CPES Assignment - I

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Total 10 questions Done

[Q1](), [Q2](), [Q3](), [Q4](), [Q5](), [Q6](), [Q7](), [Q8](), [Q9](), [Q10]()

Chapter 2
Q4:
Consider the helicopter of Example 2.1, but with a slightly different definition of the input and output. Suppose that, as in the example, the input is Ty : R → R, as in the example, but the output is the position of the tail relative to the main rotor shaft. Specifically, let the x-y plane be the plane orthogonal to the rotor shaft, and let the position of the tail at time t be given by a tuple ((x(t), y(t)). Is this model LTI? Is it BIBO stable?

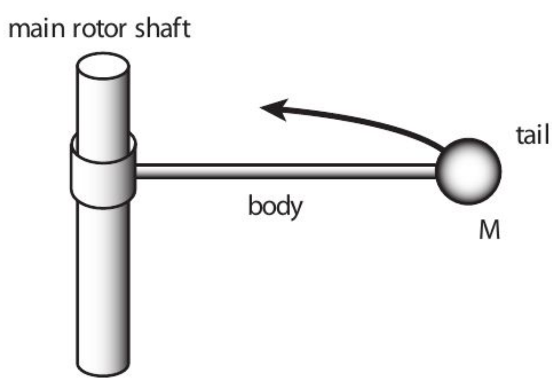


Figure 2.2: Simplified model of a helicopter.

Answer:

We can model this system with 2 output signals

$$(\mathbb{R}\square\mathbb{R}) \ \square \ (\mathbb{R} \ \square \ \mathbb{R})^2$$

where

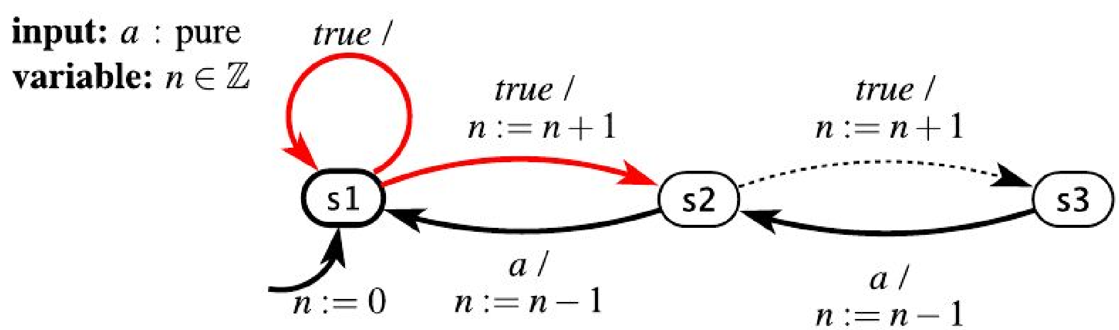
$$(\ S(T_y)\ )(t) = (\ x(t)\ ,\ y(t)\ ),$$

x(t) and y(t) represent the position of the tail in the x-y plane

LTI : We can see that this model is not linear. Thus not LTI
BIBO Stable: Yes this is Bibo Stable

Chapter 3:
Q4

How many reachable states does the following state machine have?



Ans: 3 states

Chapter 3
Q8:
This exercise studies properties of discrete signals as formally defined in the sidebar on page 44. Specifically, we will show that discreteness is not a compositional property. That is, when combining two discrete behaviors in a single system, the resulting combination is not necessarily discrete.

Consider a pure signal $x\colon \mathbb{R} \to \{present, absent\}$ given by

$$x(t) = \begin{cases} present & \text{if } t \text{ is a non-negative integer} \\ absent & \text{otherwise} \end{cases}$$

for all $t \in \mathbb{R}$. Show that this signal is discrete.

Ans:
We can show the signal is discrete by giving an order preserving one-to-one function of the form **f : T □ N.**
Here, the set of times when the signal is present is **T = N**, so we can choose the identity function for **f**

b. Consider a pure signal y : R → {present, absent} given by

$$y(t) = \begin{cases} present & \text{if } t = 1 - 1/n \text{ for any positive integer } n \\ absent & \text{otherwise} \end{cases}$$

for all $t \in \mathbb{R}$. Show that this signal is discrete.

Ans:
Similar to part (a), here we can give T as $(1 - 1/2)$ **,** $($$1 - 1/2)$ **⇒** 0, $1/2$, ⅔ ...

Thus: $f(t) = n \quad where\ t = 1 - 1/n$

Thus y is discrete

C. Consider a signal w that is the merge of x and y in the previous two parts. That is, w(t) = present if either x(t) = present or y(t) = present, and is absent otherwise. Show that w is not discrete.

