DIGITAL IMAGE PROCESSING

Image Restoration : Session 2

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Today's Lecture

- Image Restoration
  - Spatial Filtering
  - Adaptive Filtering
  - Periodic Noise Reduction

Image Restoration

Spatial Filtering: Order-Statistic Filters(1)

Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Max filter

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

Min filter

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

Spatial Filtering: Order-Statistic Filters(2)

Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

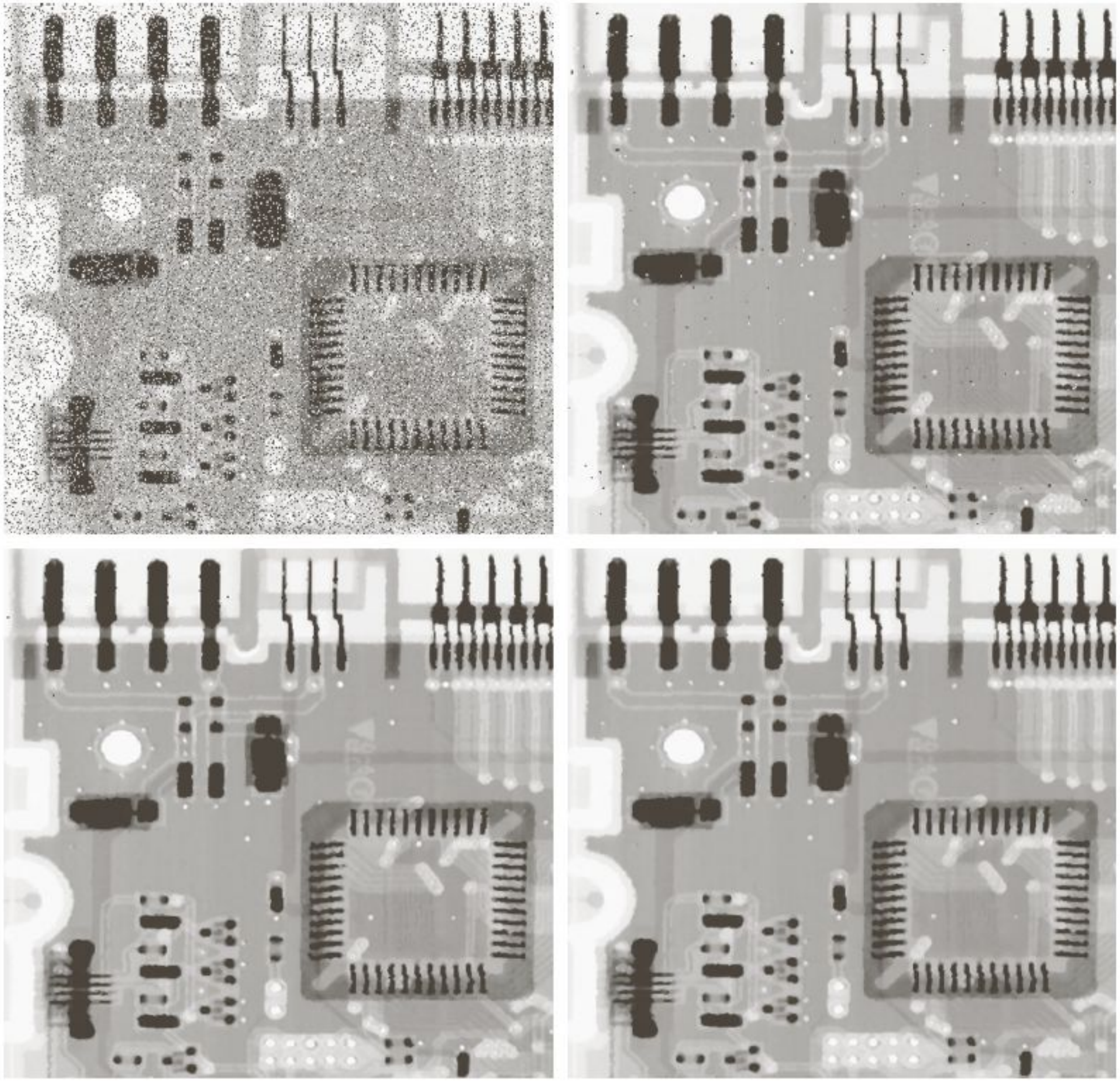
Spatial Filtering: Order-Statistic Filters(3)

Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} \{g_r(s,t)\}$$

We delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy}. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels.

