Computer Graphics and Multimedia

Tutorial 3 : Interpolation and Approximating Curves

1. Four points $P_0, \ldots, P_3$ are given. Find a polynomial curve (PC) that passes through these points and has the form

$$\begin{aligned} P(t) &= at^3 + bt^2 + ct + d \qquad (1) \\ &= [t^3\ t^2\ t\ 1][a\ b\ c\ d]^T \\ &= T(t)A\ for\ t \in [0, 1]. \end{aligned}$$

   Plot the polynomial blending functions.

2. Hermite interpolation is based on two points P_1 and P_2 and two tangent vectors P_1^t and P_2^t. It computes a curve segment that starts at P_1, going in direction P_1^t and ends at P_2 moving in the direction P_2^t. Derive the expression of the Hermite interpolation $P(t) = T(t)HB$, where B is the column $[P_1\ P_2\ P_1^t\ P_2^t]^T$. Matrix H is called the Hermite basis matrix. Plot the Hermite blending functions.

3. The first approach to the Bezier curve expresses it as a weighted sum of the points. Each control point is multiplied by a weight and the products are added. We denote, the control points by $P_0, \ldots, P_n$ (n is therefore defined as 1 less than the number of points) and the weights by B_i[1]. The expression of weighted sum is

$$P(t) = \sum_{t=0}^{n} P_i B_{n,i}(t),\ 0 \le t \le 1, \qquad (2)$$

   where $B_{n,i}(t) = \binom{n}{i} t^i (1-t)^{n-i}$.

   Calculate the Bezier curve for the four points $P_0 = (0, 0, 0)$, $P_1 = (1, 0, 0)$, $P_2 = (2, 1, 0)$ and $P_3 = (3, 0, 1)$. Notice that this is a space curve since the first three points are in the $z = 0$ plane, while the fourth one is outside that plane. Calculate the (unnormalized) principal normal vector of the curve and find its values for $t = 0,\ 0.5$, and 1. Calculate the osculating plane of the curve and find its equations for $t = 0,\ 0.5$, and 1 as above.

4. Calculate the corner points and boundary curves of the surface patch

$$P(u, v) = ((c - a)u + a, (d - b)w + b, 0), \qquad (3)$$

   where $a,\ b,\ c$ and d are given constants and the parameters u and w vary independently in the range $[0, 1]$. What kind of a surface is this?

[1] Berntein's polynomials