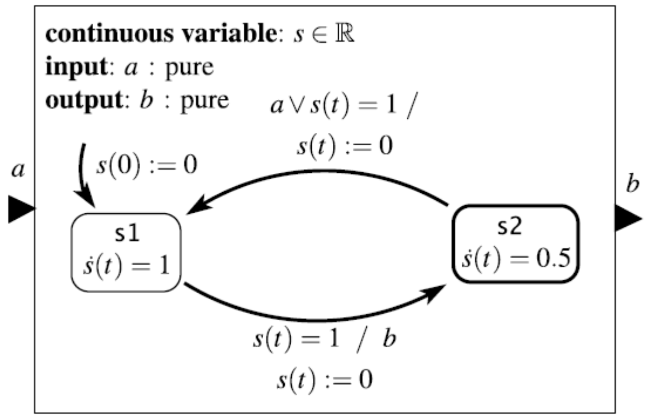
Chapter 4 Hybrid Systems

4. Consider the following timed automaton:



Assume that the input signals a and b are discrete continuous-time signals, meaning that each can be given as a function of form $a\colon \mathbb{R} \to \{present, absent\}$, where at almost all times $t \in \mathbb{R}$, $a(t) = absent$. Assume that the state machine can take at most one transition at each distinct time t, and that machine begins executing at time $t = 0$.

(a) Sketch the output b if the input a is present only at times

$$t = 0.75, 1.5, 2.25, 3, 3.75, 4.5, \cdots$$

Include at least times from $t = 0$ to $t = 5$.

Solution: The output is present at times $t = 1, 2.5, 4, \cdots$.

(b) Sketch the output b if the input a is present only at times $t = 0, 1, 2, 3, \cdots$.

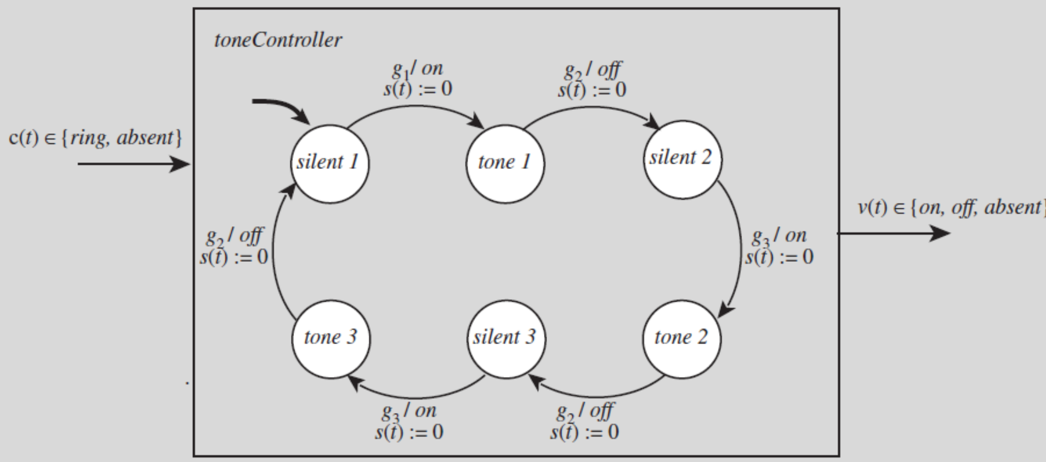
Solution: The output is present at times $t = 1, 3, 5, 7, \cdots$.

(c) Assuming that the input a can be any discrete signal at all, find a lower bound on the amount of time between events b. What input signal a (if any) achieves this lower bound?

Solution: The lower bound is 1. There is no input a that achieves this bound, but an input that comes arbitrarily close is where a is present at times $t = 1+\varepsilon, 2+2\varepsilon, 3+3\varepsilon, \cdots$, for any $\varepsilon > 0$.

5. You have an analog source that produces a pure tone. You can switch the source on or off by the input event *on* or *off*. Construct a timed automaton that provides the *on* and *off* signals as outputs, to be connected to the inputs of the tone generator. Your system should behave as follows. Upon receiving an input event *ring*, it should produce an 80 ms-long sound consisting of three 20 ms-long bursts of the pure tone separated by two 10 ms intervals of silence. What does your system do if it receives two *ring* events that are 50 ms apart?

