Test of Hypothesis 2

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Algorithm

Hypothesis testing algorithm.

1. Identify H_0 and H_1
2. Find appropriate test statistic, θ
3. Obtain sampling distⁿ of θ when H_0 is true.
4. Select significance level α and compute tabulated θ, i.e., θ_t.
5. Compare tabulated and observed value of θ and take decision.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Test on Mean

Tests on the Mean of a Normal Dist^n.

Variance known

i.i.d. random sample → $X_1, X_2, \ldots, X_n$ from $N(\mu, \sigma^2)$ pop^n.

σ^2 known.

$$\left.\begin{array}{l} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{array}\right] \rightarrow \text{Test the hypothesis at } \alpha \text{ significance level.}$$

μ_0 ← constant

If the null hypothesis H_0 is true,

$$\bar{X} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right) \rightarrow \text{sampling dist}^n.$$

Convenient to standardize $\bar{X}$ and use test statistic based on the s.r.v.

$$\boxed{\text{Test statistic: } Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}}$$

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Test on Mean

If $H_0: \mu = \mu_0$ is true, $Z_0 \sim N(0,1)$.

Hyp. Testing procedure:

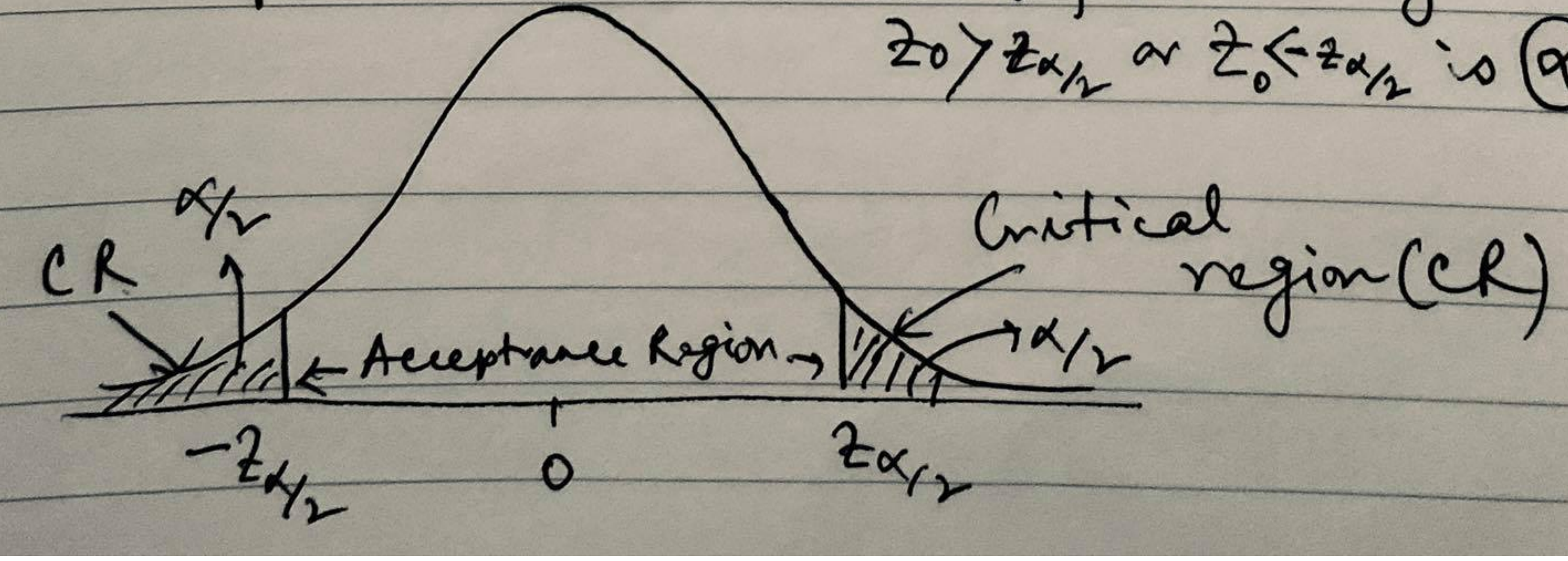
Take a random sample of size n and find sample mean $\bar{x}$.

fixed significance level approach: $\boxed{Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$

If H_0 is true, S.n. z value corr. to $\bar{x}$

Where to place critical region?
(Two sided case)

If $H_0: \mu = \mu_0$ is true, The probability is $1-\alpha$ that the test statistic Z_0 falls between $-z_{\alpha/2}$ and $z_{\alpha/2}$. If $H_0: \mu = \mu_0$ is true, prob. that test statistic Z_0 fall in region $Z_0 > z_{\alpha/2}$ or $Z_0 < -z_{\alpha/2}$ is α.



