DIGITAL IMAGE PROCESSING

Image Enhancement in Spatial Domain : Session 3

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Today's Lecture

- **Image Enhancement in Spatial Domain**
  - **Local Histogram Processing**
  - **Using Histogram Statistics for Image Enhancement**

Image Enhancement in Spatial Doma

Local Histogram Processing

- Define a neighborhood and move its center from pixel to pixel
- At each location, the histogram of the points in the neighborhood is computed. Either histogram equalization or histogram specification transformation function is obtained
- Map the intensity of the pixel centered in the neighborhood
- Move to the next location and repeat the procedure

Image Enhancement in Spatial Doma

Local Histogram Processing

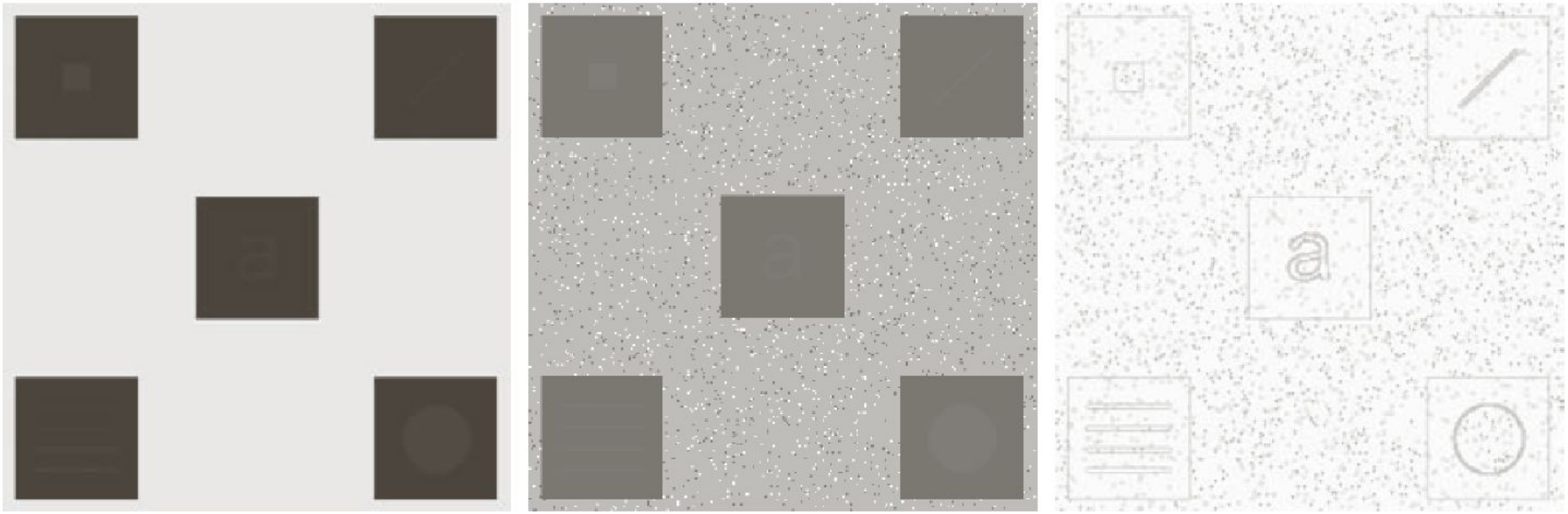


FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Image Enhancement in Spatial Doma