Solution: Assume the input alphabet is $\{ring, absent\}$ and the output alphabet is $\{on, off, absent\}$. Then the following timed automaton will control the source of the tone:



All states except *silent 1* have as a refinement the system given by

$$\dot{s}(t) = 1.$$

The guards are given by

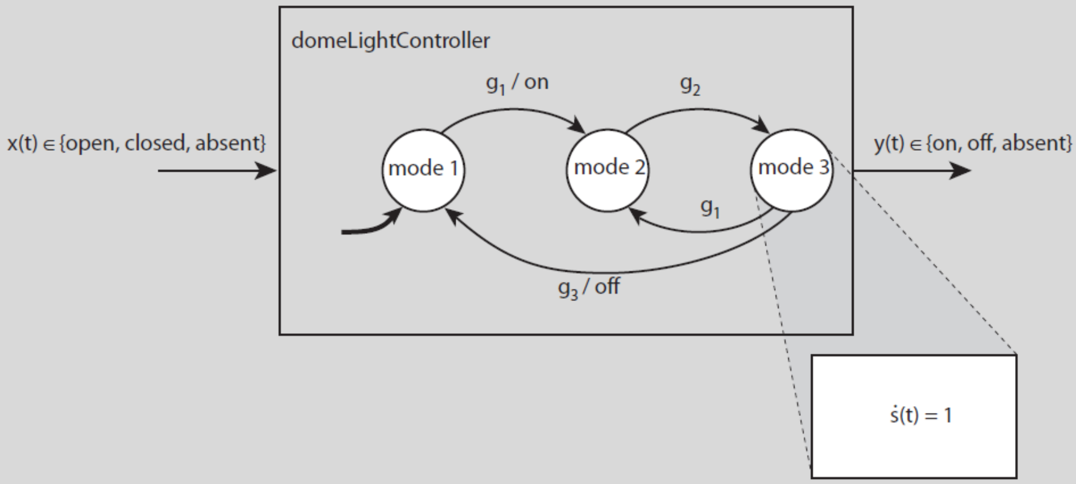
$$\begin{aligned} g_1 &= \{(c(t), s(t)) \mid c(t) = ring\} \\ g_2 &= \{(c(t), s(t)) \mid s(t) = 20\} \\ g_3 &= \{(c(t), s(t)) \mid s(t) = 10\}. \end{aligned}$$

This system ignores a *ring* input event that occurs less than 80 ms after the previous *ring* event.

6. Automobiles today have the features listed below. Implement each feature as a timed automaton.

(a) The dome light is turned on as soon as any door is opened. It stays on for 30 seconds after all doors are shut. What sensors are needed?

Solution: Assume that the automobile provides the input *open* when the first door is opened and *closed* when the last open door is closed. The following machine provides *on* to turn on the dome light and *off* to turn it off:



The guards are given by

$$
\begin{aligned}
g_1 &= \{(x(t), s(t)) \mid x(t) = open\} \\
g_2 &= \{(x(t), s(t)) \mid x(t) = closed\} \\
g_3 &= \{(x(t), s(t)) \mid s(t) = 30\}.
\end{aligned}
$$

Such sensors can be easily implemented as a **daisy chain**, a series connection of switches so that if any door is open, the electrical circuit is open.

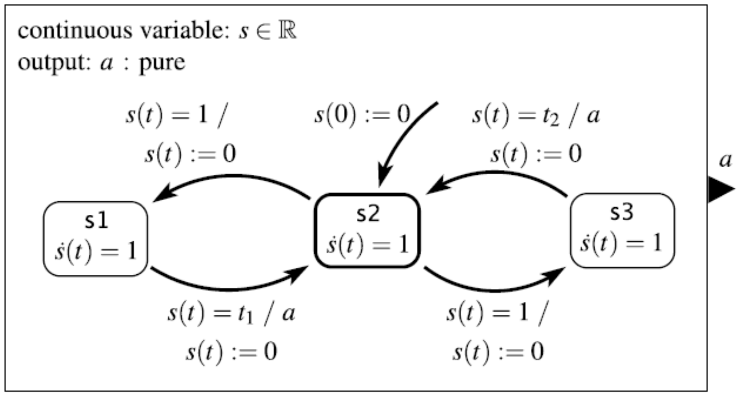
(b) Once the engine is started, a beeper is sounded and a red light warning is indicated if there are passengers that have not buckled their seat belt. The beeper stops sounding after 30 seconds, or as soon the seat belts are buckled, whichever is sooner. The warning light is on all the time the seat belt is unbuckled. **Hint:** Assume the sensors provide a *warn* event when the ignition is turned on and there is a seat with passenger not buckled in, or if the ignition is already on and a passenger sits in a seat without buckling the seatbelt. Assume further that the sensors provide a *noWarn* event when a passenger departs from a seat, or when the buckle is buckled, or when the ignition is turned off.

Solution: Assume that the vehicle sensors provide the following input alphabet,

$$Inputs = \{warn, noWarn, absent\},$$

as suggested in the hint. The following model provides the requisite control:

8. Consider the following timed automaton:



Assume t_1 and t_2 are positive real numbers. What is the minimum amount of time between events a? That is, what is the smallest possible time between two times when the signal a is present?

Solution: $1 + \min(t_1, t_2)$.