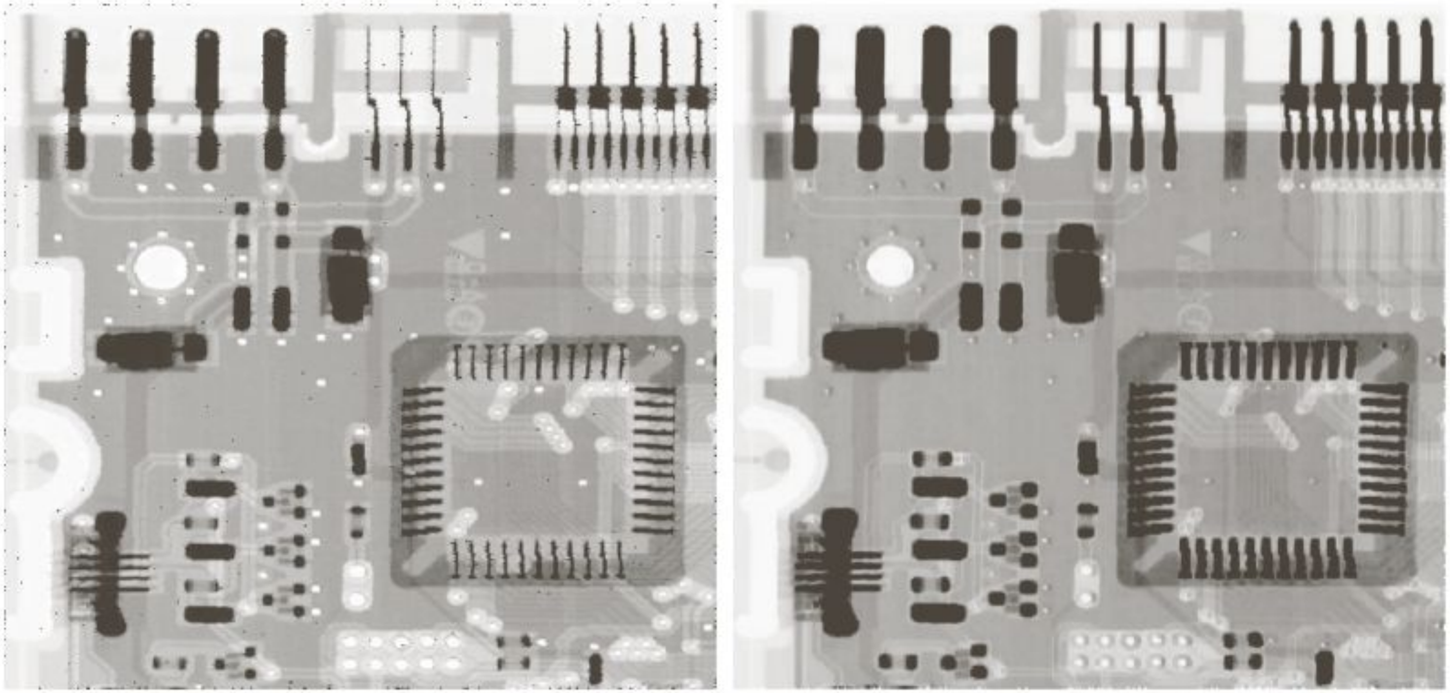
Spatial Filtering: Example(1)



a b
c d

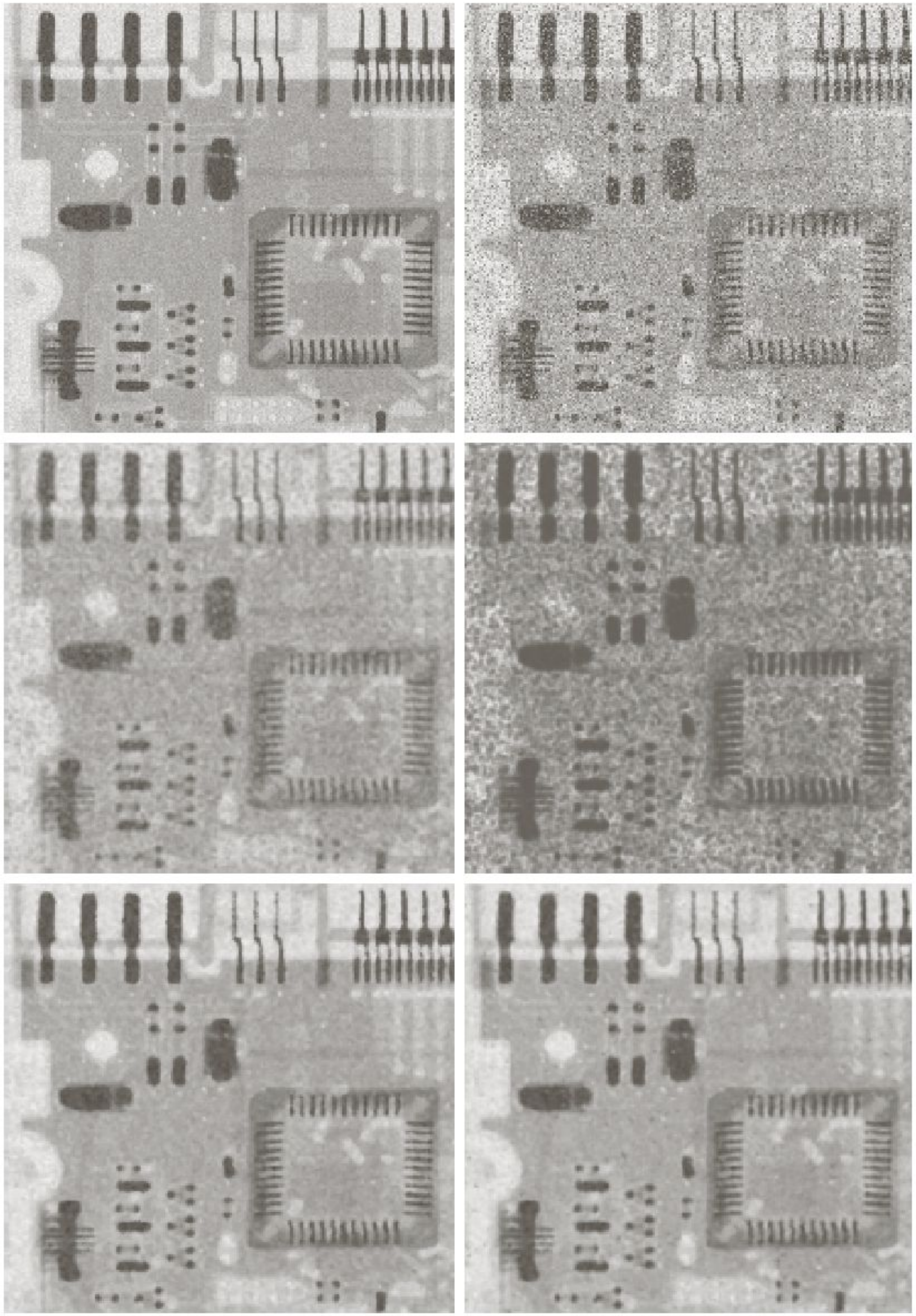
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Spatial Filtering: Example(1)



