CO: Computer Organization

Day5

Indian Institute of Information Technology, Sri City

Jan - May - 2018

http://co-iiits.blogspot.in/

Multiplication of Two Integers (unsigned)

Input $X = X_3X_2X_1X_0$
Input $Y = Y_3Y_2Y_1Y_0$
$Z = X \times Y$, where X is Multiplicand and Y is Multiplier.
$Z = \sum_{i=0}^{n-1} X.Y_i.2^i = \sum_{i=0}^{n-1} PP_i$

PP_0	0	0	0	$X_3.Y_0$	$X_2.Y_0$	$X_1.Y_0$	$X_0.Y_0$
PP_1	0	0	$X_3.Y_1$	$X_2.Y_1$	$X_1.Y_1$	$X_0.Y_1$	0
PP_2	0	$X_3.Y_2$	$X_2.Y_2$	$X_1.Y_2$	$X_0.Y_2$	0	0
PP_3	$X_3.Y_3$	$X_2.Y_3$	$X_1.Y_3$	$X_0.Y_3$	0	0	0

Result $Z = P_7P_6P_5P_4P_3P_2P_1P_0$

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PP_0	0	0	0	$X_3.Y_0$	$X_2.Y_0$	$X_1.Y_0$	$X_0.Y_0$
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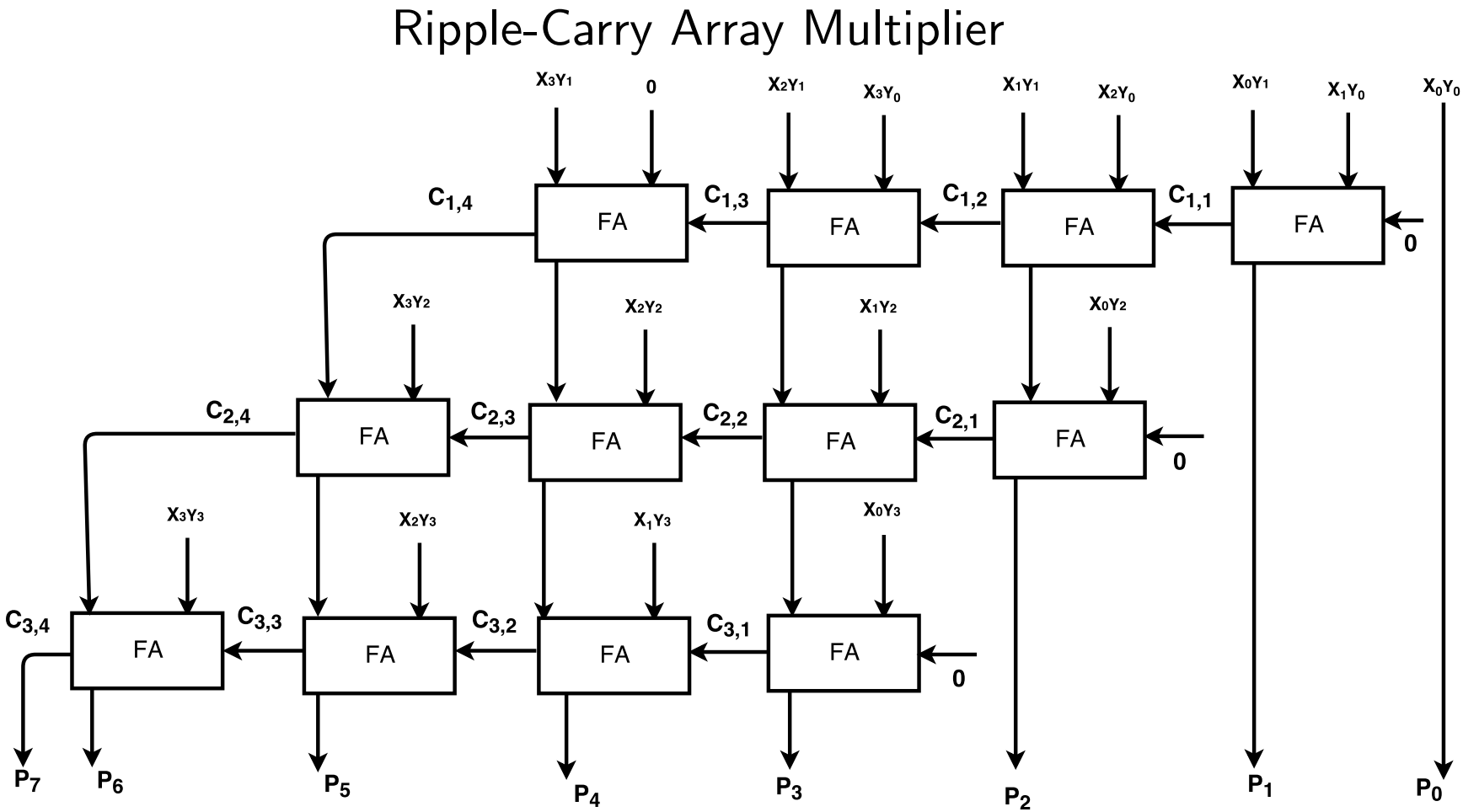
PP_2	0	$X_3.Y_2$	$X_2.Y_2$	$X_1.Y_2$	$X_0.Y_2$	0	0
PP_3	$X_3.Y_3$	$X_2.Y_3$	$X_1.Y_3$	$X_0.Y_3$	0	0	0