Thus, we reject H_0 if either

$$z_0 > z_{\alpha/2} \quad \text{or} \quad z < -z_{\alpha/2}$$

We fail to reject H_0 if

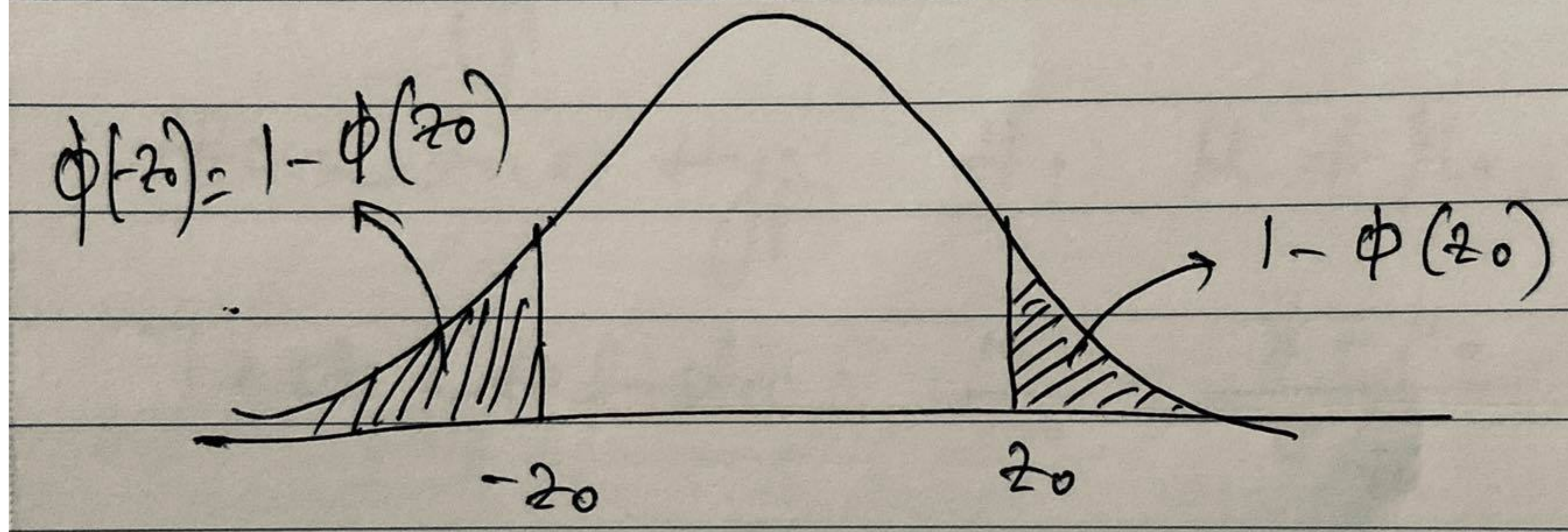
$$-z_{\alpha/2} \le z_0 \le z_{\alpha/2}.$$

Note:

Test on Mean

P-value Approach:

Compute s.n. z value corresponding to $\bar{x}$, $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ if H_0 is true.



P value $= 2[1 - \phi(|z_0|)]$.

Compare P value with significance level α and decide.

$\boxed{\text{This is } z\text{-test.}}$

Test on Mean

Summary:

Test of Hypothesis on the Mean, Variance known (Z test).

Null Hypothesis: $H_0: \mu = \mu_0$

Alternative Hyp: $H_1: \mu \neq \mu_0$.

Test Statistic: $Z_0 = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$.

