# DIGITAL IMAGE PROCESSING

Image Enhancement in Frequency Domain: Session 3

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# Today’s Lecture

- **Image Enhancement in Frequency Domain**
  - **Filtering in Frequency Domain**

# Image Enhancement in Frequency Domain

## 2-D Convolution Theorem

1-D convolution

$$f(x)\star h(x) = \sum_{m=0}^{M-1} f(m)h(x-m)$$

2-D convolution

$$f(x,y)\star h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m,y-n)$$

$$x = 0,1,2,\ldots,M-1;\ y = 0,1,2,\ldots,N-1.$$

$$f(x,y)\star h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow F(u,v)\star H(u,v)$$

# Image Enhancement in Frequency Domain

## 1-D Impulses and the Shifting Property: Continuous

A *unit impulse* of a continuous variable $t$ located at $t$=0, denoted $\delta(t)$, defined as

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

The *sifting property* $\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

# Image Enhancement in Frequency Domain

## 1- D Impulses and the Shifting Property: Discrete

A *unit impulse* of a discrete variable $x$ located at $x$=0, denoted $\delta(x)$, defined as

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\sum_{x=-\infty}^{\infty} \delta(x) = 1$$

The *sifting property*

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x - x_0) = f(x_0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$$

# Image Enhancement in Frequency Domain

## 2-D Impulses and the Shifting Property: Continuous

The impulse $\delta(t, z)$, $\quad \delta(t,z) = \begin{cases} \infty & \text{if } t = z = 0 \\ 0 & \text{otherwise} \end{cases}$

and $\quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(t,z)\,dt\,dz = 1$

The sifting property

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)\delta(t,z)\,dt\,dz = f(0,0)$$

and

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)\delta(t-t_0, z-z_0)\,dt\,dz = f(t_0, z_0)$$

# Image Enhancement in Frequency Domain

## 2-D Impulses and the Shifting Property: Discrete

The impulse $\delta(x, y)$, $\delta(x,y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$

The sifting property

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x,y)\delta(x,y) = f(0,0)$$

and

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x,y)\delta(x-x_0, y-y_0) = f(x_0, y_0)$$

# Image Enhancement in Frequency Domain

## 2-D Impulses and the Shifting Property

For a unit impulse located at origin (0,0),

$$\sum_{x=0}^{m-1}\sum_{y=0}^{n-1} f(x,y)\delta(x,y) = f(0,0)$$

Fourier transform of a unit impulse located at origin

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \delta(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$= \frac{1}{MN}$$

# Image Enhancement in Frequency Domain

## Relation between Spatial and Fourier domain

Let $f(x, y) = \delta(x, y)$.

$$f(x,y) * h(x,y) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta(m,n)h(x-m,y-n)$$