Ans: Let's assume that w is discrete. There should exist a function f:T->N that is order preserving and one to one

Since 1∈ T and 1- (1/n) ∈ T

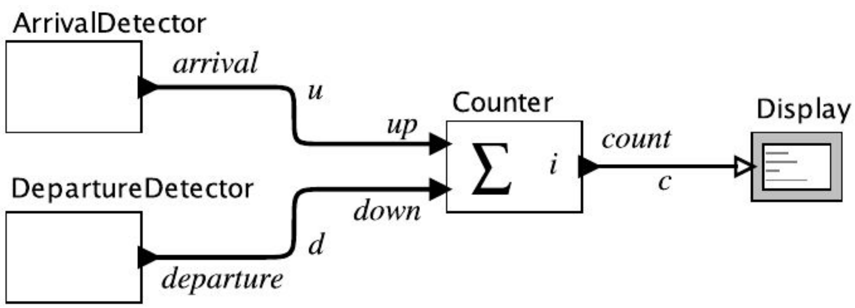
f (1) > f (1 − 1/n) (as f is one to one and order preserving)

But there is no upper bound for 1 - 1/n
Thus this is a contradiction

Thus our assumption that w is discrete is incorrect
Thus W is not discrete

D.

Consider the example shown in Figure 3.1. Assume that each of the two signals arrival and departure is discrete. Show that this does not imply that the output count is a discrete signal.



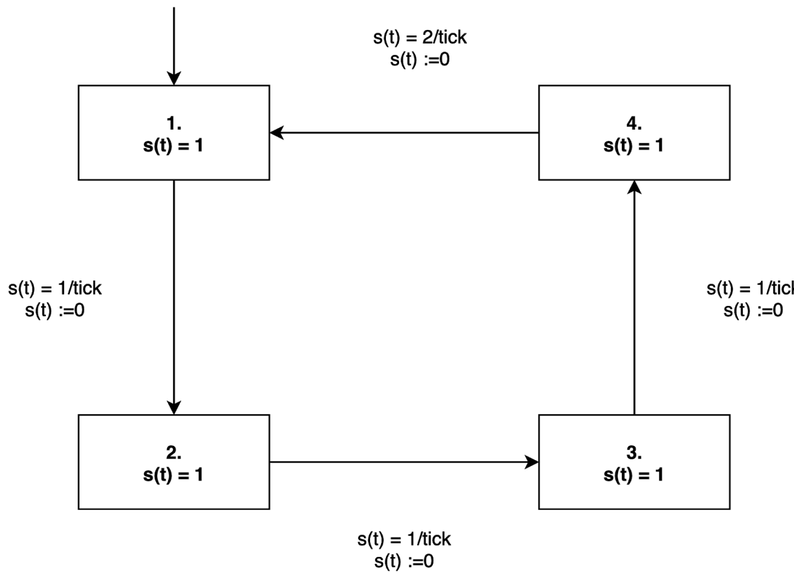
Ans:
If arrival and departure are discrete. SImilar to part c, the set of times count is present is the same.
Thus count is not discrete.

Chapter 4
Q1:

Construct (on paper is sufficient) a timed automaton similar to that of Figure 4.7

which produces tick at times 1, 2, 3, 5, 6, 7, 8, 10, 11, · · · . That is, ticks are produced with intervals between them of 1 second (three times) and 2 seconds (once).

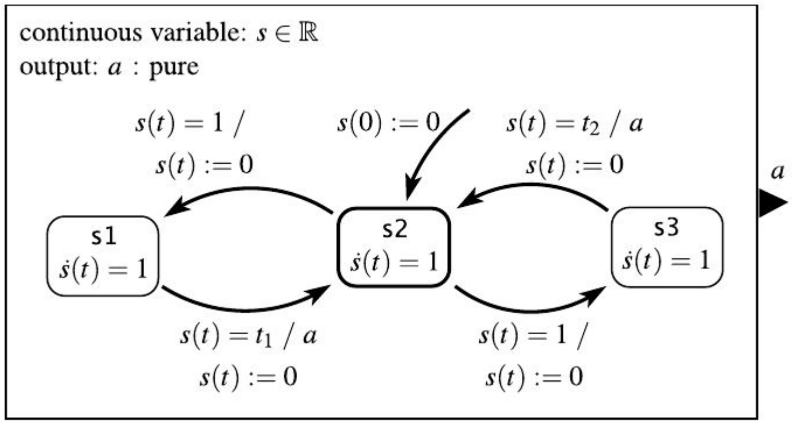
Ans



Chapter 4: Q8:

Consider the following timed automaton:

Assume t1 and t2 are positive real numbers. What is the minimum amount of time between events a? That
is, what is the smallest possible time between two times when the signal a is present?



