DIGITAL IMAGE PROCESSING

Image Enhancement in Frequency Domain: Session 2

Dr. Mrinmoy Ghorai

Indian Institute of Information Technology
Sri City, Andhra Pradesh

Today's Lecture

- **Image Enhancement in Frequency Domain**
  - **Filtering in Frequency Domain**

Image Enhancement in Frequency Domai

Properties of 2-D DFT: **Fourier Spectrum and Phase Angle**

2-D DFT in polar form

$$F(u,v) = |F(u,v)|\, e^{j\phi(u,v)}$$

Fourier spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

Power spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

Phase angle

$$\phi(\mathrm{u,v}) = \arctan\left[\frac{I(u,v)}{R(u,v)}\right]$$

Image Enhancement in Frequency Domain

Filtering in Frequency Domain: Steps

1. Multiply the input image by $-1^{(x+y)}$ to center the transform
2. Compute $F(u, v)$, the DFT of the image
3. Multiply $F(u, v)$ by a filter function $\mathrm{H}(u, v)$
4. Compute the inverse DFT of the result in (3)
5. Obtain the real part of the result in (4)
6. Multiply the result in (5) by $-1^{(x+y)}$

Image Enhancement in Frequency Domain

Filtering in Frequency Domain

- Let $f(x,y)$ is the input image and $F(u,v)$ its Fourier transform. Then the Fourier transform the output image is given by