Histogram Statistics for Image Enhancement

Average Intensity

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

Variance

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

Image Enhancement in Spatial Doma

Histogram Statistics for Image Enhancement

Local average intensity

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i)$$

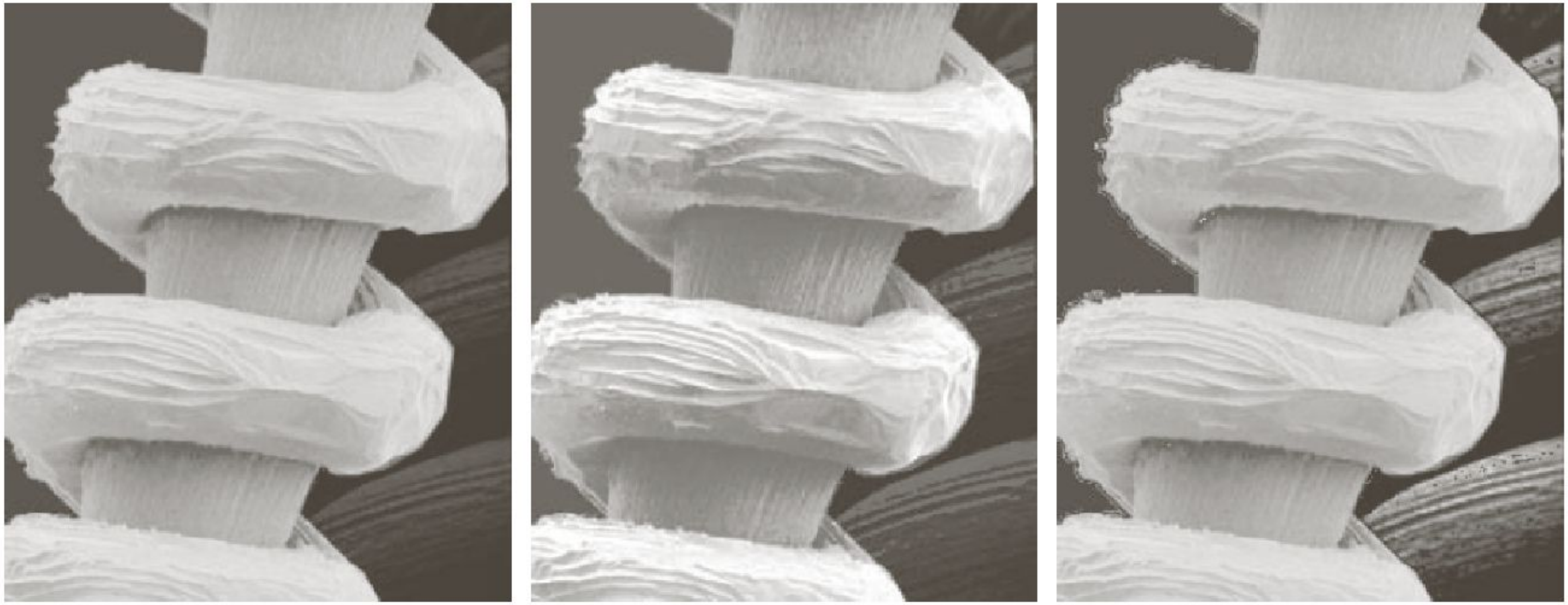
s_{xy} denotes a neighborhood

Local variance

$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 p_{s_{xy}}(r_i)$$

Image Enhancement in Spatial Doma

Histogram Statistics for Image Enhancement



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Image Enhancement in Spatial Doma

Spatial Filtering

- A spatial filter consists of (a) **a neighborhood**, and (b) **a predefined operation**

- Linear spatial filtering of an image of size $M \times N$ with a filter of size $\mathrm{m} \times n$ is given by the expression

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

where $a = \frac{m-1}{2}$ and $\mathrm{b} = \frac{n-1}{2}$.