P-value: $P = 2[1 - \Phi(|z_0|)]$

Rejection Criterion for fixed-level approach:

$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$

In terms of $\bar{x}$: Reject H_0 if $\bar{x} > a$ or $\bar{x} < b$

$a = \mu_0 + z_{\alpha/2}\,\sigma/\sqrt{n}$ and $b = \mu_0 - z_{\alpha/2}\,\sigma/\sqrt{n}$.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Example

Example: Propellant Burning Rate

Air crew escape systems get power from propellant whose mean burning rate must be 50 cm/sec. We know that s.d. of burning rate is $\sigma = 2$. cm/sec. Analyst decides to specify sig. level of $\alpha = 0.05$ and selects a random sample of $n = 25$. The ~~and~~ sample mean burning rate of $\bar{x} = 51.3$ cm/sec is found. What conclusion should be drawn?

Example

Soln. 1. Parameter of interest : μ

2. $H_0: \mu = 50$ cm/sec.

$H_1: \mu \neq 50$ cm/sec.

3. Test statistic : $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

4. Reject H_0 if P-value is less than 0.05.

or

fixed sig. level appr.: Reject H_0 if $z_0 < -z_{0.025} = -1.96$

or, $z_0 > z_{0.025} = 1.96$.

5. Compute $z_0 = \dfrac{51.3 - 50}{2/\sqrt{25}} = 3.25$.

P-value $= 2[1 - \Phi(3.25)]$

$= 0.0012$

We reject $H_0: \mu = 50$ at the 0.05 level of s

Int: There is a strong evidence that μ exceeds 50 cm/sec.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Other Cases

Large Sample test. (Normal pop^n)

If σ^2 is unknown, $n > 30/40$

Substitute σ^2 by s^2 and dist^n is normal

If σ^2 is unknown, $n < 30/40$

sub σ^2 by s^2 and dist^n is t.

Large Sample test. (Non-normal pop^n)

σ - known → dist^n is Normal

σ - unknown → dist^n is "

~~If σ^2 is un~~

If $n < 30$, non-normal pop^n

σ known / σ unknown

dist^n undetermined.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

t-Test

Testing Hypothesis on the Mean of a Normal Dist^n, Variance Unknown.

Null hypothesis: $H_0 : \mu = \mu_0$

Alternative " : $H_1 : \mu \neq \mu_0$

Test at α sig. level

Test Statistic: $T_0 = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$.

P-value: Prob. above $|t_0|$ and prob below $-|t_0|$

Rejection Criterion for fixed-level tests: $t_0 > t_{\alpha/2, n-1}$ or $t_0 < -t_{\alpha/2, n-1}$