Result $Z = P_7P_6P_5P_4P_3P_2P_1P_0$

Multiplication of Two Integers (unsigned)



Multiplication of Two Integers (unsigned)

Assume that FA takes $2\mathcal{T}$ time to generate Sum and Carry

All $X_i.Y_i$s are available at $1\mathcal{T}$.

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Condition for Overflow(OF):?

Write the generalized formula for total time:?

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Condition for Overflow(OF):**?**
Write the generalized formula for total time:**?**

Multiplication of Two Signed Integers

Input $X = X_3X_2X_1X_0$
Input $Y = Y_3Y_2Y_1Y_0$
$Z = X \times Y$, where X is Multiplicand and Y is Multiplier.
$Z = -X.Y_{n-1}.2^{n-1} + \sum_{i=0}^{n-2} X.Y_i.2^i = -PP_{n-1} + \sum_{i=0}^{n-2} PP_i$
Represent each PP using '2n' bits.

$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_2.Y_0$	$X_1.Y_0$	$X_0.Y_0$
$X_3.Y_1$	$X_3.Y_1$	$X_3.Y_1$	$X_3.Y_1$	$X_2.Y_1$	$X_1.Y_1$	$X_0.Y_1$	0
$X_3.Y_2$	$X_3.Y_2$	$X_3.Y_2$	$X_2.Y_2$	$X_1.Y_2$	$X_0.Y_2$	0	0
$\overline{X_3.Y_3}$	$\overline{X_3.Y_3}$	$\overline{X_2.Y_3}$	$\overline{X_1.Y_3}$	$\overline{X_0.Y_3}$	0	0	0
0	0	0	0	1	0	0	0

Try with example: If $X = 1110 = -2$ and $Y = 1101 = -3$, then $X.Y = 6$.

Multiplication of Two Signed Integers

Input $X = X_3X_2X_1X_0$
Input $Y = Y_3Y_2Y_1Y_0$
$Z = X \times Y$, where X is Multiplicand and Y is Multiplier.
$Z = -X.Y_{n-1}.2^{n-1} + \sum_{i=0}^{n-2} X.Y_i.2^i = -PP_{n-1} + \sum_{i=0}^{n-2} PP_i$
Represent each PP using '2n' bits.