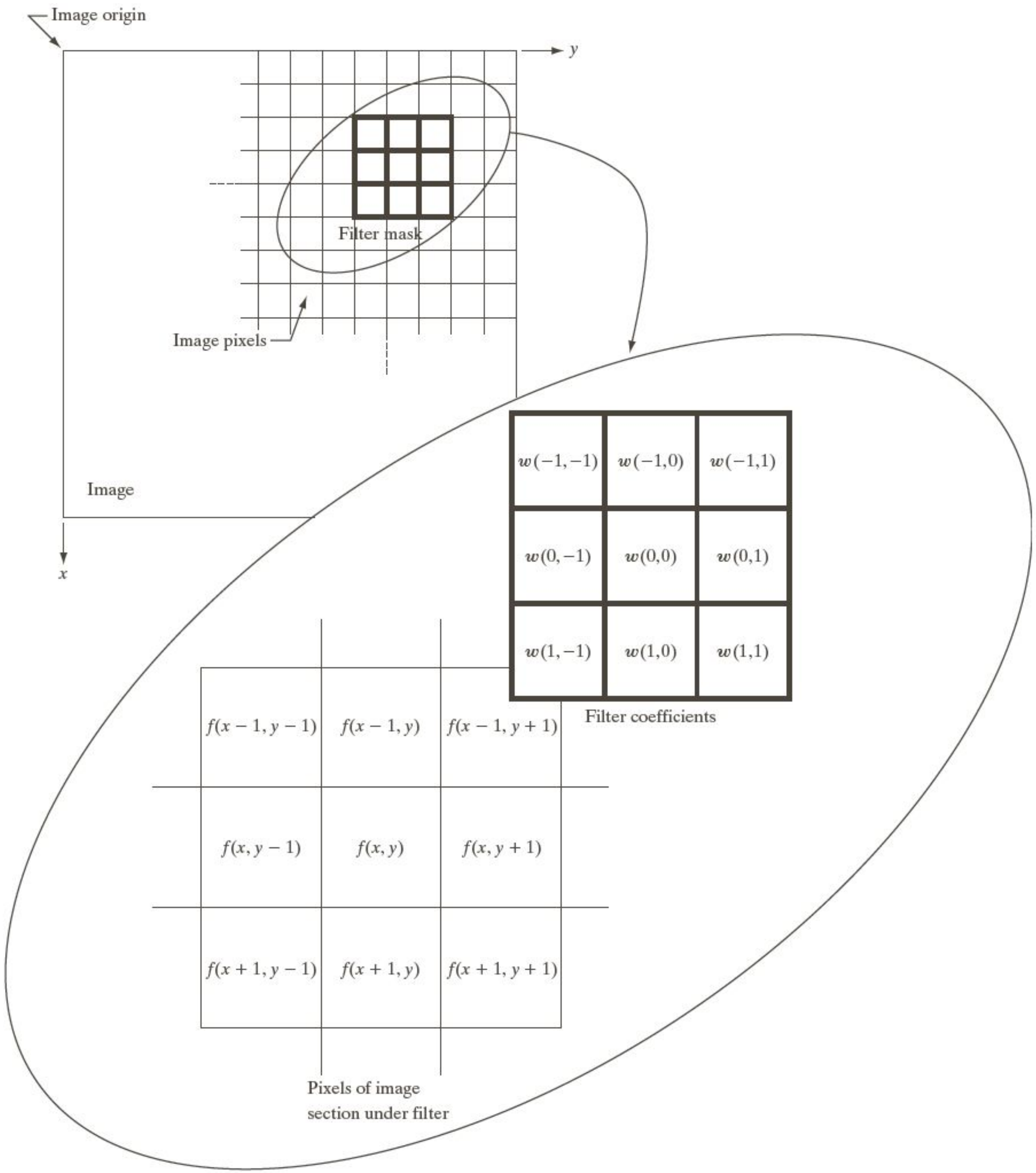


Image origin
y
Filter mask
Image pixels
Image
x
w(−1,−1)
w(−1,0)
w(−1,1)
w(0,−1)
w(0,0)
w(0,1)
w(1,−1)
w(1,0)
w(1,1)
Filter coefficients
f(x − 1, y − 1)
f(x − 1, y)
f(x − 1, y + 1)
f(x, y − 1)
f(x, y)
f(x, y + 1)
f(x + 1, y − 1)
f(x + 1, y)
f(x + 1, y + 1)
Pixels of image
section under filter

Image Enhancement in Spatial Doma

Spatial Filtering

- The process of linear filtering is called convolution.
- The filter masks are sometimes called convolution masks or convolution kernel.

Image Enhancement in Spatial Doma

Smoothing Spatial Filters

- Smoothing filters are used for blurring and for noise reduction.
- Blurring is used in removal of small details and bridging of small gaps in lines or curves.
- Smoothing spatial filters include linear filters and nonlinear filters.
- These filters sometimes are called ***averaging filters.***
- They are also referred to a ***lowpass filters*** in frequency domain.

Image Enhancement in Spatial Doma

Smoothing Linear Spatial Filters

The general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ is given

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

where $m = 2a + 1, \quad n = 2b + 1.$

Image Enhancement in Spatial Doma

Two Smoothing Averaging Filter Masks

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

average

$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

weighted average

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

Two Smoothing Averaging Filter Masks

a	b
c	d
e	f

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15,$ and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Image Enhancement in Spatial Doma

