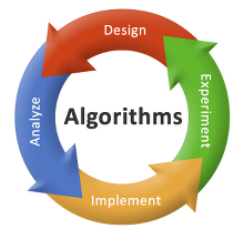


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# Course: Algorithms

# Faculty: Dr. Rajendra Prasath



# Autumn 2018

# Algorithms - Examples and The Best Practices

This class covers a few **Examples** with their complexity analysis and **the Best Practices**. This lecture illustrates the derivation of the **Worst Case Complexity** of a few selected problems and their analysis in detail.

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# Recap: Complexity Analysis

- Computational Complexity of the Algorithms
  - Best Case Analysis
  - Average Case Analysis and
  - Worst Case Analysis
- Big O – Notation, Omega Notation, Theta Notation and the choice to Data Structures
- How do we optimize the running time and space needed to solve the given problem?
  - Running Time - **Minimize** or Maximize?
  - Space Needed - **Minimize** or Maximize?

# Simple / Efficient Algorithms

- Running Times of the algorithms may vary based on  $n$
- Estimating runtime
  - Considering the input size
  - Growth in the order of magnitude of the input
  - Perform tight / upper bound analysis
- Compute the space requirements and choose appropriate data structures
- Efficient implementations with the right choice of the programming language for the given task

# Estimating Runtime – 1

- What is the runtime of  $g(n)$ ?

```
void g(int n) {  
    for (int i = 0; i < n; ++i)  
        f(i);  
}
```

Runtime  $g(n) \approx n \cdot \text{runtime}(f(n))$

# Estimating Runtime – 2

- What is the runtime of  $g(n)$ ?

```
void g(int n) {  
    for (int i = 0; i < n; ++i)  
        for (int j = 0; j < n; ++j)  
            f();  
}
```

Runtime  $g(n) \approx n^2 \cdot \text{runtime}(f(n))$

# Estimating Runtime – 3

- What is the runtime of  $g(n)$ ?

```
void g(int n) {  
    for (int i = 0; i < n; ++i)  
        for(int j = 0; j <= i; ++j)  
            f();  
}
```

$$\begin{aligned}\text{Runtime } g(n) &\approx (1 + 2 + 3 + \dots + n) \cdot \text{runtime } (f(n)) \\ &\approx (n(n+1) / 2) \cdot \text{runtime } (f(n))\end{aligned}$$

Complexity:  $O(n^2)$

# Big O - Algorithms

A method to characterize the execution time of an algorithm:

- Adding two square matrices is  $O(n^2)$
- Searching in a dictionary is  $O(\log n)$
- Sorting a vector is  $O(n \log n)$
- Solving Towers of Hanoi is  $O(2^n)$
- Multiplying two square matrices is  $O(n^3)$
- 
- 
- 
- Count the dominating terms ... discard constants



# Ranking Complexity

Function	Common name
$n!$	factorial
$2^n$	exponential
$n^d, d > 3$	polynomial
$n^3$	cubic
$n^2$	quadratic
$n\sqrt{n}$	
$n \log n$	quasi-linear
$n$	linear
$\sqrt{n}$	root - $n$
$\log n$	logarithmic
1	constant

# Big-O: Examples

$T(n)$	Complexity
$5n^3 + 200n^2 + 15$	$O(n^3)$
$3n^2 + 2^{300}$	$O(n^2)$
$5 \log_2 n + 15 \ln n$	$O(\log n)$
$2 \log n^3$	$O(\log n)$
$4n + \log n$	$O(n)$
$2^{64}$	$O(1)$
$\log n^{10} + 2\sqrt{n}$	$O(\sqrt{n})$
$2^n + n^{1000}$	$O(2^n)$

- Count dominating terms ... discard constants 10

# Complexity Analysis

- Let us assume that  $f()$  has complexity  $O(1)$
- `for(int i= 0; i< n; ++i) f();`  
    →  $O(n)$
- `for(int i= 0; i< n; ++i)`  
    `for(int j = 0; j < n; ++j) f();`  
    →  $O(n^2)$
- `for(int i= 0; i< n; ++i)`  
    `for(int j = 0; j <= i; ++j) f();`  
    →  $O(n^2)$

# Complexity Analysis

- Let us assume that  $f()$  has complexity  $O(1)$
- ```
for(int i= 0; i< n; ++i)
    for(int j = 0; j < n; ++j)
        for(int k = 0; k < n; ++k) f();
```

  
→  $O(n^3)$
- ```
for(int i= 0; i< m; ++i)
    for(int j = 0; j < n; ++j)
        for(int k = 0; k < p; ++k) f();
```

  
→  $O(mnp)$

# Best Practices

- Understanding the given problem
- Choosing right approach for solving the given problem
  - Perform tight bound analysis
  - Adaptable algorithm with increasing size of input
- Know the limitations
  - Explore ways to handle the optimization of the limitations
- Code Profiling
  - In terms of runtime... Representative Simulation
- Handling Memory Management
  - How to handle increasing number of variables?
- How to avoid wasting CPU Cycles?
  - Efficient use of CPU cycles ... even in Parallel executions
- Handle Loops efficiently
  - Avoid branching and I/O operations inside a loop
  - Process Data in blocks

# Pitfalls

- How do we use the available memory efficiently?
  - Compute the required CPU cycles and plan your algorithm execution
  - Use of excessive memory will decrease the performance and increase the run time
  - Avoid performing I/O operations
    - Avoid writing to and/or reading from Disks too often
  - Perform trade-off between memory and space available in the given system
  - Avoid overloading the file systems
    - This would slowdown the simulation or the execution of the algorithm

