IIIT Sri City.ToC -- Quiz-3;

For each statement find out whether it is **True** or **False**. Write your answer in the given space only. Correct answer will get +1 mark, wrong answer will get –0.75 mark.

1. $\overline{HALT_{TM}} \in RE$
2. **The language consisting of prime numbers is in** R.
3. $(L \in RE) \Rightarrow (L \in R)$.
4. $(L \in R) \Rightarrow (L \in RE)$.
5. $\overline{A}_{TM} \leq_m E_{TM}$
6. $(L_1 \in R \text{ and } L_2 \in RE - R) \Rightarrow (L_2 - L_1$ **is in** $RE)$.
7. $(L_3 \in RE - R) \Rightarrow (\overline{L_3}$ **is in** $RE)$.
8. $(L_1 \in R \text{ and } L_3 \in RE - R) \Rightarrow (L_1 - L_3$ **is in** $RE)$.
9. $(L_1 \in R \text{ and } L_2 \in RE - R) \Rightarrow (L_2 \cup L_1$ **is in** $RE)$.
10. RE **is closed under intersection.**
11. $(L_1 \in R \text{ and } L_2 \in RE - R) \Rightarrow (L_2 L_1$ **is in** $R)$.
12. $(L_2$ **not in** $RE) \Rightarrow (\overline{L_2}$ **is not in** $RE)$.
13. $(L_1 \in R) \Rightarrow (\overline{L_1}$ **is not in** $R)$.
14. $(L_4$ **not in** $RE) \Rightarrow (\overline{L_4}$ **is in** $RE)$.
15. $(L \in CFL) \Rightarrow \overline{(L} \in R)$.

Big Answer Questions (2.5 + 2.5 = 5 Marks)

16. Big Answer Question: Prove or disprove: ($A \leq_m B$ and B is regular) ⇒ (A is regular)

17. Big Answer Question: Prove or disprove: ($A \in RE$ and $A \leq_m \overline{A}$) ⇒ (A is undecidable)

Do not write anything in this page. Your answers should be written backside of this sheet.

Write your answers below.

Clearly write **TRUE** or **FALSE** (Do not write T or F or 1 or 0. Do not overwrite.)

Q.No	Answer	Q.No	Answer	Q.No	Answer	Q.No	Answer
1	FALSE	2	TRUE	3	FALSE	4	TRUE
5	TRUE	6	TRUE	7	FALSE	8	FALSE
9	TRUE	10	TRUE	11	FALSE	12	FALSE
13	FALSE	14	FALSE	15	TRUE		

16. No. For example, Let $A = \{0^n 1^n \mid n \geq 0\}$ and $B = \{0^n \mid n \geq 0\}$, here A is context free language (non regular language) and B is regular language. Then $\boldsymbol{A \leq_m B}$, by the mapping $\Sigma^* \longrightarrow \Sigma^*$, where $\sum = \{0, 1\}$. The function defined by,

$$f(s) = \begin{cases} 0\,; \; if \; s = \; 0^n 1^n \; for \; n \; \geq 0 \\ 1; \qquad\qquad\qquad Otherwise \end{cases}$$

17. No. A is decidable.
Given $A \leq_m \bar{A}$ and $A \in R.E$
$A \leq_m \bar{A} \Longrightarrow \bar{A} \leq_m A$

Since $A \in RE$, So $\bar{A} \in R.E$,

Now, we have $A \in R.E \; and \; \bar{A} \in R.E$

$$\Rightarrow A \in R$$

Hence A is decidable.