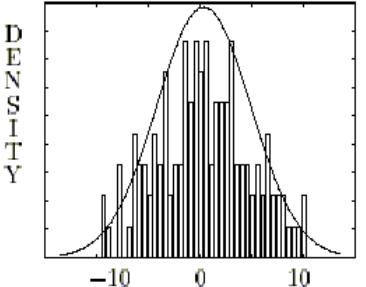
Scale 2 wavelet coefficients

Empirical PDF
Gaussian model superimposed



Scale 3 wavelet coefficients

Empirical PDF
Gaussian model superimposed



The noise $n(t)$ or is modeled for
each wavelet level as :

$$d_{j,n}(n) = N(0, \sigma_j^2)$$

Gaussian noise with a mean of zero
and a standard deviation of σ_j^2



multiscale threshold test :

hypothesize that for any event to be detected in the seismic signal

$$|d_{j,n}(s)| > 4\sigma_j$$

the occurrence of an event in the signal is true if any of the wavelet level amplitude falls outside the boundary of the noise model

Earthquakes considered:

Earthquake data:

- available from the IRIS Consortium
- full sampling rate of 40 samples/sec

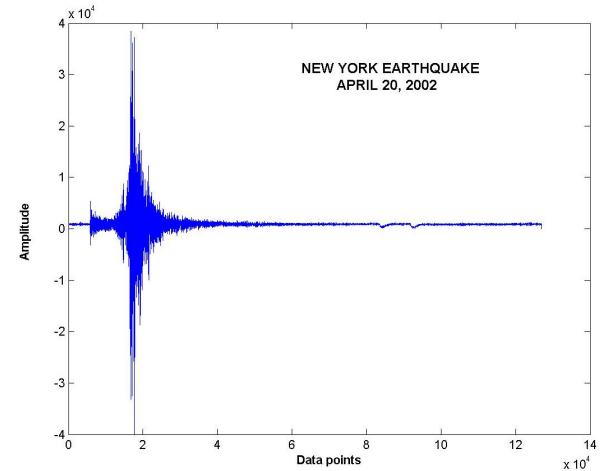
Earthquake	Magnitude	Time in UTC	Date
Central Alaska	7.9	22:12:40	Nov 3, 2002
Southern Indiana	5.0	17:37:13	Jun 18, 2002
New York	5.1	10:50:44	Apr 20, 2002
Washington	6.8	18:54:32	Feb 28, 2001
South India	7.7	03:16:40	Jan 26, 2001



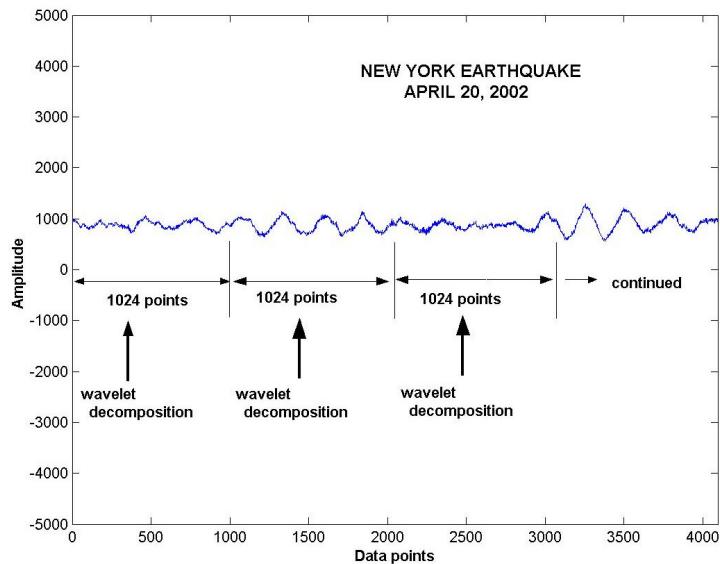
New York Earthquake:

55 minutes of seismic data with 127050 data points

Oxford, Mississippi seismic station



Wavelet transform window



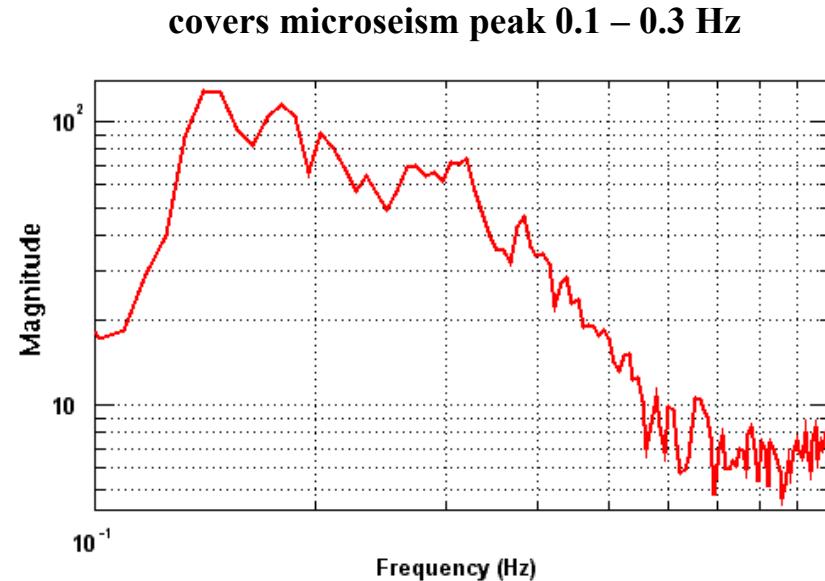


For sampling rate : 40 samples/sec

Select number of data points N: 1024

two wavelet levels 2 and 3, whose frequency ranges from 0.075 to 0.31 Hz

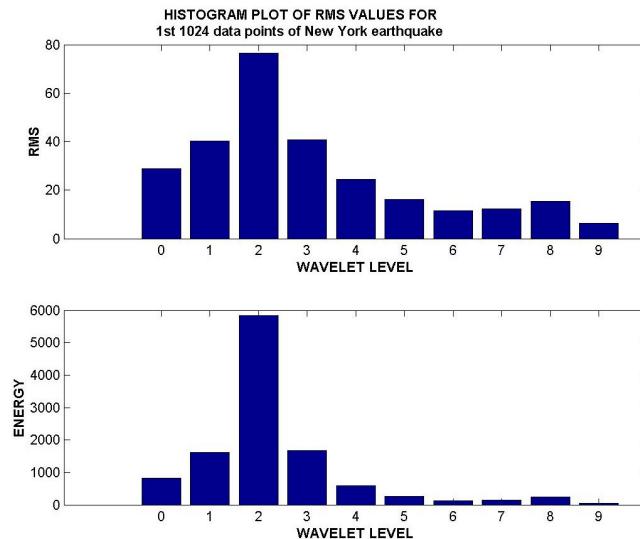
Level	Wavelet Scale	Frequency range (Hz)	Center Frequency (Hz)
0	a9	0 – 0.037	0.018
1	d9	0.037 – 0.075	0.058
2	d8	0.075 – 0.15	0.117
3	d7	0.15 – 0.31	0.234
4	d6	0.31 – 0.62	0.468
5	d5	0.625 – 1.25	0.937
6	d4	1.25 – 2.50	1.875
7	d3	2.5 – 5.0	3.75
8	d2	5 - 10	7.5
9	d1	10 - 20	15



