DIGITAL IMAGE PROCESSING

Image Enhancement in Spatial Domain : Session 2

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Today's Lecture

- Image Enhancement in Spatial Domain
  - Histogram Equalization
  - Histogram Matching

Image Enhancement in Spatial Doma

Histogram Processing

Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity value

n_k is the number of pixels in the image with intensity r_k

Normalized histogram $p(r_k) = \dfrac{n_k}{MN}$

n_k: the number of pixels in the image of

size $\mathrm{M} \times \mathrm{N}$ with intensity r_k

Image Enhancement in Spatial Doma

Histogram Processing

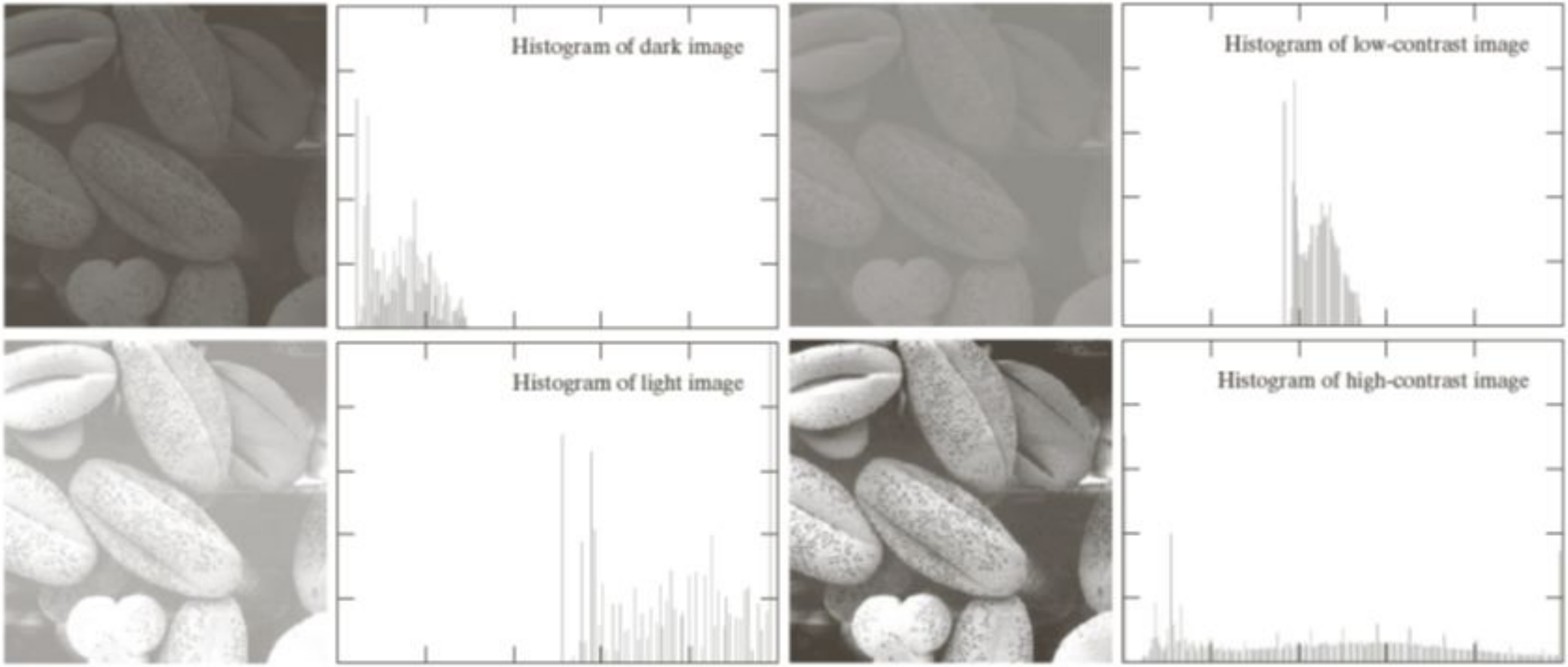


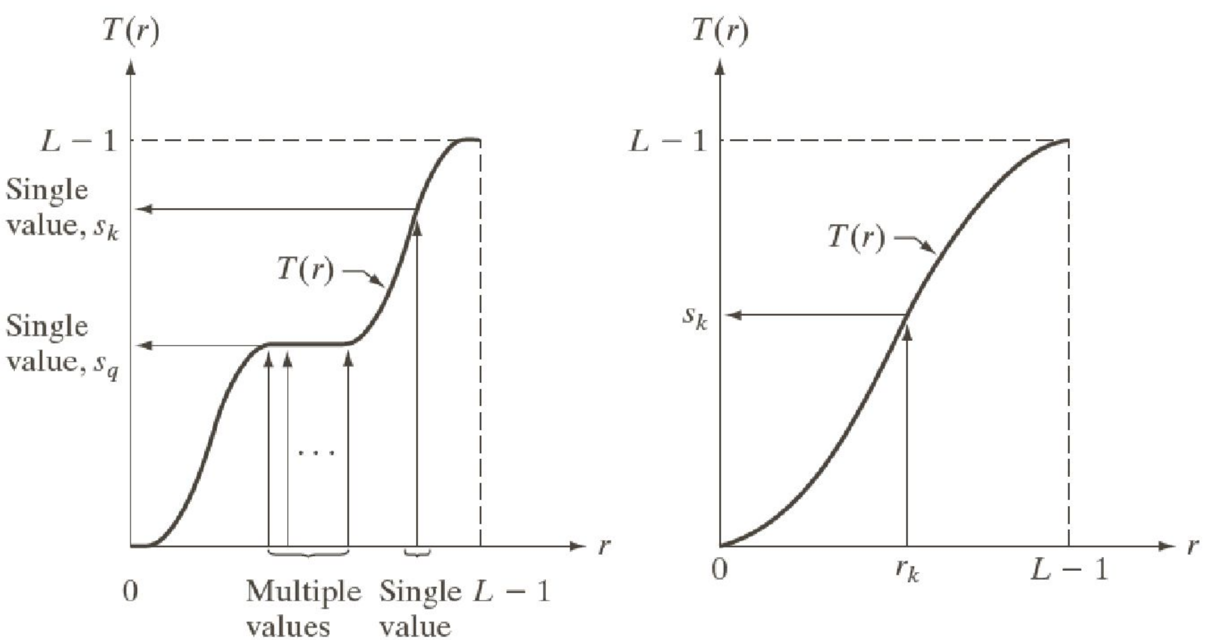
Image Enhancement in Spatial Doma

Histogram Equalization

$$s = T(r) \quad 0 \le r \le L - 1$$

a) $T(r)$ is single-valued and strictly monotonically increasing function in the interval $\mathbf{0 \le r \le L - 1}$.

b) $0 \le T(r) \le L - 1$ for $0 \le r \le L - 1$.



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

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Histogram Equalization

- The objective is to get a uniform histogram of the resultant image $T(r)$.
- The intensity level in an image may be viewed as random variables in the interval $[0\ L-1]$.
- Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s.
- **$T(r)$ is continuous and differentiable**
- **If $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies condition (a)** then the PDF $p_s(s)$ of the transformed random variable s can be obtained by

$$\boldsymbol{p_s(s) = p_r(r)\frac{dr}{ds}}$$

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Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

Cumulative Distribution Function
Satisfies condition (a) and (b)

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right]$$

$$= (L-1)\,p_r(r)$$

$$p_s(s) = \frac{p_r(r)\,dr}{ds} = p_r(r) \Big/ \left(\frac{ds}{dr}\right) = p_r(r) \Big/ \left((L-1)\,p_r(r)\right) = \frac{1}{L-1}$$