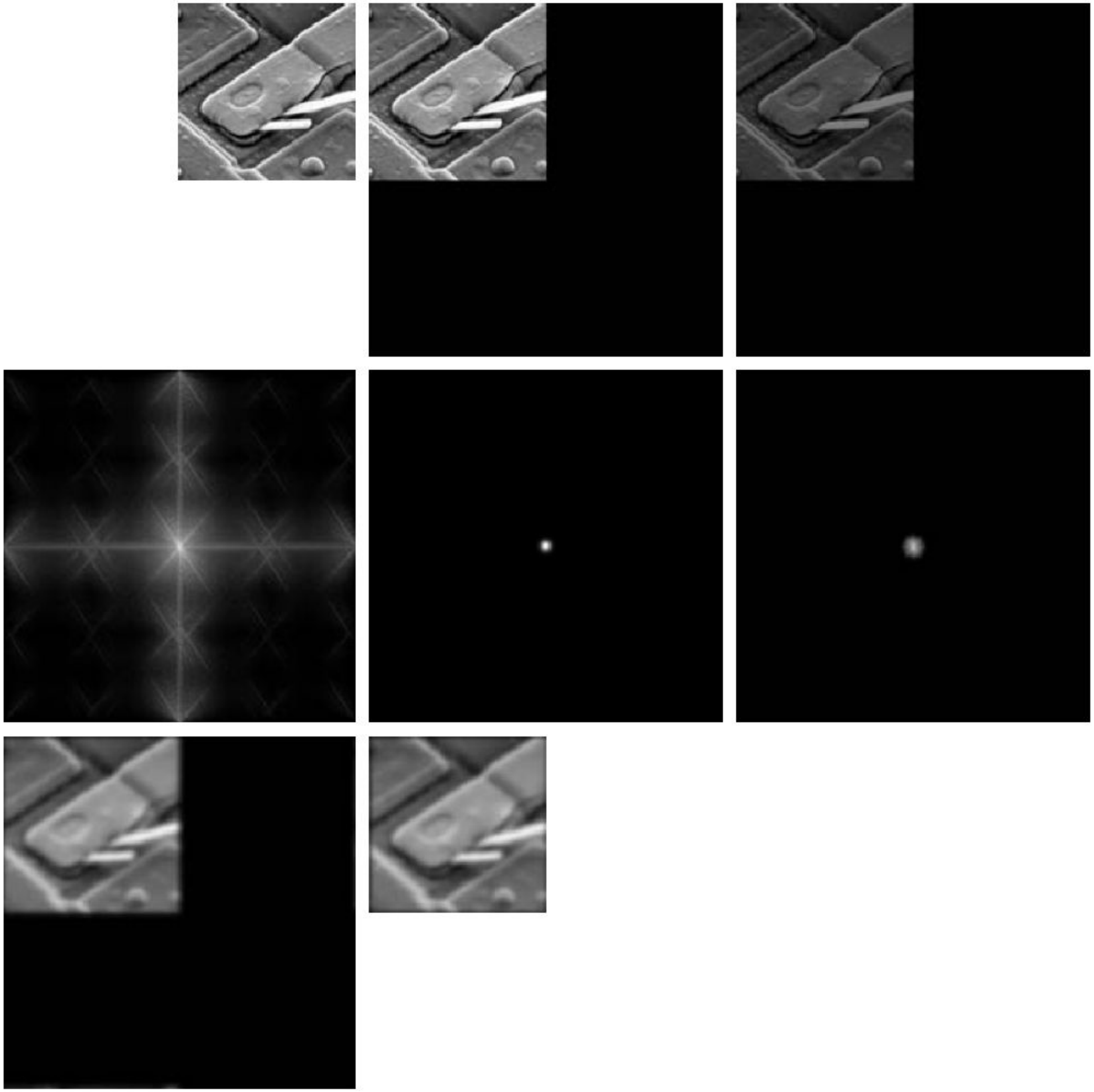
$$= \frac{1}{MN}h(x,y)$$

$$f(x,y) * h(x,y) \Longleftrightarrow F(u,v)H(u,v)$$

$$\delta(x,y) * h(x,y) \Longleftrightarrow \Im[\delta(x,y)]H(u,v)$$

$$h(x,y) \Longleftrightarrow H(u,v)$$

Given a filter in the frequency domain, we can obtain the corresponding filter in the spatial domain by taking the inverse Fourier transform of the former. The reverse is also true.



| | | |
|---|---|---|
| a | b | c |
| d | e | f |
| g | h | |

**FIGURE 4.36** (a) An $M \times N$ image, $f$. (b) Padded image, $f_p$ of size $P \times Q$. (c) Result of multiplying $f_p$ by $(-1)^{x+y}$. (d) Spectrum of $F_p$. (e) Centered Gaussian lowpass filter, $H$, of size $P \times Q$. (f) Spectrum of the product $HF_p$. (g) $g_p$, the product of $(-1)^{x+y}$ and the real part of the IDFT of $HF_p$. (h) Final result, $g$, obtained by cropping the first $M$ rows and $N$ columns of $g_p$.

# Image Enhancement in Frequency Domain

## Spatial Domain vs. Frequency Domain Filtering

Let H(u) denote the 1-D frequency domain Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma^2}$$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$$

1. Both components are Gaussian and real
2. The functions behave reciprocally

# Image Enhancement in Frequency Domain

## Spatial Domain vs. Frequency Domain Filtering

Let $H(u)$ denote the difference of Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

with $A \geq B$ and $\sigma_1 \geq \sigma_2$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 Ae^{-2\pi^2\sigma_2^2x^2}$$

High-pass filter or low-pass filter ?

