DIGITAL IMAGE PROCESSING

Digital Image Fundamentals : Session 3

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Today's Lecture

- **Digital Image Fundamentals**
  - **Mathematical Operations in DIP**

Mathematical Operations in DIP

Array vs. Matrix Operation

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \; .* \; B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Mathematical Operations in DIP

Linear vs. Nonlinear Operation

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

- H is said to be a ***linear operator***
- H is said to be a ***nonlinear operator*** if it does not meet the above qualification.

Mathematical Operations in DIP

Arithmetic Operations

Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x, y) = f(x, y) + g(x, y)$$
$$s(x, y) = f(x, y) - g(x, y)$$
$$s(x, y) = f(x, y) \times g(x, y)$$
$$s(x, y) = f(x, y) \div g(x, y)$$

Mathematical Operations in DIP

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$

Noise: n$(\boldsymbol{x}, \boldsymbol{y})$ (at every pair of coordinates $(\boldsymbol{x}, \boldsymbol{y})$, the noise is uncorrelated and has zero average value)

Noisy image: $\mathbf{g}(\boldsymbol{x}, \boldsymbol{y})$

$$\mathbf{g}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{n}(\boldsymbol{x}, \boldsymbol{y})$$

Reducing the noise by adding a set of noisy images, $\{\boldsymbol{g_i}(\boldsymbol{x}, \boldsymbol{y})\},\ i = 1,2, \dots, K$

$$\overline{g}(x, y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x, y)$$

Mathematical Operations in DIP

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$E\{\bar{g}(x,y)\} = E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\}$$

$$= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y)+n_i(x,y)\right]\right\}$$

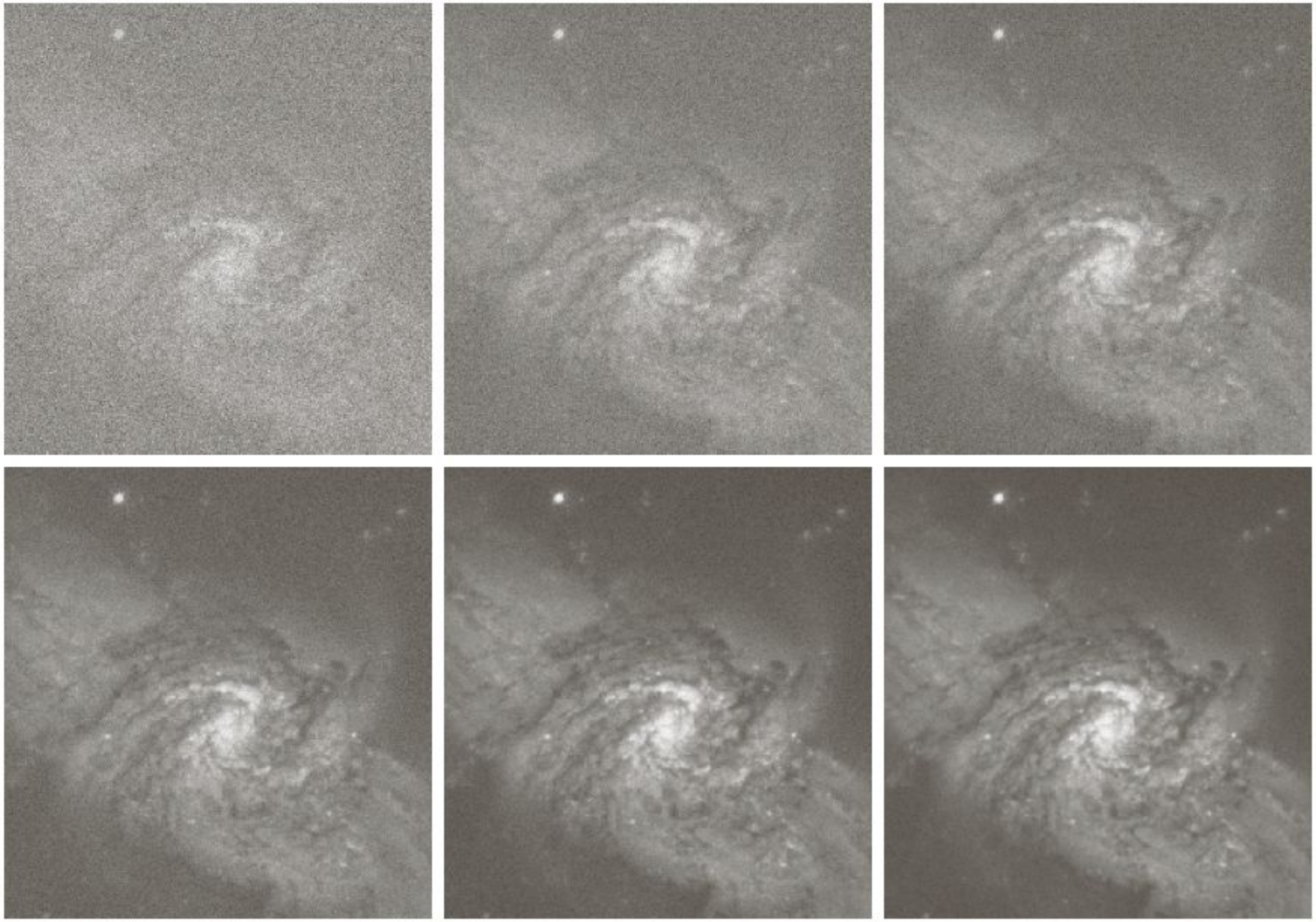
$$= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\}$$