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



a	b
c	d
e	f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Spatial Filtering: Order-Statistic Filters(3)

Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} \{g_r(s,t)\}$$

We delete the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy}. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels.

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Spatial Filtering: Adaptive Filters

- The behavior changes based on statistical characteristics of the image inside the filter region defined by the mxn rectangular window.

- The performance is superior to that of the filters discussed.

Adaptive Filtering: Adaptive, Local Noise Reduction Filters

S_{xy}: local region

The response of the filter at the center point (x,y) of S_{xy} is based on four quantities:

(a) $g(x, y)$, the value of the noisy image at (x, y);

(b) σ_η^2, the variance of the noise corrupting $f(x, y)$ to form $g(x, y)$;

(c) m_L, the local mean of the pixels in S_{xy};

(d) σ_L^2, the local variance of the pixels in S_{xy}.

Adaptive Filtering: Adaptive, Local Noise Reduction Filters

The behavior of the filter:

(a) if σ_η^2 is zero, the filter should return simply the value of $g(x, y)$.

(b) if the local variance is high relative to σ_η^2, the filter should return a value close to $g(x, y)$;

(c) if the two variances are equal, the filter returns the arithmetic mean value of the pixels in S_{xy}.

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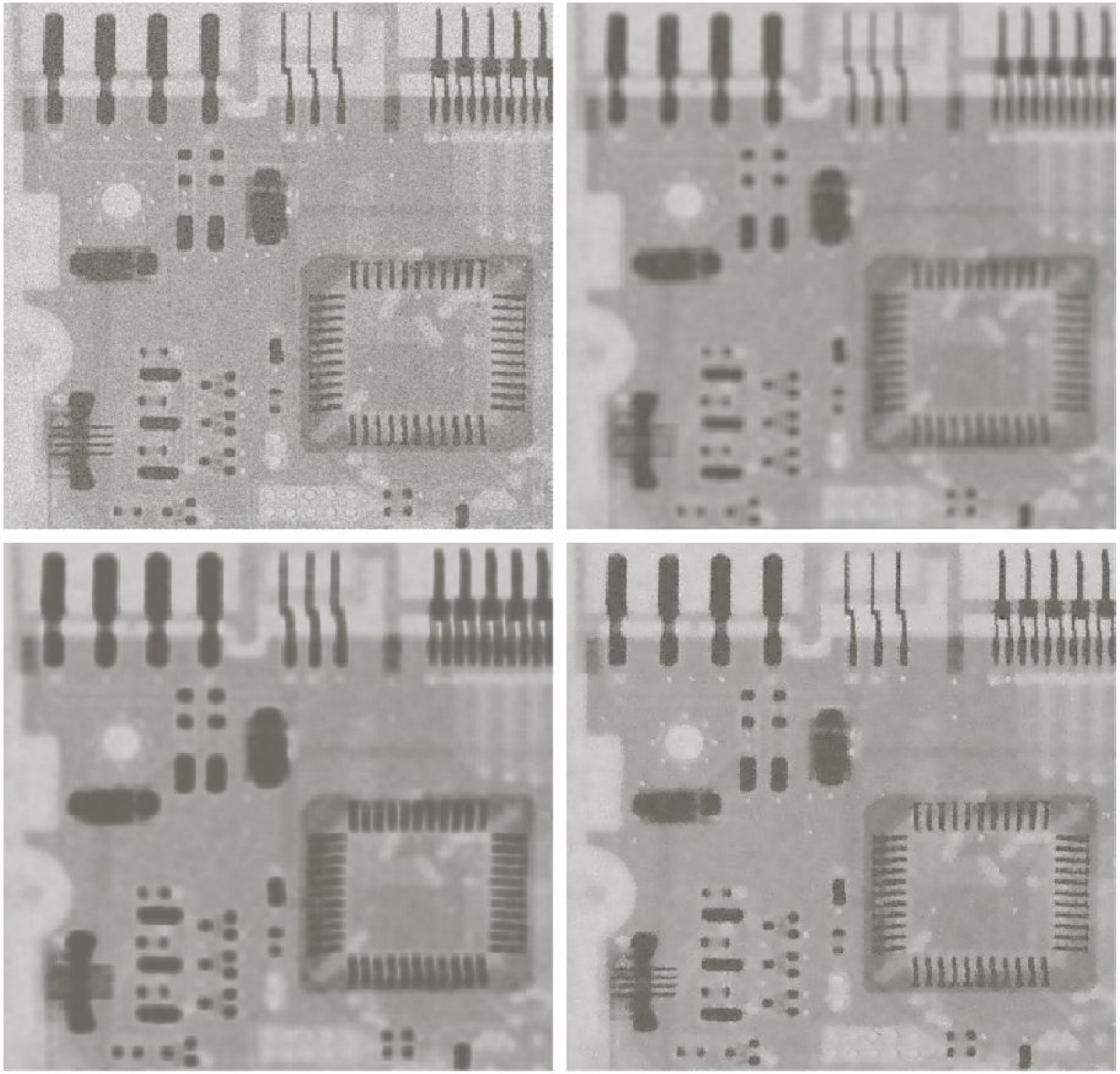
Adaptive Filtering: Adaptive, Local Noise Reduction Filters

An adaptive expression for obtaining $\hat{f}(x, y)$ based on the assumptions:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L \right]$$



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive Filtering: Adaptive Median Filters

The notation:

$$z_{\min} = \text{minimum intensity value in } S_{xy}$$

$$z_{\max} = \text{maximum intensity value in } S_{xy}$$

$$z_{\text{med}} = \text{median intensity value in } S_{xy}$$

$$z_{xy} = \text{ intensity value at coordinates } (x, y)$$

$$S_{\max} = \text{maximum allowed size of } S_{xy}$$

Adaptive Filtering: Adaptive Median Filters

The adaptive median-filtering works in two stages:

Stage A:

$$A1 = z_{\text{med}} - z_{\min}; \qquad A2 = z_{\text{med}} - z_{\max}$$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{\max}$, repeat stage A; Else output z_{med}

Stage B:

$$B1 = z_{xy} - z_{\min}; \qquad B2 = z_{xy} - z_{\max}$$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

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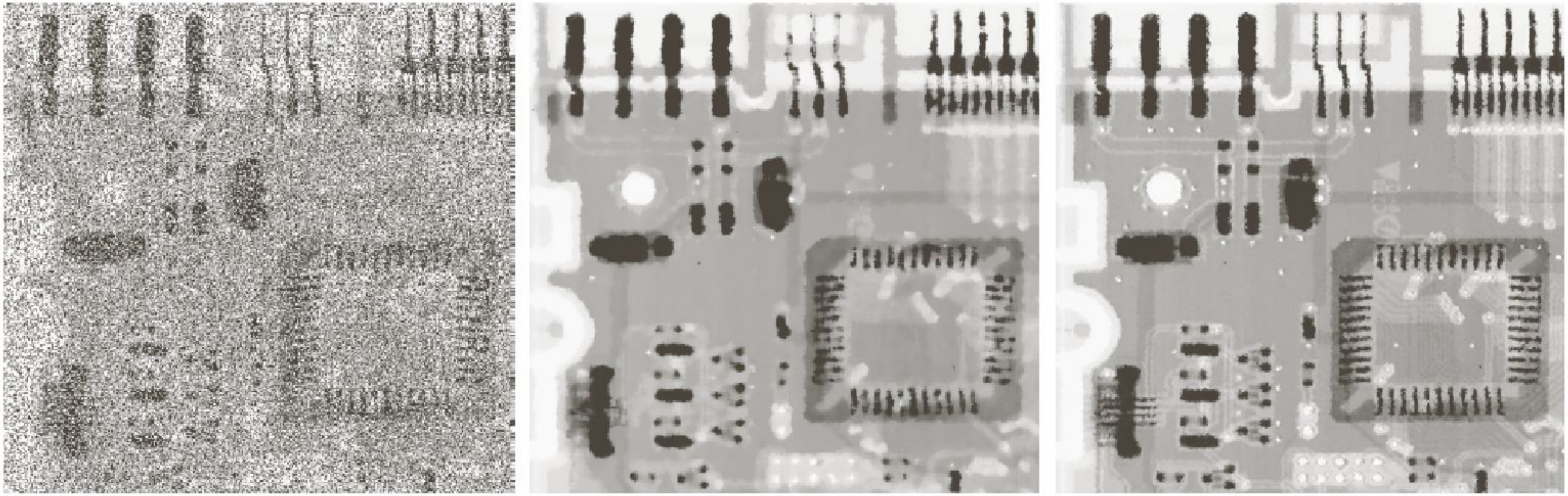
Example: Adaptive Median Filters