Fourier Transform of seismic signal



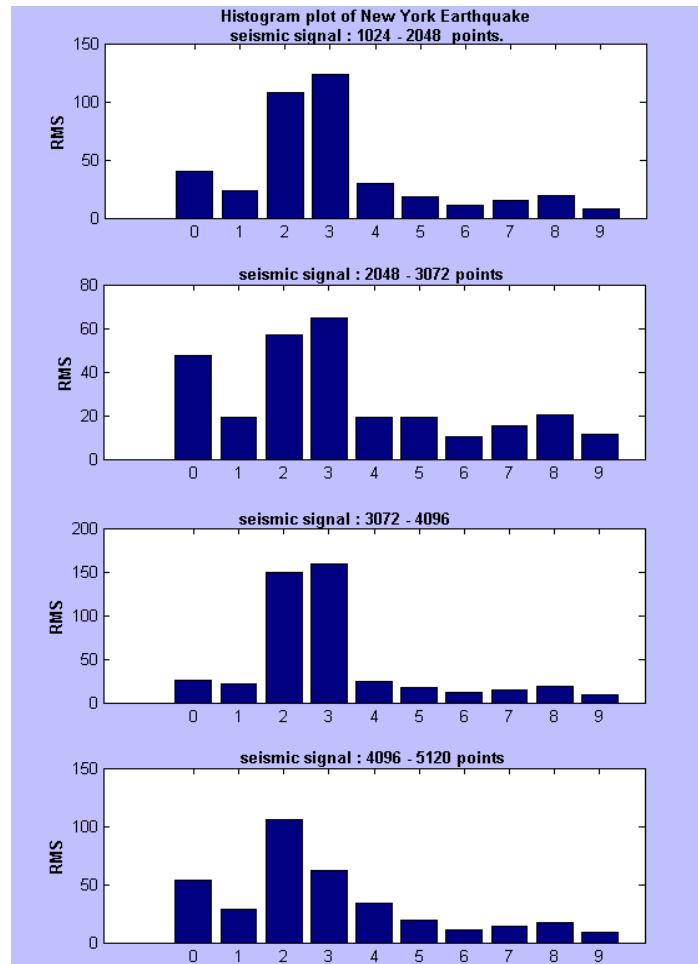
First 1024 points – rms values of wavelet transform



Before the arrival of P wave

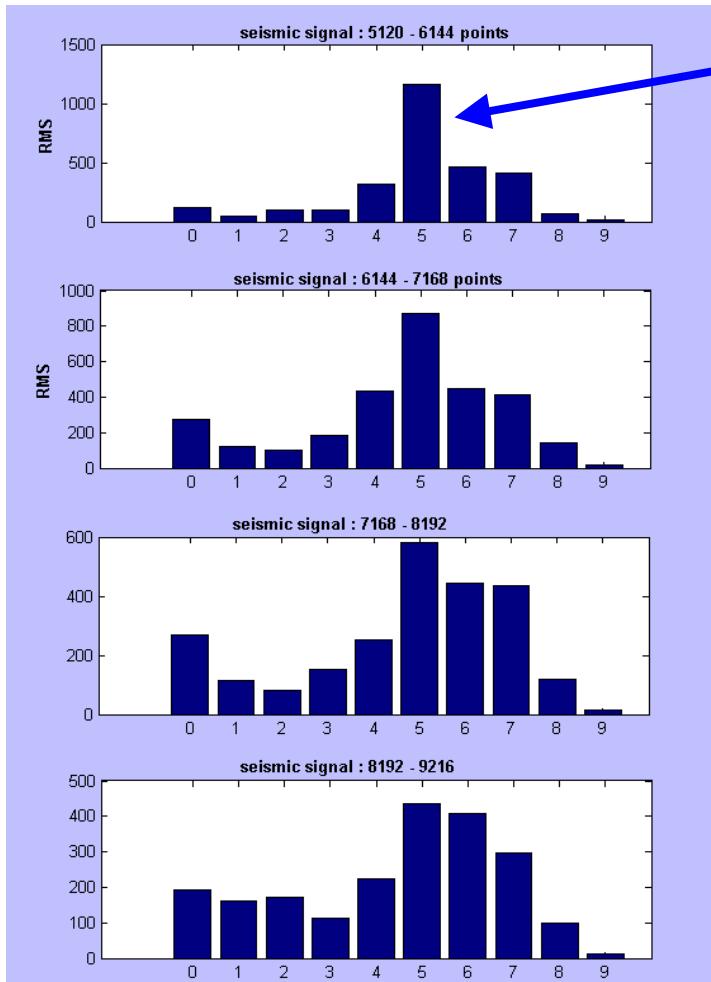
- Perform wavelet transform - Daubechies D20
- Calculate the rms values for each window
- Plot