$$G(u,v) = H(u,v)F(u,v)$$

- Computing the inverse transform to obtain the processed result

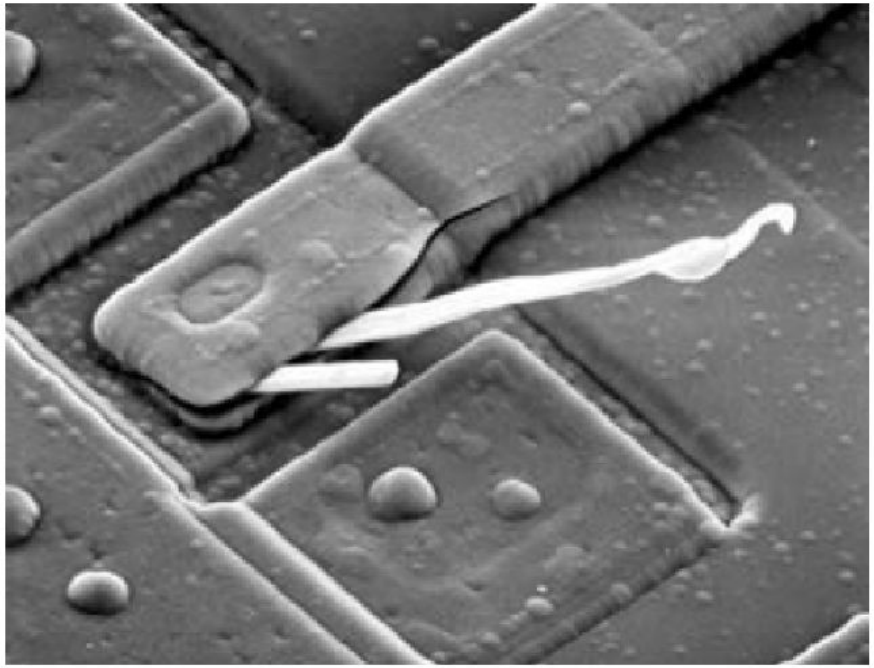
$$g(x,y) = \Im^{-1}\{H(u,v)F(u,v)\}$$

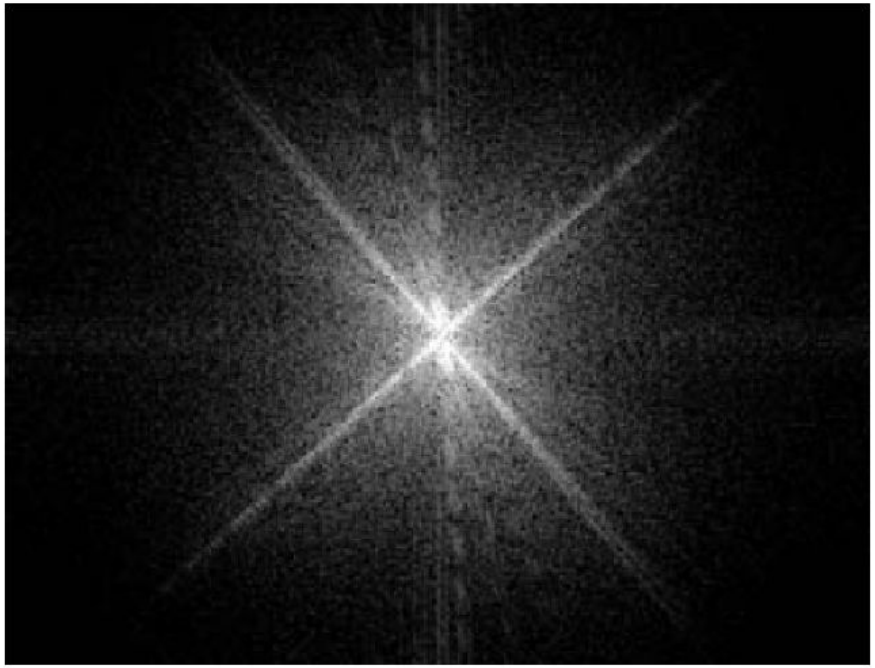
$F(u,v)$ is the DFT of the input image

$H(u,v)$ is a filter function.

Image Enhancement in Frequency Domain

Filtering in Frequency Domain





a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Image Enhancement in Frequency Domain

Filtering in Frequency Domain: **Notch Filter**

In a filter H(u,v) that is 0 at the center of the transform and 1 elsewhere, what's the output image?

