CGM 2020 | UG4 | 3-08-2020 | Adwait Thattey | S20170010004

Ans

Q1>

$C(t) := (x(t), y(t)) = \cos t + t\sin t,\ \sin t - t\cos t.$

$C'(t) = \frac{dC(t)}{dt} = -\sin t + t\cos t + \sin t,\ \cos t - (t\sin t + \cos t)$

$= t\cos t,\ t\sin t.$

$\|C'(t)\| = \sqrt{(t\cos t)^2 + (t\sin t)^2} = t\sqrt{\cos^2 t + \sin^2 t} = t.$

$\therefore$ speed $= \|C'(t)\| = t.$

arclength $= L_c(t) = \int_a^b t\,dt = \frac{t^2}{2}\Big|_a^b = \frac{1}{2}(b^2 - a^2)$

To obtain unit speed parametrization, divide by $\|C'(t)\|$

$S(t) = \frac{\cos t + t\sin t}{t},\ \frac{\sin t - t\cos t}{t}.$

$C'(s) = \frac{t(t\cos t) - (\cos t + t\sin t)}{t^2},\ \frac{t(t\sin t) - (\sin t - t\cos t)}{t^2}.$

$\|C'(s)\| = \sqrt{\left(\frac{t^2\cos t - \cos t + t\sin t}{t^4}\right)^2 + \left(\frac{t^2\sin t - \sin t + t\cos t}{t^4}\right)^2}$

$= \frac{1}{t^2}\sqrt{(t^2\cos t - \cos t + t\sin t + (-t^2\sin t + \sin t - t\cos t)}$

$* (t^2\cos t - \cos t + t\sin t + t^2\sin t - \sin t + t\cos t)$

$$= \frac{1}{t^2}\sqrt{t^4} = 1.$$

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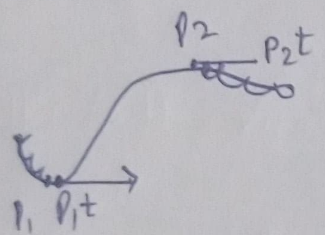
Ex 2

for a given curve



$P(t=0) = P_1$ $\quad P'(t=0) = P_1'$

$P(t=1) = P_2.$ $\quad P'(t=0) = P_2'$

$$P(t) = at^3 + bt^2 + ct + d$$

$$P(0) = a0 + b0 + c0 + d$$

$$P(1) = a.1 + b.1 + c.1 + d$$

$$P'(0) = 3a.0 + 2b.0 + c$$

$$P'(1) = 3a.1 + 2b.1 + c$$

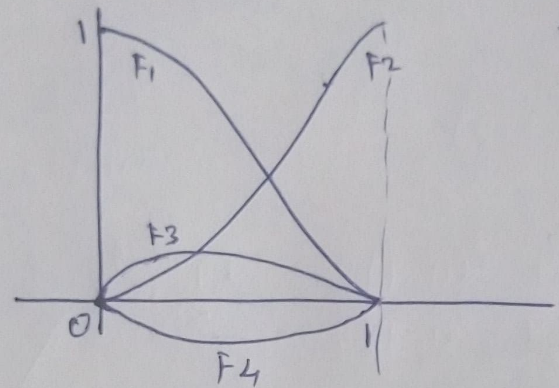
$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_1t \\ P_2t \end{bmatrix}$$

$$P(t) = (t^3\ t^2\ t\ 1) \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{H} \begin{bmatrix} P_1 \\ P_2 \\ P_1t \\ P_2t \end{bmatrix}$$

$F_1(t) = 2t^3 - 3t^2 + 1$ $\quad F_2(t) = -2t^3 + 3t^2$, $\quad F_3(t) = t^3 - 2t^2 + t$

$F_4(t) = t^3 - t^2$.

Sketch



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Sum is: $\cancel{2t^3} - \cancel{3t^2} + 1 - \cancel{2t^3} + \cancel{3t^2} + t^3 - 2t^2 + t$
$+ t^3 - t^2$

Sum $= \sum F_i(t) \neq 1$

$\therefore$ This is not a Barycentric sum

Due to Rotation, the shape of the curve will not change.
& translation