CO: Computer Organization

Day6

Indian Institute of Information Technology, Sri City

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http://co-iiits.blogspot.in/

Multiplication of Two Integers using CSAs

Input $M = m_3 m_2 m_1 m_0$
Input $Q = q_3 q_2 q_1 q_0$
$P = M \times Q$, where M is Multiplicand and Q is Multiplier.

PP_0	0	0	0	0	$m_3.q_0$	$m_2.q_0$	$m_1.q_0$	$m_0.q_0$
PP_1	0	0	0	$m_3.q_1$	$m_2.q_1$	$m_1.q_1$	$m_0.q_1$	0
PP_2	0	0	$m_3.q_2$	$m_2.q_2$	$m_1.q_2$	$m_0.q_2$	0	0
PP_3	0	$m_3.q_3$	$m_2.q_3$	$m_1.q_3$	$m_0.q_3$	0	0	0

PP_i stands for parallel product 'i'.
Result $Z = P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0$
where $P_0 = m_0.q_0$

Multiplication of Two Integers using CSAs

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Input $Q = q_3 q_2 q_1 q_0$
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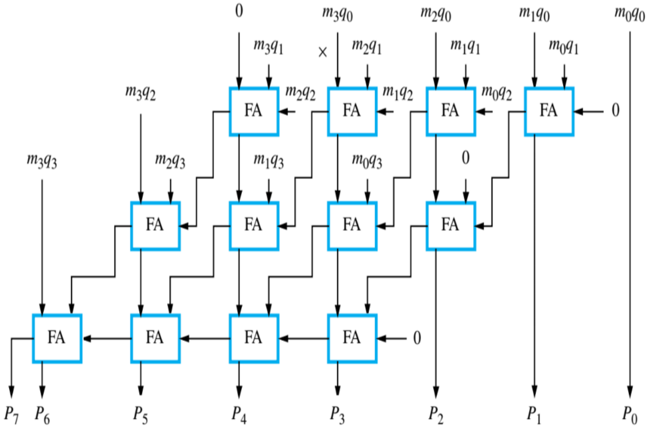


Multiplication of Two Integers using CSAs

Multiplication of Two 4-bit numbers

Level Num	No. of Summands	No. of Groups	Remaining Summands
1	4	1	1
2	3	1	0
3	2	0	0

Time=$1+2\times2+6$(using 4-bit CLAs)$=11\mathcal{T}$.

Multiplication of Two Integers using CSAs
Multiplication of Two 8-bit numbers

Level Num	No. of Summands	No. of Groups	Remaining Summands
1	8	2	2
2	6	2	0
3	4	1	1
4	3	1	0
5	2	0	0

Time=$1+2\times4+10$(using 4-bit CLAs)$=19\mathcal{T}$.

Multiplication of Two Integers using CSAs

Time= Time to generate $PPs+$ (2 x No. of levels of CSA)+(Time to perform final addition)

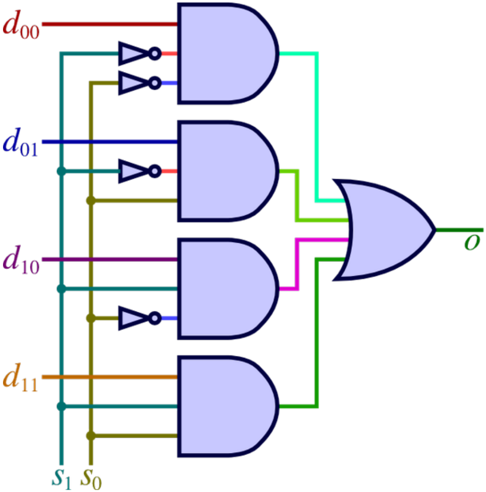
Leading Zero Detector

s_3	s_2	s_1	s_0	d_1	d_0
0	0	0	1	1	1
0	0	1	X	1	0
0	1	X	X	0	1
1	X	X	X	0	0

Leading Zero Detector takes $2\mathcal{T}$.

Assume that input and its complement are available at the same time.