Example: Gross Representation of Objects

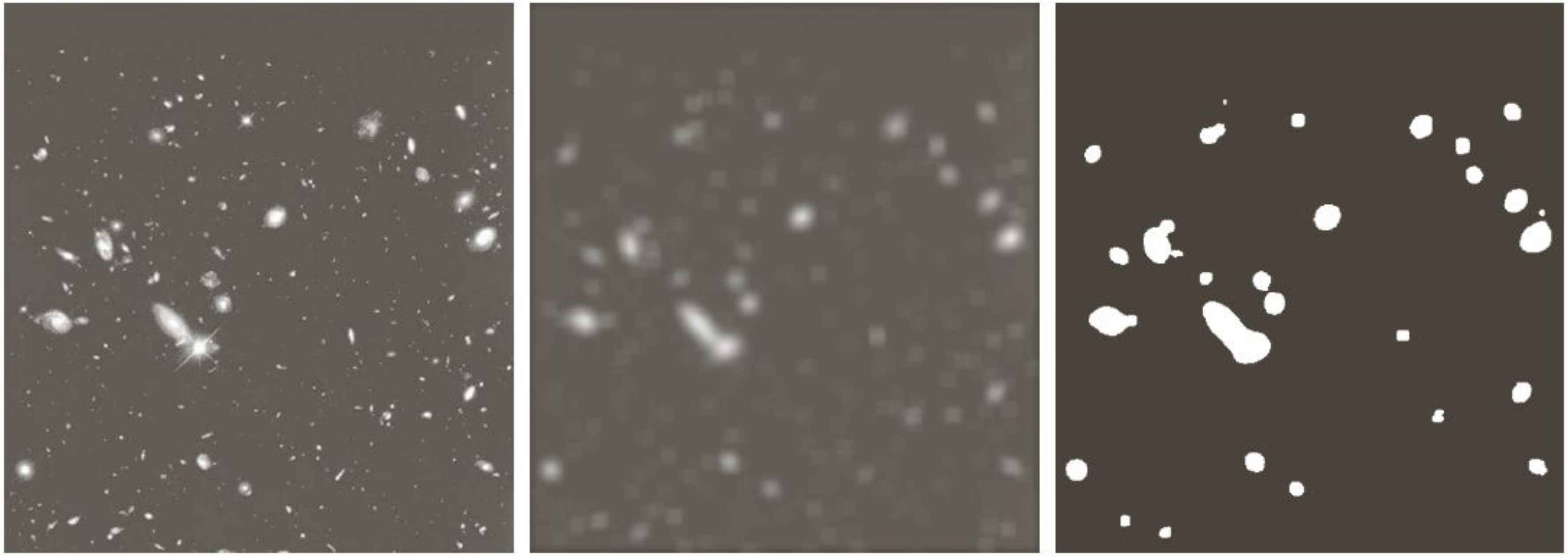

a b c

FIGURE 3.34 (a) Image of size 528×485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

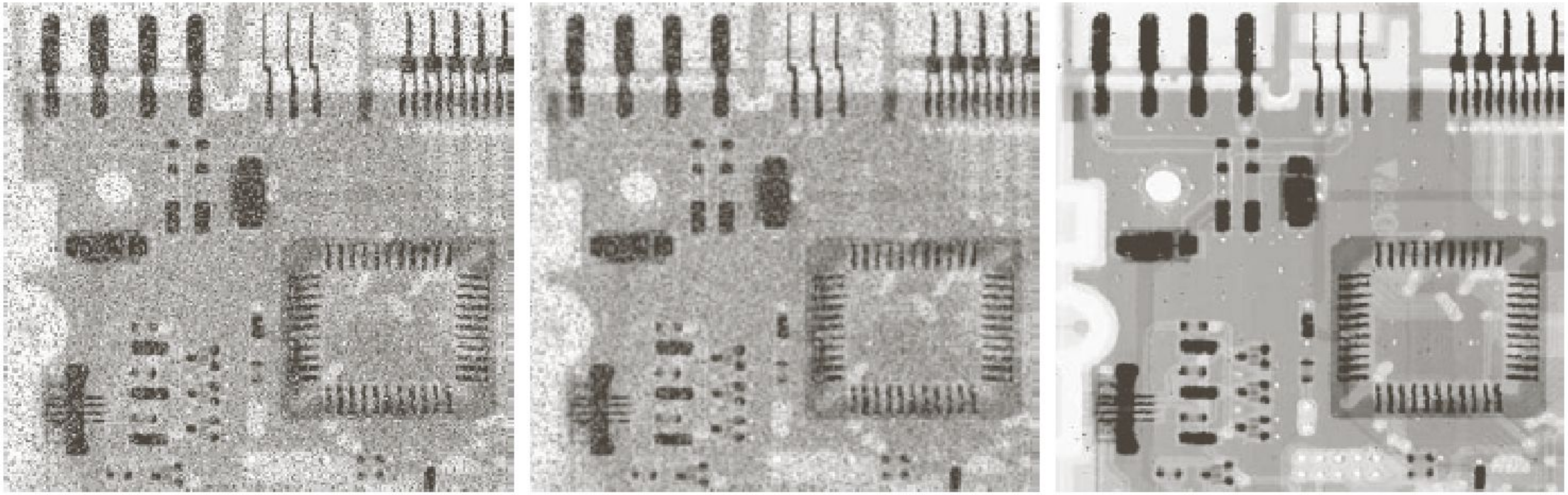
Image Enhancement in Spatial Doma

Order-Statistic (Nonlinear) Filters

- Nonlinear
- Based on ordering (ranking) the pixels contained in the filter mask
- Replacing the value of the center pixel with the value determined by the ranking result
- E.g., median filter, max filter, min filter

Image Enhancement in Spatial Doma

Example: Use of Median Filtering for Noise Reduction



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Image Enhancement in Spatial Doma

Sharpening Spatial Filters

- Foundation
- Laplacian Operator
- Unsharp Masking and Highboost Filtering
- Using First-Order Derivatives for Nonlinear Image Sharpening — The Gradient