# Image Enhancement in Frequency Domain

## Spatial Domain vs. Frequency Domain Filtering

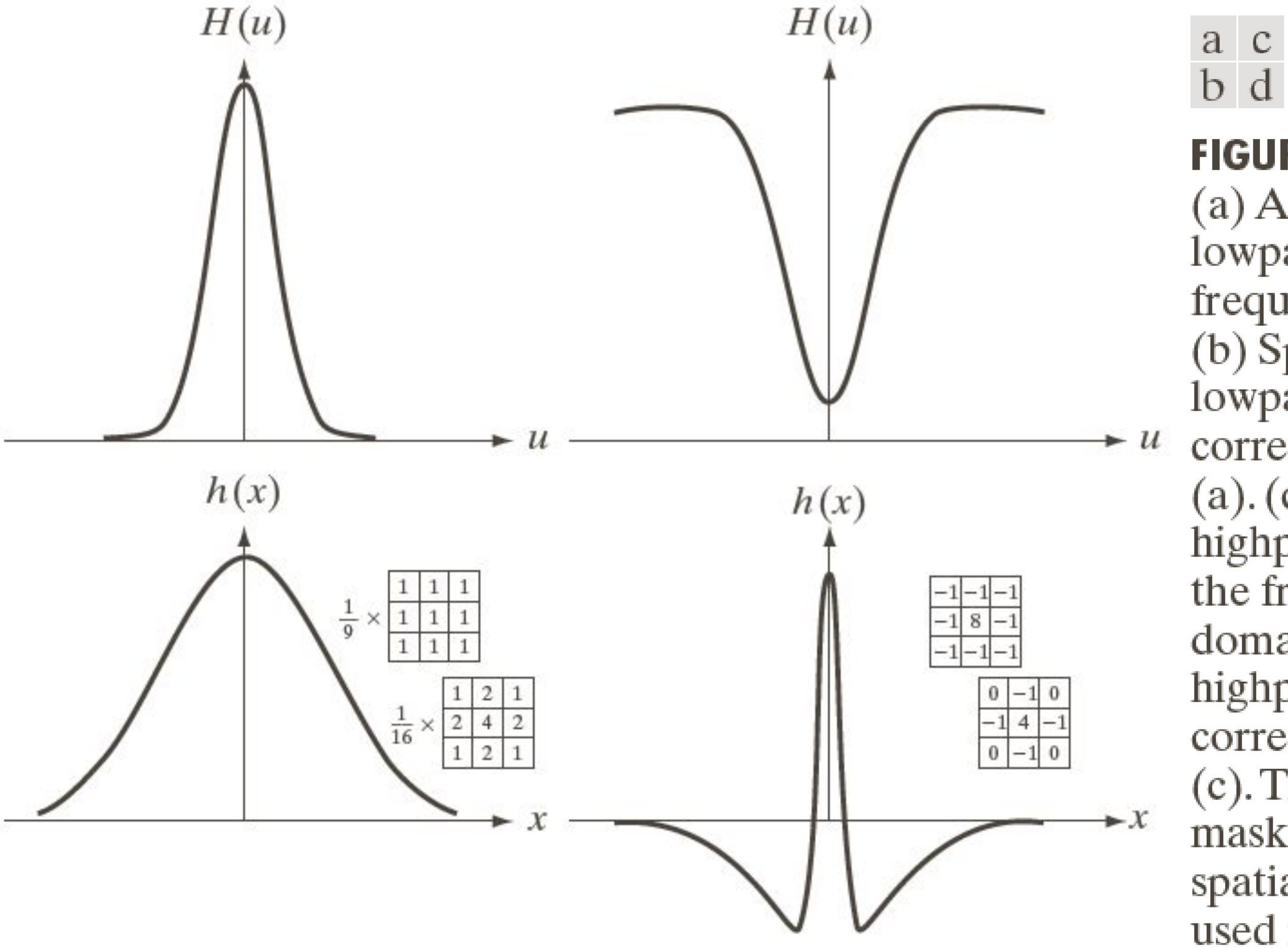
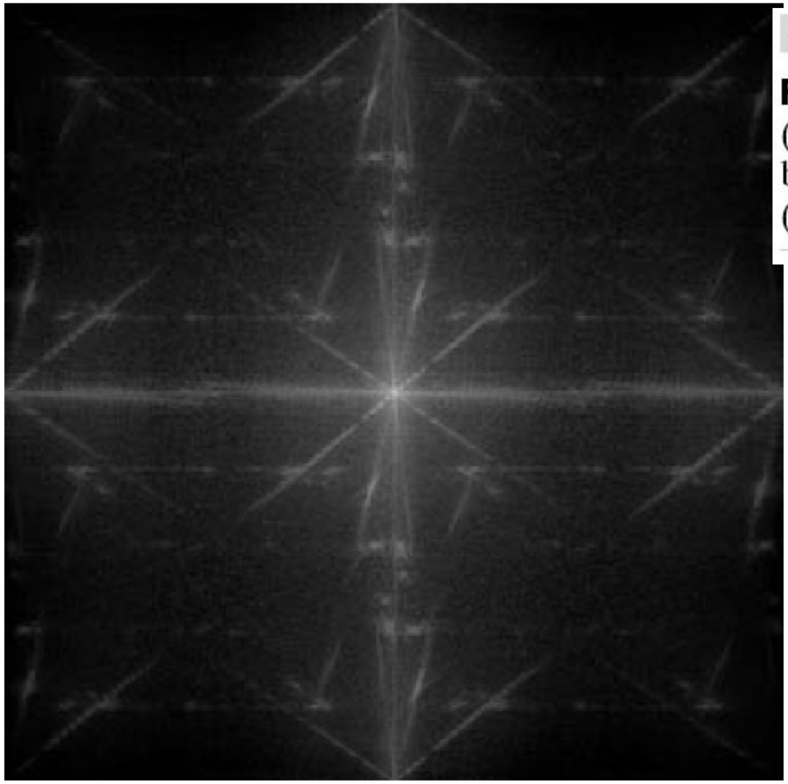