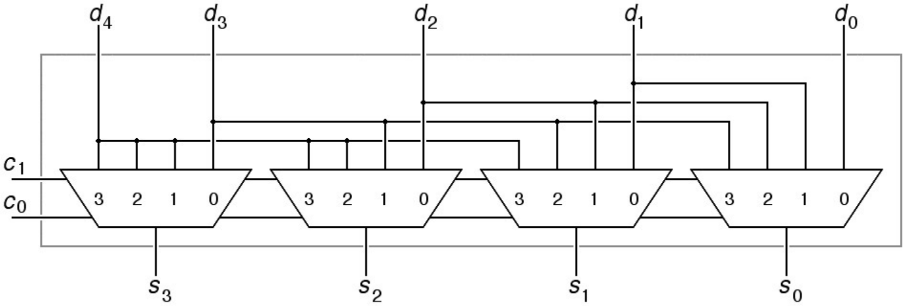
4 X 1 MUX



4 x 1 MUX takes $2\mathcal{T}$.

Assume that input and its complement are available at the same time.

Shifter



Shifter takes $2\mathcal{T}$.

Floating Point Addition/Subtraction

$$Z = X \pm Y$$

$$X = (-1)^{X_s}.(1.X_m).2^{X_e - bias}$$

$$Y = (-1)^{Y_s}.(1.Y_m).2^{Y_e - bias}$$

$$Z = (-1)^{Z_s}.(1.Z_m).2^{Z_e - bias}$$

Example1:

$X = 3.75 = (11.11) = (1.111).2^1$

$Y = 2.25 = (10.01) = (1.001).2^1$

$Z = X + Y$

$Z = (1.111).2^1 + (1.001).2^1$

$Z = (11.000).2^1 = (1.1000).2^2$

Floating Point Addition/Subtraction

Example2:

$X = 16.75 = (10000.11) = (1.000011).2^4$

$Y = 3.25 = (11.01) = (1.101).2^1 = (0.001101).2^4$

$Z = X + Y$

$Z = (1.000011).2^4 + (0.001101).2^4$

$Z = (1.010000).2^4$

Example3:

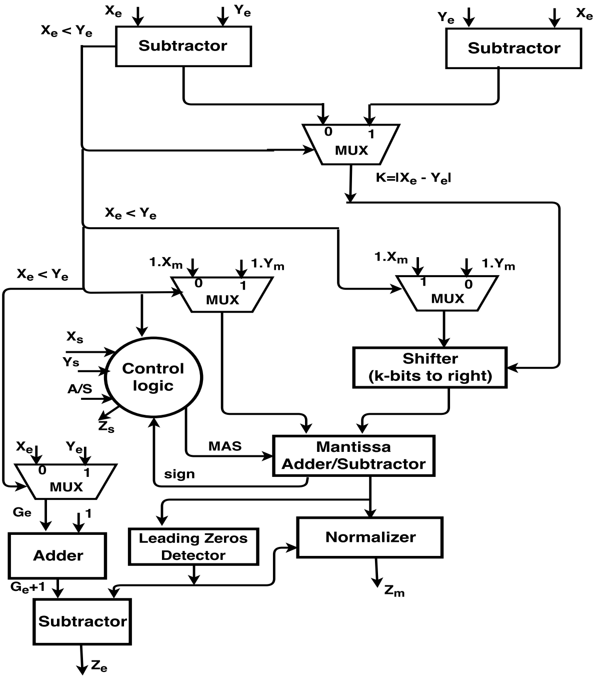
$X = 16.75 = (10000.11) = (1.000011).2^4$

