# DIGITAL IMAGE PROCESSING

Image Restoration : Session 3

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# Image Restoration

## Optimum Notch Filtering

It minimizes local variances of the restored estimated $\hat{f}(x,y)$

**Procedure for restoration tasks in multiple periodic interference**

Isolate the principal contributions of the interference pattern

Subtract a variable, weighted portion of the pattern from the corrupted image

# Image Restoration

## Optimum Notch Filtering

**Extract the principal frequency components of the interference pattern**

Place a notch pass filter at the location of each spike.

$$N(u,v) = H_{NP}(u,v)G(u,v)$$

$$\eta(x,y) = \Im^{-1}\left\{H_{NP}(u,v)G(u,v)\right\}$$

# Image Restoration

## Optimum Notch Filtering

Filtering procedure usually yields only an approximation of the true pattern. The effect of components not present in the estimate of $\eta(x, y)$ can be minimized instead by subtracting from $g(x, y)$ a weighted portion of $\eta(x, y)$ to obtain an estimate of $f(x, y)$:

$$\hat{f}(x, y) = g(x, y) - w(x, y)\eta(x, y)$$

One approach is to select $w(x, y)$ so that the variance of the estimate $\hat{f}(x, y)$ is minimized over a specified neighborhood of every point $(x, y)$.

# Image Restoration

## Optimum Notch Filtering

The local variance of $\hat{f}(x, y)$:

$$\sigma^2(x, y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[ \hat{f}(x+s, y+t) - \overline{\hat{f}}(x, y) \right]^2$$

Assume that w(x,y) remains essentially constant over the neighborhood gives the approximation
w(x+s, y+t) = w(x,y)

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# ch Filtering

$y$):

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[ \hat{f}(x+s, y+t) - \overline{\hat{f}}(x,y) \right]^2$$

$$= \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left\{ \begin{array}{l} \left[ g(x+s, y+t) - w(x+s, y+t)\eta(x+s, y+s) \right] \\ - \left[ \overline{g}(x,y) - \overline{w(x,y)\eta(x,y)} \right] \end{array} \right\}^2$$

$$= \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left\{ \begin{array}{l} \left[ g(x+s, y+t) - w(x,y)\eta(x+s, y+s) \right] \\ - \left[ \overline{g}(x,y) - w(x,y)\overline{\eta(x,y)} \right] \end{array} \right\}^2$$

# Image Restoration

# Optimum Notch Filtering

The local variance of $\hat{f}(x, y)$:

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left\{ \begin{array}{l} \left[ g(x+s, y+t) - w(x,y)\eta(x+s, y+s) \right] \\ - \left[ \bar{g}(x,y) - w(x,y)\overline{\eta(x,y)} \right] \end{array} \right\}^2$$