# A few problems

Let us try a Few Problems:

- Write an algorithm to add two numbers
- Write an algorithm to find the largest among three numbers
- Write an algorithm to find all the roots of quadratic equation
- Write an algorithm to find the factorial
- Write an algorithm to check prime number

# Add two numbers

- Write an algorithm to add two numbers

Begin

1: Declare variables num1, num2 and sum

2: Read values num1 and num2

3: Add num1 and num2 and assign result to sum

$\text{sum} \leftarrow \text{num1} + \text{num2}$

4: Print sum

End



# Largest among 3 numbers

- Write an algorithm to find the largest among three different numbers

Begin

- 1: Declare variables a, b and c
- 2: Read variables a, b and c
- 3: If  $a > b$ 
  - If  $a > c$ 
    - Display a is the largest number
  - Else
    - Display c is the largest number.
- Else
  - If  $b > c$ 
    - Display b is the largest number.
  - Else
    - Display c is the greatest number.

End

# Solve a Quadratic Equation

- Write an algorithm to find all roots of a quadratic equation  $ax^2+bx+c=0$

Begin

1: Declare variables a, b, c, D, x1, x2, rp and ip;

2: Calculate discriminant

$D \leftarrow b^2 - 4ac$

3: If  $D \geq 0$

$r1 \leftarrow (-b + \sqrt{D}) / 2a$

$r2 \leftarrow (-b - \sqrt{D}) / 2a$

Display r1 and r2 as roots.

Else

Calculate real part and imaginary part

$rp \leftarrow -b / 2a$

$ip \leftarrow \sqrt{-D} / 2a$

print  $rp + j(ip)$  and  $rp - j(ip)$  as roots

End

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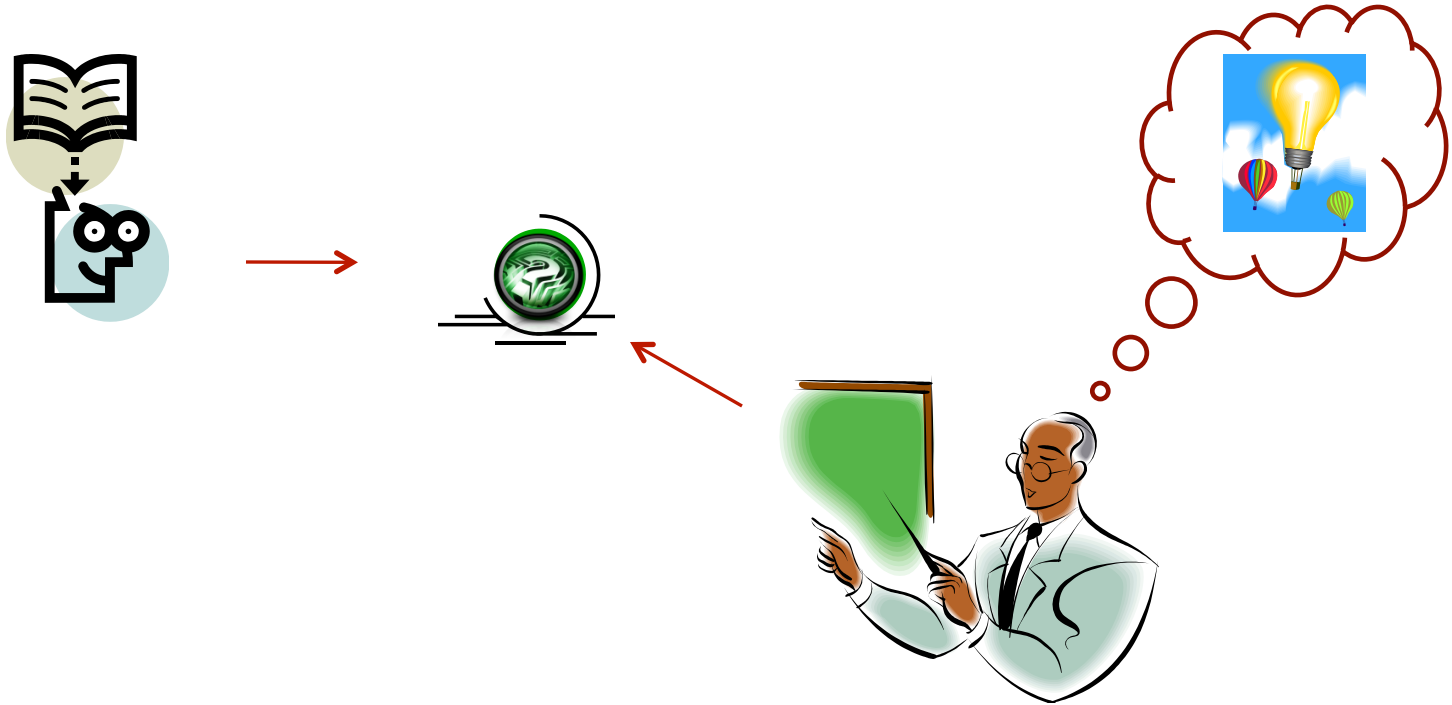
# Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)
- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)
- You may grow a culture of **collaborative learning** by helping the needy students

# Assistance

- You may post your questions to me at any time
- You may meet me in person on available time or with an appointment
- TA s would assist you to clear your doubts.
- You may leave me an email any time (email is the best way to reach me faster)

# Thanks ...



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... Questions ???