Next 4 windows – rms values



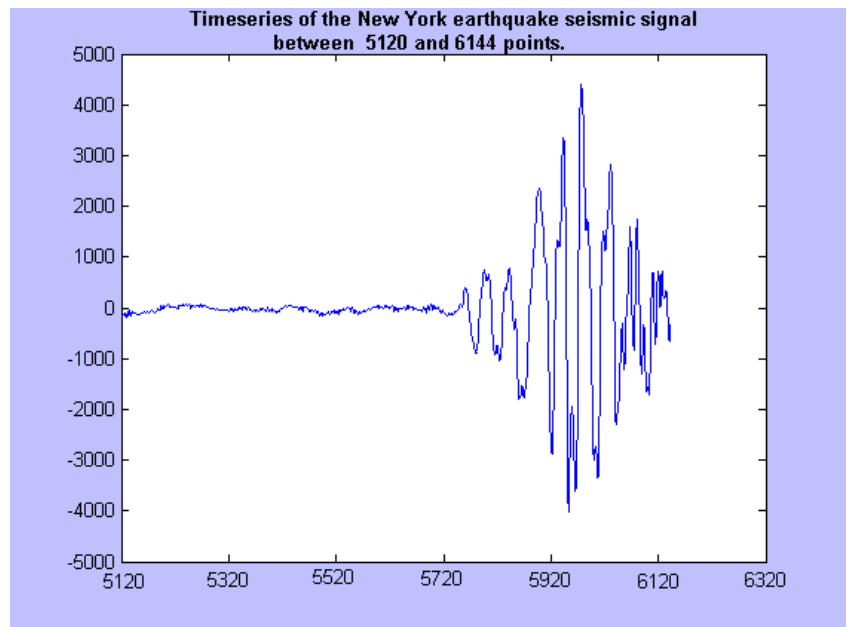


Next 4 windows



**Level 5 (d5) wavelet level's rms
values crosses threshold 200**

Corresponding time series window

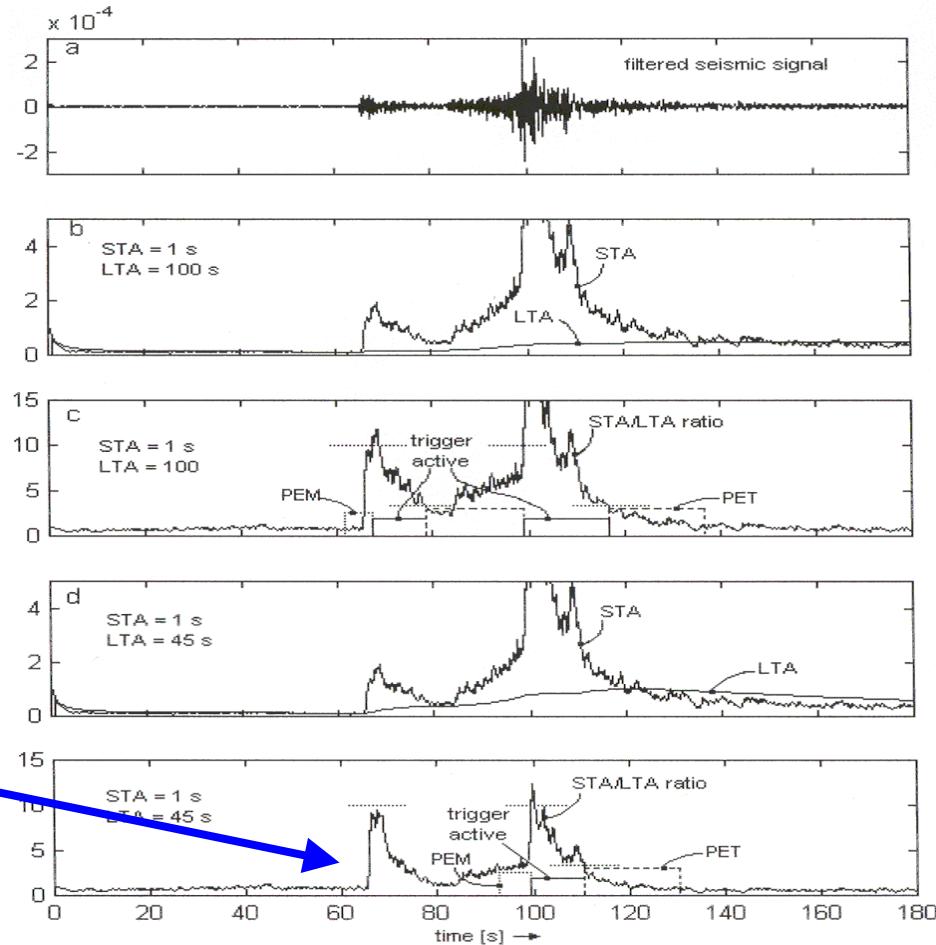




New York Earthquake:

- P phase was detected at the sixth window
- time of arrival of 153.6 seconds - 25.6 seconds/window x 6
- good 150 seconds earlier detection than conventional method used.
- inherent delay of 25.6 seconds – reduced by over lapping windows.

STA – LTA method

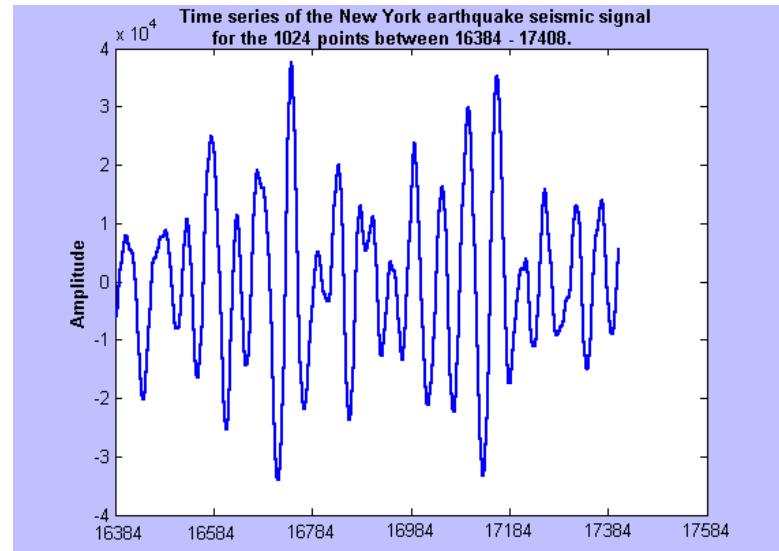
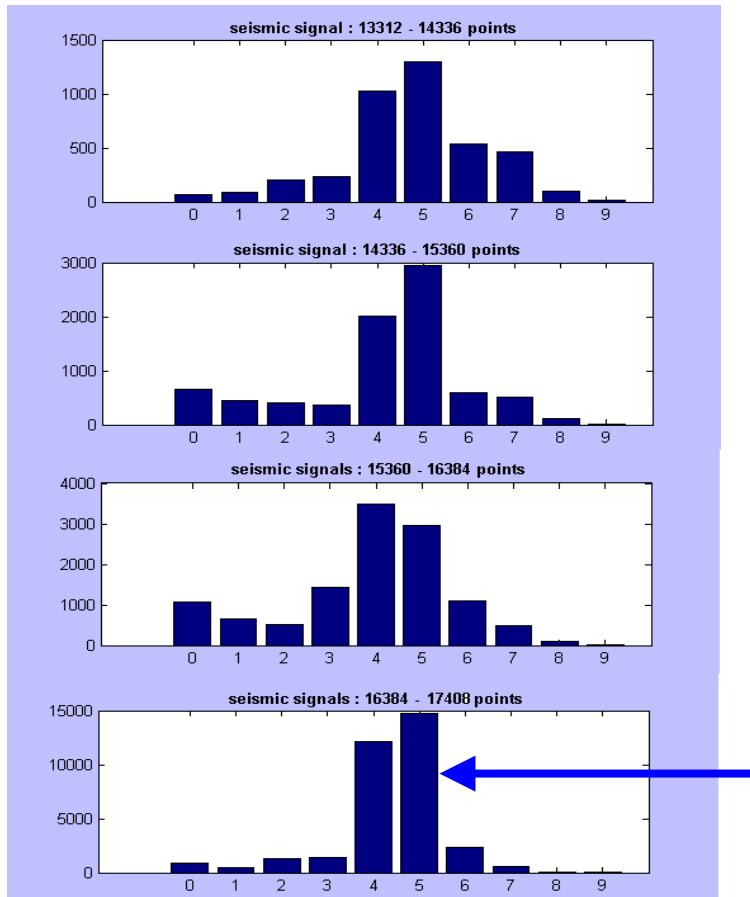