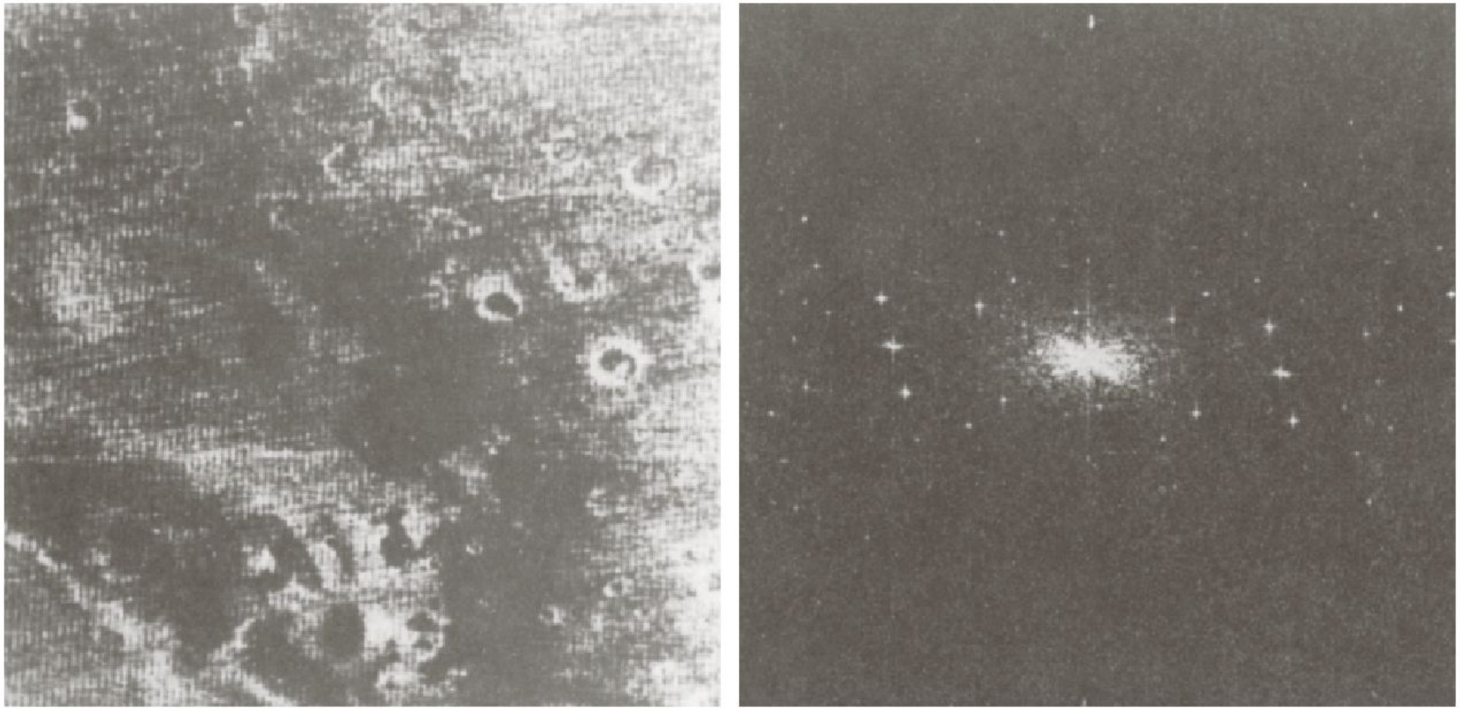
To minimize $\sigma^2(x,y)$ , $\dfrac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$