Image Enhancement in Spatial Doma

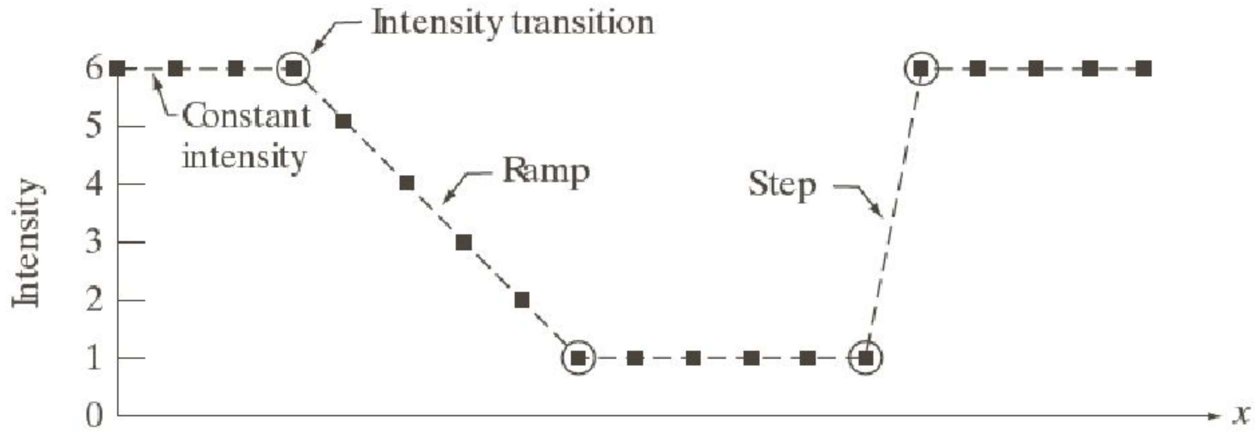
Sharpening Spatial Filters: **Foundation**

- The first-order derivative of a one-dimensional function f(x) is the difference

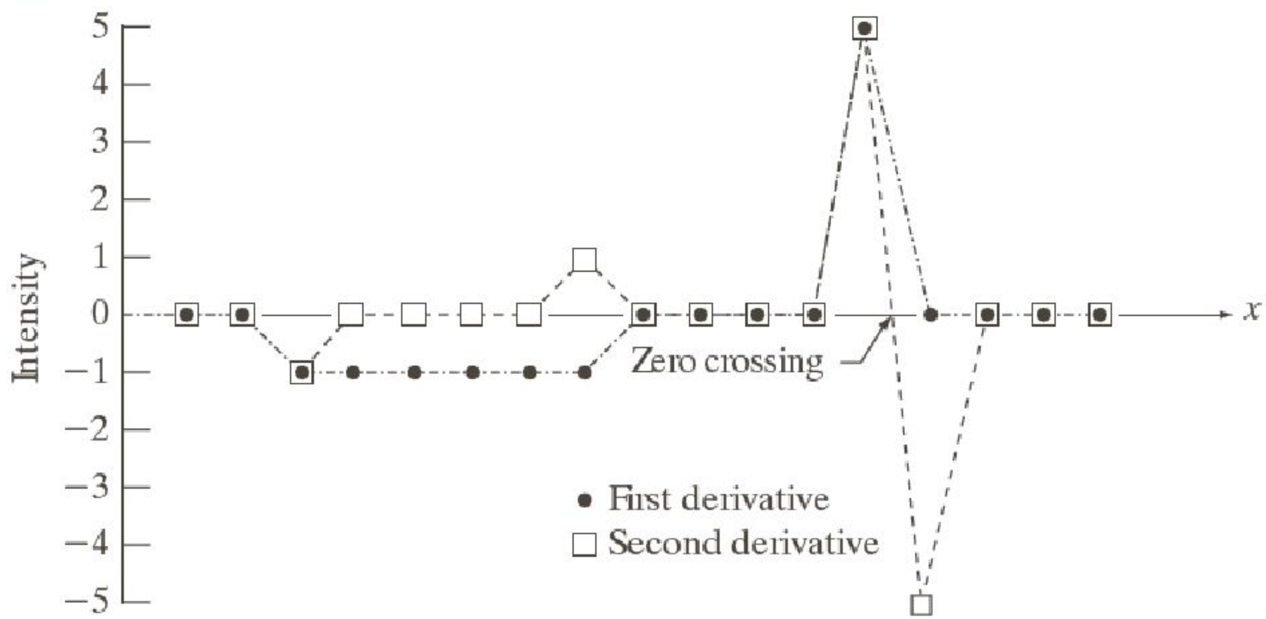
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative of f(x) as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Image Enhancement in Spatial Doma

Sharpening Spatial Filters: **Laplacian Operator**

The second-order isotropic derivative operator is the Laplacian for a function (image) $\boldsymbol{f(x,y)}$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Image Enhancement in Spatial Doma

Sharpening Spatial Filters: **Laplacian Operator**

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Image Enhancement in Spatial Doma

Sharpening Spatial Filters: **Laplacian Operator**

Image sharpening in the way of using the Laplacian:

