



# DIGITAL IMAGE PROCESSING

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Digital Image Fundamentals : Session 2

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# Today's Lecture



- Image sampling and quantization –  
image interpolation
- Relationship between pixels

# Image Interpolation

- **Interpolation:** Process of using known data to estimate unknown values  
e.g., zooming, shrinking, rotating, and geometric correction
- **Interpolation** (or **resampling**): an imaging method to increase (or decrease) the number of pixels in a digital image.

**Note:** Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

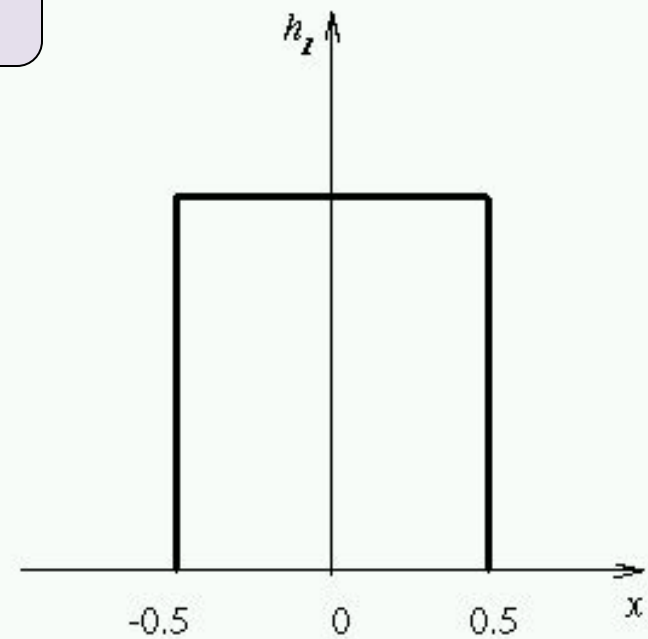
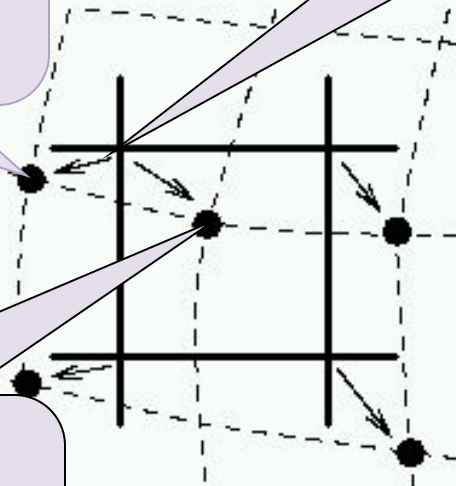
# Image Interpolation

## Nearest Neighbor Interpolation

$$\begin{aligned} f_1(x_2, y_2) &= \\ f(\text{round}(x_2), \text{round}(y_2)) &= \\ = f(x_1, y_1) \end{aligned}$$

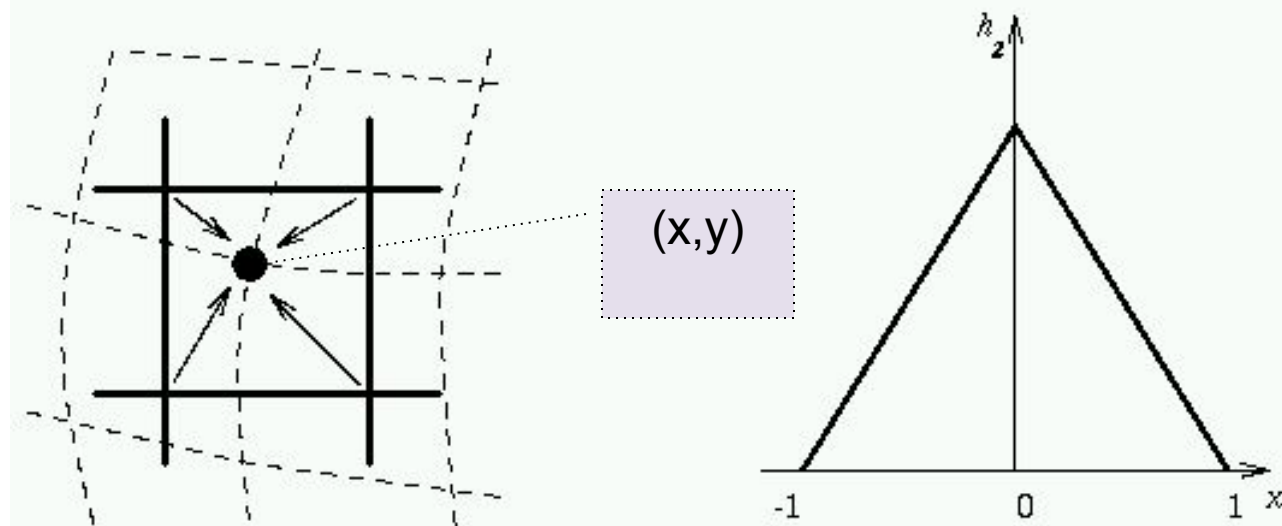
$$f(x_1, y_1)$$

$$\begin{aligned} f_1(x_3, y_3) &= \\ f(\text{round}(x_3), \text{round}(y_3)) &= \\ = f(x_1, y_1) \end{aligned}$$