for $w(x, y)$, the result is

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

# Image Restoration

# Optimum Notch Filtering: Example



a b

**FIGURE 5.20** (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

# Image Restoration

## Optimum Notch Filtering: Example

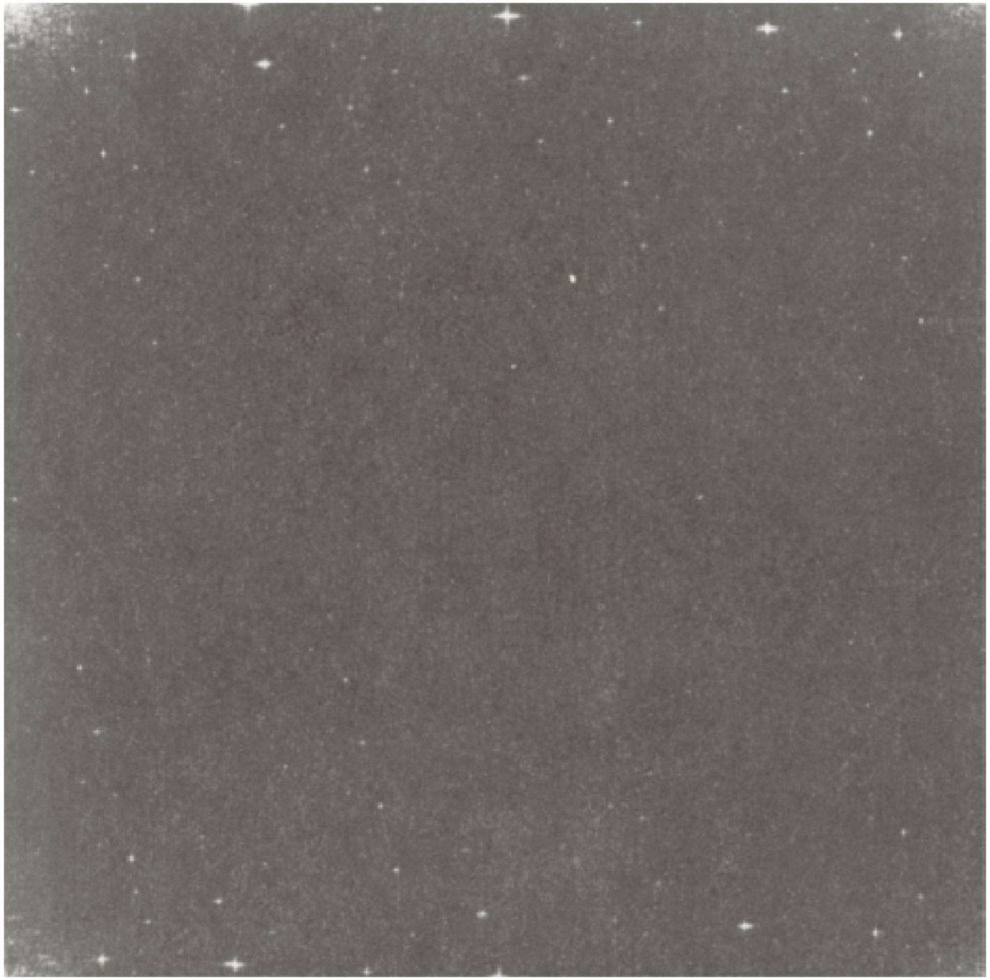
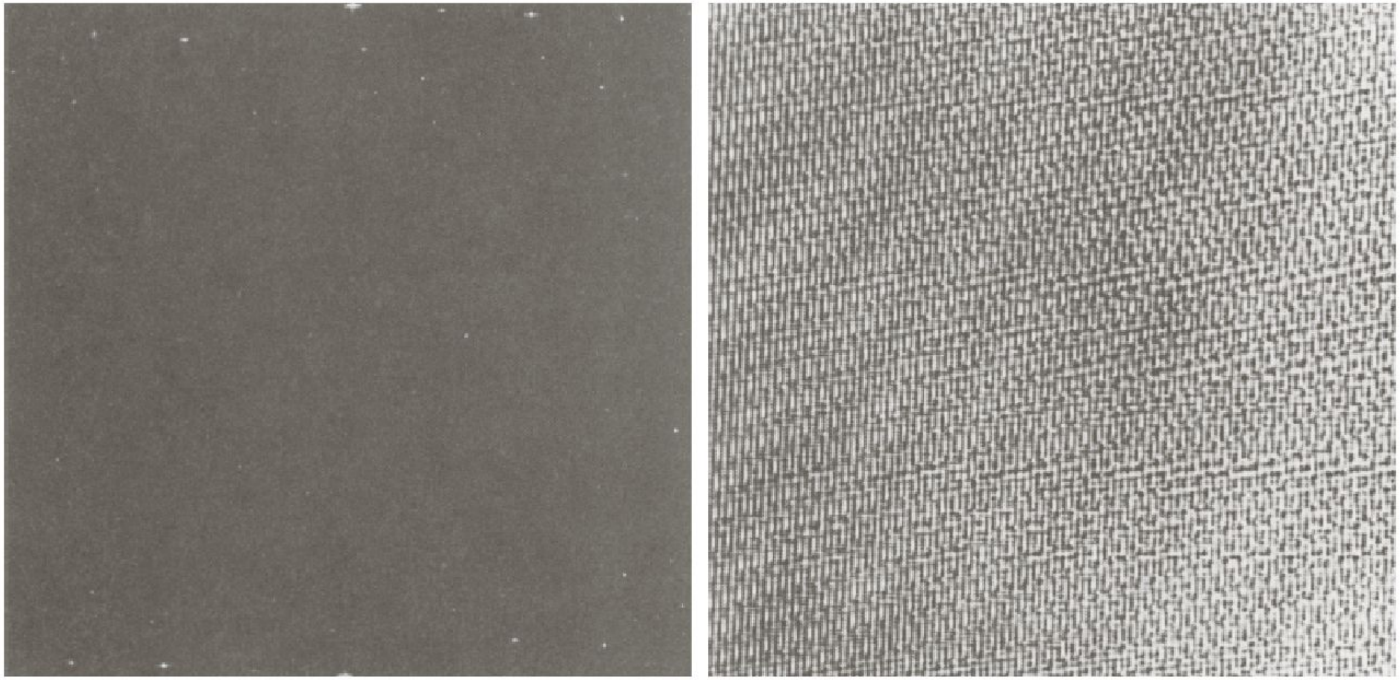