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Periodic Noise Reduction by Frequency Domain Filtering

The basic idea

Periodic noise appears as concentrated bursts of energy in the Fourier transform, at locations corresponding to the frequencies of the periodic interference

Approach

A selective filter is used to isolate the noise

Non-Selective Filters

Operate over the entire frequency rectangle

Selective Filters

- operate over some part, not entire frequency rectangle
- **bandreject or bandpass**: process specific bands
- **notch filters**: process small regions of the frequency rectangle

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Selective Filtering: Bandreject and Bandpass Filters

TABLE 4.6

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

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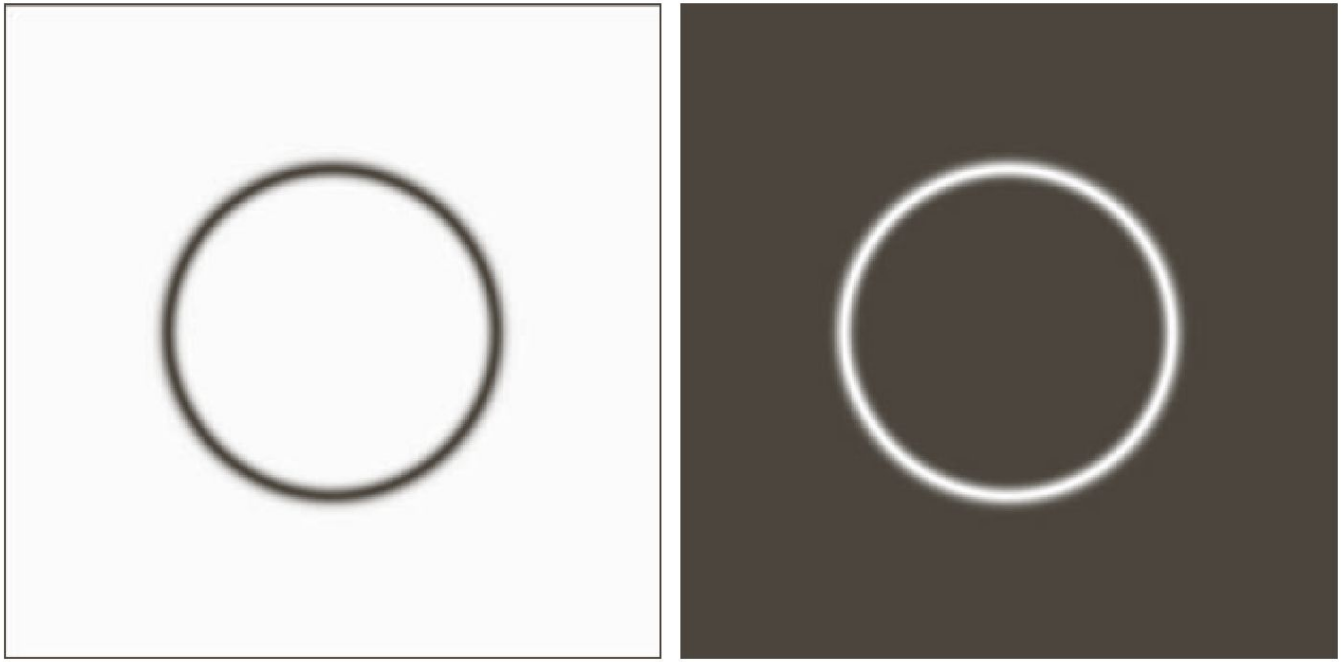
Selective Filtering: Bandreject and Bandpass Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