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

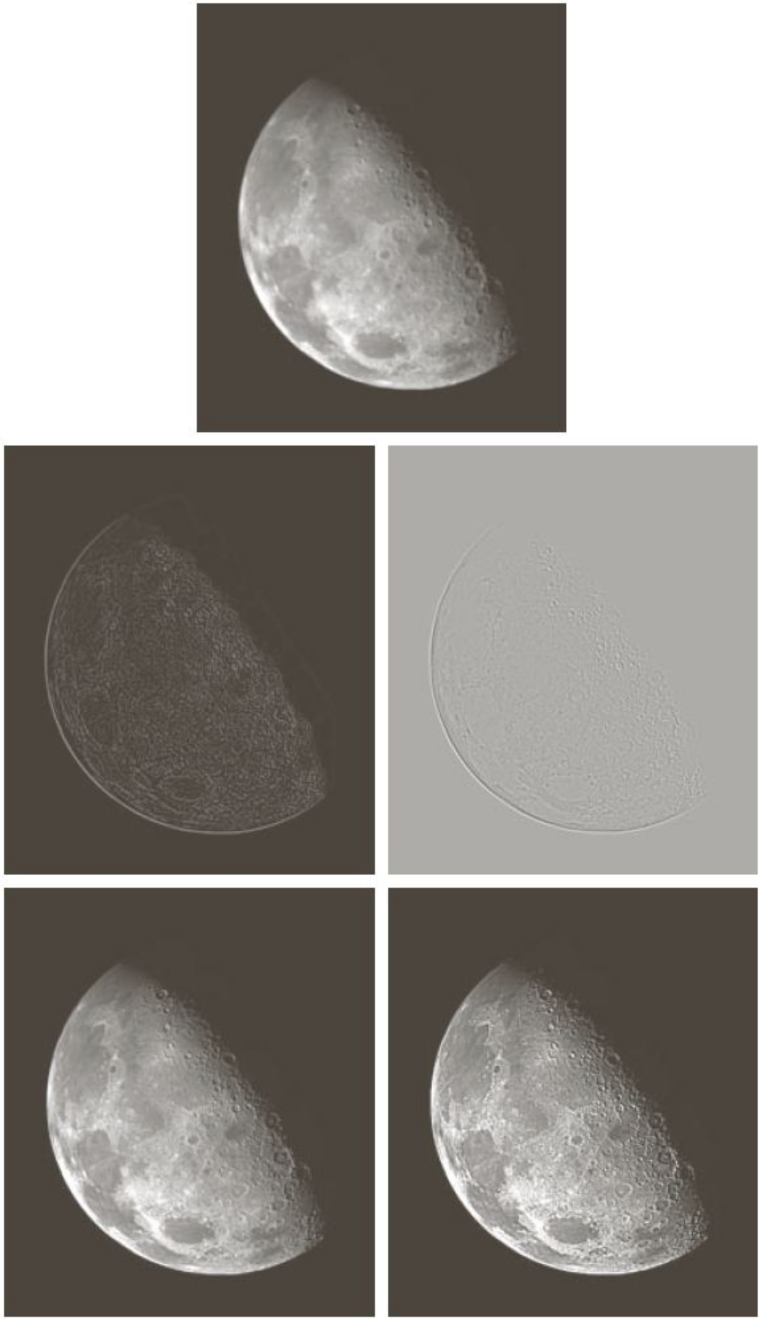
where,

$f(x,y)$ is input image,

$g(x,y)$ is sharpenend images,

$c = -1$ if $\nabla^2 f(x,y)$ corresponding to Fig. 3.37(a) or (b)

and $c = 1$ if either of the other two filters is used.



a
b c
d e

FIGURE 3.38
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Image Enhancement in Spatial Doma

Unsharp Masking and Highboost Filtering

- **Unsharp masking**

  Sharpen images consists of subtracting an unsharp (smoothed) version of an image from the original image

  e.g., printing and publishing industry

- **Steps**

  I. Blur the original image

  II. Subtract the blurred image from the original

  III. Add the mask to the original

Image Enhancement in Spatial Doma

Unsharp Masking and Highboost Filtering

Let $\overline{f}(x, y)$ denote the blurred image, unsharp masking is

$$g_{mask}(x, y) = f(x, y) - \overline{f}(x, y)$$

Then add a weighted portion of the mask back to the original

$$g(x, y) = f(x, y) + k * g_{mask}(x, y) \qquad k \geq 0$$

when $k > 1$, the process is referred to as highboost filtering.