**FIGURE 5.21** Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

# Image Restoration

## Optimum Notch Filtering: Example



a b

**FIGURE 5.22** (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

# Image Restoration

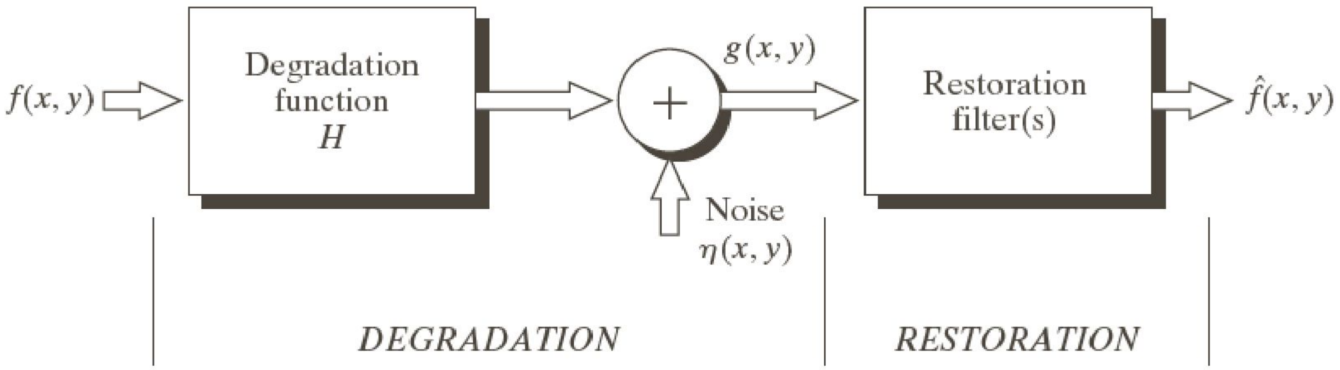
# Optimum Notch Filtering: Example

**FIGURE 5.23** Processed image. (Courtesy of NASA.)

# Image Restoration

## Linear, Position-Invariant Degradations

**FIGURE 5.1**
A model of the image degradation/ restoration process.




$$g(x, y) = H\left[f(x, y)\right] + \eta(x, y)$$

# Image Restoration

## Linear, Position-Invariant Degradations

$H$ is linear

$$H\left[af_1(x,y)+bf_2(x,y)\right]=aH\left[f_1(x,y)\right]+bH\left[f_2(x,y)\right]$$

$f_1$ and $f_2$ are any two input images.

An operator having the input-output relationship

$g(x,y)=H\left[f(x,y)\right]$ is said to be position invariant

if

$$H\left[f(x-\alpha,y-\beta)\right]=g(x-\alpha,y-\beta)$$

for any $f(x,y)$ and any $\alpha$ and $\beta$.

# Image Restoration

## Linear, Position-Invariant Degradations: Continuous Impulse Function

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)\,d\alpha\,d\beta$$

Assume for a moment that $\eta(x,y)=0$

if $H$ is a linear operator,

$$g(x,y)=H\left[f(x,y)\right]$$

$$=H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)\,d\alpha\,d\beta\right]$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H\left[f(\alpha,\beta)\delta(x-\alpha,y-\beta)\right]d\alpha\,d\beta$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H\left[\delta(x-\alpha,y-\beta)\right]d\alpha\,d\beta$$