Ans:

$$1 + min(t1, \ t2)$$

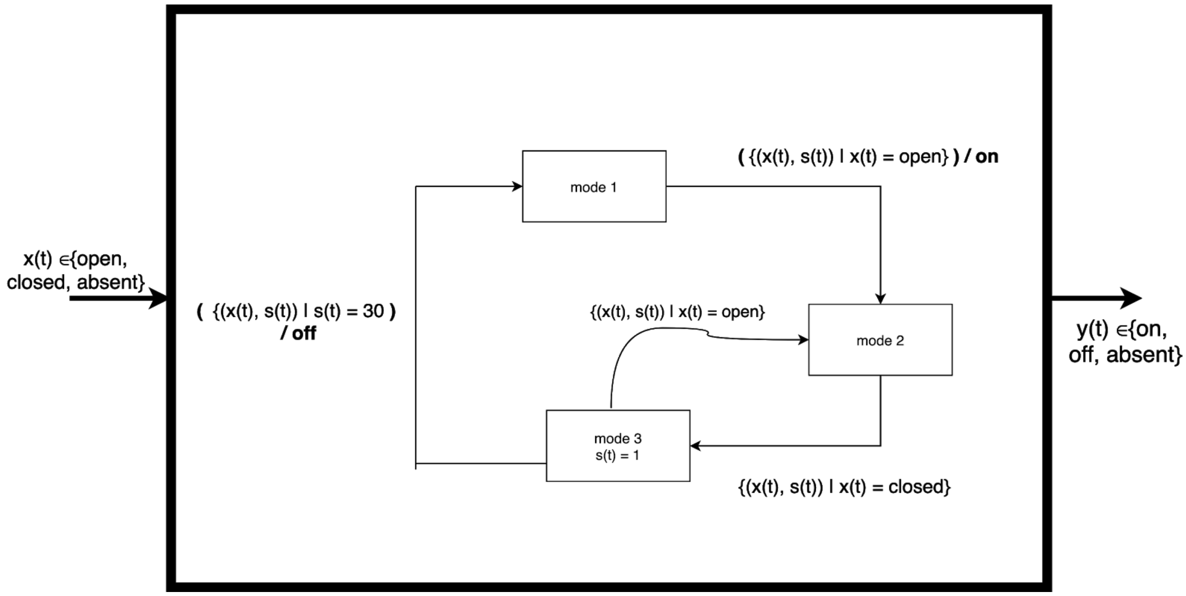
Chapter 4: Q6

Automobiles today have the features listed below. Implement each feature as a timed automaton.

(a) The dome light is turned on as soon as any door is opened. It stays on for 30 seconds after all doors are shut. What sensors are needed?

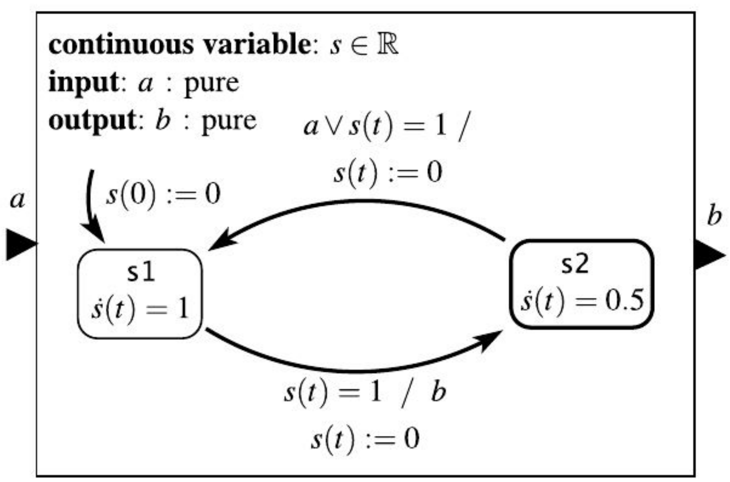
Ans:

The following machine provides on to turn on the dome light and off to turn it off:



Chapter 4: Q4:

Consider the following timed automaton:



Assume that the input signals a and b are discrete continuous-time signals, meaning that each can be given as a function of form a : R → {present, absent}, where at almost all times t ∈ R, a(t) = absent. Assume that the state machine can take at most one transition at each distinct time t, and that machine begins executing at time t = 0.

