DIGITAL IMAGE PROCESSING

Image Enhancement in Frequency Domain: Session 1

Dr. Mrinmoy Ghorai

Indian Institute of Information Technology
Sri City, Andhra Pradesh

Today's Lecture

- Image Enhancement in Frequency Domain
    - Fourier Transform

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Introduction

- **Fourier Series**

  Any periodic function can be expressed as the sum of sines and /or cosines of different frequencies, each multiplied by a different coefficients

- **Fourier Transform**

  Any function that is not periodic can be expressed as the integral of sines and /or cosines multiplied by a weighing function

Jean Baptiste Joseph Fourier, French mathematician and physicist (03/21/1768-05/16/1830)

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Example of Fourier Series

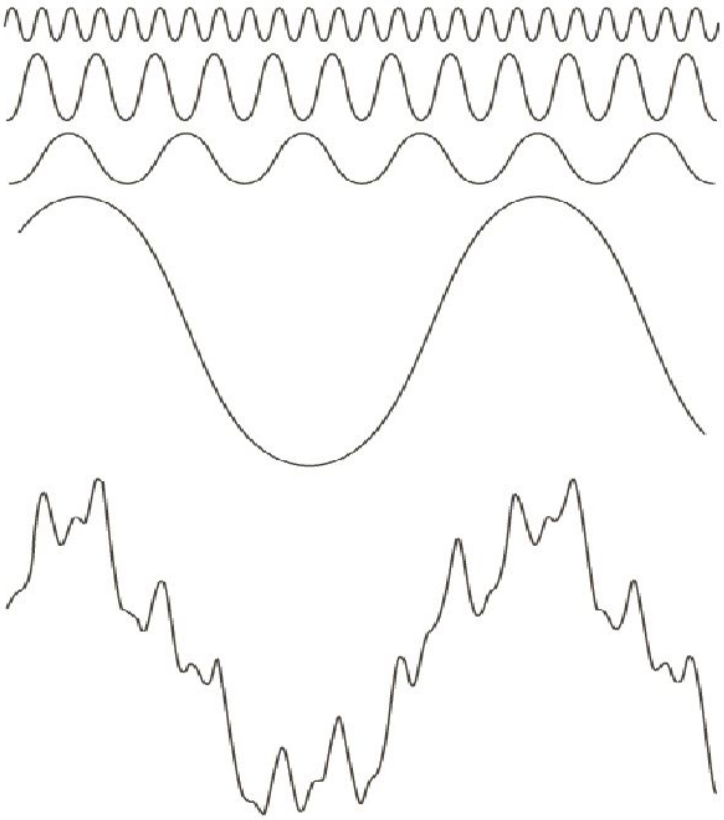

FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

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Preliminary Concepts

$$j = \sqrt{-1}, \text{ a complex number}$$

$$C = R + jI$$

the conjugate

$$C^* = R - jI$$

$$|C| = \sqrt{R^2 + I^2} \text{ and } \theta = \arctan(I / R)$$

$$C = |C|(\cos\theta + j\sin\theta)$$

Using Euler's formula,

$$C = |C| e^{j\theta}$$

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Fourier Series

A function $f(t)$ of a continuous variable t that is periodic with period, T, can be expressed as the sum of sines and cosines multiplied by appropriate coefficients

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi n}{T}t} dt \qquad \text{for } n = 0, \pm 1, \pm 2, \ldots$$

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1-D Fourier Transform: **Continuous Variable**

The *Fourier Transform* of a continous function $f(t)$

$$F(\mu) = \Im\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

The *Inverse Fourier Transform* of $F(\mu)$

$$f(t) = \Im^{-1}\{F(\mu)\} = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

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1-D Fourier Transform: **Continuous Variable**

