SDA Mid Sem-I Date: 29 Sept 2020. Batch: UG4

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Q5

Since $n = 49 > 30$, therefore we will take ~~z~~ Normal distribution ~~by~~

where
$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim Z$$

$1 - \alpha = 90.9$ $\therefore$ $\alpha = 0.1$ $\frac{\alpha}{2} = 0.05$

For 90% CI → we know that limits will be

$$L = \left(\bar{x} - Z_{0.05} * \frac{s}{\sqrt{n}}\right) \qquad U = \left(\bar{x} + Z_{0.05} * \frac{s}{\sqrt{n}}\right)$$

Here $s^2 = 0.5625$ $s = 0.75$ $\sqrt{n} = \sqrt{49} = 7$.
$\bar{x} = 89$

$$\therefore L = \left(89 - Z_{0.05} * \frac{0.75}{0.7}\right) \qquad U = \left(\cancel{\bar{x}}\, 89 + Z_{0.05} * \frac{0.75}{\sqrt{n}}\right.$$

$Z_{0.05} = -1.64.$

$$\therefore L = 89 - 1.645 * \frac{0.75}{7} \qquad U = \left(89 + 1.645 * \frac{0.75}{7}\right.$$

$L = 89 - 0.17$ $U = 89 + 0.17$

88.83 $= 89.17$

$\therefore$ 90% CI → $[88.83, 89.17]$.

Q1) $n = 100$. $\mu = 6.525$ $\sigma = 1.25$.

To find $P(6.275 \le \bar{x} \le 6.775)$

since n is large, we can take normal dist

∴ ~~Pt~~ normalizing

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

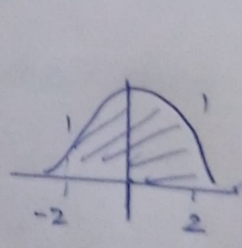
normalizing both sides

$$P\left(\frac{6.275-\mu}{\sigma/\sqrt{n}} < z < \frac{6.775-\mu}{\sigma/\sqrt{n}}\right)$$

~~Pt~~ substituting values

$$P\left(\frac{6.275-6.525}{1.25/10} < z < \frac{6.775-6.525}{1.25/10}\right)$$

$$P(-2.0 < z < 2.0)$$



~~Pt~~ ~~= P~~

$= P(z < 2) - P(z < -2)$

By normal table $\Rightarrow 0.9772 - 0.0228$

$= 0.9544$

∴ Probability of getting $\bar{x}$ betn 6.275 & 6.775 is 0.9544

Q2

We need to calculate bias of each estimator.

$$\text{Bias} = E[\bar{X_i}] = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$E[X_i] = \frac{X_1 + X_2 + X_3}{3}$$

$$\theta_1 = \frac{X_1 + X_2 + X_3}{3}$$

$$\therefore \text{Bias}(\theta_1) = \theta_1 - E(X_i) = 0$$

$$\text{Bias}(\theta_2) \quad \theta_2 - E(X_i) = \frac{5X_1 + 3X_2 + X_3}{9} - \frac{X_1 + X_2 + X_3}{3}$$

$$= \frac{2X_1 - 2X_3}{9}$$

As θ_1 is unbiased whereas θ_2 has a bias

$\therefore$ θ_1 is a better estimator. We should choose θ_1

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Q3

Given

X	0	1	2	3
P(x)	$\frac{1-\theta}{3}$	$\frac{\theta}{3}$	$\frac{2(1-\theta)}{3}$	$\frac{2\theta}{3}$

Sample : [2,0,3,1,0,1,3,2,1,2].

∴ for MLE

$$L(\theta) = P(X=2).P(X=0).P(X=3).P(X=1).P(X=0).P(X=1)$$
$$P.(X=3).P(X=2).P(X=1).P(X=2)$$

$$L(\theta) = \left(\frac{1-\theta}{3}\right)^2 \left(\frac{\theta}{3}\right)^3 \left(\frac{2(1-\theta)}{3}\right)^3 \left(\frac{2\theta}{3}\right)^2$$

Taking log likelihood

$$\ln(L(\theta)) = 2\ln\left(\frac{1-\theta}{3}\right) + 3\ln\left(\frac{\theta}{3}\right) + 3\ln\left(\frac{2(1-\theta)}{3}\right) + 2\ln\left(\frac{2\theta}{3}\right)$$

~~Taking derivation~~

$$\cancel{\frac{d}{d\theta}(\ln(L(\theta)))} = 2\ln(1-\theta) + 3\ln(\theta) + 3\ln(1-\theta) + 2\ln\theta + \underline{\underline{C}}$$

Taking derivative

$$\ln(L(\theta) = -\frac{5}{1-\theta} + \frac{5}{\theta}$$

Equate to 0

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$$\frac{5}{\hat{\theta}} = \frac{5}{1-\hat{\theta}} \qquad 2\hat{\theta} = 1 \qquad \hat{\theta} = 1/2$$

$\therefore$ MLE $\hat{\theta} = \frac{1}{2} = 0.5$.

(b) Using Moment Estimator

$$E[X] = \sum_{i=1}^{3} x\,P(x) = 0 \times \frac{1-\theta}{3} + 1 \times \frac{\theta}{3} + 2 \times \frac{2(1-\theta)}{3} + 3 \times \frac{2\theta}{3}$$

$$= \frac{\theta}{3} + \frac{4}{3} - \frac{4\theta}{3} + 2\theta = \frac{3\theta+4}{3}$$

The value of mean based on samples is

$$\bar{X} = \frac{2+0+3+1+0+1+3+2+1+2}{10} = \frac{15}{10} = 1.5$$

$\therefore$ equating both

$$\frac{3\theta+4}{3} = 1.5 \qquad 3\theta + 4 = 4.5$$

$$3\theta = 0.5 \qquad \theta = \frac{0.5}{3} = 0.1666$$

$\therefore$ $\hat{\theta} = 0.1666$..

Q4

a) acceptance region is [98.7, 101.3]

$P(98.7 \leq \bar{x} \leq 101.3)$

Normalizing to z.

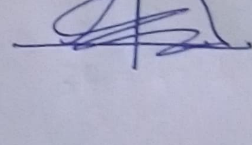
$n = 9$, $\mu = 100$, $\sigma^2 = 4$.

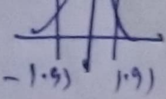
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$P\left(\frac{98.7-100}{2/3} \leq z \leq \frac{101.3 - 100}{2/3}\right)$$

$$P\left(\frac{-1.3}{2/3} \leq z \leq \frac{1.3}{2/3}\right)$$

$$P(-1.95 \leq z \leq 1.95)$$





-1.95 1.95

$\Rightarrow P(z < 1.95) - P(z < -1.95)$

$= 0.9744 - 0.0256$

$= 0.9488$

$\therefore \boxed{\text{Type I error} = 1 - 0.9488 = 0.0512}$