

$$X = [x_1 \ x_2 \ x_3]^T. \quad X_0 = (0 \ 0 \ 0)^T$$

$$f(x) = 0.5x_1^2 + x_2^2 + 1.5x_3^2 + x_1 + 2x_2 + 3x_3 + 2$$

$$\Delta f(x) = (x_1 + 1 \quad 2x_2 + 2 \quad 3x_3 + 3)^T$$

$$\Delta f(x_0) = (1 \ 2 \ 3)^T$$

Newton Descent:

$$\Rightarrow X_1 = x_0 - \eta * \Delta f(x_0)$$

$$\Rightarrow X_1 = (0 \ 0 \ 0)^T - 0.1 * (1 \ 2 \ 3)^T = -0.1 * (0.1 \ 0.2 \ 0.3)^T = (-0.1 \ -0.2 \ -0.3)^T$$

Gradient Descent:

$$\Rightarrow X_1 = x_0 - H^{-1} * \Delta f(x_0)$$

$$\Rightarrow X_1 = x_0 - H^{-1} * \left(\frac{\partial f}{\partial x_1} \ \frac{\partial f}{\partial x_2} \ \frac{\partial f}{\partial x_3} \right)^T$$

$$\Rightarrow \text{Hessian matrix} = H =$$

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$H^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} * \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$