(a) Sketch the output b if the input a is present only at times t = 0.75, 1.5, 2.25, 3, 3.75, 4.5, · · ·

Ans: The output is present at times t = 1, 2.5, 4, · · · .

(b) Sketch the output b if the input a is present only at times t = 0, 1, 2, 3, · · · .

Ans: The output is present at times t = 1, 3, 5, 7, · · · .

(c) Assuming that the input a can be any discrete signal at all, find a lower bound on the amount of time between events b. What input signal a (if any) achieves this lower bound?

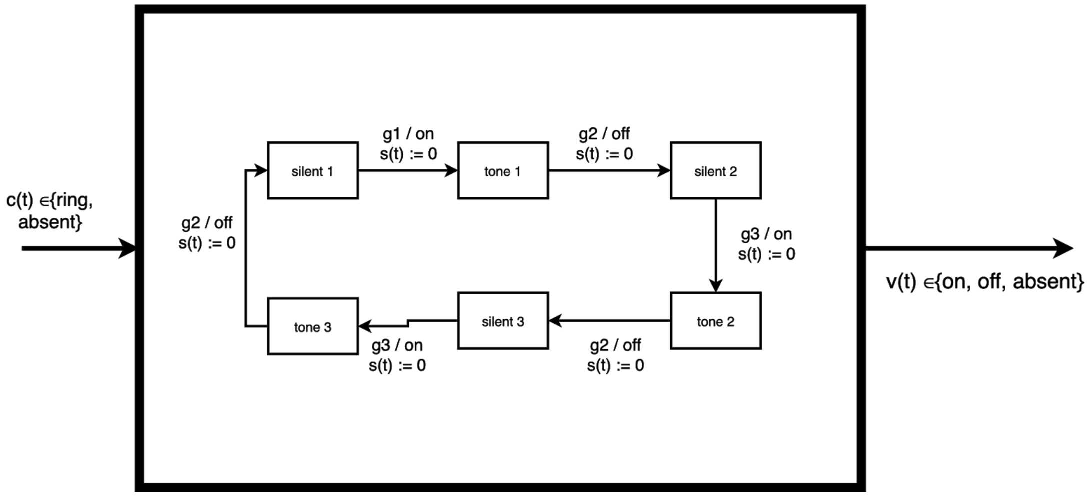
Ans: The lower bound is 1. There is no input a that achieves this bound, but an input that comes arbitrarily close is where a is present at times t = 1 + ε, 2 + 2ε, 3 + 3ε, · · · , for any ε > 0.

Chapter 4, Q5:

You have an analog source that produces a pure tone. You can switch the source on or off by the input event on or off. Construct a timed automaton that provides the on and off signals as outputs, to be connected to the inputs of the tone generator. Your system should behave as follows. Upon receiving an input event ring, it should produce an 80 ms-long sound consisting of three 20 ms-long bursts of the pure tone separated by two 10 ms intervals of silence. What does your system do if it receives two ring events that are 50 ms apart?

Ans:

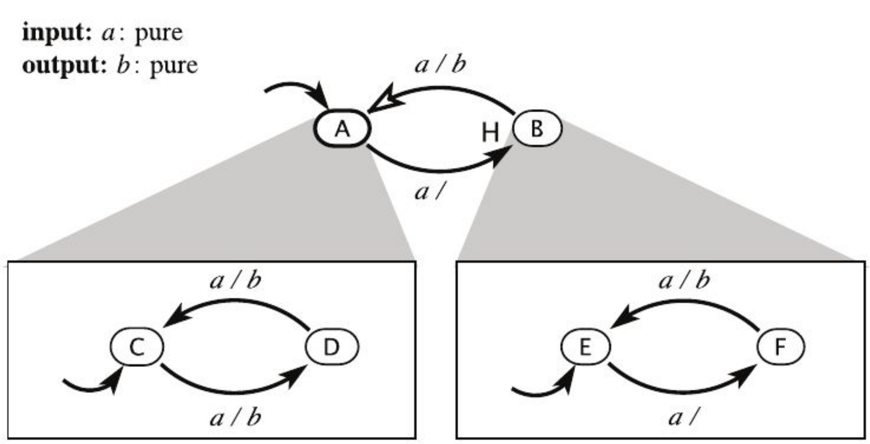
Input Alphabet: {ring, absent}
Output Alphabet: {on, off, absent}



The guards are given by
g1 = {(c(t), s(t)) | c(t) = ring}
g2 = {(c(t), s(t)) | s(t) = 20}
g3 = {(c(t), s(t)) | s(t) = 10}.