$Y = 3.25 = (1.101).2^1 = (0.001101).2^4$

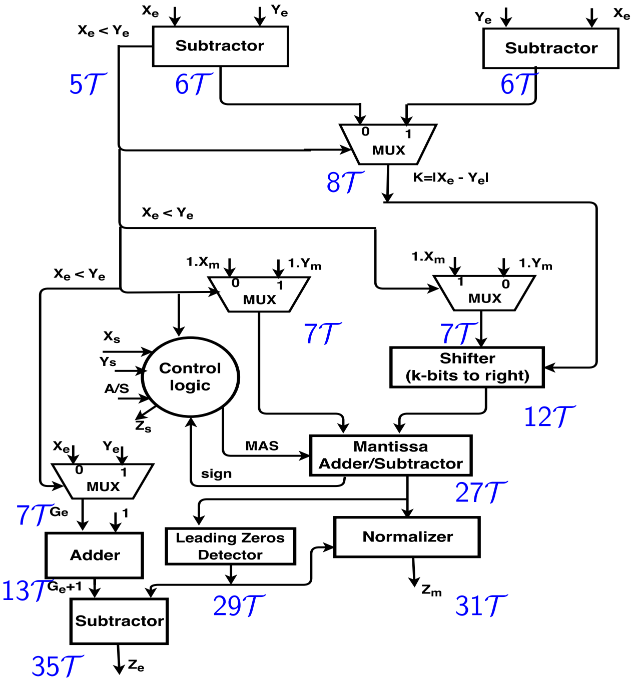
$Z = X - Y$

$Z = (1.000011).2^4 - (0.001101).2^4$

$Z = (0.110110).2^4 = (1.10110).2^3$



X_e
Y_e
$X_e < Y_e$
Subtractor
Y_e
X_e
Subtractor
0
1
MUX
$K = |X_e - Y_e|$
$X_e < Y_e$
$X_e < Y_e$
$1.X_m$
$1.Y_m$
0
1
MUX
$1.X_m$
$1.Y_m$
1
0
MUX
X_s
Y_s
A/S
Control logic
Z_s
Shifter (k-bits to right)
MAS
Mantissa Adder/Subtractor
sign
X_e
Y_e
0
1
MUX
Ge
1
Adder
Leading Zeros Detector
Normalizer
Z_m
G_e+1
Subtractor
Z_e


Xe
Ye
Xe < Ye
Subtractor
5T
6T
Ye
Xe
Subtractor
6T
0
1
MUX
8T
K=|Xe - Ye|
Xe < Ye
1.Xm
1.Ym
0
1
MUX
1.Xm
1.Ym
1
0
MUX
Xe < Ye
7T
7T
Xs
Ys
A/S
Zs
Control logic
Shifter
(k-bits to right)
12T
MAS
Mantissa Adder/Subtractor
sign
Xe
Ye
0
1
MUX
27T
7T
Ge
1
Adder
Leading Zeros Detector
Normalizer
13T
Ge+1
Subtractor
29T
Zm
31T
35T
Ze

Floating Point Multiplication

$$Z = X \times Y$$

$$X = (-1)^{X_s}.(1.X_m).2^{X_e - bias}$$

$$Y = (-1)^{Y_s}.(1.Y_m).2^{Y_e - bias}$$

$$Z = (-1)^{Z_s}.(1.Z_m).2^{Z_e - bias}$$

Floating Point Multiplication

$$Z = X \times Y = (-1)^{X_s \bigoplus Y_s}.(1.X_m \times 1.Y_m).2^{(X_e+Y_e-(2\times bias))}$$

Example1:

$$X = 3.75 = (11.11) = (1.111).2^1$$

$$Y = 3.75 = (11.11) = (1.111).2^1$$

$$Z = (1.111).2^1 \times (1.111).2^1$$

$$Z = (11.100001).2^2 = (1.1100001).2^2$$

Floating Point Multiplication

$$Z = X \times Y = (-1)^{X_s \bigoplus Y_s}.(1.X_m \times 1.Y_m).2^{(X_e + Y_e - (2 \times bias))}$$

Example1:

$$X = 3.75 = (11.11) = (1.111).2^1$$

$$Y = 3.75 = (11.11) = (1.111).2^1$$

$$Z = (1.111).2^1 \times (1.111).2^1$$

$$Z = (11.100001).2^2 = (1.1100001).2^2$$

Floating Point Multiplication

$$Z = X \times Y = (-1)^{X_s \bigoplus Y_s}.(1.X_m \times 1.Y_m).2^{(X_e + Y_e - (2 \times bias))}$$

Example2:

$$X = 7.875 = (-1)^0.(1.11111) = (-1)^0.(1.11111).2^2$$

$$Y = -3.0 = (-1)^1.(11.00) = (-1)^1.(1.100).2^1$$

$$Z = (-1)^{0 \bigoplus 1}.(1.11111 \times 1.100).2^{2+1}$$

$$Z = (-1)^1.(1.0111101).2^4 = -23.625$$

Floating Point Multiplication

$$Z = X \times Y = (-1)^{X_s \bigoplus Y_s}.(1.X_m \times 1.Y_m).2^{(X_e + Y_e - (2 \times bias))}$$

Example2:

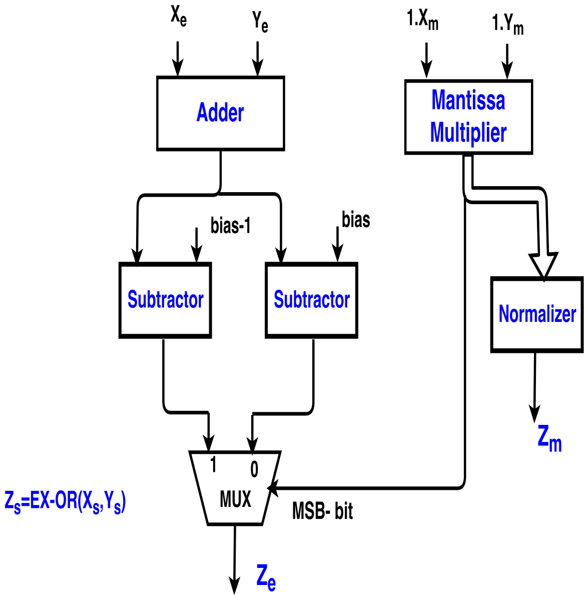
$$X = 7.875 = (-1)^0.(1.11111) = (-1)^0.(1.11111).2^2$$