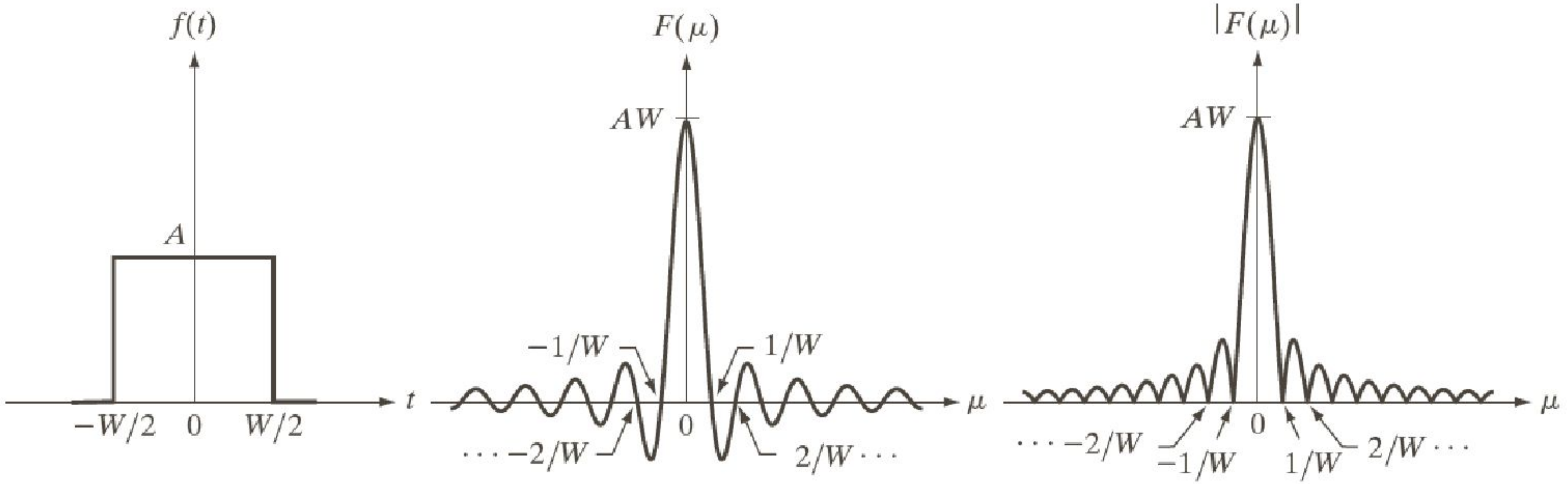

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FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

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1-D Discrete Fourier Transform

$$F(\mu) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi\mu x/M}, \qquad \mu = 0, 1, \ldots, M-1$$

$$f(x) = \frac{1}{M} \sum_{\mu=0}^{M-1} F(\mu) e^{j2\pi\mu x/M}, \qquad x = 0, 1, 2, \ldots, M-1$$

1. The domain (values of μ) over which the value of $F(\mu)$ range is called the Frequency Domain.
2. Each of the *M* terms of $F(\mu)$ is called a Frequency Component of the transform.

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2-D Fourier Transform: **Continuous Variable**

$$F(\mu,\nu)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(\mu t+\nu z)}dt\,dz$$

and

$$f(t,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mu,\nu)e^{j2\pi(\mu t+\nu z)}d\mu\,d\nu$$

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2-D Discrete Fourier Transform and Its Inverse

DFT:

$$F(\mu,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(\mu x/M+vy/N)}$$

$$\mu=0,1,2,\ldots,M-1; v=0,1,2,\ldots,N-1;$$

$f(x,y)$ is a digital image of size $\mathrm{M}\times\mathrm{N}$.

IDFT:

$$f(x,y)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}F(\mu,v)e^{j2\pi(\mu x/M+vy/N)}$$

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Properties of 2-D DFT: **Relationships between Samples in the Frequency and Spatial Domains**

Let ΔT and ΔZ denote the separations between samples, then the seperations between the corresponding discrete, frequency domain variables are given by

$$\Delta\mu = \frac{1}{M\Delta T}$$

and $$\Delta v = \frac{1}{N\Delta Z}$$

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Properties of 2-D DFT: **Translation and Rotation**

$$f(x,y)e^{j2\pi(\mu_0 x/M+v_0 y/N)} \Leftrightarrow F(\mu-\mu_0, v-v_0)$$

and

$$f(x-x_0, y-y_0) \Leftrightarrow F(\mu,v)e^{-j2\pi(\mu x_0/M+v y_0/N)}$$

Using the polar coordinates

$$x = r\cos\theta \quad y = r\sin\theta \quad \mu = \omega\cos\varphi \quad v = \omega\sin\varphi$$

results in the following transform pair:

$$f(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0)$$

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Properties of 2-D DFT: **Periodicity**