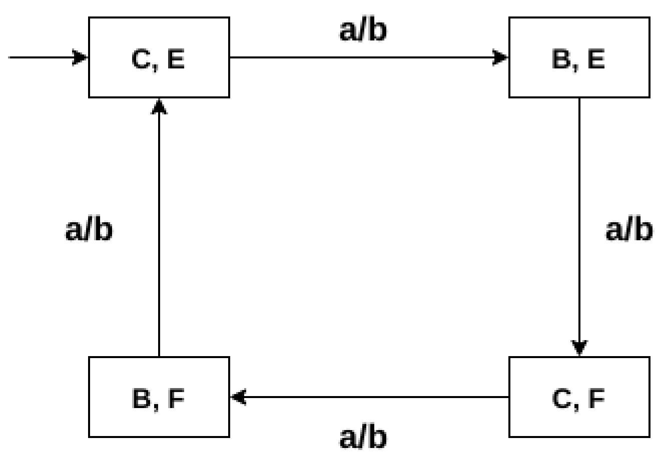
Chapter 7, Q5:

Consider the following hierarchical state machine:

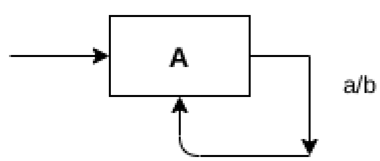


Construct an equivalent flat FSM giving the semantics of the hierarchy. Describe in words the input/output behavior of this machine. Is there a simpler machine that exhibits the same behavior? (Note that equivalence relations between state machines are considered in Chapter 14, but here, you can use intuition and just consider what the state machine does when it reacts.)

the flattened state machine looks like this:



which can be simplified to

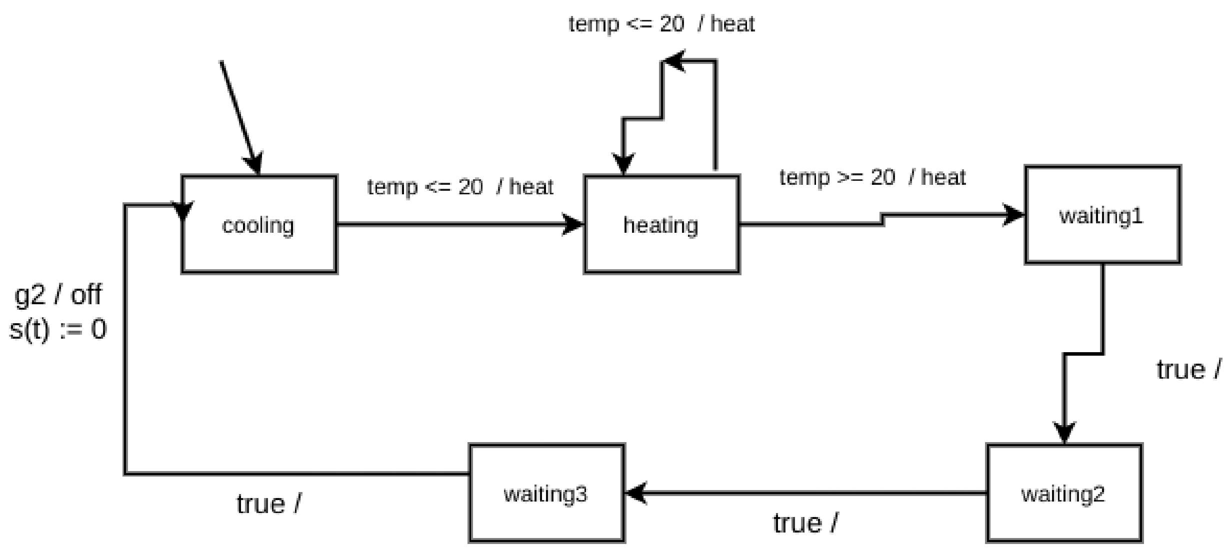


Chapter 3, Q2

Consider a variant of the thermostat of example 3.5. In this variant, there is only one temperature threshold,and to avoid chattering the thermostat simply leaves the heat on or off for at least a fixed amount of time.In the initial state, if the temperature is less than or equal to 20 degrees Celsius, it turns the heater on, and leaves it on for at least 30 seconds. After that, if the temperature is greater than 20 degrees, it turns the heater off and leaves it off for at least 2 minutes. It turns it on again only if the temperature is less than or equal to 20 degrees

(a) Design an FSM that behaves as described, assuming it reacts exactly once every 30 seconds.

Ans:



(b) How many possible states does your thermostat have? Is this the smallest number of states possible?

Ans: 5 states. Yes

c) Does this model thermostat have the time-scale invariance property?

Ans: No