CO: Computer Organization

Day2

Indian Institute of Information Technology, Sri City

Jan - May - 2018

http://co-iiits.blogspot.in/

Number Systems

- Representation of Integer Numbers
  - Signed Magnitude Representation
  - 1's Complement Representation
  - 2's Complement Representation
- Representation of Real Numbers
  - Fixed Point Representation
  - Floating Point Representation

Resolution is difference between two successive numbers.

Representation of Integer Numbers

Let A $= a_{n-1}a_{n-2}a_{n-3}...a_0$ is an n-bit binary number
if A is an **unsigned integer**, then value of A is : $\sum_{i=0}^{n-1}(2^i \times a_i)$.
if A is a **signed integer**:

- Signed Magnitude Representation:
  - $A = \sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 0$
  - $A = -\sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 1$
- 1's Complement Rep.: $A = -(2^{n-1} - 1) \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$
- 2's Complement Rep.: $A = -2^{n-1} \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$

Resolution: 1

Representation of Integer Numbers

Write all possible 4-bit numbers and write its equivalent value using the three rep.

Representation of Integer Numbers

Write all possible 4-bit numbers and write its equivalent value using the three rep.

Binary Number	Signed Magnitude	1's Complement	2's Complement
0 0 0 0	+0	+0	+0
0 0 0 1	+1	+1	+1
0 0 1 0	+2	+2	+2
0 0 1 1	+3	+3	+3
0 1 0 0	+4	+4	+4
0 1 0 1	+5	+5	+5
0 1 1 0	+6	+6	+6
0 1 1 1	+7	+7	+7
1 0 0 0	-0	-7	-8
1 0 0 1	-1	-6	-7
1 0 1 0	-2	-5	-6
1 0 1 1	-3	-4	-5
1 1 0 0	-4	-3	-4
1 1 0 1	-5	-2	-3
1 1 1 0	-6	-1	-2
1 1 1 1	-7	-0	-1

Range of Numbers

Let A $= a_{n-1}a_{n-2}a_{n-3}...a_0$ is an n bit binary number
if A is an **unsigned integer**, then range of A is : 0 to $(2^n - 1)$.
if A is a **signed integer**:

- Signed Magnitude Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- 1's Complement Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- 2's Complement Rep., range of A is : -2^{n-1} to $(2^{n-1} - 1)$.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.
Let A = **1 0 1 0** is a 4-bit binary number,
Representation of A using 8-bits (i.e. B): **1 1 1 1 1 0 1 0**.
is A=B ?

- In 2's Complement Rep.: **Yes**.
- In 1's Complement Rep.: **Yes**.
- In Signed Magnitude Rep.: **No**.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.
Let A = **1 0 1 0** is a 4-bit binary number,
Representation of A using 8-bits (i.e. B): **1 1 1 1 1 0 1 0**.
is A=B ?

- In 2's Complement Rep.: **Yes**.
- In 1's Complement Rep.: **Yes**.
- In Signed Magnitude Rep.: **No**.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.
Let A = **1 0 1 0** is a 4-bit binary number,
Representation of A using 8-bits (i.e. B): **1 1 1 1 1 0 1 0**.
is A=B ?

- In 2's Complement Rep.: **Yes**.
- In 1's Complement Rep.: **Yes**.
- In Signed Magnitude Rep.: **No**.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.
Let A = **1 0 1 0** is a 4-bit binary number,
Representation of A using 8-bits (i.e. B): **1 1 1 1 1 0 1 0**.
is A=B ?