$$= f(x,y)$$

$$\sigma^2_{\bar{g}(x,y)} = \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)}$$

$$= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)} = \frac{1}{K}\sigma^2_{n(x,y)}$$

Mathematical Operations in DIP



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Mathematical Operations in DIP

Example: Image Subtraction for Mask Mode Radiography

Mask $\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})$**:** an X-ray image of a region of a patient's body

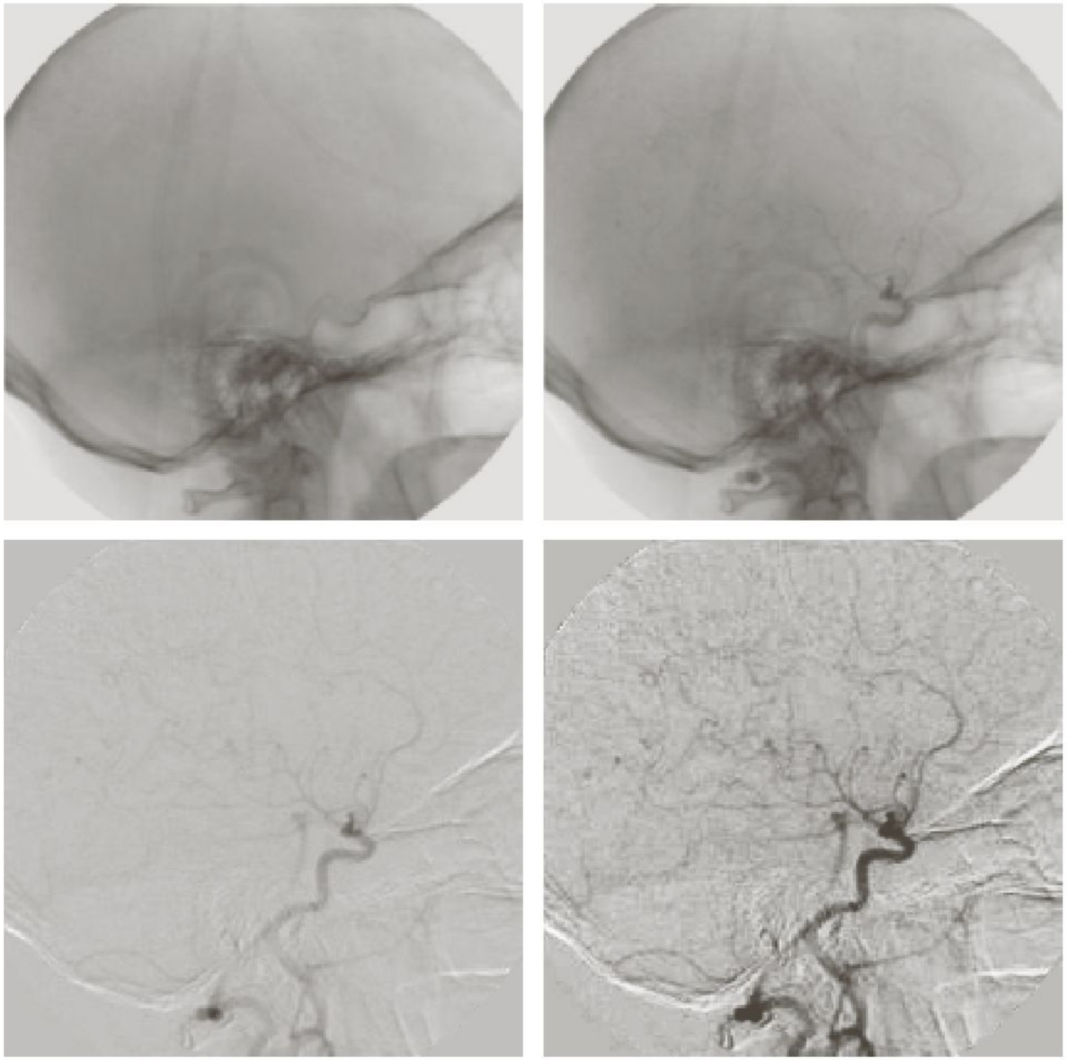
Live images $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$**:** X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $\mathbf{g}(\boldsymbol{x},\boldsymbol{y})$:

$$\mathbf{g}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.

Mathematical Operations in DIP



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

Mathematical Operations in DIP

Example: Image Multiplication for Shading Correction

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Mathematical Operations in DIP

Set and Logical Operations

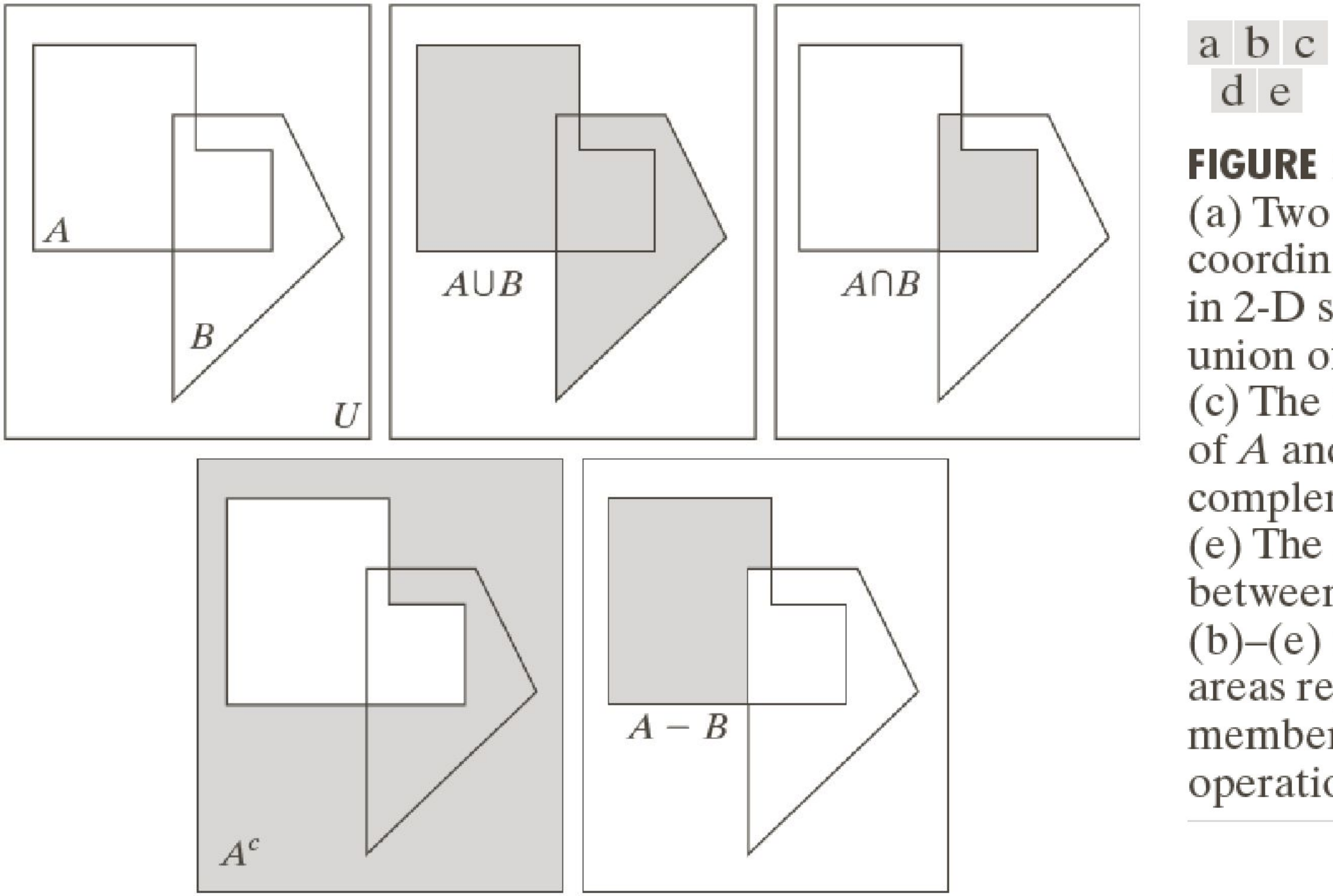


FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Mathematical Operations in DIP

Set and Logical Operations: Complement

Let A be the elements of a gray-scale image. The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$K = 2^k - 1$; k is the number of intensity bits used to represent z