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Test on Variance

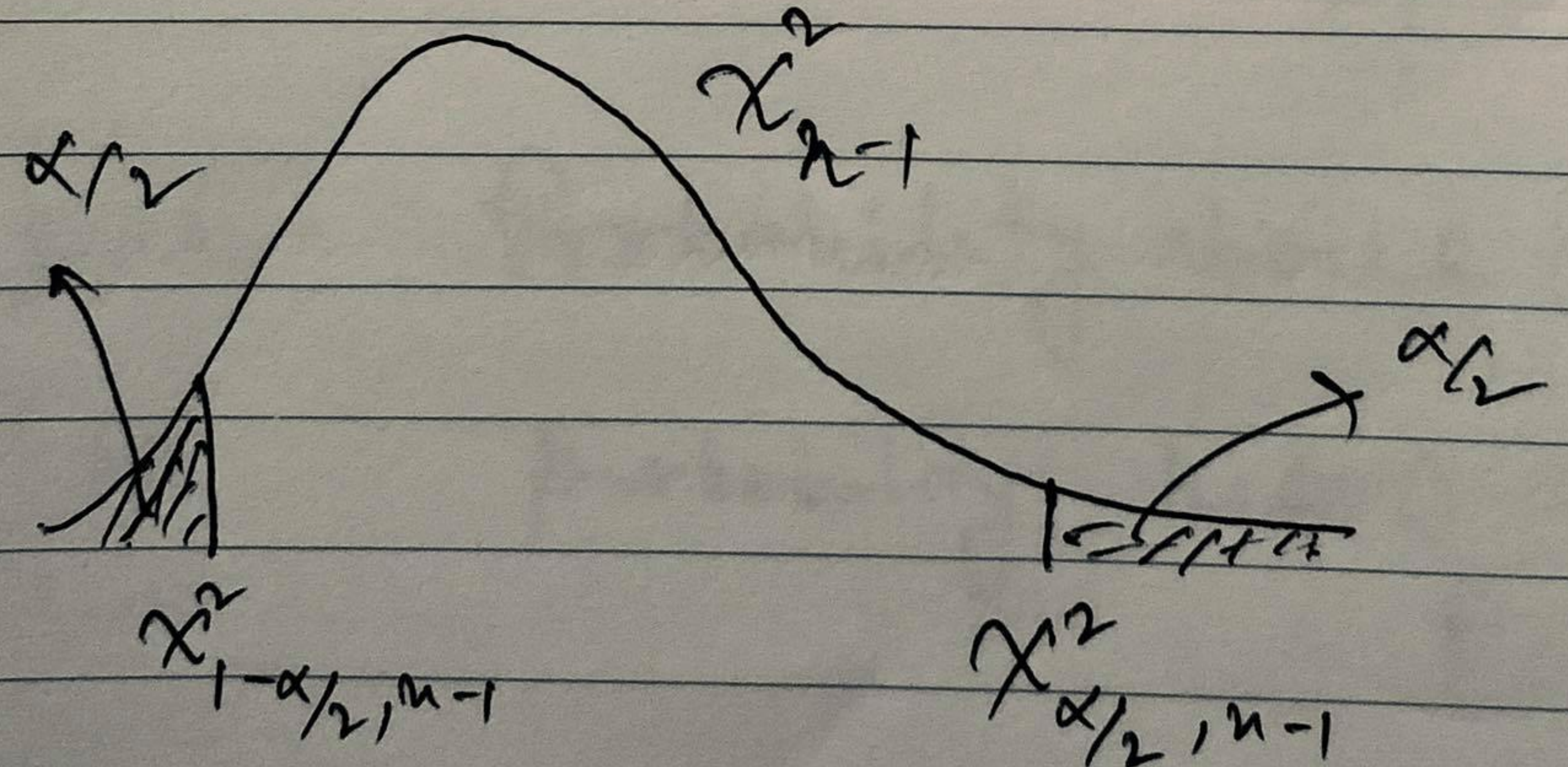
Tests on variance of a Normal Dist^n

$$\left.\begin{aligned} H_0 &: \sigma^2 = \sigma_0^2 \\ H_1 &: \sigma^2 \neq \sigma_0^2 \end{aligned}\right] \text{ Test at } \alpha \text{ sig. leve}$$

Test statistic: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Rejection Criteria: $\chi_0^2 > \chi^2_{\alpha/2,\, n-1}$

$\chi_0^2 < \chi^2_{1-\alpha/2,\, n-1}$



Test on Difference in Mean

Inference on the Difference in Means of two Normal Dist^n's, Variance Known

$$H_0: \mu_1 - \mu_0 = \Delta_0$$
$$H_1: \mu_1 - \mu_0 \neq \Delta_0$$

$\rightarrow$ Test at α sig. level

~~P-value: Probability above $|z_0|$~~

Test statistic:

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

P-value: Probability above $|z_0|$ and probability below $-|z_0|$,

$$P = 2[1 - \Phi(|z_0|)].$$

Rejection Criterion for fixed-level tests: $\longrightarrow$ $z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R.,

Other Cases

Inference on the Difference in Means of Two Normal Dist^n's, Variances unknown

Case 1: $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

$H_0: \mu_1 - \mu_0 = \Delta_0$

$H_1: \mu_1 - \mu_0 \neq \Delta_0$

Test statistic: $$T_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

P-value: Prob. above $|t_0|$ and below $-|t_0|$.

Rejection Criterion: $t_0 > t_{\alpha/2, n_1+n_2-2}$ or

$t_0 < -t_{\alpha/2, n_1+n_2-2}$.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Test on Ratio of Variances

Tests on the Ratio of the Variances of

Two Normal Distributions

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$ → test at α sig. level.

Test Statistic: $f_0 = \dfrac{S_1^2}{S_2^2}$

Rejection Criterion: $f_0 > f_{\alpha/2, n_1-1, n_2-1}$ or

$f_0 < f_{1-\alpha/2, n_1-1, n_2-1}$.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..