NUMBERS

DRILL – 1 – TO FIND THE NUMBER OF FACTORS

Steps: Express the number as $N = a^p \times b^q \times c^r$

No. of factors = $(p+1)(q+1)(r+1)$

Sum of the factors = $[a^{p+1} - 1 / a - 1]\ [b^{q+1} - 1 / b - 1]\ [\ c^{r+1} - 1 / c - 1]$

Product of the factors = $N^{(p+1)(q+1)(r+1)/2}$ (Including 1 & itself)

Cross check for the number 15.

Now complete the following table:

Number	No. Of Factors	Sum of divisors	Product of divisors
60			
36 x 36			
126 x 440			
52900			

DRILL 2 – TO FIND THE NUMBER OF ENDING ZEROES

The number of zeroes at the end of any product is the number of actual 2's or 5's whichever is less.
In the case of n!, the number of ending zeroes is $n/5 + n/5^2 + n/5^3 + \ldots n/5^n$ where $n >= 5^n$

Numbers	Zeroes	Numbers	Zeroes
25!		100!	
50!		200!	
25! + 50!		100! + 200!	
25! x 50!		100! x 200!	
136!		252!	
140!		244!	
136! + 140!		252! + 244!	
136! x 140!		252! x 244!	

DRILL 3 – TO FIND THE LAST DIGIT

Let the number be $(xyz)^n$

Divide n by 4 and check the table to find the last digit. Also complete the next table based on the same strategy.

Remainder	Last digit
0, z is even	6
0, z is odd except 5	1
1	Z
2	Z^2
3	Z^3

Expression	Last Digit	Expression	Last Digit
2^9		15743^{577}	
12^4		6525^{899}	
336^{21}		$(ab......2)^{4n+1}$	
$(ab......3)^{4n+3}$		$45^{25} \times 36^{45}$	
$99^{11} \times 11^{99} \times 34^{43}$		$100^{21} \times 21^{103}$	

DRILL 4 – TO FIND THE REMAINDER

- $X^n + 1$ will always be divisible by X + 1 only when n is odd.
- $X^n – 1$ will always be divisible by X + 1 only when n is even.
- $x^n – a^n$ is always divisible by x – a for all values of n.
- $x^n – a^n$ is always divisible by x + a for even values of n.
- $x^n + a^n$ is always divisible by x + a for odd values of n.
- $x^n + a^n$ is not divisible by x – a for any value of n.
- For any value of n, if any number $(kx + 1)^n$ divided by x will leave a remainder 1^n
- When p is a prime number and N is any natural number not divisible by p, then N^{p-1} if divided by p will leave a remainder 1.

➢ What will be the remainder when $(67^{67} + 67)$ is divided by 68?

a. 1 b. 63 c. 66 d. 67

➢ Which of the following numbers will completely divide $(49^{15} - 1)$?

a. 8 b. 14 c. 46 d. 50

➢ What will be remainder when 17^{200} is divided by 18?

a. 17 b. 16 c. 1 d. 2

DRILL 5 - TO FIND THE REMAINDER

X ÷ D = R1 (Remainder)

X ÷ d =? (Remainder – R2), where d is a factor of D.

Then the required remainder R2 is the remainder when the larger remainder R1 is divided by smaller divisor‘d’.

- On dividing a number by 56, we get 29 as remainder. On dividing the same number by 8, what will be the remainder?
  a. 4 b. 5 c. 6 d. 7

- On dividing a number by 357, we get 39 as remainder. On dividing the same number by 17, what will be the remainder?
  a. 0 b. 3 c. 5 d. 11

- On dividing a number by 527, we get 42 as remainder. On dividing the same number by 17, what will be the remainder?
  a. 4 b. 6 c. 8 d. 14

DRILL 6 – ALGEBRAIC FORMULAE

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = a^3 + b^3 + 3ab(a+b)$
- $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2 = a^3 - b^3 - 3ab(a-b)$
- $(a^2 - b^2) = (a + b)(a - b)$
- $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$
- $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$
- $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

- [(753 x 753) + (247 x 247) – (753 x 247)] / [(753 x 753 x 753) + (247 x 247 x 247)]
  a. 1 / 1000 b. 1/506 c. 253 / 500 d. NOTA

- $(963 + 476)^2 + (963 - 476)^2$ / 963 x 963 + 476 x 476
  a. 1449 b. 497 c. 2 d. 4

- $(489 + 375)^2 – (489 - 375)^2$ / 489 x 375
  a. 144 b. 864 c. 2 d. 4

- (397 x 397) + (104 x 104) + 2 x 397 x 104
  a. 250001 b. 251001 c. 260101 d. 261001

- (768 x 768 x 768) + (232 x 232 x 232) / (768 x 768) – (768 x 232) + (232 x 232)
  a. 1000 b. 536 c. 500 d. 268

- (854 x 854 x 854) – (276 x 276 x 276) / (854 x 854) + (854 x 276) + (276 x 276)
  a. 1130 b. 578 c. 565 d. 1156

DRILL 7 – DIVISIBILITY TEST

- If the number 517a324 is divisible by 3, then the smallest whole number to replace ‘a’ is
  a. 0 b. 1 c. 2 d. NOTA

- Which of the following number is divisible by 24?
  a. 35718 b. 63810 c. 537804 d. 3125736

- If the product 4862 x 9P2 is divisible by 12, then the value of P is
  a. 1 b. 5 c. 6 d. 8

- 476ab0 is divisible by both 3 and 11. The non zero values of a & b are
  a. 7 & 4 b. 7 & 5 c. 8 & 5 d. NOTA

- If the number 42573x is divisible by 72, then the least value of x is
  a. 4 b. 5 c. 6 d. 7

DRILL 8 - PROGRESSIONS

Arithmetic Progression
$t_n = a + (n – 1)d$

$S_n = n[2a + (n – 1)d] / 2$
where 'a' is the first term, 'd' is the common difference, t_n is the nth term and S_n is the sum of n terms.