STA/LTA Method:

- misses the P phase wave – LTA 45 secs



Continuing:



corresponding time window

S phase wave - Signal peaked at 17th window

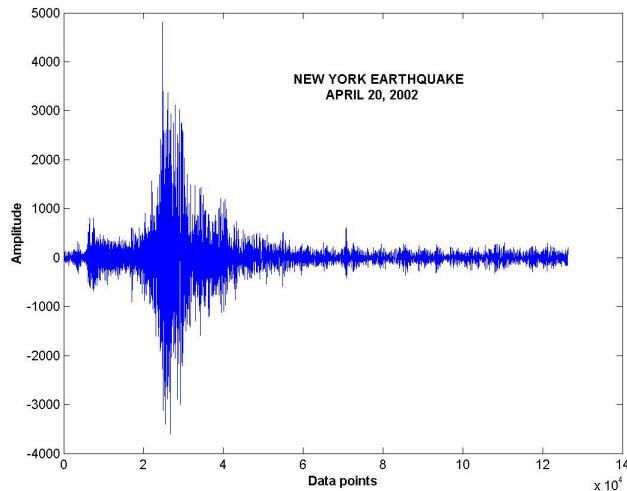
Time of arrival : 435.2 secs

difference S phase arrival time - P phase arrival time is: 435.2 (7.25 min) – 153.6 (2.56 min) = 281.6 secs (4.69min)

approx equal to the distance between the epicenter of the earthquake and Oxford, Mississippi = 1583 miles

