Chapter 13 Temporal Logic

1. For each of the following questions, give a short answer and justification.

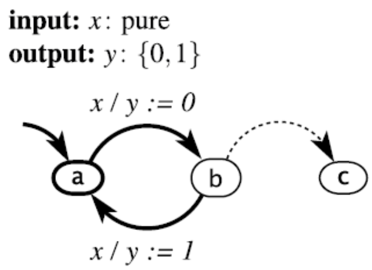
   (a) TRUE or FALSE: If $\mathbf{GF}p$ holds for a state machine A, then so does $\mathbf{FG}p$.

   Solution: FALSE. Consider a trace where p holds for every second reaction. This satisfies $\mathbf{GF}p$ but not $\mathbf{FG}p$.

   (b) TRUE or FALSE: $\mathbf{G}(\mathbf{G}p)$ holds for a trace if and only if $\mathbf{G}p$ holds.

   Solution: TRUE. "if" part: If $\mathbf{G}p$ holds, then p holds for every element of the trace, so $\mathbf{G}(\mathbf{G}p)$ holds; "only if" part: If $\mathbf{G}(\mathbf{G}p)$ holds, then for every suffix of the trace, $\mathbf{G}p$ holds, which means that p holds for every element of every suffix of the trace. Hence, it holds for every element of the trace.

2. Consider the following state machine:



(Recall that the dashed line represents a default transition.) For each of the following LTL formulas, determine whether it is true or false, and if it is false, give a counterexample:

(a) $x \implies \mathbf{F}\mathsf{b}$

Solution: *true*

(b) $\mathbf{G}(x \implies \mathbf{F}(y=1))$

Solution: *false*. Counterexample: If the input sequence begins with $(x, absent, \cdots)$, then the machine will be in state c. From that point on, even if x is *present*, it is not true that eventually $y = 1$ will appear on the output.

(c) $(\mathbf{G}x) \implies \mathbf{F}(y=1)$

Solution: *true*. In this case, the input x is always present, so $y = 1$ will be produced on every second reaction.

(d) $(\mathbf{G}x) \implies \mathbf{GF}(y=1)$

Solution: *true*. In this case, the input x is always present, so $y = 1$ will be produced on every second reaction, which is infinitely often.

(e) $\mathbf{G}((\mathsf{b} \wedge \neg x) \implies \mathbf{FG}\mathsf{c})$

Solution: *true*

(f) $\mathbf{G}((\mathsf{b} \wedge \neg x) \implies \mathbf{G}\mathsf{c})$

Solution: *false*. Unlike the previous example, for this to be true, it requires that the state machine be in c in the *same* reaction in which it is in b, which cannot happen.

(g) $(\mathbf{GF}\neg x) \implies \mathbf{FG}\mathsf{c}$

Solution: *false*. A counterexample is the reaction to the input sequence $(x, x, \neg x, x, x, \neg x, \cdots)$, where the pattern repeats. In this case, x is absent infinitely often, so the left side is true. However, the right side not true because state c is never reached.

4. This problem is concerned with specifying in linear temporal logic tasks to be performed by a robot. Suppose the robot must visit a set of n locations $l_1, l_2, \ldots, l_n$. Let p_i be an atomic formula that is *true* if and only if the robot visits location l_i.

   Give LTL formulas specifying the following tasks:

   (a) The robot must eventually visit at least one of the n locations.

Solution:

$$\bigvee_{i=1}^{n} \mathbf{F}\, p_i$$

   (b) The robot must eventually visit all n locations, but in any order.

Solution:

$$\bigwedge_{i=1}^{n} \mathbf{F}\, p_i$$

   (c) The robot must eventually visit all n locations, in the order $l_1, l_2, \ldots, l_n$.

Solution:

$$\mathbf{F}\,(p_1 \wedge \mathbf{F}\,(p_2 \wedge \mathbf{F}\,(p_3 \wedge \ldots \mathbf{F}\, p_n)))$$