1. Consider a one dimensional two class problem. Let $p(x|\omega_1)$ be a uniform distribution with parameters (3, 7). That is for $3 \le x \le 7$, the class conditional density $p(x|\omega_1) = \frac{1}{4}$ and anywhere outside to this, it is zero. Similarly $p(x|\omega_2)$ is also uniform with parameters (5, 8). Let the classifier be, if $x < 6$ decide ω_1, else decide ω_2. Let the apriori probabilities be $P(\omega_1) = \frac{1}{4}, P(\omega_2) = \frac{3}{4}$ Find error rate for this classifier.

(3,7) (5,8)

$$p(\boldsymbol{x}|\boldsymbol{\omega_1}) = \begin{cases} \frac{1}{4} & for \quad 3 \le x \le 7 \\ 0 & otherwise \end{cases}$$

$$p(\boldsymbol{x}|\boldsymbol{\omega_2}) = \begin{cases} \frac{1}{8-5} & for \quad 5 \le x \le 8 \\ 0 & otherwise \end{cases}$$

P(error) = $\int_{-\infty}^{\infty} P(\boldsymbol{error}\ |\boldsymbol{x})\ p(x)dx$

= $\int_{-\infty}^{6} P(\boldsymbol{\omega_2}|\boldsymbol{x})\ p(x)dx \ + \ \int_{6}^{\infty} P(\boldsymbol{\omega_1}|\boldsymbol{x})\ p(x)dx$

[∵ if $x < 6$ decide ω_1, else decide ω_2]

[$\because P(\boldsymbol{error}\ |\boldsymbol{x}) = P(\boldsymbol{\omega_2}|\boldsymbol{x})\ \ if\ we\ decide\ \omega_1$]

= $\int_{-\infty}^{6} \frac{p(x\ |\ \omega_2)\ P(\omega_2)}{\cancel{p(x)}} \cancel{p(x)}dx \ + \ \int_{6}^{\infty} \frac{\boldsymbol{p}(x|\omega_1)\ P(\omega_1)}{\cancel{p(x)}} \cancel{p(x)}dx$

= $P(\boldsymbol{\omega_2}) \left[\int_{-\infty}^{5} p(\boldsymbol{x}\ |\ \boldsymbol{\omega_2})\ dx + \int_{5}^{6} p(\boldsymbol{x}\ |\ \boldsymbol{\omega_2})\ dx\right] + \ P(\boldsymbol{\omega_1}) \left[\int_{6}^{7} p(\boldsymbol{x}\ |\ \boldsymbol{\omega_1})\ dx + \int_{7}^{\infty} p(\boldsymbol{x}\ |\ \boldsymbol{\omega_1})\ dx\right]$

= $\frac{3}{4}$ [0 + $\int_{5}^{6} \frac{1}{3}\ dx$] + $\frac{1}{4}$ [$\int_{6}^{7} \frac{1}{4}\ dx\ + 0$]

= ¾ $(\frac{x}{3}]_5^6) + \frac{1}{4}\ (\frac{x}{4}]_6^7)$

= $\frac{3}{4} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{4}$

= 5/ 16 = 0.3125

1. Consider a one dimensional two class problem. Let $p(x|\omega_1)$ be a uniform distribution with parameters (3, 7). That is for $3 \le x \le 7$, the class conditional density $p(x|\omega_1) = \frac{1}{4}$ and anywhere outside to this, it is zero. Similarly $p(x|\omega_2)$ is also uniform with parameters (5, 8). Let the classifier be, if $x < 6$ decide ω_1, else decide ω_2. Let the apriori probabilities be $P(\omega_1) = \frac{3}{4}, P(\omega_2) = \frac{1}{4}$ Find error rate for this classifier.

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= $\int_{-\infty}^{6} \frac{p(x\mid\omega_2)\ P(\omega_2)}{\cancel{p(x)}} \cancel{p(x)}dx\ +\ \int_{6}^{\infty} \frac{\boldsymbol{p(x|\omega_1)\ P(\omega_1)}}{\cancel{p(x)}} \cancel{p(x)}dx$

= $P(\boldsymbol{\omega_2}) \left[\int_{-\infty}^{5} p(\boldsymbol{x}\mid\boldsymbol{\omega_2})\ dx + \int_{5}^{6} p(\boldsymbol{x}\mid\boldsymbol{\omega_2})\ dx \right] + \ P(\boldsymbol{\omega_1}) \left[\int_{6}^{7} p(\boldsymbol{x}\mid\boldsymbol{\omega_1})\ dx + \int_{7}^{\infty} p(\boldsymbol{x}\mid\boldsymbol{\omega_1})\ dx \right]$