Mathematical Operations in DIP

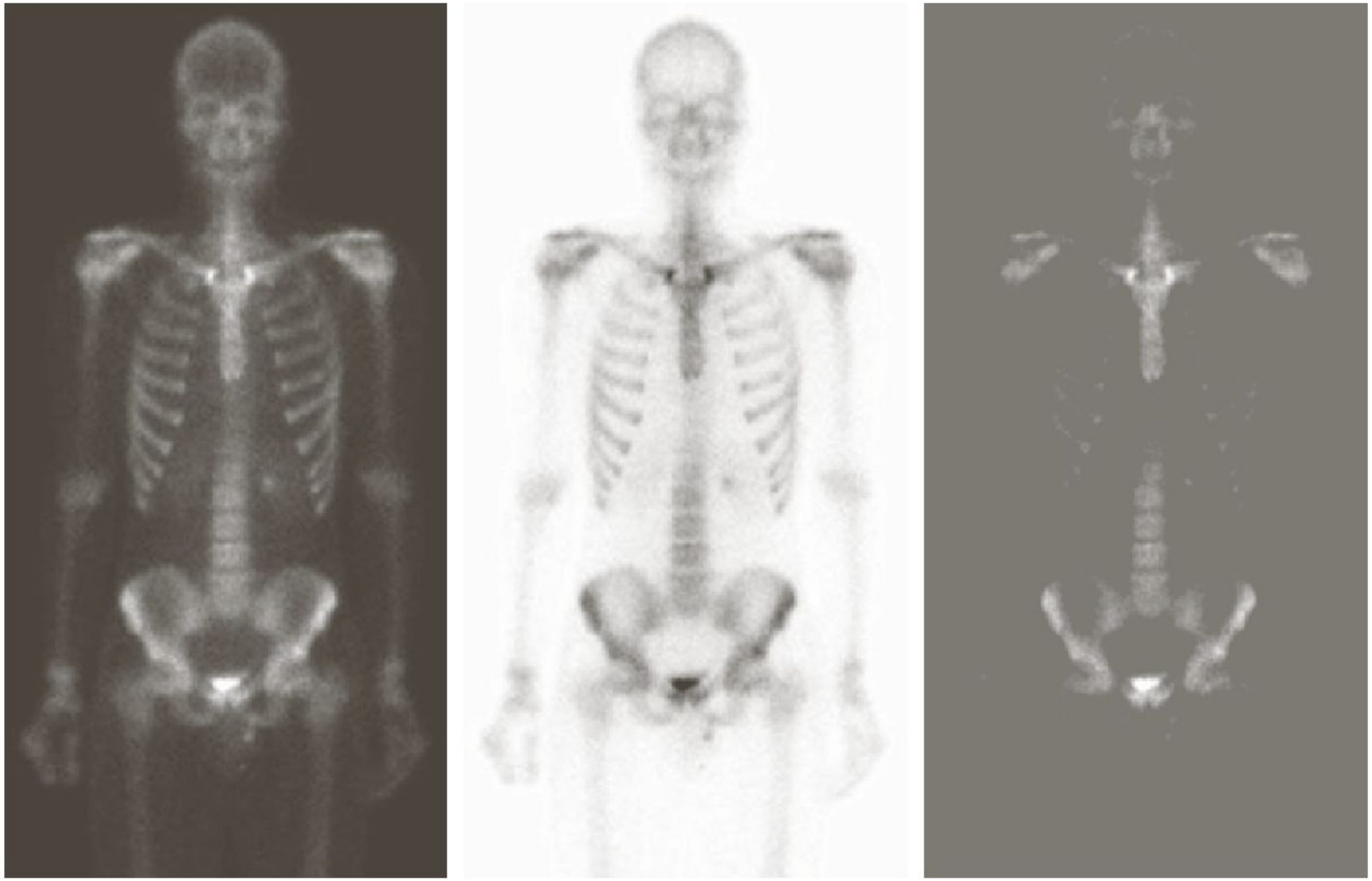
Set and Logical Operations: Union

The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max_z(a,b) \mid a \in A, b \in B\}$$

Mathematical Operations in DIP

Set and Logical Operations



