CSCI 360 ♦ Introduction to Artificial Intelligence

Homework 2

Honor Code: You must work independently on this assignment – please see the related statements in the syllabus.

Your solution must be submitted as a single PDF file via Blackboard.

Part I. From Bayesian Network to Full Joint Probability Distribution (12 points)

Recall that a Bayesian network is a data structure for representing the full joint probability distribution. Given the following Bayesian network, where A = having allergy and S = Sneezing, compute the full joint probability table that the Bayesian network represents.

P(A)
0.1

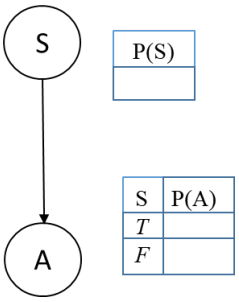
A	P(S)
T	*0.6*
F	*0.3*

A	S	P(A,S)
T	*T*	
T	*F*	
F	*T*	
F	*F*	

T T	0.1 * 0.6 = 0.06	(3 points)
T F	0.1 * 0.4 = 0.04	(3 points)
F T	0.9 * 0.3 = 0.27	(3 points)
F F	0.9 * 0.7 = 0.63	(3 points)

Part II. From Full Joint Probability Distribution to Bayesian Network (18 points)

Recall that different Bayesian networks may be used to represent the same joint probability distribution. Given the following Bayesian network, please compute its conditional probability tables (CPTs) so that it represents the same joint probabilities as the one shown in Part I.



P(S)

S	P(A)
T	
F	

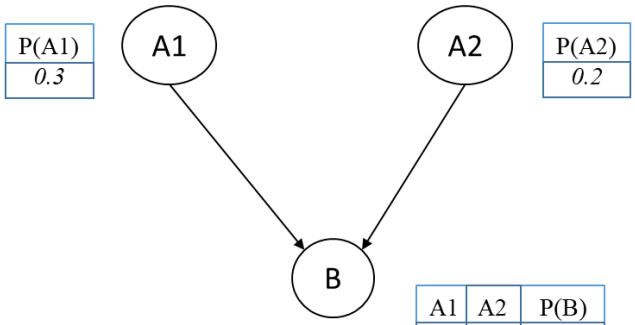
P(s) = 0.06 + 0.27 = 0.33 (6 points)

P(a|s) = P(a,s) / P(s) = 0.06/0.33 = 0.18 (6 points)

P(a|¬s) = P(a,¬s) / P(¬s) = 0.04/0.67 = 0.0597 (6 points)

Part III. Inference in Bayesian Networks (30 points)

Given the following Bayesian network, please answer the following questions:



P(A1)
0.3

P(A2)
0.2

A1	A2	P(B)
T	*T*	*0.8*
T	*F*	*0.7*
F	*T*	*0.6*
F	*F*	*0.5*

- (6 points) Compute P(a1, a2, b) = P(a1) * P(a2) * P(b|a1,a2) = 0.3 * 0.2 * 0.8 = 0.048
- (6 points) Compute P(a1, ¬a2, b) = P(a1) * P(¬a2) * P(b|a1,¬a2) = 0.3 * 0.8 * 0.7 = 0.168
- (6 points) Compute P(a1, b) = 0.048 + 0.168 = 0.216
- (12 points) Compute P(a2 | a1, b) = P(a1,a2,b) / P(a1,b) = 0.048/0.216 = 0.22

Part IV. Decision Theory (40 points)

Consider buying a house whose market value is $500K, but you can get it for $450K. However, there is a risk: if the house has structural problems (its current condition is not OK), you will have to spend $110K to fix it. The probability that currently it's in OK condition is estimated to be 75%; that is, P(OK) = 0.75.

1) (4 points) What is the expected utility (i.e., your expected net gain or loss in dollars) if you buy?

   EU = P(OK) * (Value – Price) + P(¬OK) * (Value-Price-Fix) = 0.75 * 50K + 0.25 * (-60K) = $ 22.5K

2) (4 points) You may hire someone to perform an inspection before you decide to buy. However, the inspection is not always accurate. You estimate that the probability that
   - the inspection passes given that the house is actually OK is 80%; that is, P(pass|OK) = 0.80;
   - the inspection passes given that the house is not OK is 40%; that is, P(pass|¬OK) = 0.40.

What is P(pass), the overall probability that the house passes the inspection?

P(pass) = P(OK) * P(pass|OK) + P(¬OK) * P(pass|¬OK) = 0.75 * 0.8 + 0.25 * 0.4 = 0.7

3) (4 points) Compute P(OK|pass) = P(pass|OK) * P(OK) / P(pass) = 0.8 * 0.75 / 0.7 = 0.857

4) (4 points) Compute P(OK|¬pass) = P(¬pass|OK) * P(OK) / P(¬pass) = 0.2 * 0.75 / 0.3 = 0.5

5) (8 points) If the inspection passes, and you want to buy, what is the expected utility?

   EU_pass = P(OK|pass) * (Value-Price) + P(¬OK|pass) * (Value-Price-Fix)

   = 0.857*50K + 0.143*(-60K) = $34.27K

6) (8 points) If the inspection fails, and you want to buy, what is the expected utility?

   EU_not_pass = P(OK|¬pass) * (Value-Price) + P(¬OK|¬pass) * (Value-Price-Fix)

   = 0.5 * 50K + 0.5 * (-60K) = - $5K

7) (8 points) Should you pay for the inspection? If so, at most how much should you pay the inspector?

   EU = P(pass) * EU_pass + P(¬pass) * EU_not_pass = 0.7 * 34.27K + 0.3 * (-5K) = $22.489K

   Since $22.489K < $22.5K, you should NOT pay for the inspection.