$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_3.Y_0$	$X_2.Y_0$	$X_1.Y_0$	$X_0.Y_0$
$X_3.Y_1$	$X_3.Y_1$	$X_3.Y_1$	$X_3.Y_1$	$X_2.Y_1$	$X_1.Y_1$	$X_0.Y_1$	0
$X_3.Y_2$	$X_3.Y_2$	$X_3.Y_2$	$X_2.Y_2$	$X_1.Y_2$	$X_0.Y_2$	0	0
$\overline{X_3.Y_3}$	$\overline{X_3.Y_3}$	$\overline{X_2.Y_3}$	$\overline{X_1.Y_3}$	$\overline{X_0.Y_3}$	0	0	0
0	0	0	0	1	0	0	0

Try with example: If $X = 1110 = -2$ and $Y = 1101 = -3$, then $X.Y = 6$.

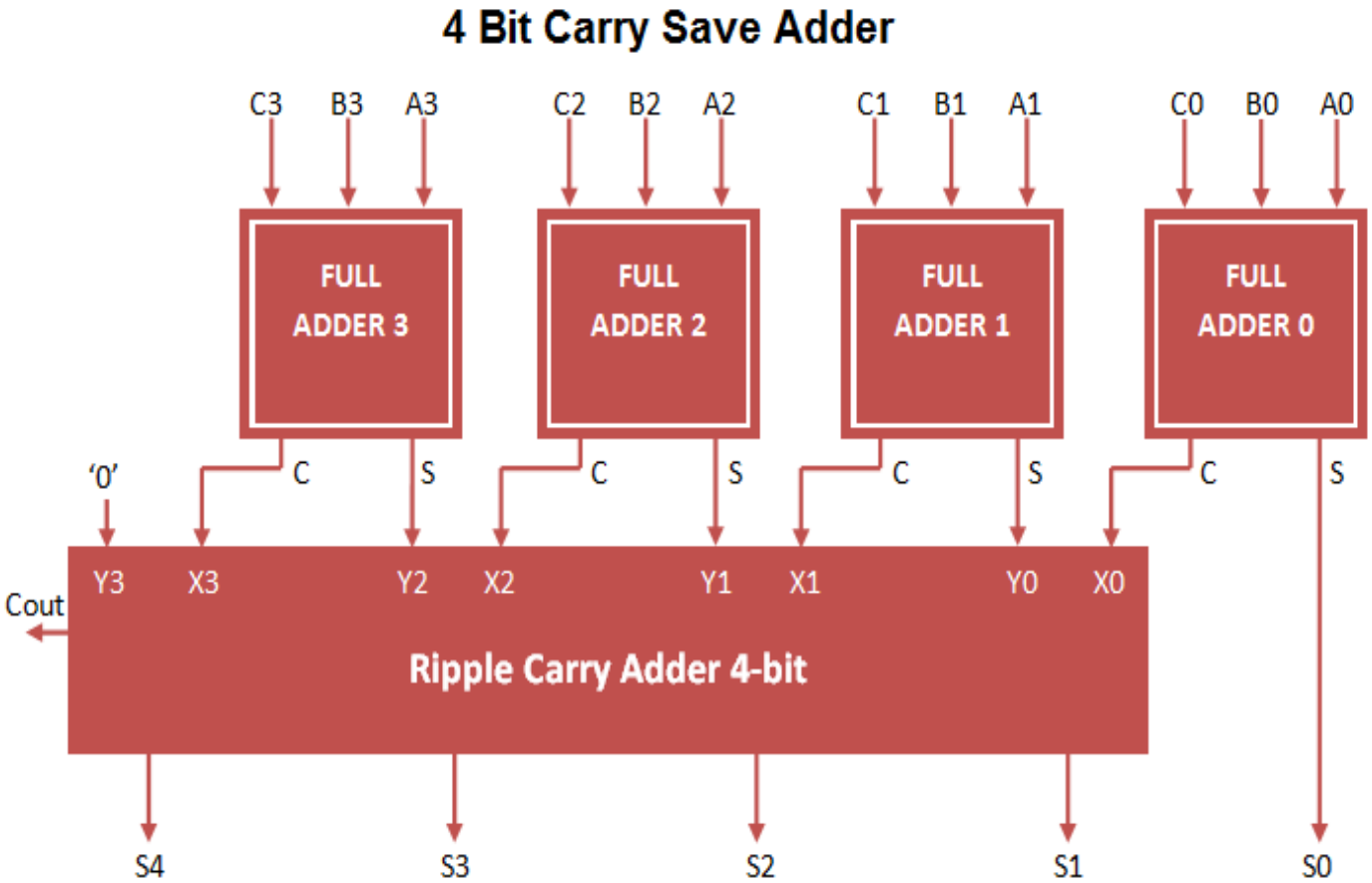
Multiplication of Two Signed Integers

Draw a Combinational Array Multiplier or Ripple-Carry Array Multiplier.

Write the generalized formula for total time:**?**

Carry Save Adder (CSA)

Input $A = A_3 A_2 A_1 A_0$
Input $B = B_3 B_2 B_1 B_0$
Input $C = C_3 C_2 C_1 C_0$