$2-D$ Fourier transform and its inverse are infinitely periodic

$$F(\mu,v) = F(\mu + k_1 M, v) = F(\mu, v + k_2 N) = F(\mu + k_1 M, v + k_2 N)$$

$$f(x,y) = f(x + k_1 M, y) = f(x, y + k_2 N) = f(x + k_1 M, y + k_2 N)$$

$$f(x)e^{j2\pi(\mu_0 x/M)} \Leftrightarrow F(\mu - \mu_0)$$

$$\mu_0 = M/2, \quad f(x)(-1)^x \Leftrightarrow F(\mu - M/2)$$

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(\mu - M/2, v - N/2)$$

Properties of 2-D DFT: **Symmetry**

	Spatial Domain†		Frequency Domain†
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

†Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

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Properties of 2-D DFT: **Fourier Spectrum and Phase Angle**

2-D DFT in polar form

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$

Fourier spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v) \right]^{1/2}$$

Power spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

Phase angle

$$\phi(\mathrm{u,v}) = \arctan\left[\frac{I(u,v)}{R(u,v)} \right]$$

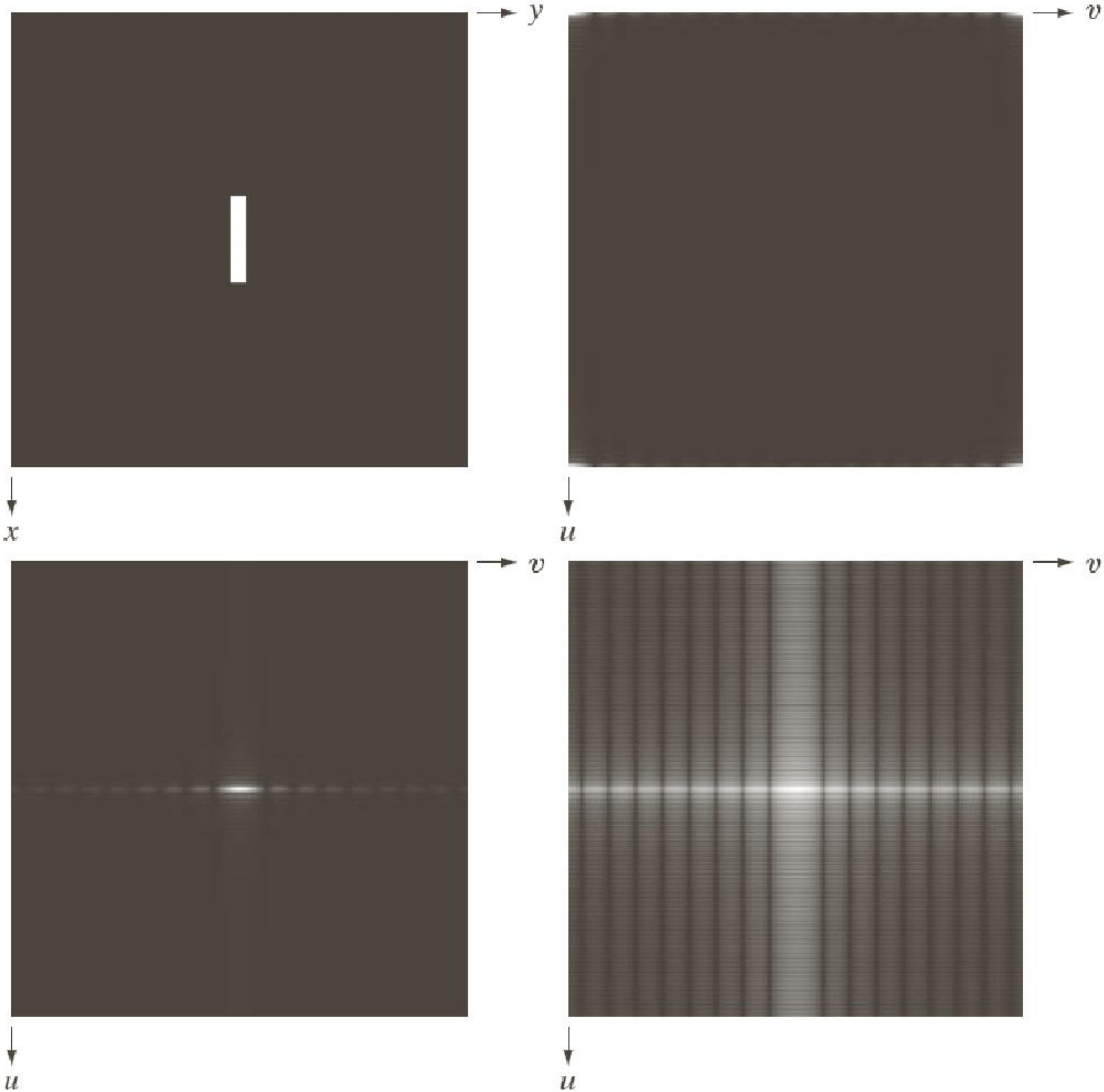
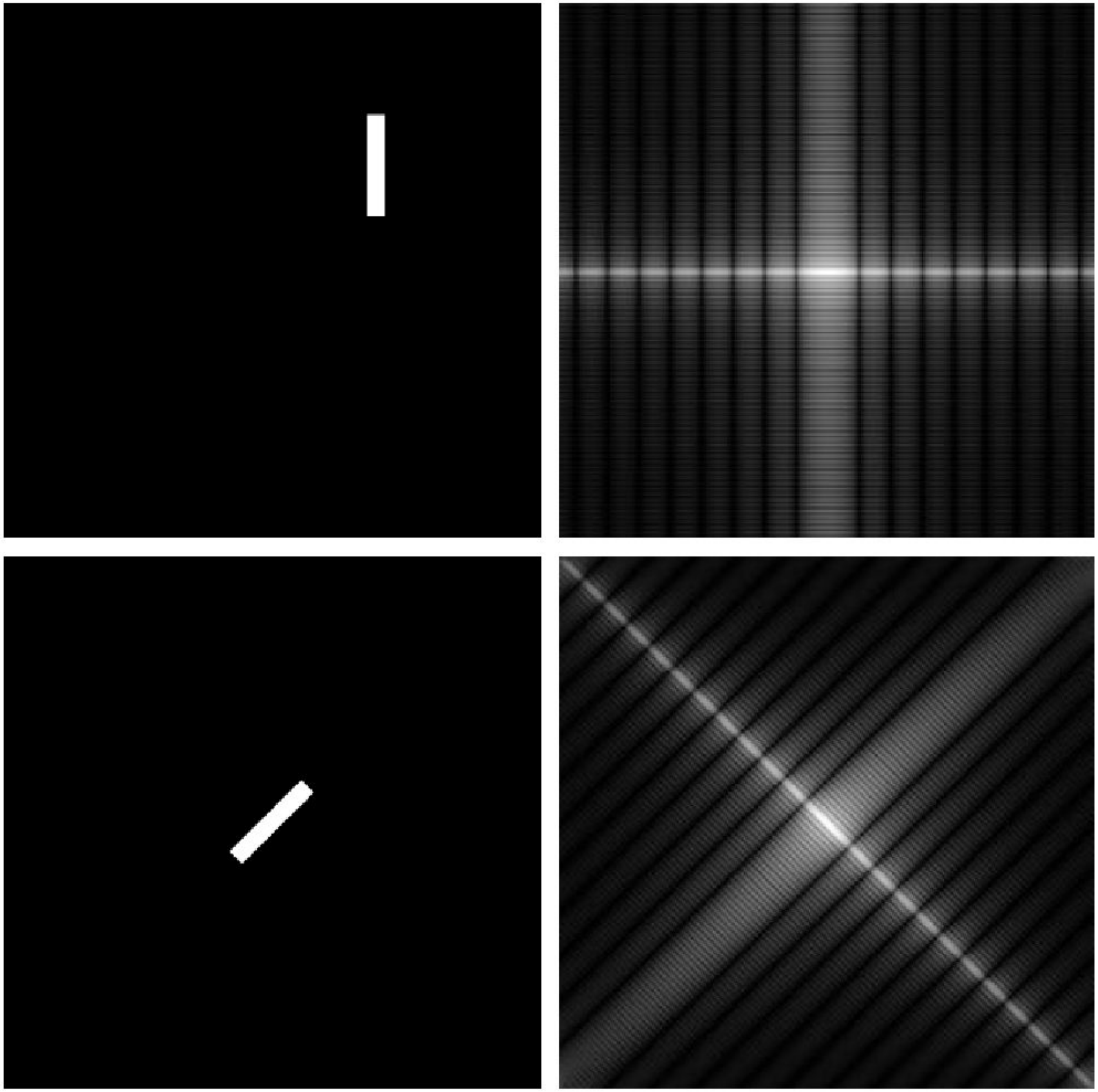


FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.



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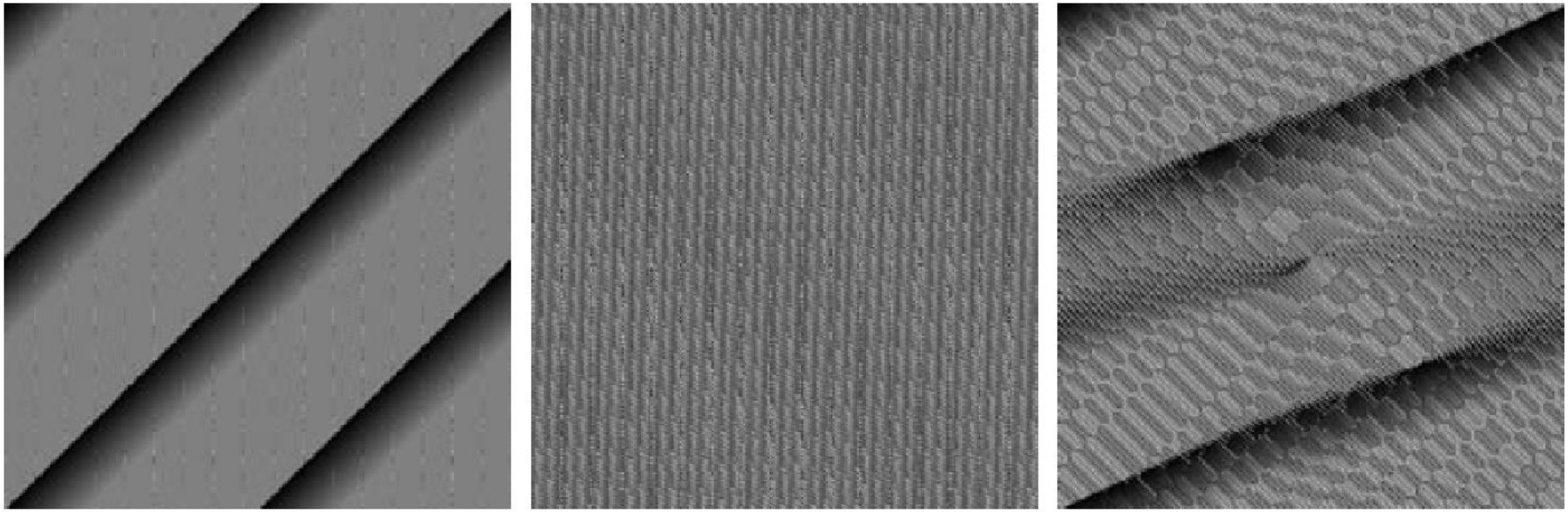
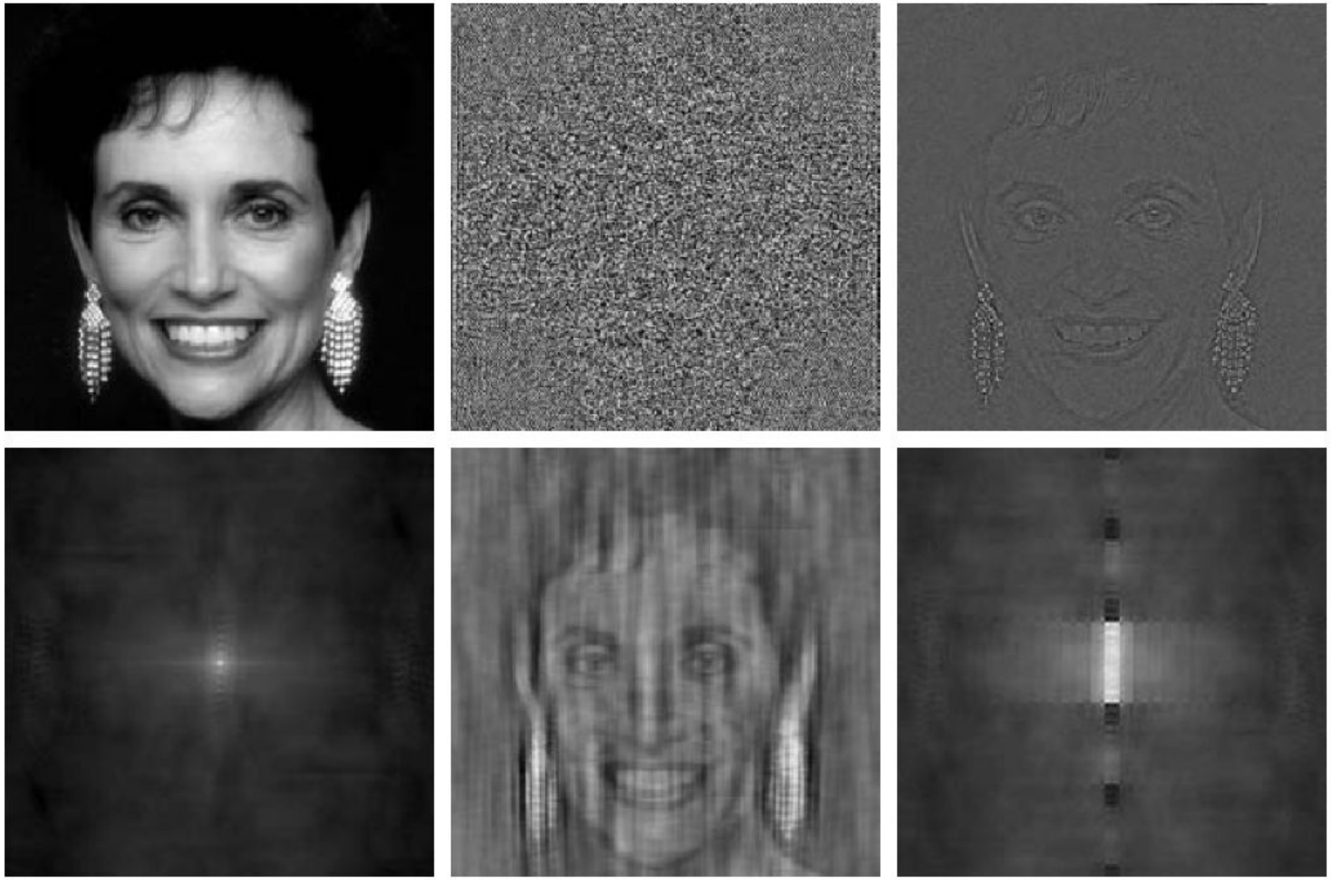
Phase Angle: **Example**



a b c

FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

Phase Angle and The Reconstructed: **Example**



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d e f

FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

Next Class

- Image Enhancement in Frequency Domain
  - Filtering in Frequency Domain

Thank you:
Question?