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Histogram Equalization

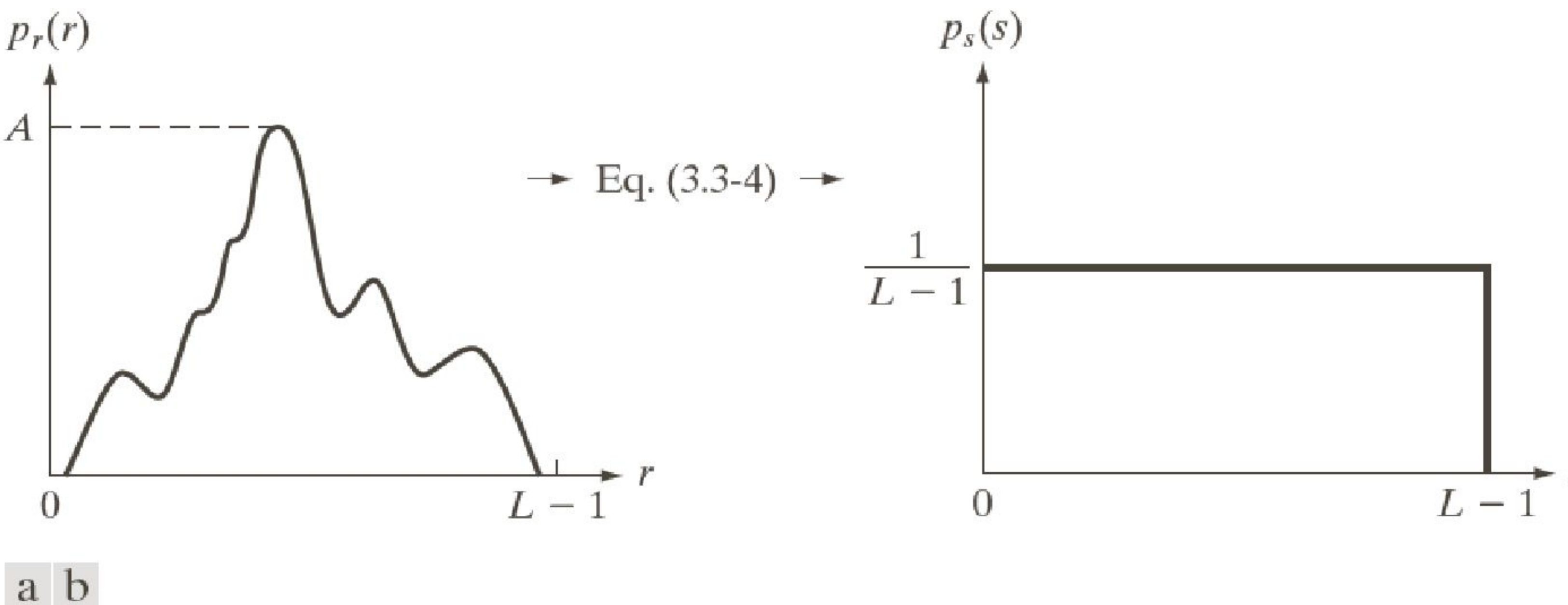

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

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Histogram Equalization: Example

Suppose that the (continuous) intensity values in an image have the PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L\text{-}1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function for equalizing the image histogram.

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Histogram Equalization: Example

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$= (L-1)\int_0^r \frac{2w}{(L-1)^2} dw$$

$$= \frac{r^2}{L-1}$$

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Histogram Equalization

Continuous case:

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad \text{k=0,1,..., L-1}$$

$p_r(r_j)$ is the probability of Occurrence of gray level r_j in an image

histogram equalization or linearization

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Histogram Equalization: Example

Suppose that a 3-bit image ($L = 8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in following table. Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

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Histogram Equalization: Example

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \quad \rightarrow 1$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \quad \rightarrow 3$$

$$s_2 = 4.55 \quad \rightarrow 5 \qquad s_3 = 5.67 \quad \rightarrow 6$$

$$s_4 = 6.23 \quad \rightarrow 6 \qquad s_5 = 6.65 \quad \rightarrow 7$$

$$s_6 = 6.86 \quad \rightarrow 7 \qquad s_7 = 7.00 \quad \rightarrow 7$$

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Histogram Equalization: Example

