Computer Graphics and Multimedia

Tutorial 2 : Change of Basis and Change of Coordinate Frames

1. Example discussing an instance of sound processing.

   (a) Record your voice for 5 seconds in variable *myvoice* with $F_s = 4000$ samples per second. Generate η, a WGN with mean zero and variance of 0.04. Play $noisymyvoice = myvoice + \eta$.

   (b) Construct a circular convolution matrix $H : \mathbb{R}^{20000} \rightarrow \mathbb{R}^{20000}$ representing the linear shift invariant system with impulse response $h(n) = \frac{1}{\sqrt{30}}$, $n = 0, 1, 2, \ldots, 29$.

   (c) Perform convolution (circular) on *noisymyvoice* using matrix multiplication $lessnoisymyvoice = H * noisymyvoice$. Play and observe the difference between the filtered and original noisy voices.

   (d) Find out eigenvalues and corresponding eigenvectors of H. Let the eigen vectors be put in columns of matrix D and eigen values on diagonals of Λ. Check the identity, $HD = D\Lambda$.

   (e) Decompose *noisymyvoice* in terms of colums of D, i.e. $noisymyvoice = D * NoisyMyVoice$. Plot magnitude and phase components of *NoisyMyVoice*. Do you see connection of *NoisyMyVoice* with the Discrete Fourier transform?

   (f) Try to argue and convince yourself that the following statement about Linear Shift Invarinat Systmes is essentially an application of change of basis.

   > ... convolution in time domain is equivalent to multiplication in frequency domain ...

2. We have been discussing about change of co-ordinate systems in class and, we also looked at the way in which translation can be included in the transformation matrix in homogeneous co-ordinate system. Let us try to use these ideas to describe motion of a tricycle shown in Figure. 1. There are three co-ordinate systems in place to describe the tricycle motion: the world co-ordinate system $B_{wo} = \{x_{wo}, y_{wo}, z_{wo}\}$, the tricycle co-ordinate system $B_{tr} = \{x_{tr}, y_{tr}, z_{tr}\}$ and the wheel co-ordinate system $B_{wh} = \{x_{wh}, y_{wh}, z_{wh}\}$. The tricycle co-ordinate system is fixed to the tricycle-frame and not to the handlebars.

   Let the initial positions of the tricycle co-ordinate system and wheel co-ordinate systems be fixed to $T_{tr}(0)$ and $T_{wh}(0)$ in the world co-ordinate system. Also assume that initially the wheel and tricycle axes are parallel to the world co-ordinate axes.

   Based on the motion that the tricycle undergoes in t seconds, the co-ordinate systems change positions to $T_{tr}(t)$ and $T_{wh}(t)$ at time t and, similarly the co-ordinate systmes them selves change to $B_{tr}(t)$ and $B_{wh}(t)$. I hope by now you see how the co-ordinate systems are connected to each other.

   (a) Suppose that the wheel rotates with an angular velocity of α radians per second and the radius of the wheel is r cm. Consider a point P located on the rim of the wheel.

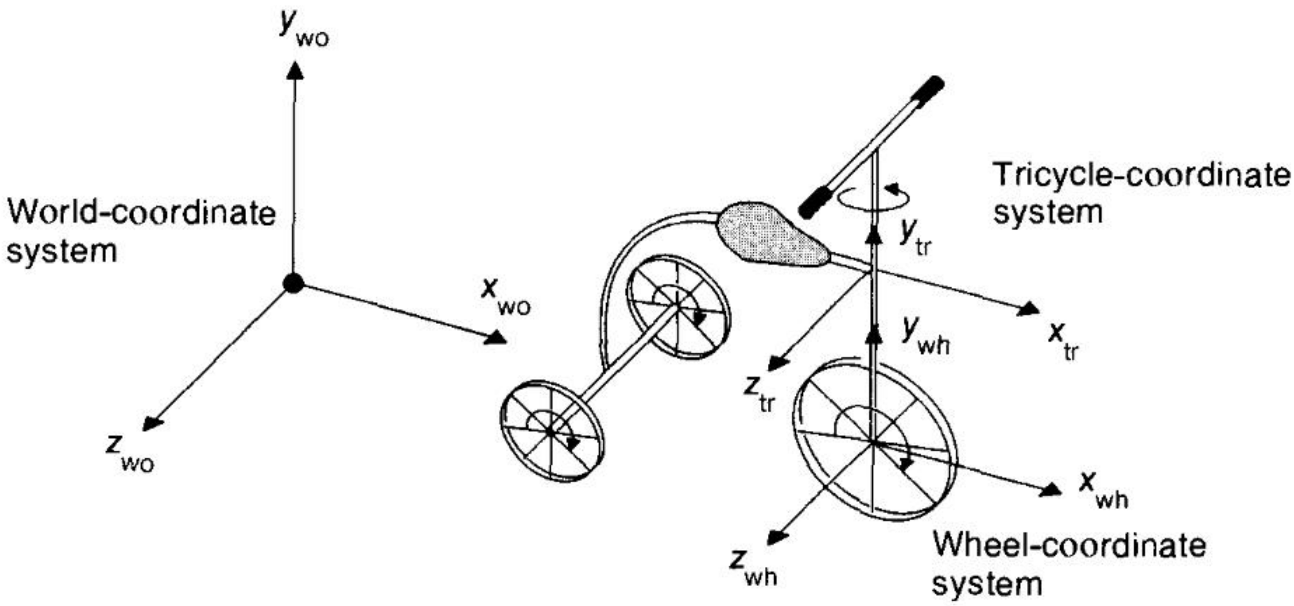

Figure 1: A stylized tricycle with three coordinate systems: reproduced from *Computer Graphics: Principles and Practice* by Foley, van Dam, Feiner and Hughes, p.no. 225

Let the co-ordinates of $P(0)$ (position at $t = 0$) w.r.t. wheel co-ordinate system be given by $(P_x^{wh}(0), P_y^{wh}(0), P_z^{wh}(0))$. Find out co-ordinates of $P(t = 1)$ in all five co-ordinate systems B_{wo}, $B_{tr}(0)$, $B_{wh}(0)$, $B_{tr}(1)$ and $B_{wh}(1)$.

(b) Instead of moving linearly from the rest at $t = 0$, suppose the tricycle handlebars are turned at β radians per second to the left, find $P(1)$ in all five co-ordinate systems B_{wo}, $B_{tr}(0)$, $B_{wh}(0)$, $B_{tr}(1)$ and $B_{wh}(1)$.