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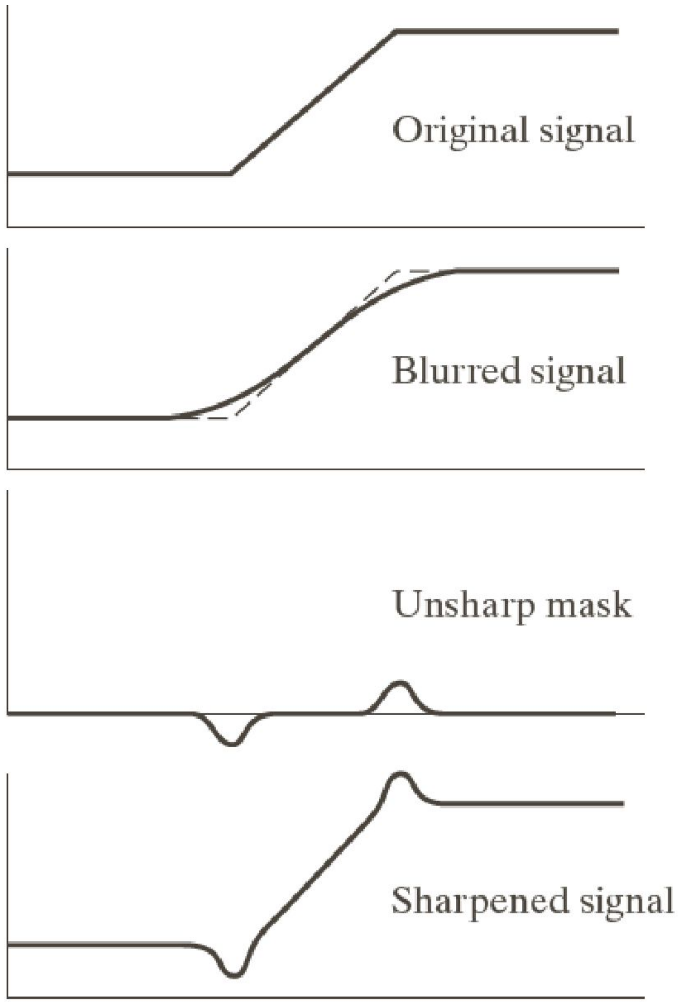
Unsharp Masking and Highboost Filtering



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Image Enhancement in Spatial Domain

Unsharp Masking and Highboost Filtering: **Example**

a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

Image Enhancement in Spatial Doma

Image Sharpening based on First-Order Derivatives

For function $f(x, y)$, the gradient of f at coordinates (x, y) is defined as

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

The *magnitude* of vector ∇f, denoted as $M(x, y)$

Gradient Image $M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x{}^2 + g_y{}^2}$

Image Enhancement in Spatial Doma

Image Sharpening based on First-Order Derivatives

The *magnitude* of vector ∇f, denoted as $M(x, y)$

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x{}^2 + g_y{}^2}$$

$$M(x, y) \approx |g_x| + |g_y|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$M(x, y) = |z_8 - z_5| + |z_6 - z_5|$$