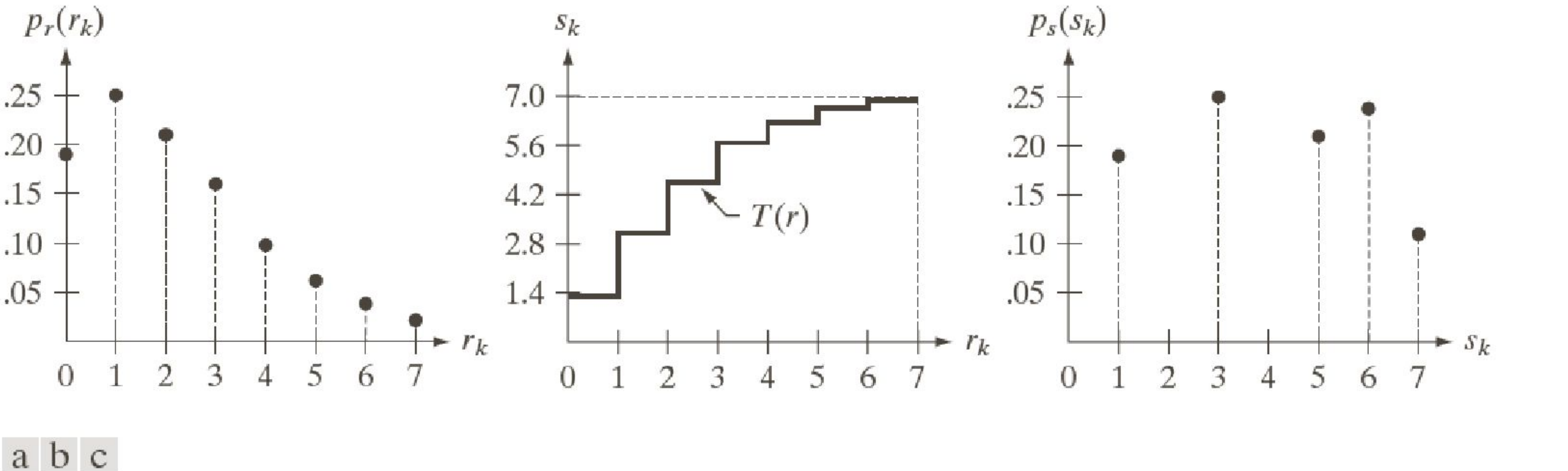

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

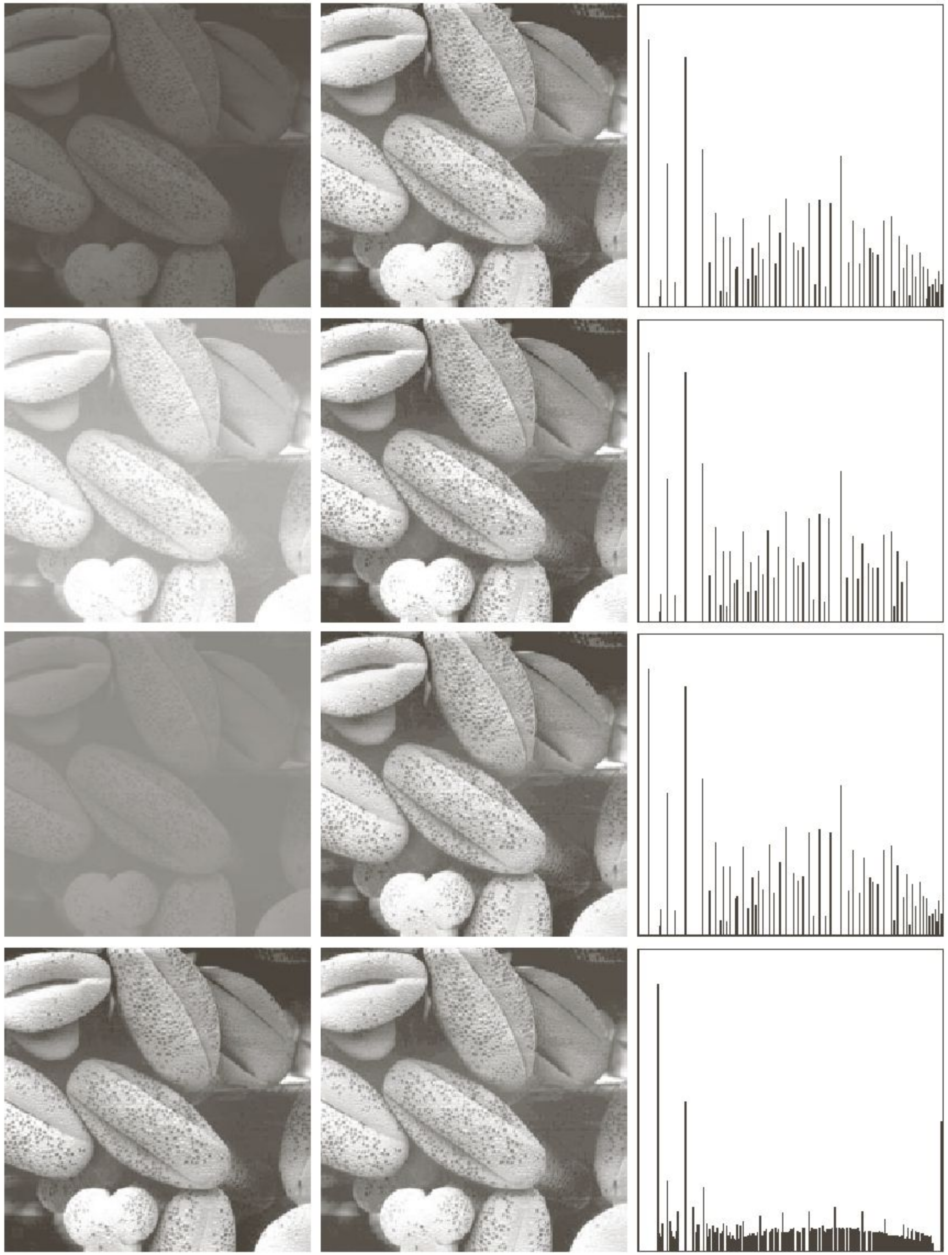

FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

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Histogram Matching (Histogram Specification)