**FIGURE 4.37**
(a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

# Image Enhancement in Frequency Domain

## Spatial Domain vs. Frequency Domain Filtering



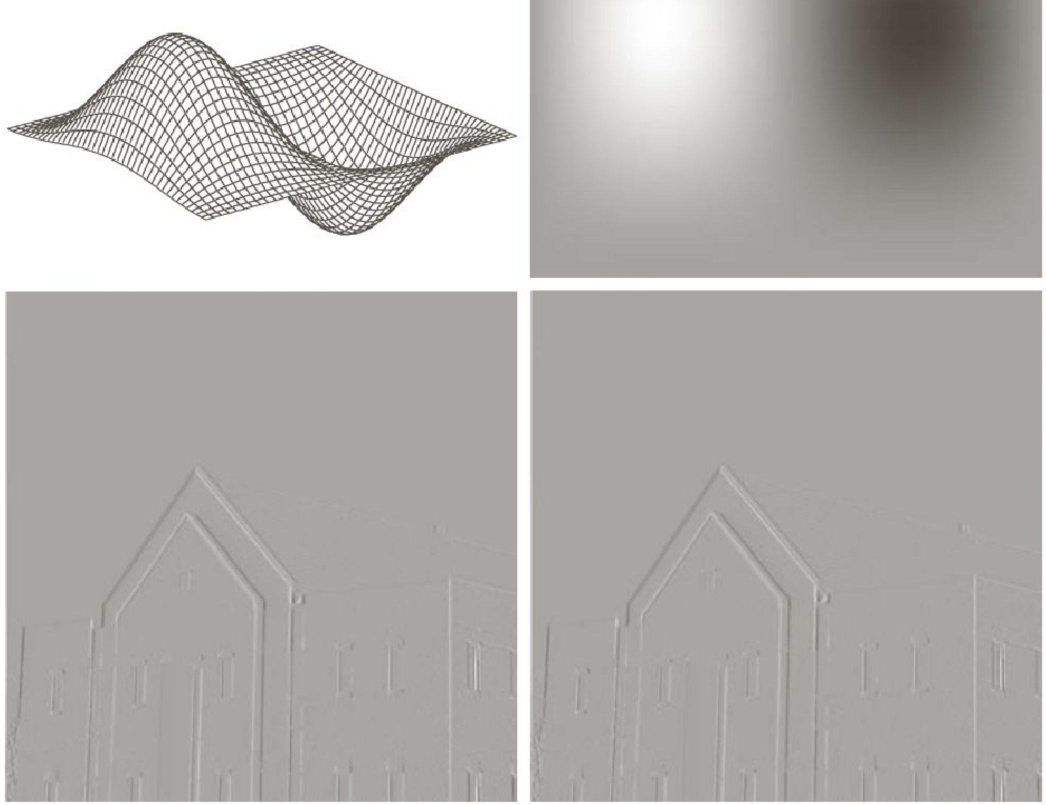
a b

**FIGURE 4.38**
(a) Image of a building, and (b) its spectrum.

# Image Enhancement in Frequency Domain

## Spatial Domain vs. Frequency Domain Filtering

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −2 | 0 | 2 |
| −1 | 0 | 1 |



a b
c d

**FIGURE 4.39** (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

# Next Class

- Image Enhancement in Frequency Domain
  - Filtering in Frequency Domain

Thank you:
Question?