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Mathematical Operations in DIP

Set and Logical Operations

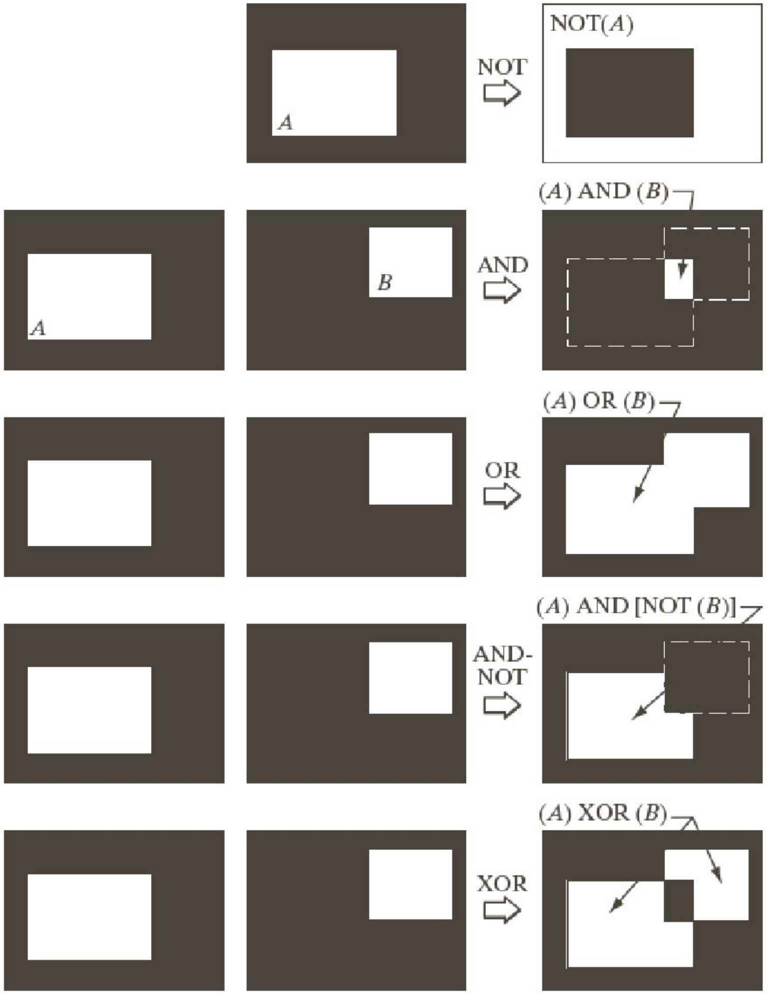


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Mathematical Operations in DIP