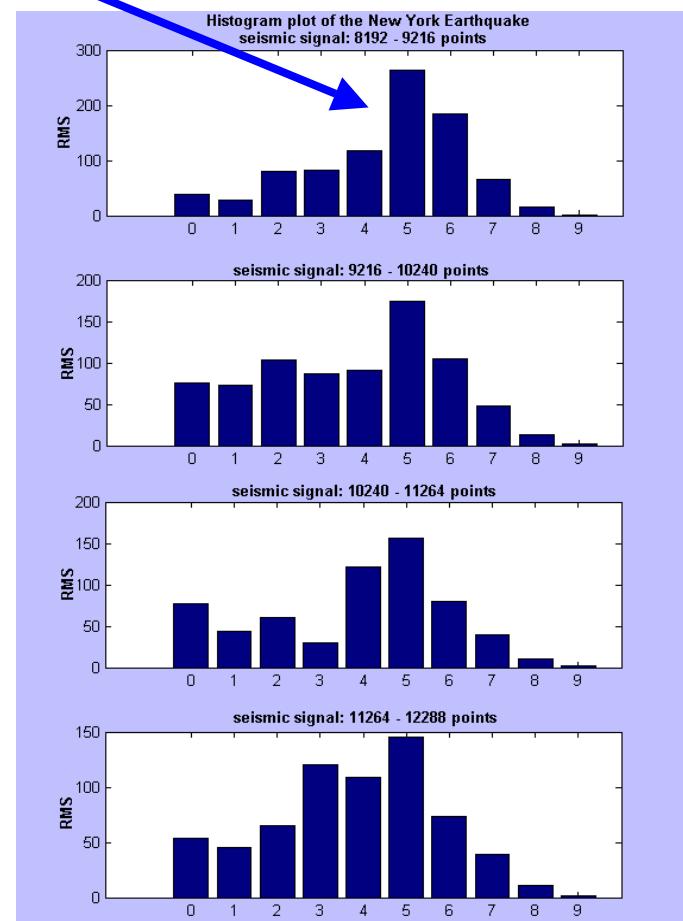


New York Earthquake: Junction City, TX station



Seismic signal recorded

P phase detected 9th window – arrival
time of 230.4 seconds





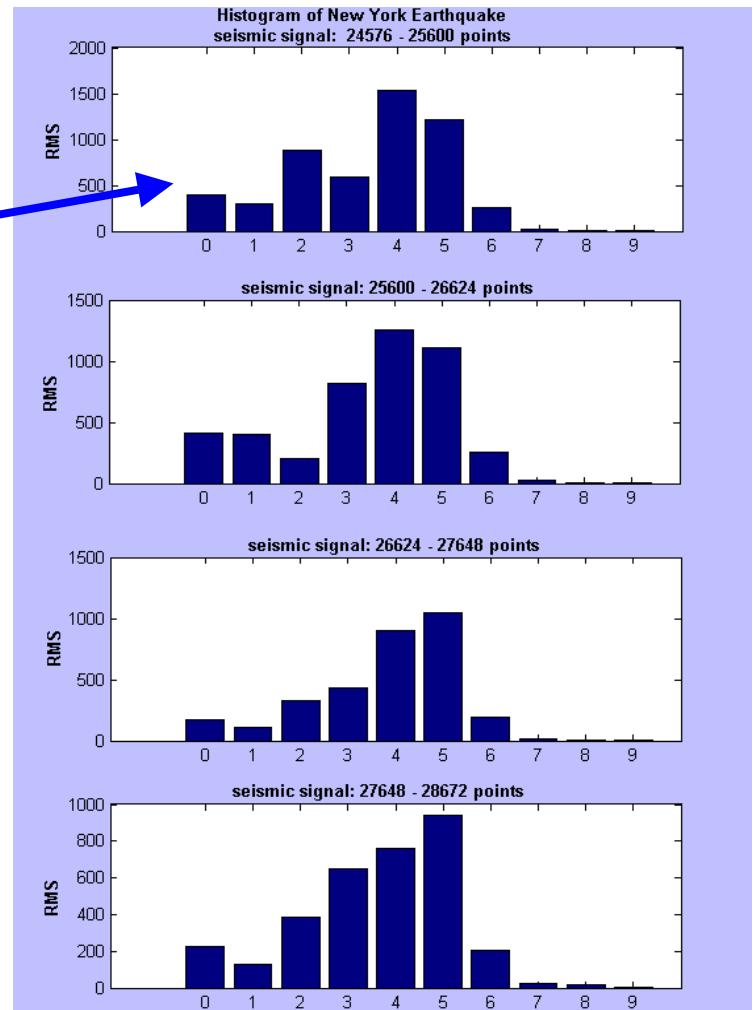
New York Earthquake:

Junction City, TX station

- S phase is detected in the 25th wavelet decomposition window
- the time of arrival is 640 seconds

$$640 \text{ (10.67 min)} - 230.4 \text{ (3.84 min)} = 409.6 \text{ seconds (6.83 min)}$$

