IIIT, Sricity

Mathematics-2 (Calculus)

Assignment-2

1. Evaluate the following integrals

(a) $\int_0^3 \int_1^2 xy(x+y)\,dx\,dy$

(b) $\int_0^a \int_x^a (x^2+y^2)\,dx\,dy$

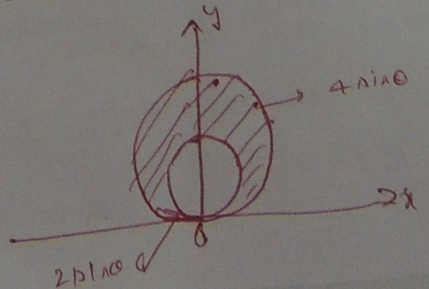
2. Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

3. Show that $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2}\,dx\,dy = 8a^2\left(\frac{\pi}{2} - \frac{5}{3}\right)$ by changing the variables to polar coordinates

4. Evaluate

(a) $\int_0^{\infty} \int_0^{\pi/2} e^{r^2} r\,d\theta\,dr$

(b) $\iint r^3\,dr\,d\theta$ over the area included between the circles $r = 2a\sin\theta$ and $r = 4a\sin\theta$



5(a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$. Plot the region of integration.

(b) Change the order of integration and evaluate the same question 5(a)

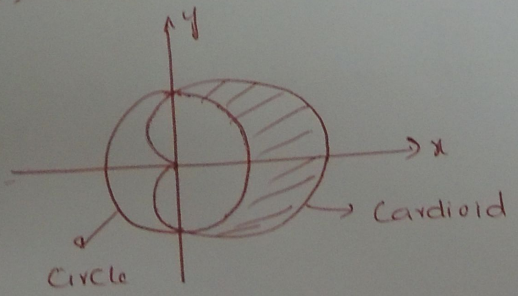
6. Evaluate triple integral

$$\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz\,dx\,dy.$$

7. Evaluate $\iiint_V z^2\,dx\,dy\,dz$ over the region V bounded by the surfaces $x^2+y^2=a^2$, $x^2+y^2=z$ and $z=0$

8. Find the area which is inside the Cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$

[Hint:- Answer $= \frac{a^2}{4}(\pi+8)$]



9. Find the volume of the solid enclosed between the two surfaces $z=x^2+9y^2$ and $z=18-x^2-9y^2$

10. Find the centre of mass of the thin surface enclosed by the parabola $y^2=4ax$, the x axis and its latus rectum ($x=a$).