Spatial Operations: Single-pixel operations

Alter the values of an image's pixels based on the intensity.

$$s = T(z)$$

e.g.,

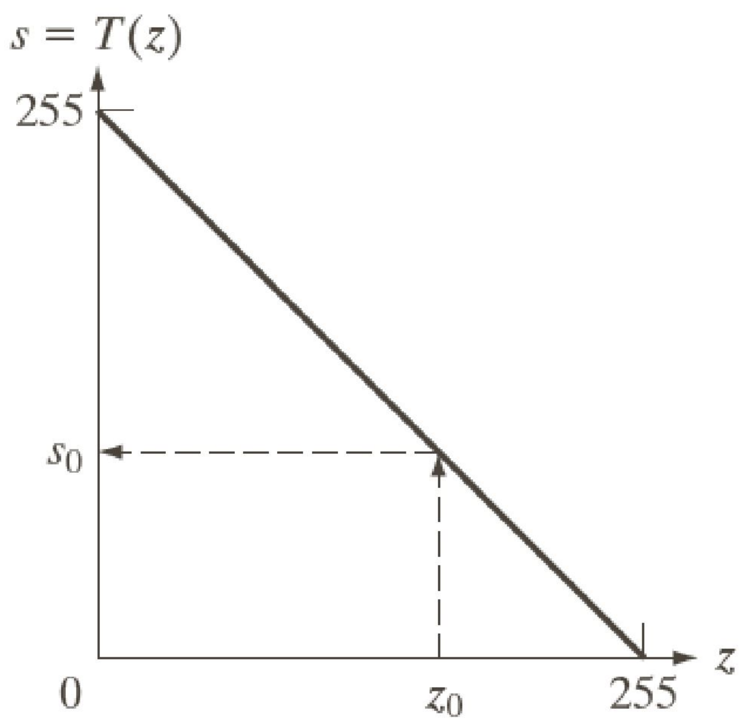
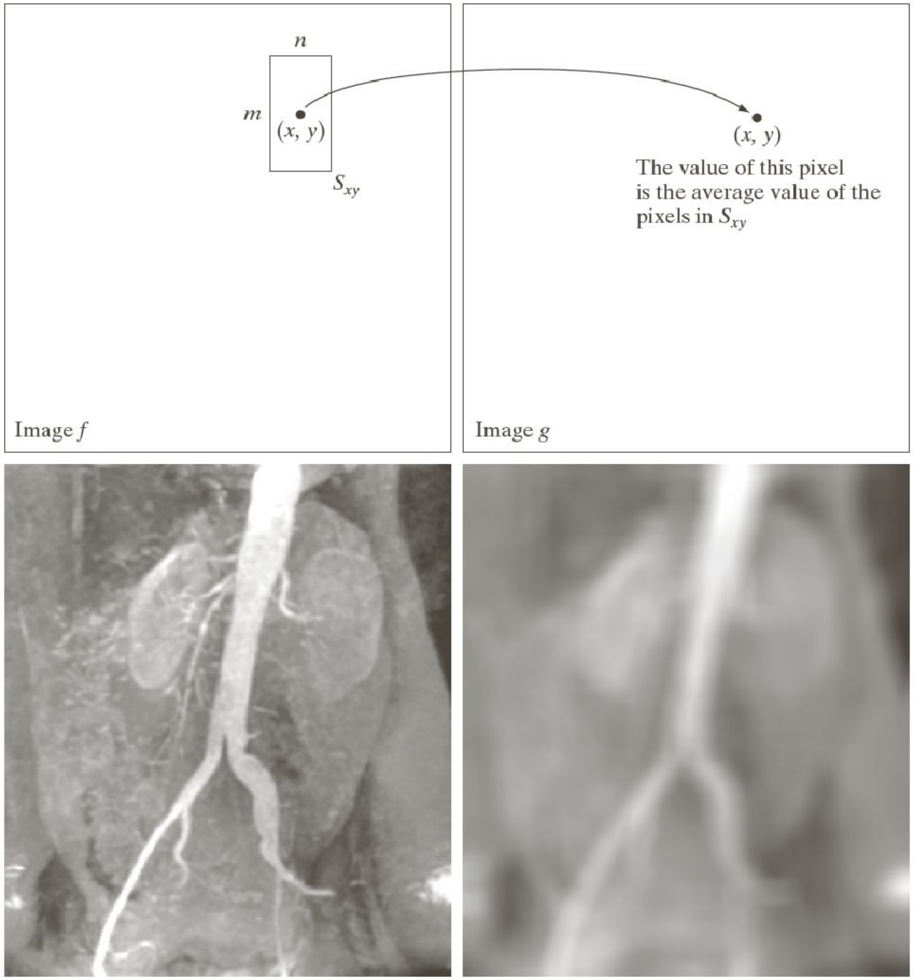