Geometric Progression
$t_n = a\ r^{(n-1)}$
$S_n = a(r^n – 1) / r – 1$
where 'a' is the first term, 'd' is the common ratio, t_n is the nth term and S_n is the sum of n terms.
Also

Sum of first n natural numbers = *n(n+1)/2*
Sum of the squares of first n natural numbers = *n(n+1)(2n+1)/6*
Sum of the cubes of first n natural numbers = $[n^2(n + 1)^2] / 4$
Sum of first n natural odd numbers = n^2

- Find the 10th term of the A. P.: 2, 4, 6, ...
  (a) 16 (b) 18 (c) 20 (d) 24

- The 10th term of an A. P. is – 15 and 31st term is –57, find the 15th term.
  (a) -25 (b) -30 (c) -34 (d) -38

- Is 600 a term of the A. P.: 2, 9, 16, ...?
  (a) Yes (b) No (c) Data Insufficient (d) CBD

- Which term of the A. P. 2 ½, 4, 5 ½, , , , ... is 31? Find also the 10th term?
  (a) 10^{th} term & 31 (b) 20^{th} term & 16 (c) 15^{th} term & 12 (d) NOTA

- The 35th term of an A. P. is 69. Find the sum of its 69 terms.
  (a) 4204 (b) 4486 (c) 4761 (d) CBD

- Find the 6th term of the G. P.: 4, 8, 16, ...
  (a) 48 (b) 64 (c) 80 (d) 128

- The 1^{st} and the 9th term of a G. P. are 1 and 256 respectively. Find the G. P.
  (a) 1, 2, 4, 8, 16 (b) 1, 4, 16, 32, 64 (c) 1, 2, 4, 16, 32 (d) 2, 4, 8, 16, 32

- Which term of the G. P.: 5, –10, 20, – 40, ... is 320?

(a) 6th term (b) 7th term (c) 8th term (d) 9th term

DRILL 9 – HCF & LCM

Important formulae:
For any 2 numbers A and B, $(HCF)_{A,B} \times (LCM)_{A,B} = A \times B$

LCM of fractions = LCM of numerators / HCF of Denominators
HCF of fractions = HCF of numerators / LCM of Denominators

Complete the table:

A	B	HCF (A,B)	LCM (A,B)
12	9		
34	50		
25		5	200
	60	6	1260
7		1	35
¾	½		
4/7	5/7		
1/3	1/6		
5/7	7/5		

DRILL 10 – HCF & LCM – RAPID INFORMATION LIST

S.No	*Type of Problem*	*Approach to Problem*
1	Find the **greatest number** that will exactly divide x, y and z.	Required number = HCF of x, y and z

2	Find the **greatest number** that will divide x, y and z leaving remainders a, b and c respectively	Required number = HCF of (x – a), (y – b) and (z – c)
3	Find the **least number** that is exactly divisible by x, y and z.	Required number = LCM of x, y and z
4	Find the **least number** which when divided by x, y and z leaves remainder a, b and c respectively.	Then it is observed that x – a = y – b = z – c = k Required number = (LCM of x, y, z) - k
5	Find the **least number** which when divided by x, y and z leaves the same remainder 'r'	Required number = (LCM of x, y, z) + r
6	Find the **greatest number** that will divide x, y and z leaving same remainder in each case	Required number = HCF of (x - y), (y – z) and (z – x)

Find the following:

- The greatest number that will exactly divide 200 and 320
- The greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively
- The least number which when divided by 27, 35, 45 and 49 leaves remainder 6 in each case
- A number when divided by 11, 13 and 17 leaves remainders of 7, 9 and 3 respectively. Find the smallest such 4-digit number.
  (a) 2427 (b) 2856 (c) 2586 (d) None of these
- Find the smallest number which when divided by 4, 11 or 13 leaves a remainder of 1 and is greater than the remainder?
  (a) 543 (b) 573 (c) 512 (d) 532
- Find the smallest number which when divided by 9 and 11 leaves remainders of 7 and 9 respectively?
  (a)88 (b) 97 (c) 94 (d) 95

Drill – 11 – BASE OF A NUMBER

BASE SYSTEM	BASE	NUMERS USED	NO.OF DIGITS USED
DECIMAL SYSTEM	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	10
OCTAL SYSTEM	8	0, 1, 2, 3, 4, 5, 6, 7	8
BINARY SYSTEM	2	0, 1	2
IN GENERAL, A NUMBER SYSTEM WITH BASE, B		0 TO B-1	B

Convert the given numbers to their equivalents in other bases across the rows:

Base 2	Base 5	Base 8	Base 10
			39
110101			
		74	

GOOGLY QUESTIONS

1. How many positive integers less than 300 are divisible by both 9 and 4?
   Solution:
   Numbers less than 300 divisible by 9 = 33
   Numbers less than 300 divisible by 4 = 75
   Numbers less than 300 divisible by both 9 and 4 = 75 + 33 = 108

2. Find the number of zeroes at the end of 250! + 300!
   Solution:
   Number of zeroes at the end of 250! = 62
   Number of zeroes at the end of 300! = 74
   Number of zeroes at the end of 250! + 300! = 62 + 74 = 136