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Perspective Plots of Bandreject Filters

a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Image Restoration

Result of Filtering

a b
c d

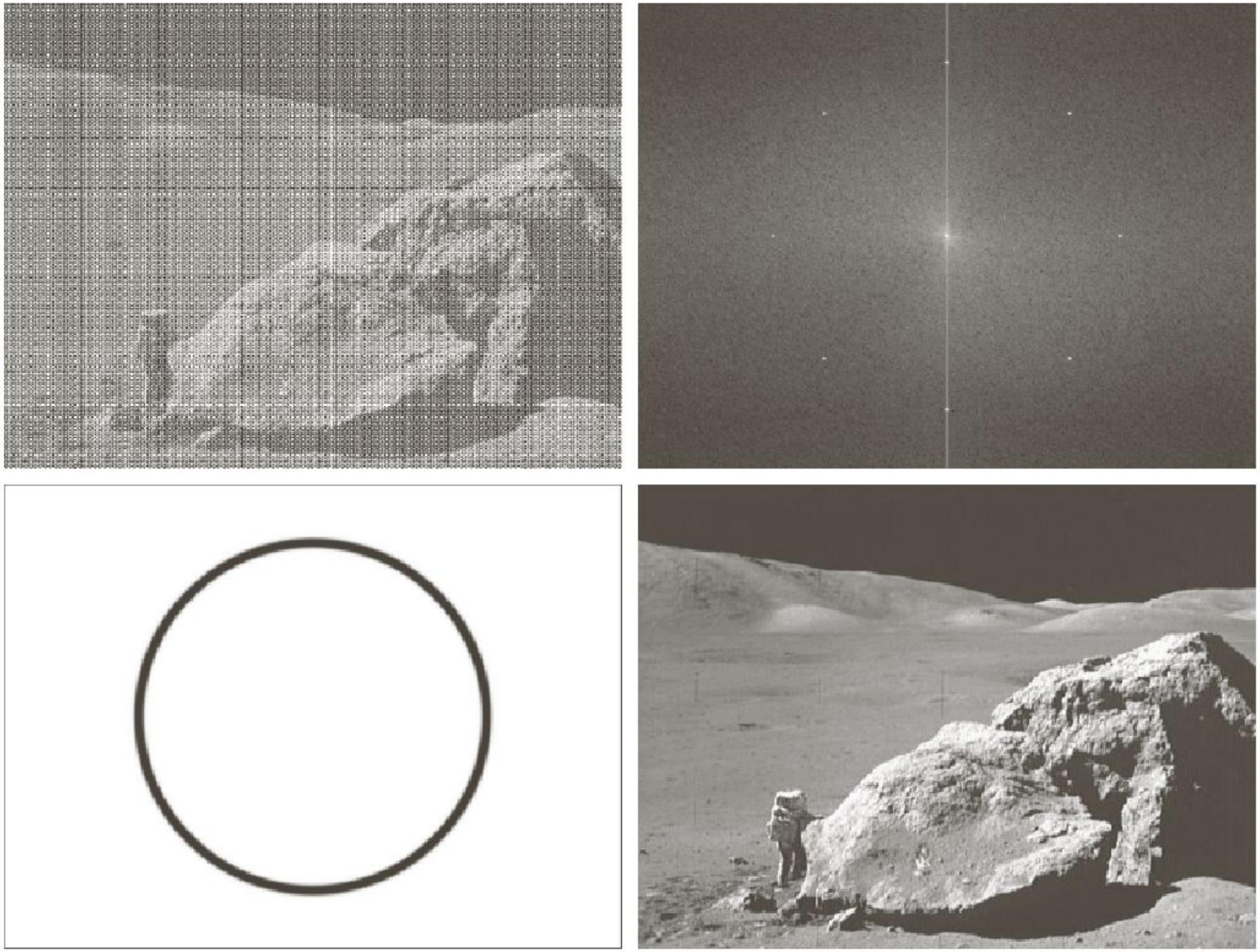

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

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Perspective Plots of Notch Filters