- **Generate a processed image that has a specified histogram**

Let $p_r(r)$ and $p_z(z)$ denote the continous probability density functions of the variables r and z. $p_z(z)$ is the specified probability density function.

Let s be the random variable with the probability

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Define a random variable z with the probability

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

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Histogram Matching (Histogram Specification)

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

Note: $T(r)$ can be obtained once $p_r(r)$ has been estimated from the input image. $G(z)$ can be obtained since $p_z(z)$ is given.

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Histogram Matching: Procedure

- Obtain $p_r(r)$ from the input image and then obtain the values of s

$$s = (L-1)\int_0^r p_r(w)dw$$

- Use the specified PDF and obtain the transformation function $G(z)$

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

- Mapping from s to z

$$z = G^{-1}(s)$$

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Histogram Matching: Example

- Assuming continuous intensity values, suppose that an image has the intensity PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L\text{-}1 \\ 0\ , & \text{otherwise} \end{cases}$$

- Find the transformation function that will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \dfrac{3z^2}{(L-1)^3}, & \text{for } 0 \le z \le (L\text{-}1) \\ 0, & \text{otherwise} \end{cases}$$

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Histogram Matching: Example

- Find the histogram equalization transformation for the input image

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = (L-1)\int_0^r \frac{2w}{(L-1)^2}dw$$

- Find the histogram equalization transformation for the specified histogram

$$G(z) = (L-1)\int_0^z p_z(t)dt = (L-1)\int_0^z \frac{3t^2}{(L-1)^3}dt = \frac{z^3}{(L-1)^2} = s$$

- The transformation function

$$z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)^2 \frac{r^2}{L-1}\right]^{1/3} = \left[(L-1)r^2\right]^{1/3}$$

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Histogram Matching: Discrete Case

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k, round the value to the integer range $[0\ L-1]$.

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range $[0\ L-1]$.

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

- Mapping from s_k to z_q

$$z_q = G^{-1}(s_k)$$

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Histogram Matching: Example (Discrete Case)

Suppose that a 3-bit image ($L = 8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in following table(on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

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Histogram Matching: Example (Discrete Case)

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_5) = 4.55 \rightarrow 5 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_7) = 7.00 \rightarrow 7$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