$$Y = -3.0 = (-1)^1.(11.00) = (-1)^1.(1.100).2^1$$

$$Z = (-1)^{0 \bigoplus 1}.(1.11111 \times 1.100).2^{2+1}$$

$$Z = (-1)^1.(1.0111101).2^4 = -23.625$$

Floating Point Multiplier



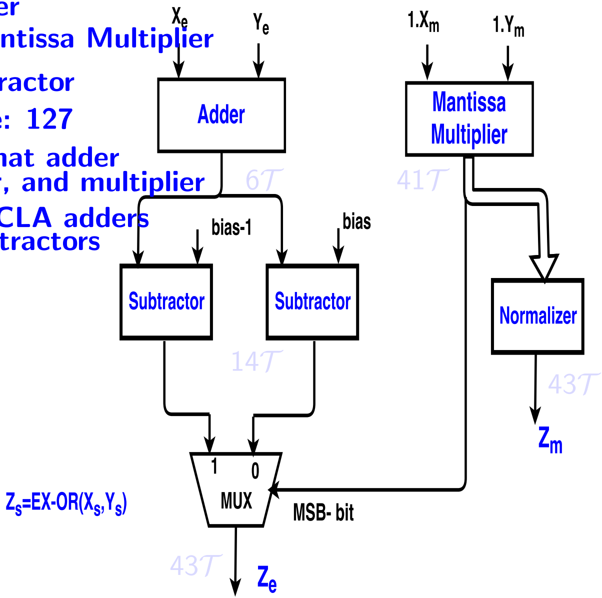
Single-precision Floating Point Multiplier

8-bit Adder
24-bit Mantissa Multiplier

9-bit subtractor

Bias value: 127

Assume that adder subtractor, and multiplier use 4-bit CLA adders /BLA subtractors



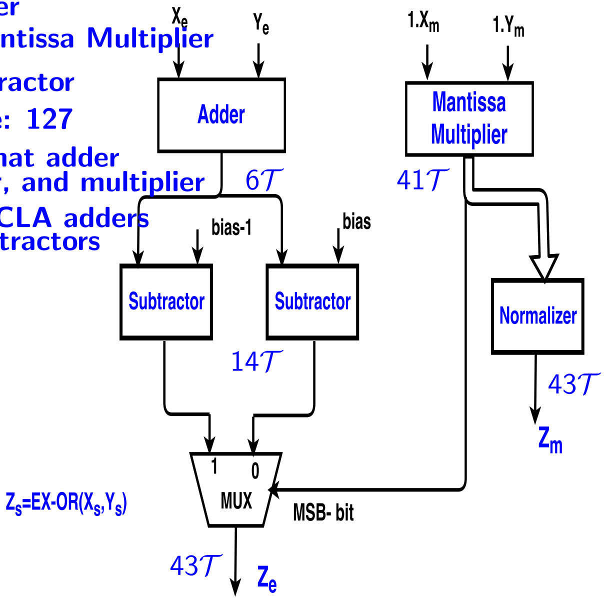
Single-precision Floating Point Multiplier

8-bit Adder
24-bit Mantissa Multiplier

9-bit subtractor

Bias value: 127

Assume that adder subtractor, and multiplier use 4-bit CLA adders /BLA subtractors



4-stage Pipeline for Single-precision FP Multiplications

1. Stage 1: Z_s Generator, Addition of Exponents, PP generation, 5-levels of CSA.
2. Stage 2: 2 levels of CSA, 12-bit addition, exponent subtractors.
3. Stage 3: 20-bit adder .
4. Stage 4: 16-bit adder, Normalizer.

4-stage Pipeline for Single-precision FP Additions

1. Stage 1: Difference between exponents, inputs selection using $X_e < Y_e$ signal.
2. Stage 2: shifter, MAS 8-bit addition, $G_e + 1$ generator.
3. Stage 3: MAS 16-bit addition.
4. Stage 4: LZD, Normalizer, Subtactor.