CSCI 360 ♦ Introduction to Artificial Intelligence

Homework 1

Honor Code: You must work independently on this assignment – please see the related statements in the syllabus.

Your solution must be submitted as a single PDF file via Blackboard.

Part I. True or False Questions (10 points)

(1) During the lecture, we explained that you can construct a knowledge base (KB) and then query KB to see if a sentence *(S)* is entailed. Is it possible for both *(KB |= S)* and *(KB |= ¬S)*? (2 points)

YES. When (KB=False), for example, since False entails any sentence.

(2) What about *(KB |= S)* and *(¬KB |= S)* – is it possible that both of them hold? (2 points)

YES. When (S=True), for example, any KB entails True.

(3) A propositional logic formula is a "tautology" if the formula is true in all models. Is the propositional formula $(P \rightarrow P) \rightarrow P$ a tautology? (2 points)

NO. Tautology means the formula holds in all models (or it can be simplified into TRUE).

(4) For A* search, the heuristic functions obtained through problem relaxation are guaranteed to be "admissible". Are they guaranteed to be "consistent"? (2 points)

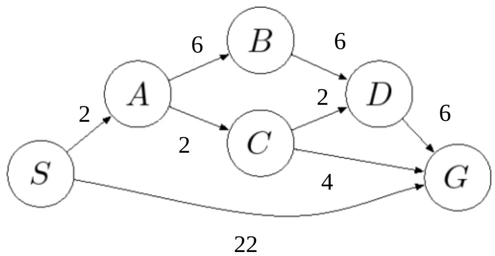
YES. They are the actual cost functions of a relaxed game.

(5) If both *h1(n)* and *h2(n)* are "admissible" heuristic functions for A* search, the composite function *h3(n) = MAX(h1(n),h2(n))* is also admissible. What about *h4(n)=MIN(h1(n),h2(n))* – is it also admissible? (2 points)

YES. By definition, h4(n) is also admissible.

Part II. Tree Search (25 points)

During the lecture, we discussed various uninformed and informed “Tree Search” algorithms. These algorithms, unlike in “Graph Search”, do not remember the visited nodes. Please run these algorithms on the following state space graph, where (S) is the initial state, (G) is the goal state, edge labels are the step costs, and the heuristic function is shown in the right-hand-side table.



State	$h(n)$
S	**8**
A	**4**
B	**12**
C	**2**
D	**6**
G	**0**

For each search strategy below, please show the order in which nodes are “expanded”, starting with node (S) and ending as soon as node (G) is found. Please also show the path from (S) to (G).

NOTE: During your search, please assume successors of each node are returned in “left-to-right” order. That is, successors of (S) are nodes (A) and (G), arranged in that order; successor of (A) are nodes (B) and (C), arranged in that order; and successors of (C) are nodes (D) and (G), arranged in that order.

(1) **Depth First Search:** Order of node expansion (2 points): S (G)

Path found (2 points): S G Path cost (1 point): 22

Another allowed solution: expansion order: S A B D (G) path found: S A B D G Path cost: 20

(2) **Uniform Cost Search:** Order of node expansion (2 points): S A C D B (G)

Path found (2 points): S A C G Path cost (1 point): 8

(3) **Greedy Best First Search:** Order of node expansion (2 points): S (G)

Path found (2 points): S G Path cost (1 point): 22

(4) **Iterative Deepening:** Order of node expansion (2 points): S (G)

IDDFS: Another allowed solution: expand Path found (2 points): S G Path cost (1 point): 22

(5) **A* Search with *h(n)*:** Order of node expansion (2 points): S A C (G)

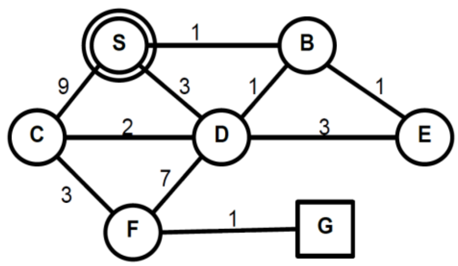
Path found (2 points): S A C G Path cost (1 point): 8

Full credit: It’s fine to miss (G) at the end (of both the node expansion and the path found).

Other than that, no partial credit will be given.

Part III. Heuristic Functions (15 points)

We discussed two properties (admissible and consistent) of a heuristic function, and their impact on A* Tree-Search and A* Graph-Search. In the following state space graph, where (S) is the initial state, (G) is the goal state, and edge labels are the actual step cost, please analyze the heuristic functions h1(n), h2(n) and h3(n) shown in the table, together with the optimal heuristic h*(n).



Node	h1	h2	h3	h* (optimal)
S-Start	5	5	5	8
B	4	4	5	7
C	5	3	2	4
D	3	1	4	6
E	4	3	4	8
F	1	0	1	1
G-Goal	0	0	0	0