- In 2's Complement Rep.: **Yes**.
- In 1's Complement Rep.: **Yes**.
- In Signed Magnitude Rep.: **No**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a**True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a**True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a**True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a**True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

Real Numbers

1. $(4.5)_{10} = (100.1)_2$
2. $(8.25)_{10} = (1000.01)_2$
3. $(16.125)_{10} = (10000.001)_2$
4. $(0.875)_{10} = (0.111)_2$
5. $(4.5)_{10} = (1.001)_2 \times 2^2$
6. $(8.25)_{10} = (1.00001)_2 \times 2^3$
7. $(16.125)_{10} = (1.0000001)_2 \times 2^4$
8. $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.
Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent = **2** + **127** = **129** = **1000 0001**
- Mantissa = **001** = **0010000 0000 0000 0000 0000**
- Sign= + ve = 0
- $(4.5)_{10} = (1.001)_2 \times 2^2$= 0 **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent = **2** + **127** = **129** = **1000 0001**
- Mantissa = **001** = **0010000 0000 0000 0000 0000**
- Sign= + ve = 0
- $(4.5)_{10} = (1.001)_2 \times 2^2$= 0 **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

  Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

- Biased Exponent = **2** + **127** = **129** = **1000 0001**

- Mantissa = **001** = **0010000 0000 0000 0000 0000**

- Sign= + ve = 0

- $(4.5)_{10} = (1.001)_2 \times 2^2$= 0 **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent = **2** + **127** = **129** = **1000 0001**
- Mantissa = **001** = **0010000 0000 0000 0000 0000**
- Sign= + ve = 0
- $(4.5)_{10} = (1.001)_2 \times 2^2$= 0 **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent $= \mathbf{2} + \mathbf{127} = \mathbf{129} = \mathbf{1000\ 0001}$
- Mantissa $= \mathbf{001} = \mathbf{001}0000\ 0000\ 0000\ 0000\ 0000$
- Sign$= +$ ve $= 0$
- $(4.5)_{10} = (1.001)_2 \times 2^2 = 0$ **10000001 001**00000000000000000000

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

Sign	Biased Exponent	Mantissa/Fraction	

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent = **2** + **127** = **129** = **1000 0001**
- Mantissa = **001** = **0010000 0000 0000 0000 0000**
- Sign= + ve = 0
- $(4.5)_{10} = (1.001)_2 \times 2^2$= 0 **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent $=$ **2** $+$ **127** $=$ **129** $=$ **1000 0001**
- Mantissa $=$ **001** $=$ **0010000 0000 0000 0000 0000**
- Sign$= +$ ve $= 0$
- $(4.5)_{10} = (1.001)_2 \times 2^2 = 0$ **10000001 00100000000000000000000**

IEEE 754

- The IEEE Standard for Floating-Point Arithmetic.

	Sign	Biased Exponent	Mantissa/Fraction	
Single Precision N=32	1 bit	8 bits	23 bits	Bias Value:+127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value :+1023

Biased Exponent=True Exponent + Bias Value

- Rep. of $(4.5)_{10}$ using Single Precision
  $(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$
  Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a**True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.
- Biased Exponent $=$ **2** $+$ **127** $=$ **129** $=$ **1000 0001**
- Mantissa $=$ **001** $=$ **0010000 0000 0000 0000 0000**
- Sign$= +$ ve $= 0$
- $(4.5)_{10} = (1.001)_2 \times 2^2 =$ 0 **10000001 00100000000000000000000**

IEEE 754

- Biased Exponent=True Exponent + Bias Value, where $1\leq$ Biased Exponent $\leq (2^{BiasedExponentBits} - 2)$.
- Single Precision (N=32), $1\leq$ Biased Exponent ≤ 254.
- Biased Exponent $= 0$,
  - Mantissa $= \pm 0$, then Value is ± 0.
  - Mantissa $\neq 0$, then Value is **not a normalized number**.
- Biased Exponent $= 255$,
  - Mantissa $= \pm 0$, then Value is $\pm\infty$.
  - Mantissa $\neq 0$, then Value is **NAN**.
- Range of positive values: $[1.0 \times 2^{-126}, (2 - 2^{-23}) \times 2^{127}]$
- Range of negative values: $[-(2 - 2^{-23}) \times 2^{127}, -1.0 \times 2^{-126}]$
- Single Precision Number Resolution: $2^{-23} \times 2^{TrueExponent}$