Homogeneity

# Image Restoration

## Linear, Position-Invariant Degradations

Assume for a moment that $\eta(x, y) = 0$

if $H$ is a linear operator and position invariant,

$$H\left[\delta(x-\alpha, y-\beta)\right] = h(x-\alpha, y-\beta)$$

$$g(x, y) = H\left[f(x, y)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta) H\left[\delta(x-\alpha, y-\beta)\right] d\alpha\, d\beta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta) h(x-\alpha, y-\beta) d\alpha\, d\beta$$

Convolution integral in 2-D

# Image Restoration

## Linear, Position-Invariant Degradations

In the presence of additive noise,

if $H$ is a linear operator and position invariant,

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\,d\beta+\eta(x,y)$$

$$=h(x,y)\star f(x,y)+\eta(x,y)$$

$$G(u,v)=H(u,v)F(u,v)+N(u,v)$$

**Since degradations are modeled as being the result of convolution, image deconvolution is used frequently to signify linear image restoration.**

# Image Restoration

## Estimating the Degradation Function

Three principal ways to estimate the degradation function

1. Observation
2. Experimentation
3. Mathematical Modeling

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Methodl)

- Environmental conditions cause degradation
  - A model about atmospheric turbulence

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

$k$ : a constant that depends on the nature of the turbulence

| a | b |
|---|---|
| c | d |

**FIGURE 5.25** Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Method2)

- Derive a mathematical model from basic principles

  E.g., An image blurred by uniform linear motion between the image and the sensor during image acquisition

Suppose that an image $f(x, y)$ undergoes planar motion, $x_0(t)$ and $y_0(t)$ are the time-varying components of motion in the $x$- and $y$-directions, respectively.

The optical imaging process is perfect. T is the duration of the exposure. The blurred image $g(x, y)$

$$g(x, y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Method2)

$$g(x,y)=\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt$$

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt\right]e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[x-x_0(t),y-y_0(t)\right]e^{-j2\pi(ux+vy)}dxdy\right]dt$$

$$=\int_0^T F(u,v)e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Method2)

$$H(u,v)=\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Suppose that the image undergoes uniform linear motion in the $x$-direction only, at a rate given by $x_0(t)=at/T$.

$$H(u,v)=\int_0^T e^{-j2\pi ux_0(t)}dt$$

$$=\int_0^T e^{-j2\pi uat/T}dt$$

$$=\frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Method2)

Suppose that the image undergoes uniform linear motion in the $x$-direction and $y$-direction, at a rate given by

$x_0(t) = at/T$ and $y_0(t) = bt/T$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$= \int_0^T e^{-j2\pi[ua+vb]t/T}dt$$

$$= \frac{T}{\pi(ua+vb)}\sin\left[\pi(ua+vb)\right]e^{-j\pi(ua+vb)}$$

# Image Restoration

## Estimating the Degradation Function: Mathematical Modeling (Example)


a b

**FIGURE 5.26**
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

# Image Restoration

## Inverse Filtering

An estimate of the transform of the original image

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$\hat{F}(u,v) = \frac{F(u,v)H(u,v) + N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

# Image Restoration

## Inverse Filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

1. We can't exactly recover the undegraded image because $N(u,v)$ is not known.
2. If the degradation function has zero or very small values, then the ratio $N(u,v)/H(u,v)$ could easily dominate the estimate $\hat{F}(u,v)$.

# Image Restoration

## Inverse Filtering

EXAMPLE

The image in Fig. 5.25(b) was inverse filtered using the exact inverse of the degradation function that generated that image. That is, the degradation function is

$$H(u,v) = e^{-k\left[(u-M/2)^2+(v-N/2)^2\right]^{5/6}}, \quad k = 0.0025$$

# Image Restoration

## Inverse Filtering

One approach is to limit the filter frequencies to values near the origin.

EXAMPLE

The image in Fig. 5.25(b) was inverse filtered using the exact inverse of the degradation function that generated that image. That is, the degradation function is

$$H(u,v) = e^{-k\left[(u-M/2)^2+(v-N/2)^2\right]^{5/6}}$$

$$k = 0.0025,\ M = N = 480.$$


**FIGURE 5.27** Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with $H$ cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

# Next Class

- **Image Restoration**
  - **More Filters**

**Thank you:**
**Question?**