# Image Interpolation

## Bilinear Interpolation



$$f_2(x, y) = (1 - a)(1 - b)f(l, k) + a(1 - b)f(l + 1, k) \\ + (1 - a)bf(l, k + 1) + abf(l + 1, k + 1) \\ l = \text{floor}(x), k = \text{floor}(y), a = x - l, b = y - k$$

# Image Interpolation

## Bicubic Interpolation

- The intensity value assigned to point  $(x, y)$  is obtained by the following equation

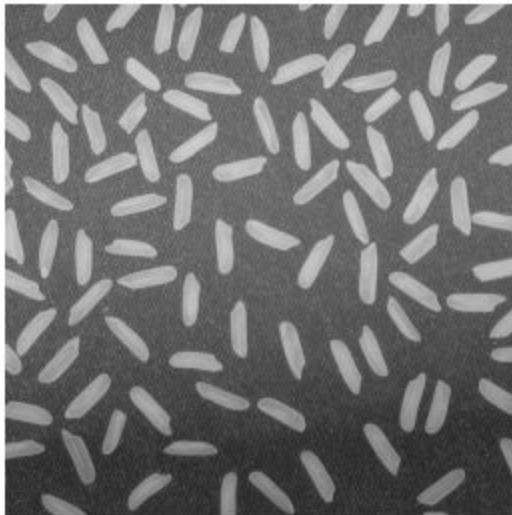
$$f_3(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