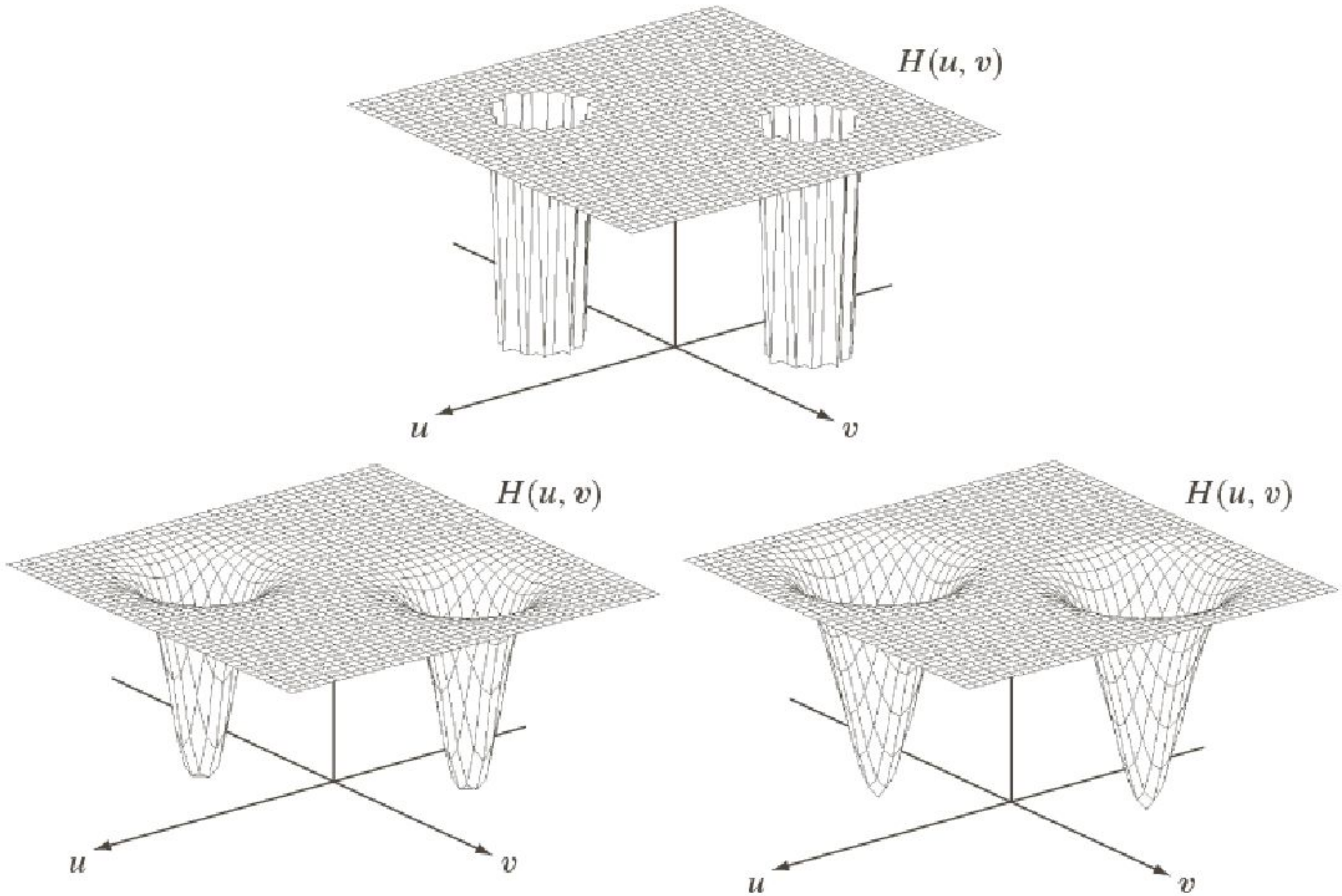


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

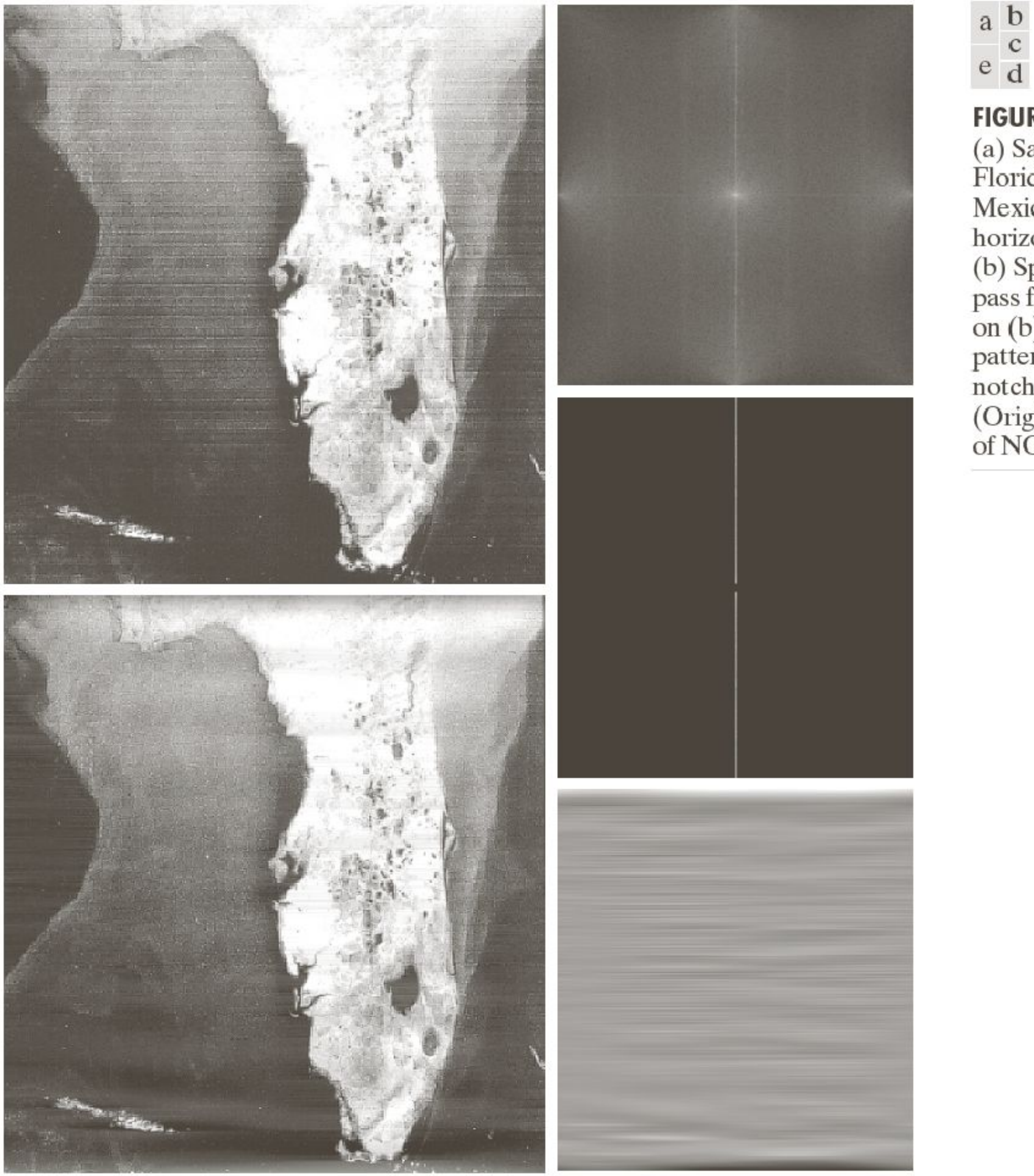
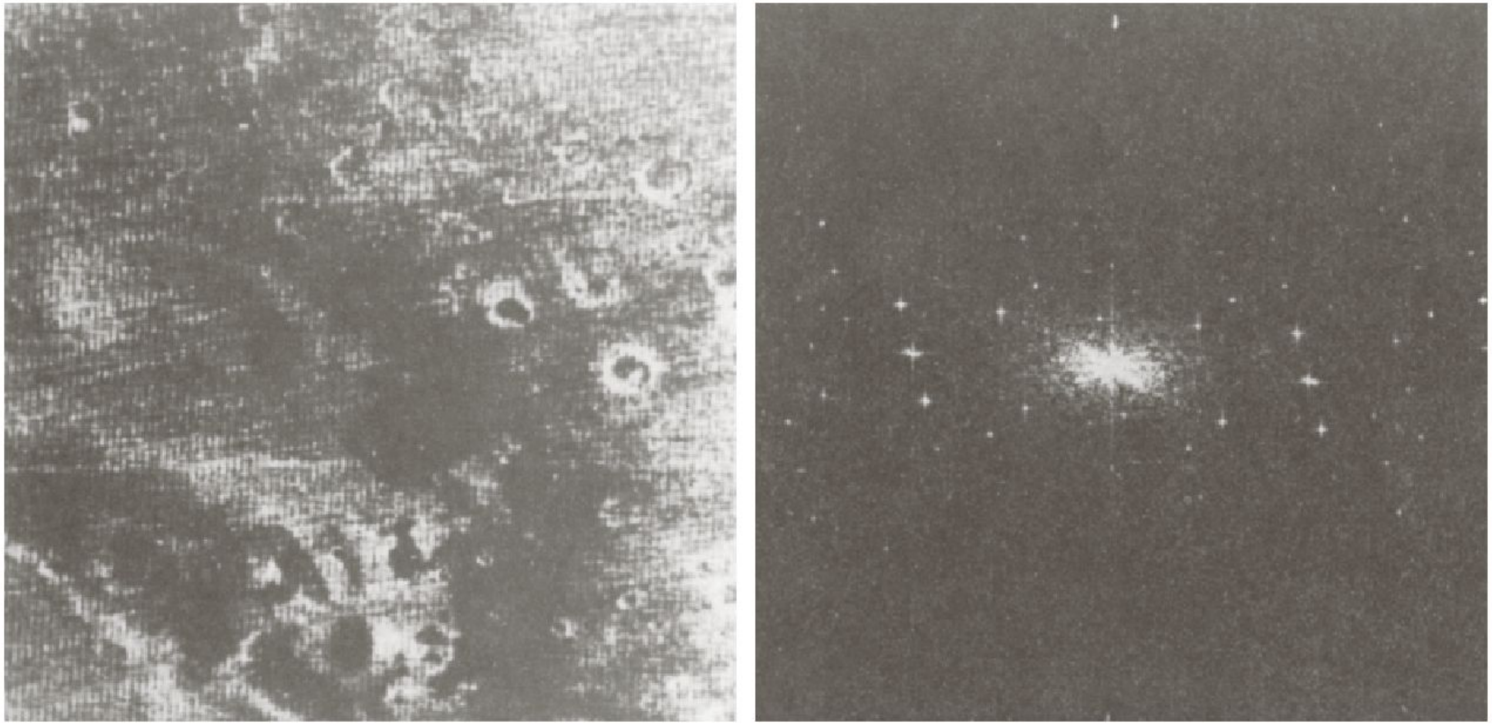


FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Image Restoration



a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

Next Class

- Image Restoration
  - More Filters

Thank you:
Question?