Mathematics - II

Extra Practice problems

1. Evaluate following improper integrals. Discuss about its kind and convergence

(a) $\int_1^{\infty} \frac{dx}{x\sqrt{x^2-1}}$ (b) $\int_{-\infty}^{0} \cosh x\,dx$ (c) $\int_1^{\infty} \ln(1/x)\,dx$

(d) $\int_{-1}^{1} \frac{dx}{x^{2/5}}$

2. Test for convergence of improper integral

(a) $\int_0^{\infty} \frac{dx}{\sqrt{x^3+1}}$

(b) $\int_0^{4} \frac{\sin^2 x}{\sqrt{x}(x-1)}\,dx$

(c) $\int_0^{1} \frac{dx}{x^2 \cos x}$

(3) Apply quotient test-1 to test for convergence

(a) $\int_1^{\infty} \frac{\log x}{x^2}$

(b) $\int_1^{\infty} \frac{dx}{x\sqrt{x^2+1}}$

(c) for what values of p $I = \int_0^{\infty} \frac{1-e^{-x}}{x^p}$ converges

④ Evaluate integral using Γ-β functions ④

(a) $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$

(b) $\int_0^1 x^2 \left(\log\left(\frac{1}{x}\right)\right)^3 dx$

(c) $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\, dx$

⑤ evaluate following integrals

(a) $\int_3^4 \int_1^2 \frac{dy\, dx}{(x+y)^2}$

(b) $\int_{-1/2}^{1} \int_{-x}^{1+x} (x^2+y)\, dx\, dy$

(c) $\int_0^1 \int_0^x \int_0^{x+y} (x+y+z)\, dz\, dy\, dx$

(d) $\int_0^a \int_0^x \int_0^y xyz\, dz\, dy\, dx$

⑥ find the area bounded by Parabola $y^2 = 4ax$ and Straight line $x+y = 3a$

⑦ Change the order of integration and evaluate $\int_0^b \int_0^{a\sqrt{b^2-y^2}/b} xy\, dx\, dy$

8) Change order of integration and evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$

9) Change to Polar coordinates and evaluate

$$\int_0^1 \int_y^a \frac{x}{x^2+y^2} \, dx \, dy$$

10) evaluate $\iint_R x^2+y^2 \, dy \, dx$ where R is region bounded by ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in quadrant I

11) Find Volume bounded by cylinders $x^2+y^2=2ax$ and $z^2=2ax$

12) Volume bounded by $z=0$, $x^2+y^2=1$, $x+y+z=3$

13) Verify Rolle's theorem $f(x)=\frac{\sin x}{e^x}$ in $[0,\pi]$

14) Verify lagrange's mean value theorem for $f(x)=\log_e x$ in $[1,e]$

15) Verify Cauchy's mean value theorem $f(x)=x^2$, $g(x)=x^3$ in $[1,2]$

16) Find grad f where $f=x^3-y^3+x^2z$

17) Find Max value of the directional derivative $\phi=x^2yz$ at $(1,4,1)$

18. If $\bar{F} = (x+y+1)i + j - (x+y)k$. find $\bar{F} \cdot \text{curl}\bar{F}$

19) Compute line integral $\int y^2 dx - x^2 dy$ round the triangle whose vertices are $(1,0), (0,1), (-1,0)$ in xy plane

20) find work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + \bar{j} + 2\bar{k}$

21) evaluate by Green's theorem $\oint_C (y - \sin x)dx + \cos x\, dy$ where C is triangle by lines $y=0, x=\frac{\pi}{2}, \pi y = 2x$

22) Verify green's theorem for $\int (3y + y^2)dx + x^2 dy$ where C is bounded by $y = x$ and $y = x^2$

Solution key

1.(a) $\frac{\pi}{2}$, converges (b) Diverges (c) diverges

(d) Converges

2.(a) converges

(b) converges

(c) Diverges

3(a) Converges (Hint $g(x) = 1/x^2$)

(b) Converges (Hint $g(x) = 1/x^{3/2}$)

(c) $1 < p < 2$ converges

4) (a) $\frac{1}{2} \frac{\pi}{\sin(\pi/4)} = \frac{\pi}{\sqrt{2}}$

(b) $\frac{2}{9}$

(c) $\frac{2\Gamma(5)\Gamma(10)}{\Gamma(15)}$

(5) evaluate following integrals

(a) $\ln(25/24)$

(b) $\frac{63}{32}$

(c) $7/8$

(d) $\frac{a^6}{48}$

(6) $\frac{10a^2}{3}$

(7) $\frac{b^2a^2}{8}$

(8) $\frac{\pi}{16}$

(9) $\frac{\pi}{4a^2}$

(10) $\frac{\pi ab(a^2+b^2)}{16}$

11) $\frac{128a^3}{15}$

12) 3π

13) $C = \frac{\pi}{4} + (0, \pi)$

14) $C = e - 1$

15) $C = 14/9$

16) $-i - 3j + 1C$

17) 9

18) 0

19) $\frac{-2}{3}$

20) $\frac{27}{2}$

$-(\frac{\pi}{2} + 2)$