How to perform the addition of 4-inputs(A, B, C, D) using CSAs

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Input $A = A_3 A_2 A_1 A_0$
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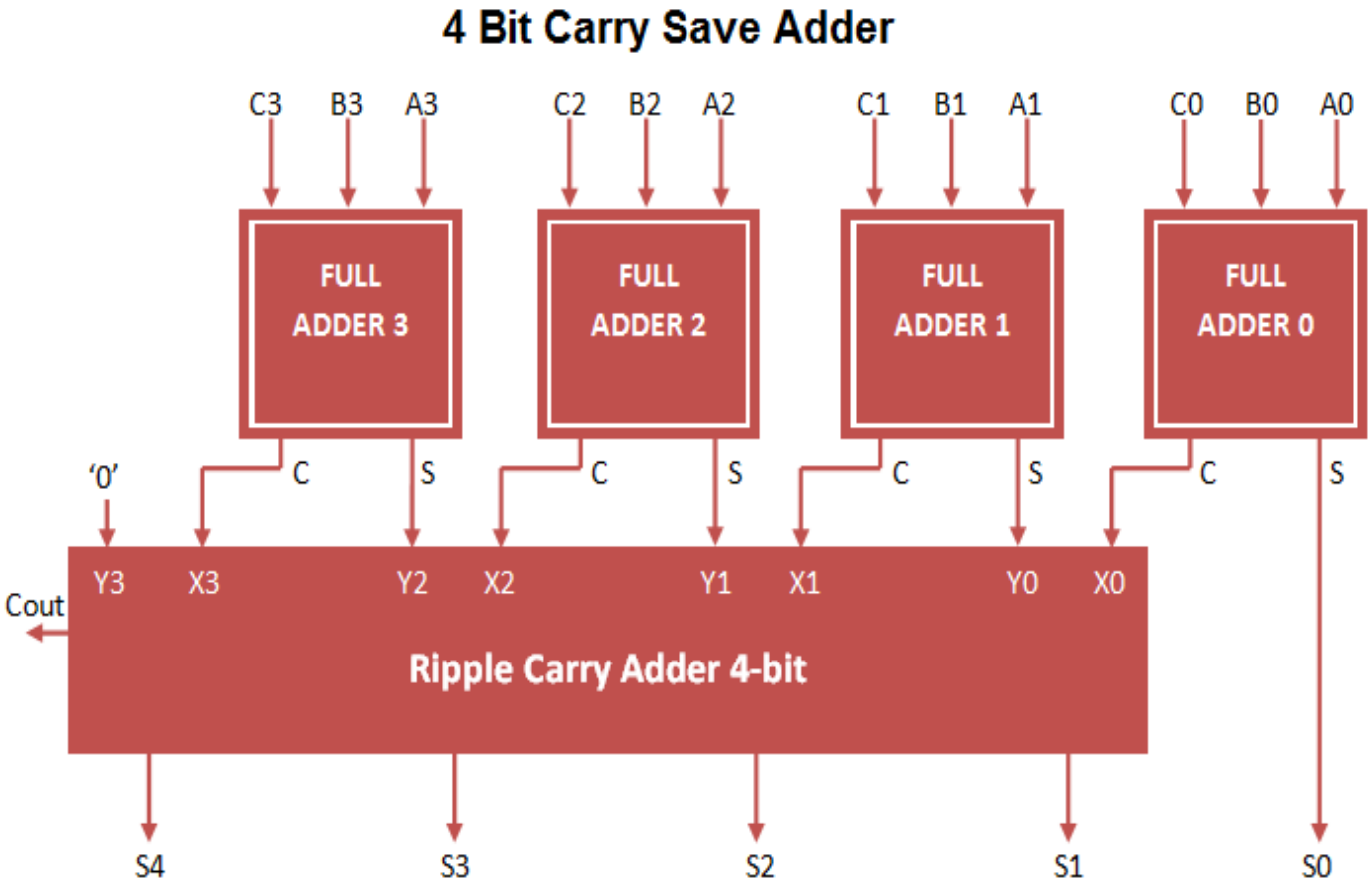
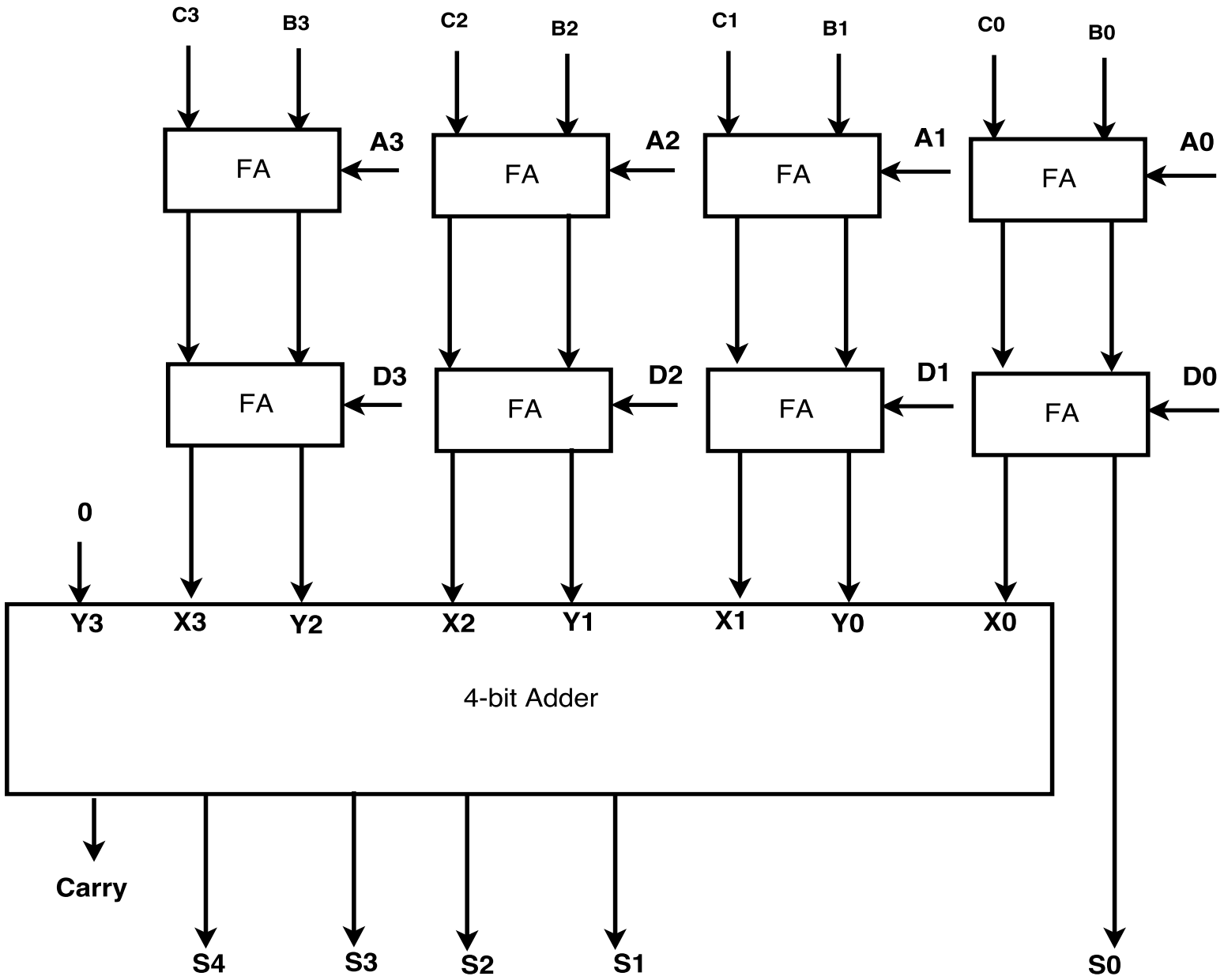