Image Sharpening based on First-Order Derivatives

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

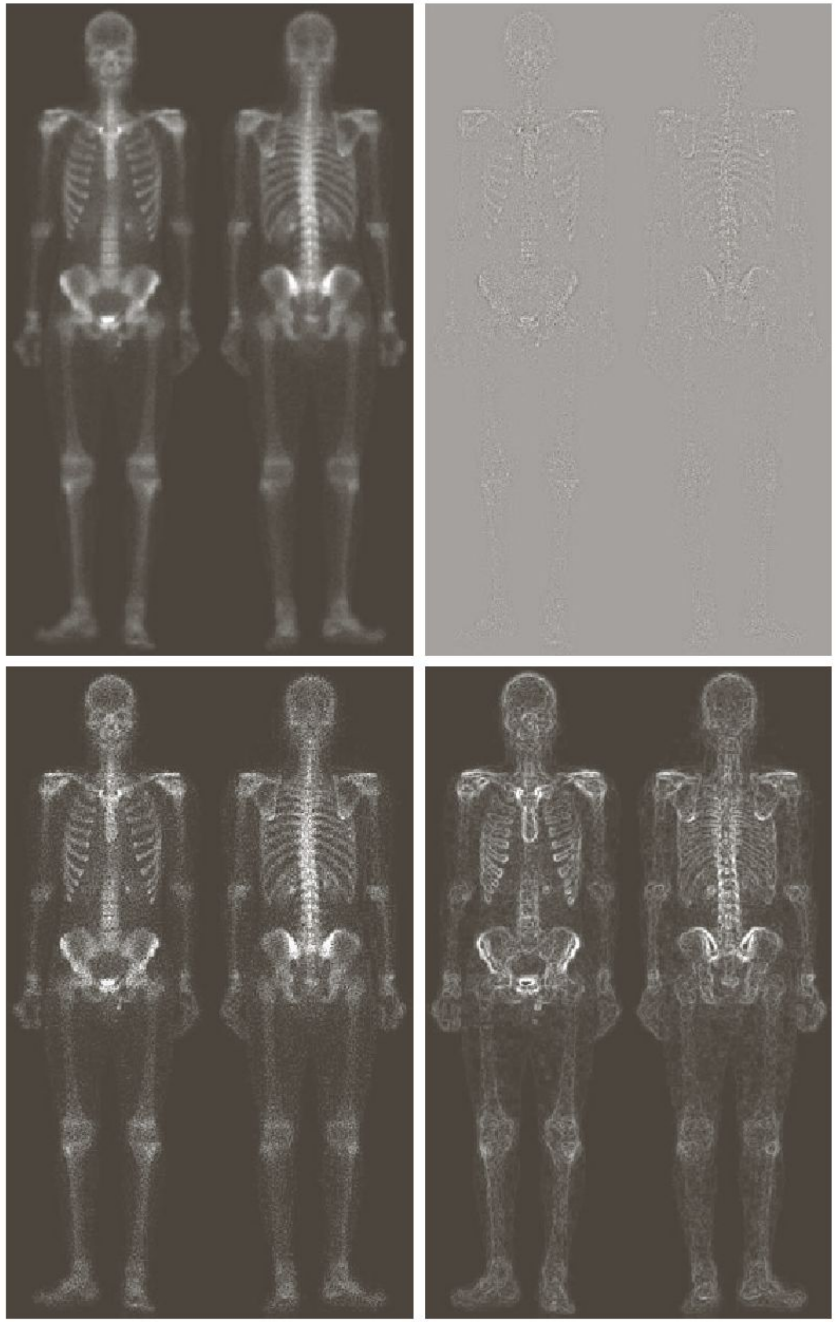
a
b c
d e

FIGURE 3.41 A 3×3 region of an image (the zs are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

Example

Combining Spatial Enhancement Methods

Goal:
Enhance the image by sharpening it and by bringing out more of the skeletal detail



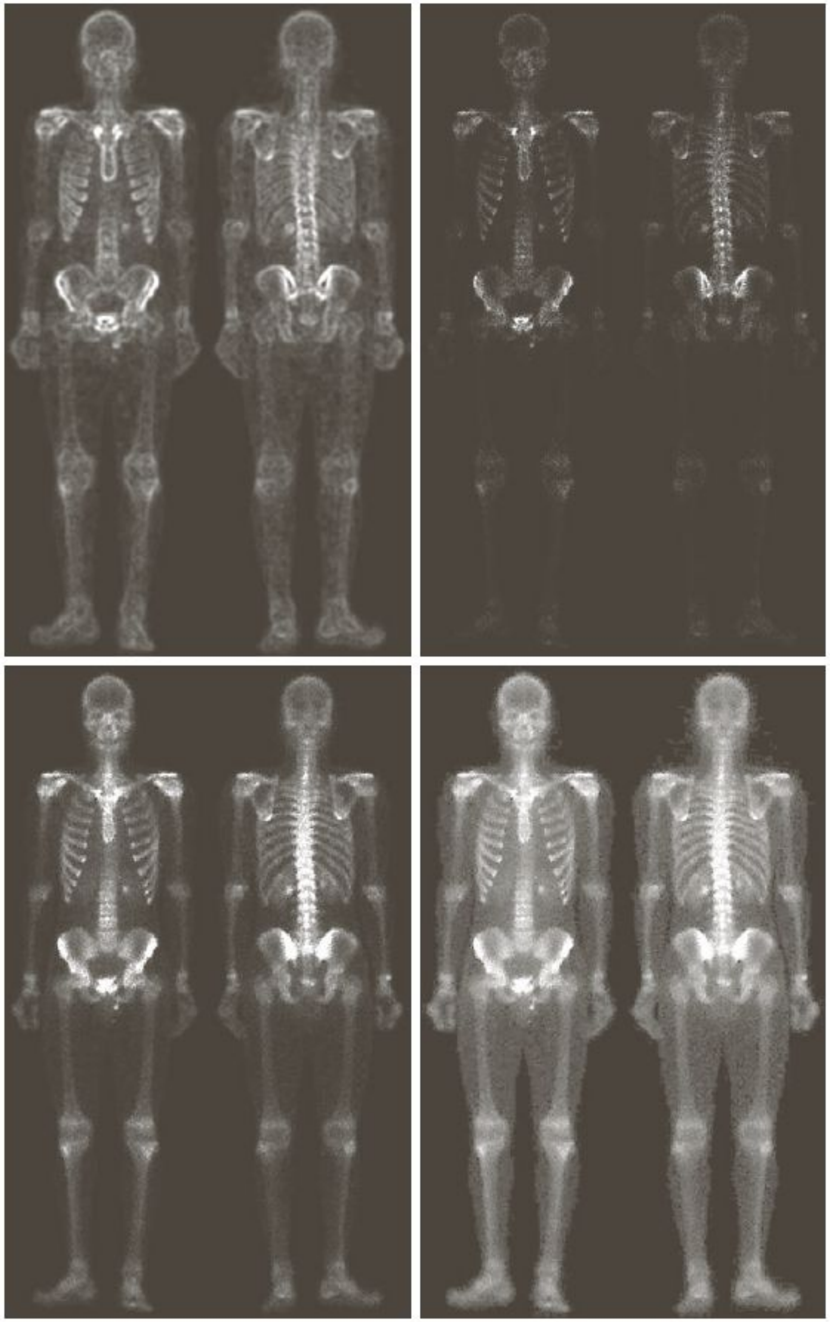
a b
c d

FIGURE 3.43
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Example

Combining Spatial Enhancement Methods

Goal:
Enhance the image by sharpening it and by bringing out more of the skeletal detail



e	f
g	h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Next Class

- **Image Enhancement in Frequency Domain**

Thank you:
Question?