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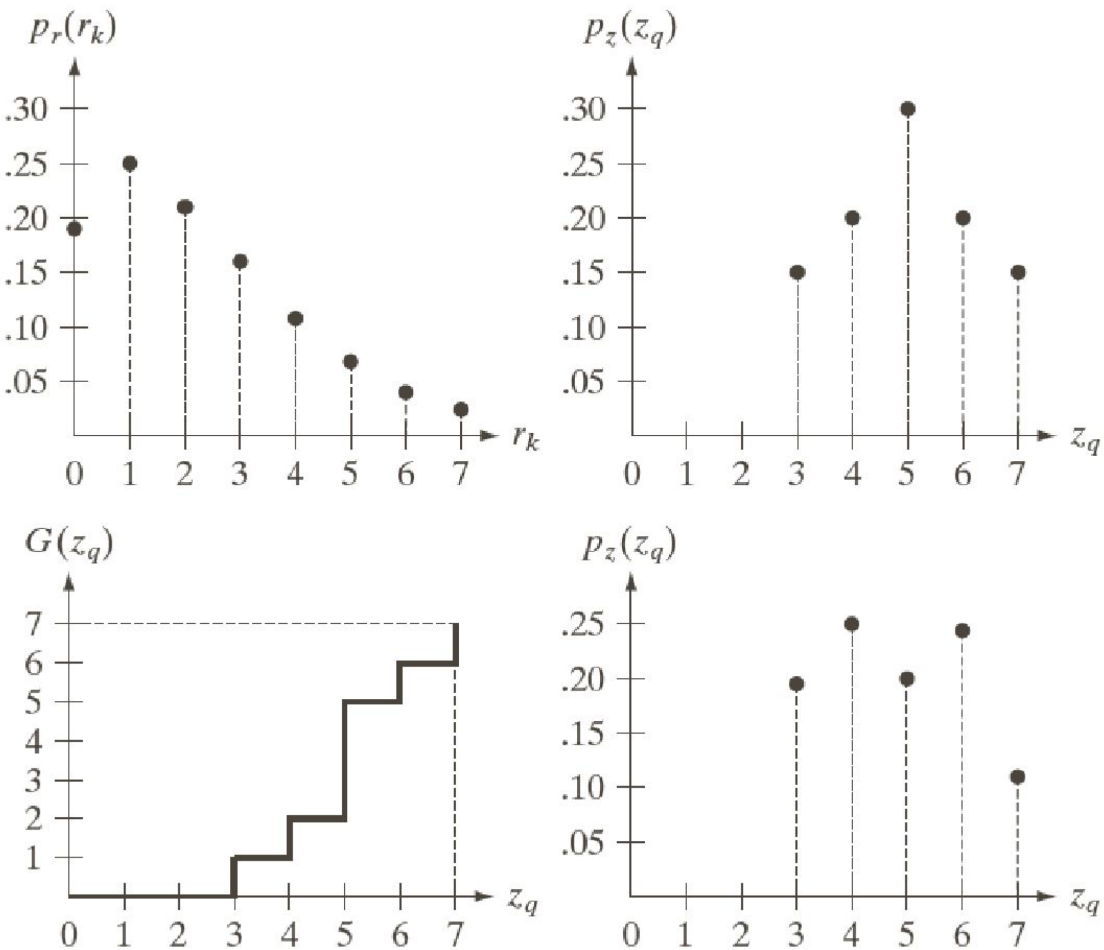
Histogram Matching: Example (Discrete Case)



a	b
c	d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Image Enhancement in Spatial Doma

Histogram Matching: Example (Discrete Case)

- Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$

$$s_5 = 7, s_6 = 7, s_7 = 7.$$

- Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1 \quad \mathbf{s_0} \qquad G(z_4) = 2.45 \rightarrow 2 \quad \mathbf{s_1}$$

$$G(z_5) = 4.55 \rightarrow 5 \quad \mathbf{s_2} \qquad G(z_6) = 5.95 \rightarrow 6 \quad \mathbf{s_3}$$

$$G(z_7) = 7.00 \rightarrow 7 \quad \mathbf{s_4} \quad \mathbf{s_5} \quad \mathbf{s_6} \quad \mathbf{s_7}$$

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Histogram Matching: Example (Discrete Case)

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$

$$s_5 = 7, s_6 = 7, s_7 = 7.$$

r_k

0

1

2

3

4

5

6

7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

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Histogram Matching: Example (Discrete Case)