How to perform the addition of 4-inputs(A, B, C, D) using CSAs.

Carry Save Adder (CSA) for 4 inputs

Input $A = A_3 A_2 A_1 A_0$
Input $B = B_3 B_2 B_1 B_0$
Input $C = C_3 C_2 C_1 C_0$
Input $D = D_3 D_2 D_1 D_0$

Carry Save Adder (CSA) for 4 inputs. is it correct!



Multiplication of Two Integers using CSAs

Input $M = m_3 m_2 m_1 m_0$
Input $Q = q_3 q_2 q_1 q_0$
$Z = M \times Q$, where M is Multiplicand and Q is Multiplier.
$Z = \sum_{i=0}^{n-1} M.q_i.2^i = \sum_{i=0}^{n-1} PP_i$
Here PPs are summands.

Try with example: If $M = 1110 = 14$ and $Q = 1101 = 13$, then $M.Q = 182$.

Multiplication of Two Integers using CSAs

Input $M = m_3 m_2 m_1 m_0$
Input $Q = q_3 q_2 q_1 q_0$
$P = M \times Q$, where M is Multiplicand and Q is Multiplier.

PP_0	0	0	0	0	$m_3.q_0$	$m_2.q_0$	$m_1.q_0$	$m_0.q_0$
PP_1	0	0	0	$m_3.q_1$	$m_2.q_1$	$m_1.q_1$	$m_0.q_1$	0
PP_2	0	0	$m_3.q_2$	$m_2.q_2$	$m_1.q_2$	$m_0.q_2$	0	0
PP_3	0	$m_3.q_3$	$m_2.q_3$	$m_1.q_3$	$m_0.q_3$	0	0	0

PP_i stands for parallel product 'i'.
Result $Z = P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0$
where $P_0 = m_0.q_0$

Multiplication of Two Integers using CSAs

Multiplication of Two 4-bit numbers

Level Num	No. of Summands	No. of Groups	Remaining Summands
1	4	1	1
2	3	1	0
3	2	0	0

Time=$1+2\times2+6$(using 4-bit CLAs)=$11\mathcal{T}$.

Multiplication of Two Integers using CSAs
Multiplication of Two 8-bit numbers

Level Num	No. of Summands	No. of Groups	Remaining Summands
1	8	2	2
2	6	2	0
3	4	1	1
4	3	1	0
5	2	0	0

Time=$1+2\times4+10$(using 4-bit CLAs)=$19\mathcal{T}$.

Multiplication of Two Integers using CSAs

Time= Time to generate *PPs*+ (2 x No. of levels of CSA)+(Time to perform final addition)

Carry Save Adder (CSA) for 16 inputs