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= 13/48 = 0.2708

1. Consider a one dimensional two class problem. Let $p(x|\omega_1)$ be a uniform distribution with parameters (1, 5). That is for $1 \le x \le 5$, the class conditional density $p(x|\omega_1) = \frac{1}{4}$ and anywhere outside to this, it is zero. Similarly $p(x|\omega_2)$ is also uniform with parameters (3, 6). Let the classifier be, if $x < 4$ decide ω_1, else decide ω_2. Let the apriori probabilities be $P(\omega_1) = \frac{3}{4}, P(\omega_2) = \frac{1}{4}$ Find error rate for this classifier.

(1,5)

$$p(\boldsymbol{x}|\boldsymbol{\omega_1}) = \begin{cases} \frac{1}{4} & for \quad 1 \le x \le 5 \\ 0 & otherwise \end{cases}$$

(3,6)

$$p(\boldsymbol{x}|\boldsymbol{\omega_1}) == \begin{cases} \frac{1}{6-3} & for \quad 3 \le x \le 6 \\ 0 & otherwise \end{cases}$$

P(error) = $\int_{-\infty}^{\infty} P(\boldsymbol{error}\ |\boldsymbol{x})\ p(x)dx$

$= \int_{-\infty}^{4} P(\boldsymbol{\omega_2}|\boldsymbol{x})\ p(x)dx\ +\ \int_{4}^{\infty} P(\boldsymbol{\omega_1}|\boldsymbol{x})\ p(x)dx$

[$\because$ if $x < 4$ decide ω_1, else decide ω_2]

[$\because P(\boldsymbol{error}\ |\boldsymbol{x}) = P(\boldsymbol{\omega_2}|\boldsymbol{x})\ \ if\ we\ decide\ \omega_1$]

$= \int_{-\infty}^{4} \frac{p(x\mid\omega_2)\,P(\omega_2)}{\cancel{p(x)}} \cancel{p(x)}dx\ +\ \int_{4}^{\infty} \frac{\boldsymbol{p}(x|\omega_1)\,P(\omega_1)}{\cancel{p(x)}} \cancel{p(x)}dx$

$= P(\boldsymbol{\omega_2})\left[\int_{-\infty}^{3} p(\boldsymbol{x}\mid\boldsymbol{\omega_2})\ dx + \int_{3}^{4} p(\boldsymbol{x}\mid\boldsymbol{\omega_2})\ dx\right] + P(\boldsymbol{\omega_1})\left[\int_{4}^{5} p(\boldsymbol{x}\mid\boldsymbol{\omega_1})\ dx + \int_{5}^{\infty} p(\boldsymbol{x}\mid\boldsymbol{\omega_1})\ dx\right]$

$= \frac{1}{4}\ [\ 0 + \int_{3}^{4}\frac{1}{3}\ dx\ ] + \frac{3}{4}\ [\int_{4}^{5}\frac{1}{4}\ dx\ + 0]$

$=$ ¼ $(\frac{x}{3}]_3^4) + \frac{3}{4}\ (\frac{x}{4}]_4^5)$

$= \frac{1}{4}\cdot\frac{1}{3} + \frac{3}{4}\cdot\frac{1}{4}$

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= $\int_{-\infty}^{4} \frac{p(x \mid \omega_2)\, P(\omega_2)}{\cancel{p(x)}} \cancel{p(x)}dx \; + \; \int_{4}^{\infty} \frac{\boldsymbol{p}(x|\omega_1)\, P(\omega_1)}{\cancel{p(x)}} \cancel{p(x)}dx$

= $P(\boldsymbol{\omega_2}) \left[\int_{-\infty}^{3} p(\boldsymbol{x} \mid \boldsymbol{\omega_2})\, dx + \int_{3}^{4} p(\boldsymbol{x} \mid \boldsymbol{\omega_2})\, dx \right] + \; P(\boldsymbol{\omega_1}) \left[\int_{4}^{5} p(\boldsymbol{x} \mid \boldsymbol{\omega_1})\, dx + \int_{5}^{\infty} p(\boldsymbol{x} \mid \boldsymbol{\omega_1})\, dx \right]$

= $\frac{3}{4}$ [0 + $\int_{3}^{4} \frac{1}{3}\, dx$] + $\frac{1}{4}$ [$\int_{4}^{5} \frac{1}{4}\, dx \; + 0$]

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