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Mathematical Operations in DIP

Neighborhood Operations



Mathematical Operations in DIP

Geometrical Spatial Transformation

- Geometric transformation (rubber-sheet transformation)
  - A spatial transformation of coordinates
  - intensity interpolation that assigns intensity values to the spatially transformed pixels.

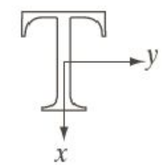
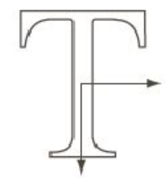
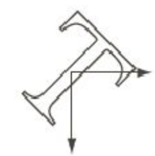
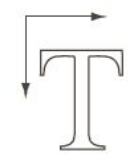
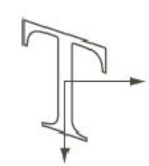
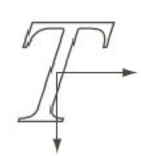
$$(x, y) = T\{(v, w)\}$$

- Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

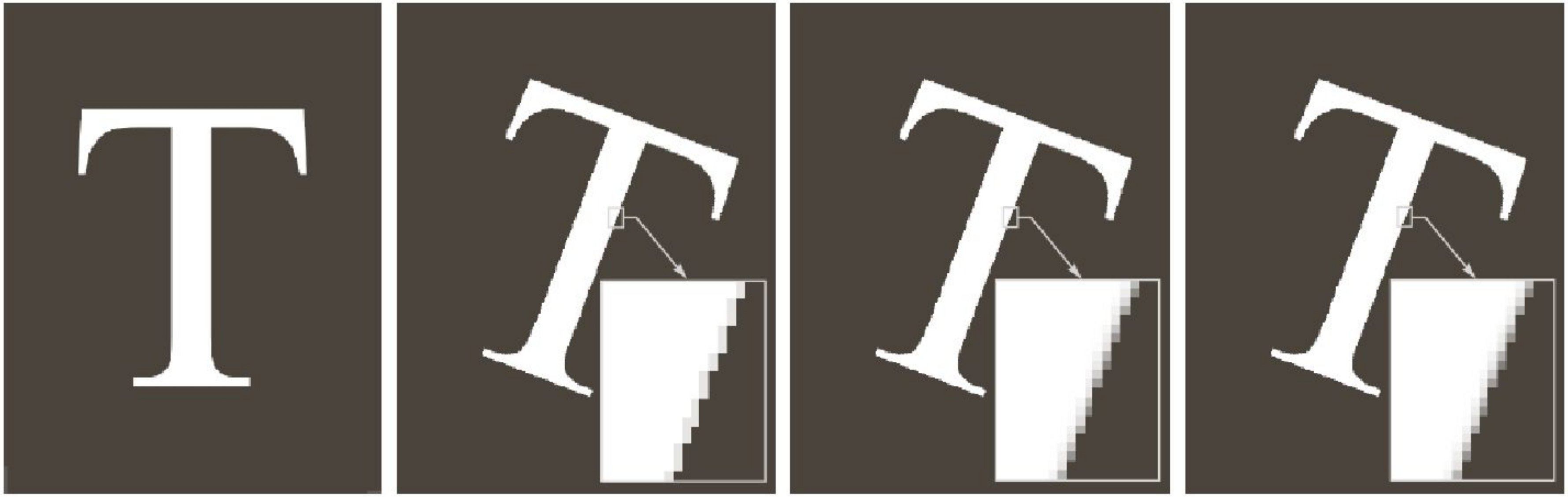
TABLE 2.2

Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Mathematical Operations in DIP

Image Rotation and Intensity Interpolation



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Mathematical Operations in DIP

Image Registration

- Input and output images are available but the transformation function is unknown.
  Goal: estimate the transformation function and use it to register the two images.