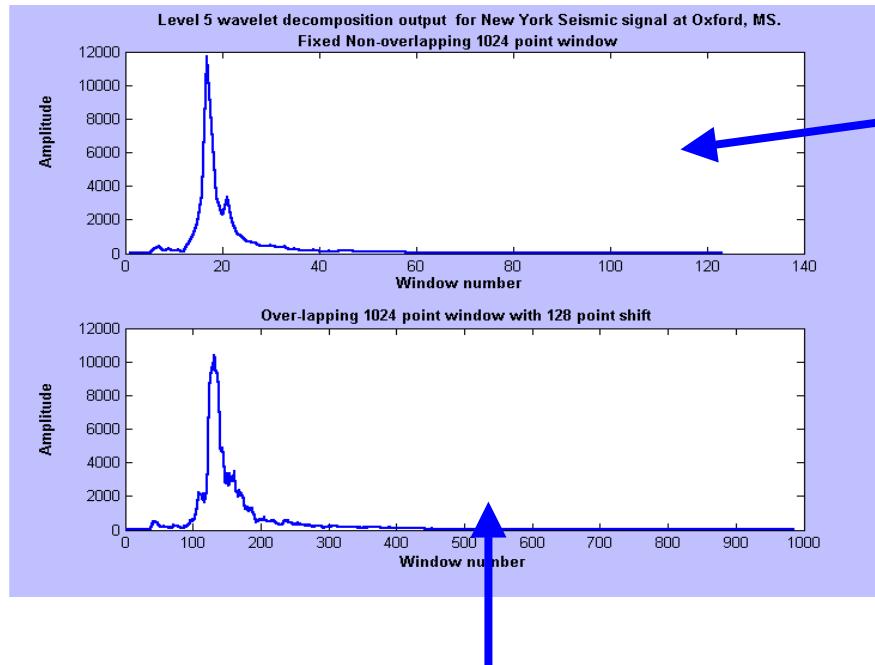
Distance to epicenter = 2545 miles





Improving accuracy with moving window

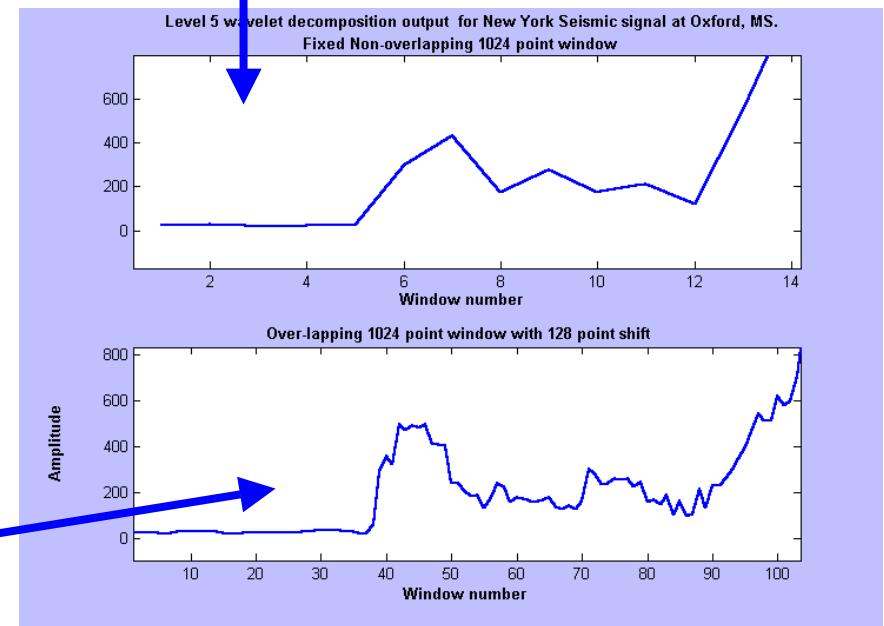
Plots of rms values for level 5 (d5) wavelet decomposition



overlapping window –
128 point shift

Fixed non-overlapping
window

Zoomed in at the P phase





Earthquakes recorded at given stations	P Phase arrival time (seconds)	S Phase arrival time (seconds)	Estimated time difference seconds (distance in kms)
New York (20 Apr 2002 – Plattsburg, NY) - Oxford, MS - Junction, TX	124.8 156.8	422.4 614.4	297.6 (2976) 457.6 (4576)
Southern Indiana (18 June 2002 - Evansville, IN) - Oxford, MS - Junction, TX	153.6 192.0	220.8 428.8	67.2 (672) 236.8 (2368)
Washington (28 Feb 2001 – Nisqually, WA) - Pine, OR - Longmire, WA	60.8 3.2	192 38.2	131.2 (1312) 35.2 (352)
South India (26 Jan 2001 – Gujarat, India) - Kislovodsk, Ru - Ala Archa, Ky	230.4 204.8	723.2 556.8	492.8 (4928) 352 (3520)
Central Alaska (3 Nov 2002 – Cantwell, AK) - Oxford, MS - Junction, TX	297.6 297.6	1562.4 1562.4	1264(12640) 1264(12640)



CONCLUSION

Summarizing the contributions of this research:

- ✓ **Understanding pole –zero and magnitude – phase characteristics of wavelet filters**
- ✓ **Shown – some wavelet filters have same magnitude characteristics, different phase response**
- ✓ **Presented - methods of identification, detection of inrush and fault currents**
- ✓ **Proposed a protection method for faults**
- ✓ **Research on energy distribution and leakage in output wavelet decompositions**
- ✓ **Study of frequency bandwidth characteristics of wavelet levels and comparison**
- ✓ **Importance of careful selection of sampling frequency and number of data points**
- ✓ **Proposing a wavelet method for detection of P-phase and S-phase waves in earthquakes**

References: Listed in the dissertation

Matlab programs: Listed in Appendix



Thanks for your patience

Questions ?