(1) Is heuristic function h1(n) admissible? (3 points) ______ NO ______

(2) Is heuristic function h1(n) consistent? (2 points) ______ NO ______

(3) Is heuristic function h2(n) admissible? (3 points) ______ YES ______

(4) Is heuristic function h2(n) consistent? (2 points) ______ NO ______

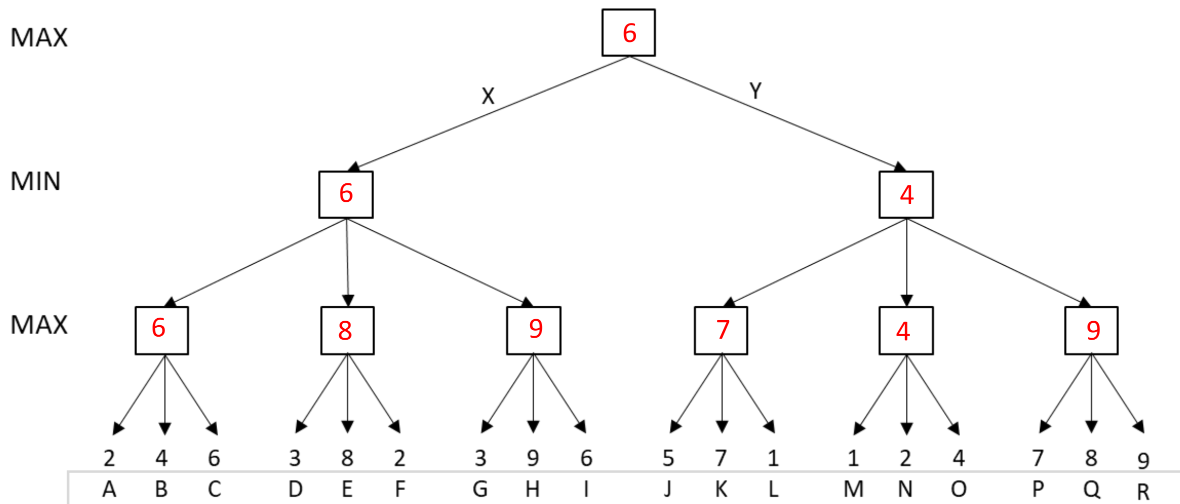
(5) Is heuristic function h3(n) admissible? (3 points) ______ YES ______

(6) Is heuristic function h3(n) consistent? (2 points) ______ YES ______

Hint: Please think about “what could go wrong?” during Graph Search, if the heuristic function used by A* algorithm is not admissible, or not consistent.

Part IV. Adversarial Search (30 points)

We discussed the Minimax algorithm for deterministic game with perfect information. It allows player "MAX" to compute the best move. We also discussed "alpha-beta pruning" which allows the search to skip part of the game tree. In the game tree below, please run the Minimax algorithm together with alpha-beta pruning, and then answer the questions.



(1) Please compute the minimax values for each node, based on the utility values given at the bottom. (3 points – 0.33333 point per minimax value)

(2) At the root node, should MAX choose "X" or "Y"? (2 points)

X

(3) Assume that the terminal nodes are visited in alphabetic order (from A, B, C, … to P, Q, R). Which of these terminal nodes will be skipped by alpha-beta pruning? (5 points)

F,I,P,Q,R

(4) To maximize the number of terminal nodes (A, B, C, …, P, Q, R) that will be skipped by alpha-beta pruning, in what order (best order) should we visit these terminal nodes? (10 points)

[C, B, A] → [E, (d), (f)] → [H, (i), (g)] → [O, N, M] → [(k), (j), (l)] → [(r), (q), (p)]

^ any order ^ 'E' first 'H' first ^ any order ^ ^ don't matter ^ ^ don't matter ^

If group [C,B,A] is not ahead of other groups, -3 points

If 'E' is not first in group [E, (d), (f)], -2 points

If 'H' is not first in group [H, (i), (g)], -2 points

If group [O, N, M] is not head of the other two groups, -3 points

(5) In the worst case, alpha-beta pruning will not be able to skip any node. In what order should the terminal nodes be visited to result in such a worst case? (10 points)

[P, Q, R] → [L, J, K] → [M, N, O] → [G, I, H] → [F, D, E] → [A, B, C]

This is perhaps only one possible solution. Other solutions are also possible.

For example: [G, I, H] → [F,D,E] → [A, B, C] → [P,Q,R] → [L,J,K] → [M,N,O]

^in any order^

Partial Credit: in the student's solution, If you detect any node that can actually be pruned away, -2 points

If you detect two nodes that can actually be pruned away, -4 points

If you detect three nodes that can actually be pruned away, -6 points, ...

NOTE to graders: You don't have to detect all nodes that may be pruned away in the student's solution.

Just try your best. Find the obvious mistakes. Deduct some points. And move on!

Part V. Resolution in Propositional Logic (20 points)

Assume that you have a knowledge base (KB) consists of the following sentences in propositional logic:

$$\neg(D \vee E) \;,\; \neg(C \rightarrow D) \;,\; (A \leftrightarrow B) \;,\; (C \rightarrow (B \vee D)) \;,\; (E \vee \neg F)$$

You would like to check whether (KB) entails (A). This can be done by using "resolution". First, you need to put (KB) in Conjunctive Normal Form (CNF). Then, you use resolution to show (KB)^(¬A) is unsatisfiable.

(1) Please represent (KB)^(¬A) in the CNF format. (10 points)

$(\neg D)$, $(\neg E)$, (C) , $(\neg D)$, $(\neg A \vee B)$, $(\neg B \vee A)$, $(\neg C \vee B \vee D)$, $(E \vee \neg F)$, $(\neg A)$

Partial credit: For each of the 9 clauses above, if it's missing, -1 point

(2) Please use resolution to prove that (KB)^(¬A) is unsatisfiable. (10 points)

$(\neg B \vee A)$, $(\neg A)$ *resolvent is* $(\neg B)$

$(\neg C \vee B \vee D)$ *resolvent is* $(\neg C \vee D)$

$(\neg D)$ *resolvent is* $(\neg C)$

(C) *resolvent is* $( )$

QED.

Partial credit: For each of the 4 resolution steps outlined above, if it's missing, -2 points