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

$$4 \rightarrow 7$$

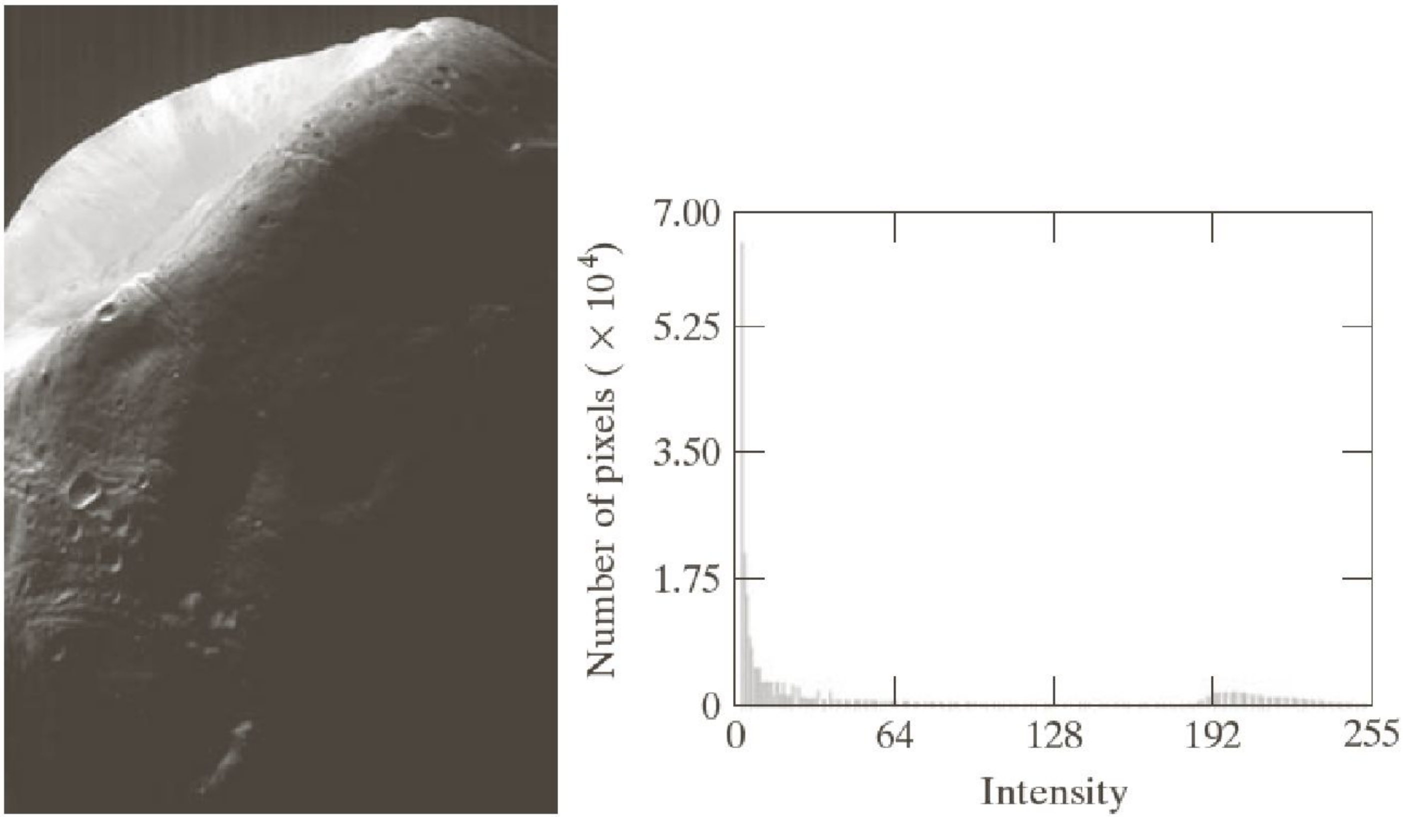
$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

Image Enhancement in Spatial Doma

Histogram Matching: Example



a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

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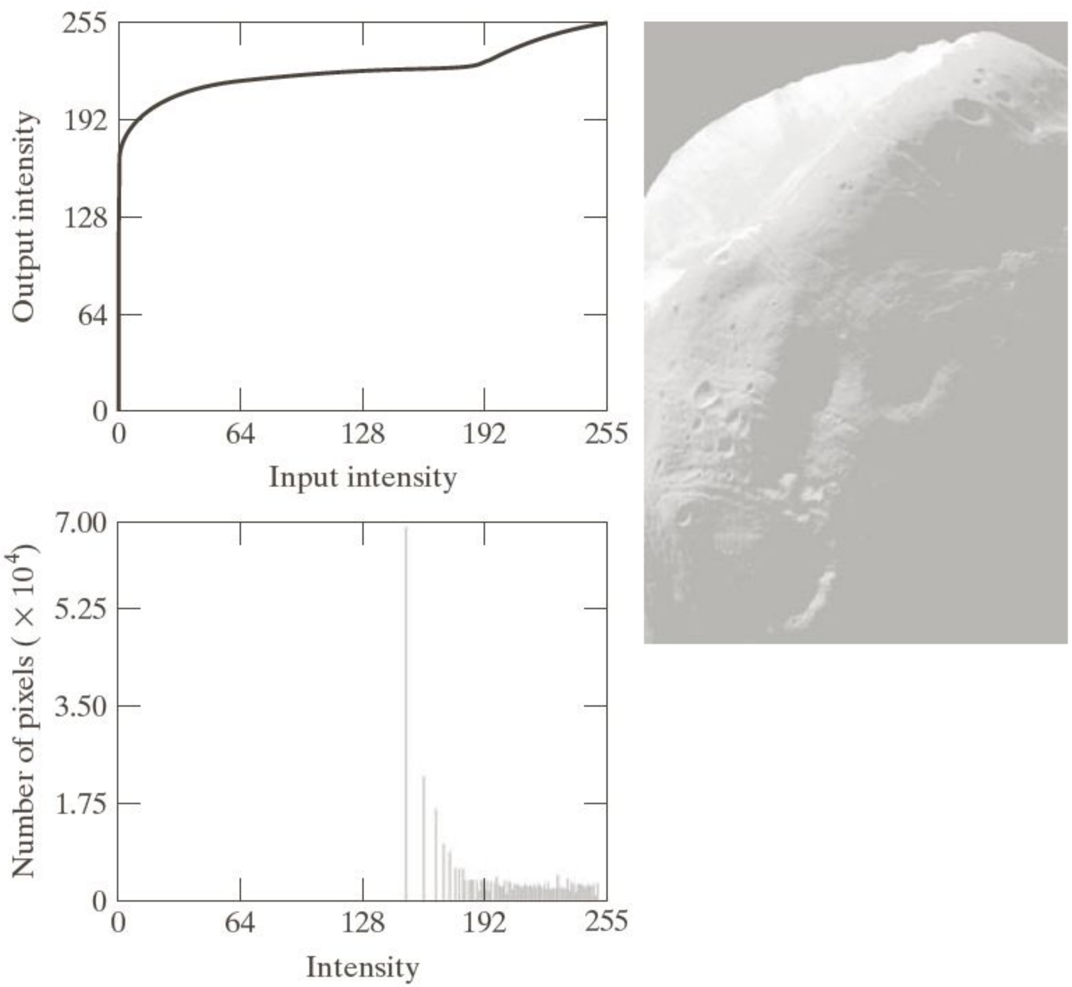
Histogram Matching: Example

