Chapter 14 Equivalence and Refinement

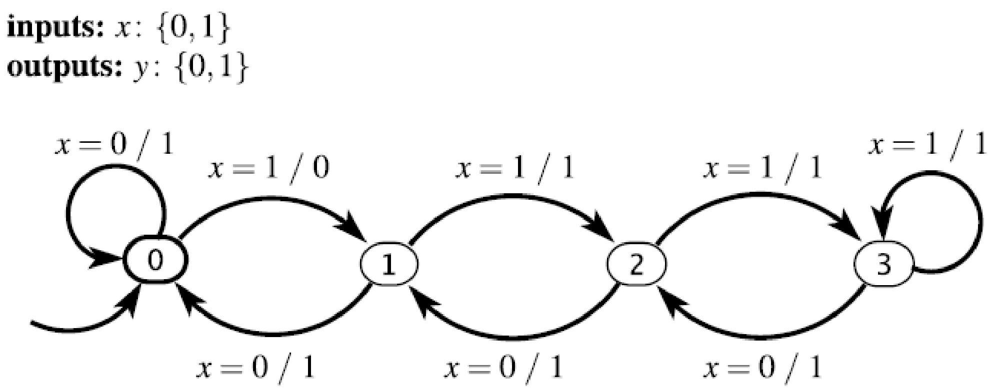
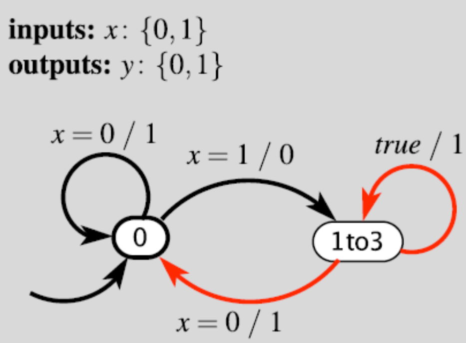


Figure 14.2: Machine that outputs at least one 1 between any two 0's.

3. The state machine in Figure 14.2 has the property that it outputs at least one 1 between any two 0's. Construct a two-state nondeterministic state machine that simulates this one and preserves that property. Give the simulation relation. Are the machines bisimilar?

Solution: The following machine does the job:



As suggested by the state names, 1to3 matches states 1, 2, and 3, while 0 matches 0. Hence, the simulation relation is

$$\{(0,0),(1,\mathsf{1to3}),(2,\mathsf{1to3}),(3,\mathsf{1to3})\}\ .$$

The machines are not bisimilar. The above machine has more observable traces than the one in Figure 14.2.

input: x: pure
output: y: $\{0,1\}$

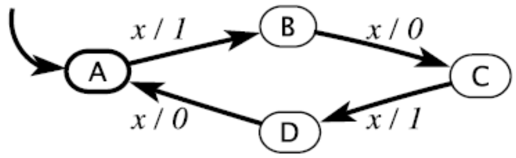
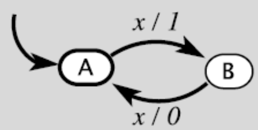


Figure 14.5: A machine that has more states than it needs.

5. Consider the state machine in Figure 14.5. Find a bisimilar state machine with only two states, and give the bisimulation relation.

Solution: A two-state bisimilar machine is shown below:

input: x: pure
output: y: $\{0,1\}$



The bisimulation relation is

$$S = \{(A,A),(B,B),(C,A),(D,B)\},$$

or equivalently,

$$S' = \{(A,A),(B,B),(A,C),(B,D)\},$$

6. You are told that state machine A has one input x, and one output y, both with type $\{1,2\}$, and that it has states $\{a,b,c,d\}$. You are told nothing further. Do you have enough information to construct a state machine B that simulates A? If so, give such a state machine, and the simulation relation.

Solution: Yes, we can give such a machine B. It has one state; let's call it e, with two self loops labeled:

$$true/1$$

$$true/2$$

The simulation relation is

$$S = \{(a,e),(b,e),(c,e),(d,e)\}.$$