- One of the principal approaches for image registration is to use ***tie points*** (also called ***control points***)
- The corresponding points are known precisely in the input and output (**reference**) images.

Mathematical Operations in DIP

Image Registration

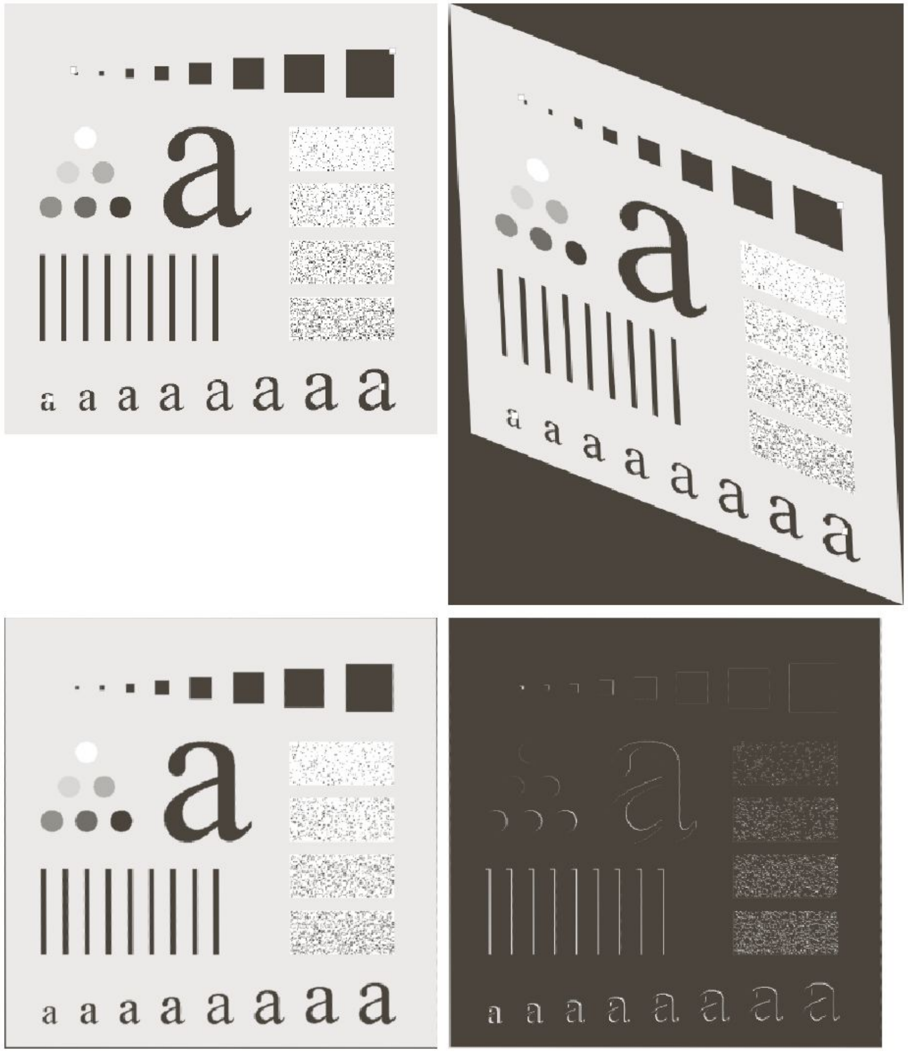
A simple model based on bilinear approximation:

$$\begin{cases} x = c_1 v + c_2 w + c_3 vw + c_4 \\ y = c_5 v + c_6 w + c_7 vw + c_8 \end{cases}$$

where (v, w) and (x, y) are the coordinates of tie points in input and reference images.

Mathematical Operations in DIP

Image Registration



a	b
c	d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

Next Class

- Image Enhancement in Spatial Domain
  - Intensity Transform
  - Spatial Filtering

Thank you:
Question?