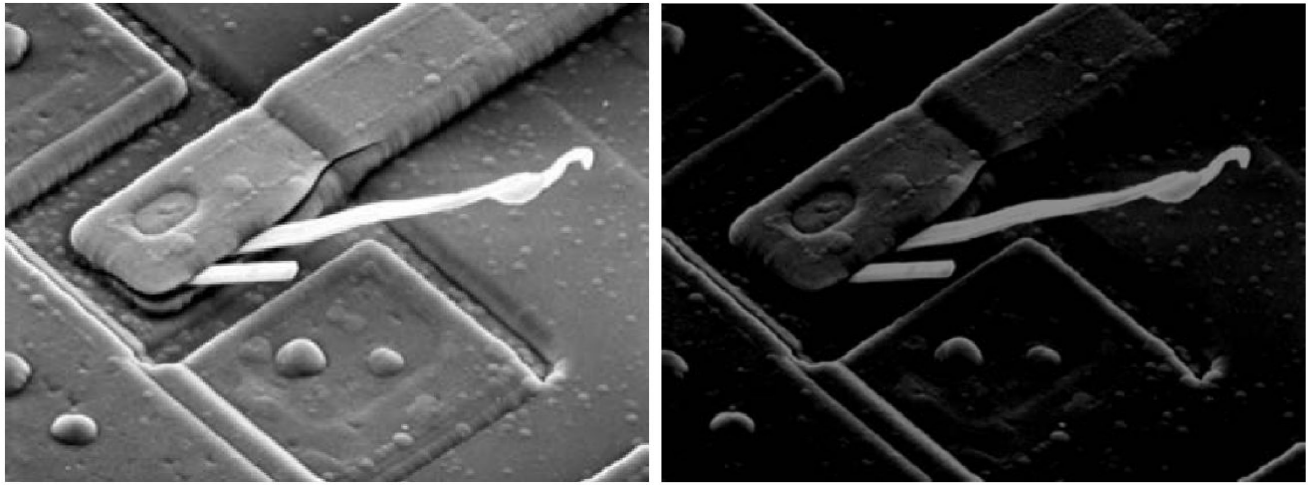


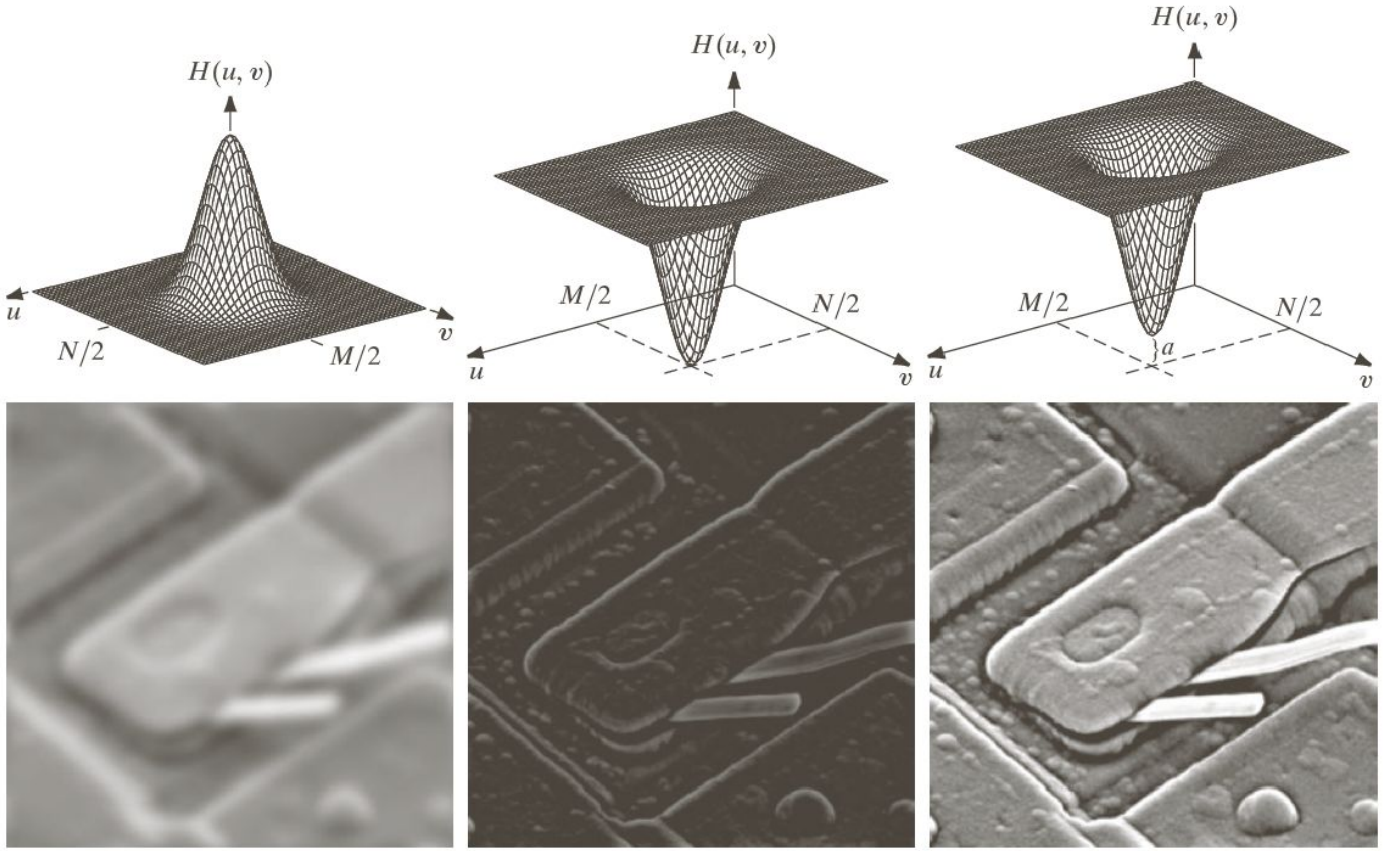
Image Enhancement in Frequency Domain

Filtering in Frequency Domain: **Properties**

- Low frequencies in the Fourier transform are responsible for the general gray-level appearance of an image over smooth regions.
- High frequencies are responsible for detail, such as edges and noise.
- A filter that attenuates high frequencies while “passing” the low frequencies is called *lowpass filter.*
- A filter that attenuates low frequencies while “passing” the high frequencies is called *highpass filter.*

Image Enhancement in Frequency Domain

Filtering in Frequency Domain



a b c
d e f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Image Enhancement in Frequency Domain

2-D Convolution Theorem

1-D convolution

$$f(x)\star h(x) = \sum_{m=0}^{M-1} f(m)h(x-m)$$

2-D convolution

$$f(x,y)\star h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m,y-n)$$

$$x = 0,1,2,...,M-1; y = 0,1,2,...,N-1.$$

$$f(x,y)\star h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow F(u,v)\star H(u,v)$$

Next Class

- Image Enhancement in Frequency Domain
  - Filtering in Frequency Domain

Thank you:
Question?