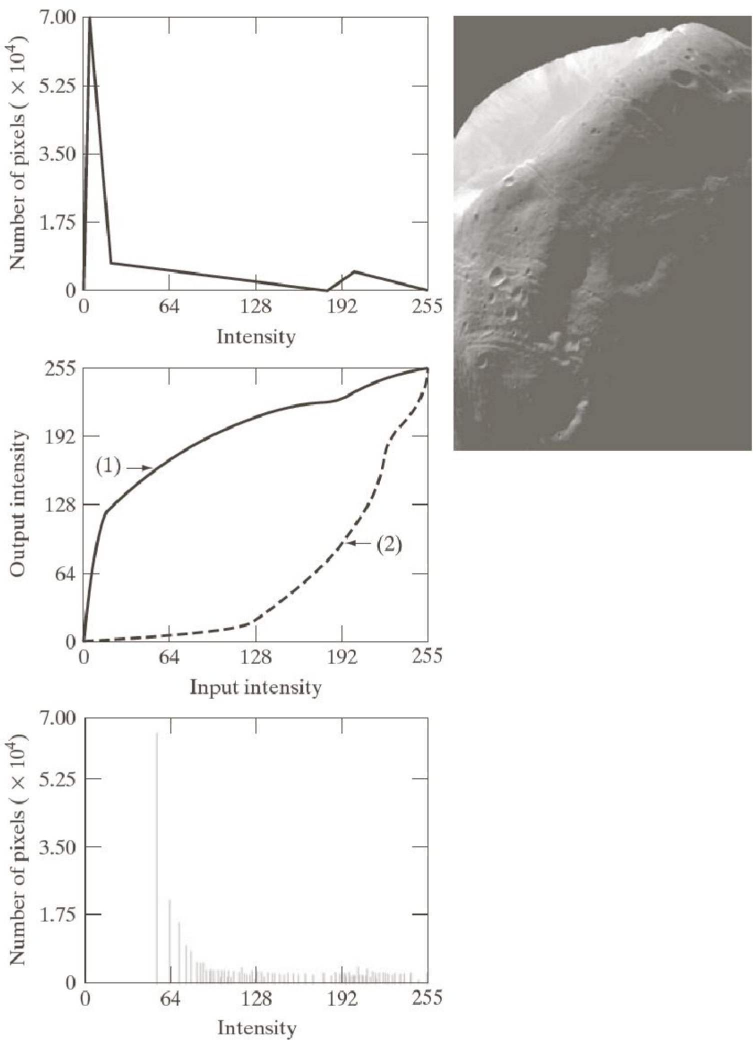


a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

Image Enhancement in Spatial Doma

Histogram Matching: Example



a	c
b	
d	

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

Next Class

- Image Enhancement in Spatial Domain
  - Local Histogram Processing
  - Using Histogram Statistics for Image Enhancement

Thank you:
Question?