Chapter 7 Sensors and Actuators

1. Show that the composition $f \circ g$ of two affine functions f and g is affine.

Solution: Assume

$$f(x) = a_1 x + b_1$$

and

$$g(x) = a_2 x + b_2.$$

Then

$$(f \circ g)(x) = a_1(a_2 x + b_2) + b_1 = (a_1 a_2)x + (a_1 b_2 + b_1),$$

which is an affine function

$$(f \circ g)(x) = a_3 x + b_3,$$

where $a_3 = a_1 a_2$ and $b_3 = a_1 b_2 + b_1$.

2. The dynamic range of human hearing is approximately 100 decibels. Assume that the smallest difference in sound levels that humans can effectively discern is a sound pressure of about 20 μPa (micropascals).

   (a) Assuming a dynamic range of 100 decibels, what is the sound pressure of the loudest sound that humans can effectively discriminate?

Solution: We have

$$100 = 20 \log_{10}\left(\frac{H - L}{p}\right)$$

where p is 20 μPa. Assume $L = 0$ (the lowest sound pressure represents no sound at all) and solve for H to get

$$H = p 10^{100/20} = 2Pa.$$

   (b) Assume a perfect microphone with a range that matches the human hearing range. What is the minimum number of bits that an ADC should have to match the dynamic range of human hearing?

Solution: At 6 dB per bit, to match 100dB, we need at least 100/6 = 16.7 bits. Since fractional bits are not possible, we need at least 17 bits.

3. The following questions are about how to determine the function

$$f\colon (L,H) \to \{0,\ldots,2^B-1\},$$

for an accelerometer, which given a proper acceleration x yields a digital number $f(x)$. We will assume that x has units of "g's," where 1g is the acceleration of gravity, approximately $g = 9.8\text{meters}/\text{second}^2$.

(a) Let the bias $b \in \{0,\ldots,2^B-1\}$ be the output of the ADC when the accelerometer measures no proper acceleration. How can you measure b?

Solution: Place the accelerometer horizontally so that there is no component of gravity along the axis being measured. In theory, you could also put the accelerometer in free fall in a vacuum, but this would require a rather complicated experimental setup, and it would also require that the accelerometer not be twisting while it falls.

(b) Let $a \in \{0,\ldots,2^B-1\}$ be the *difference* in output of the ADC when the accelerometer measures 0g and 1g of acceleration. This is the ADC conversion of the sensitivity of the accelerometer. How can you measure a?

Solution: Place the accelerometer at rest so that gravity is along the axis being measured, then subtract b.

(c) Suppose you have measurements of a and b from parts (3b) and (3a). Give an affine function model for the accelerometer, assuming the proper acceleration is x in units of g's. Discuss how accurate this model is.

Solution: The affine function model is

$$f(x) = ax + b.$$

This function has two sources of inaccuracy. First, $f(x)$ can only take on integer values in the set $\{0,\ldots,2^B-1\}$, so there will be quantization errors. Second, any proper acceleration outside the measurable range will be saturated at either 0 or 2^B-1.

(d) Given a measurement $f(x)$ (under the affine model), find x, the proper acceleration in g's.

Solution:

$$x = \frac{f(x)-b}{a}.$$

(e) The process of determining a and b by measurement is called **calibration** of the sensor. Discuss why it might be useful to individually calibrate each particular accelerometer, rather than assume fixed calibration parameters a and b for a collection of accelerometers.

Solution: Sensors vary from device to device due to manufacturing variability, so even accelerometers with identical designs may exhibit different calibration parameters.

(f) Suppose you have an ideal 8-bit digital accelerometer that produces the value $f(x) = 128$ when the proper acceleration is 0g, value $f(x) = 1$ when the proper acceleration is 3g to the right, and value

$f(x) = 255$ when the proper acceleration is 3g to the left. Find the sensitivity a and bias b. What is the dynamic range (in decibels) of this accelerometer? Assume the accelerometer never yields $f(x) = 0$.

Solution: The sensitivity is $a = 127/3$ and the bias is $b = 128$. The precision is $p = 3/127 \approx 0.024$g. The range is given by $H = 3$g and $L = -3$g. The dynamic range is therefore

$$D_{dB} = 20\log_{10}(6/0.024) = 48\text{dB}.$$