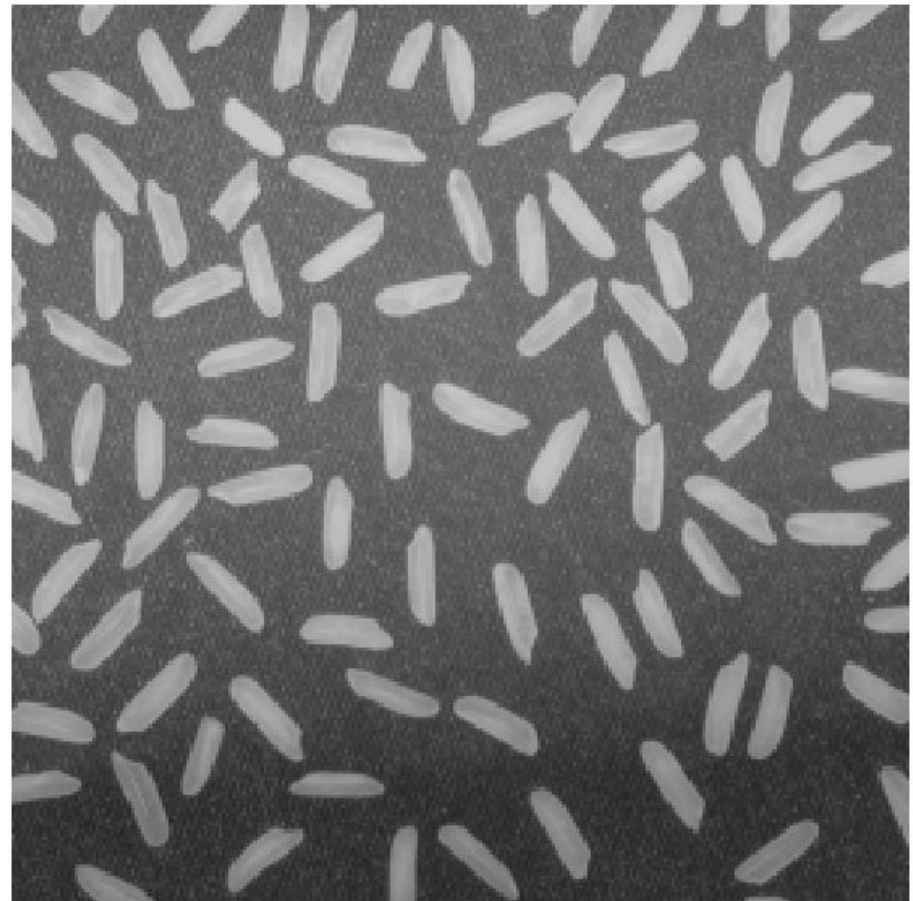
# Image Interpolation

## Example

original image



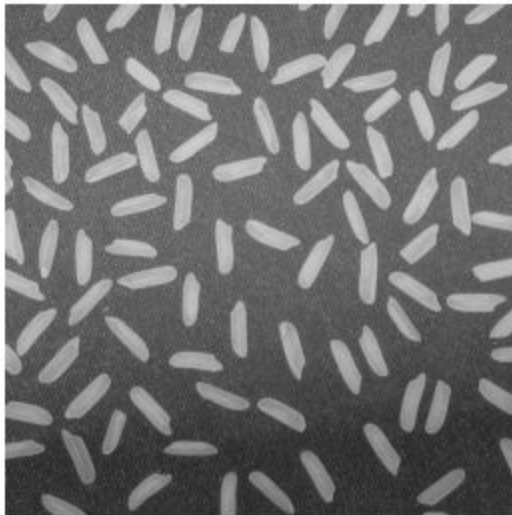
nearest



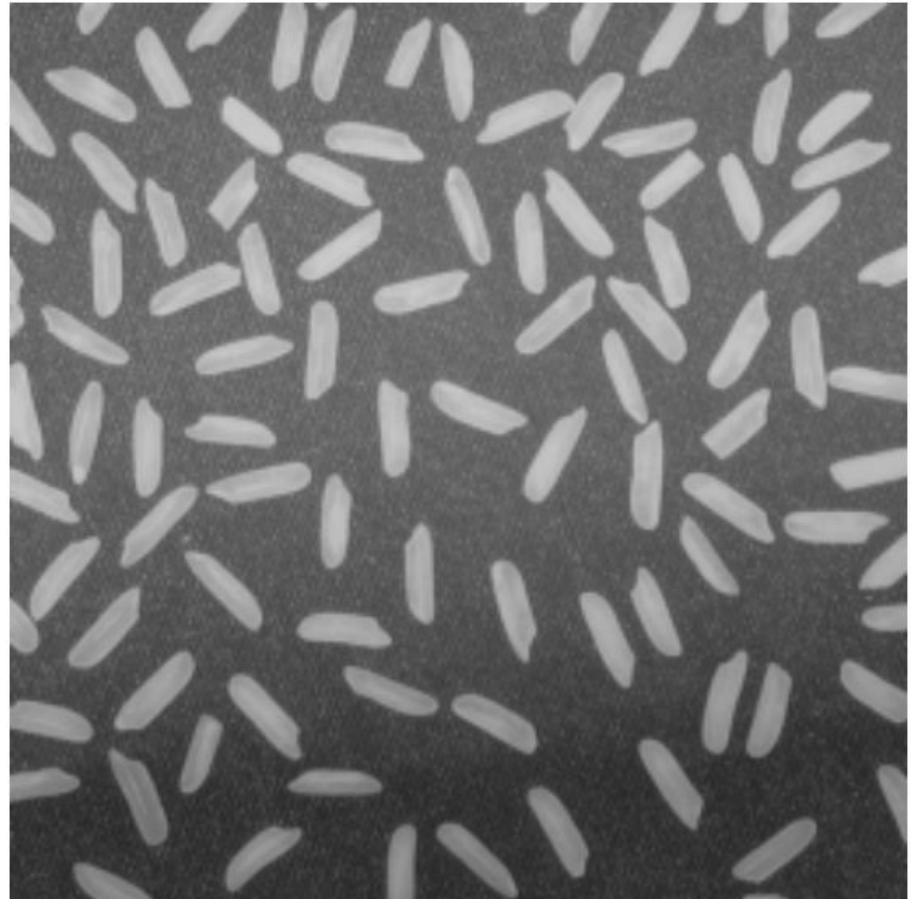
# Image Interpolation

## Example

original image



bilinear

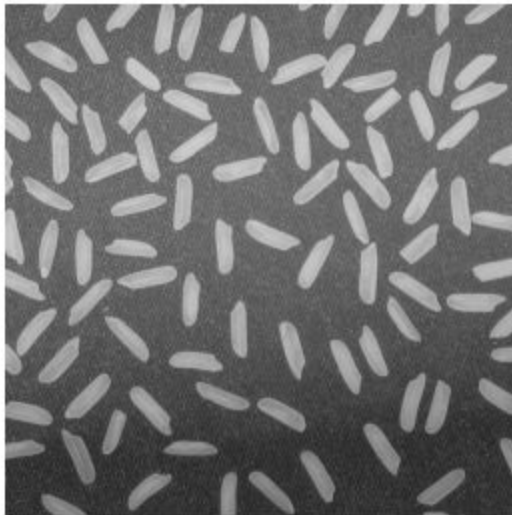




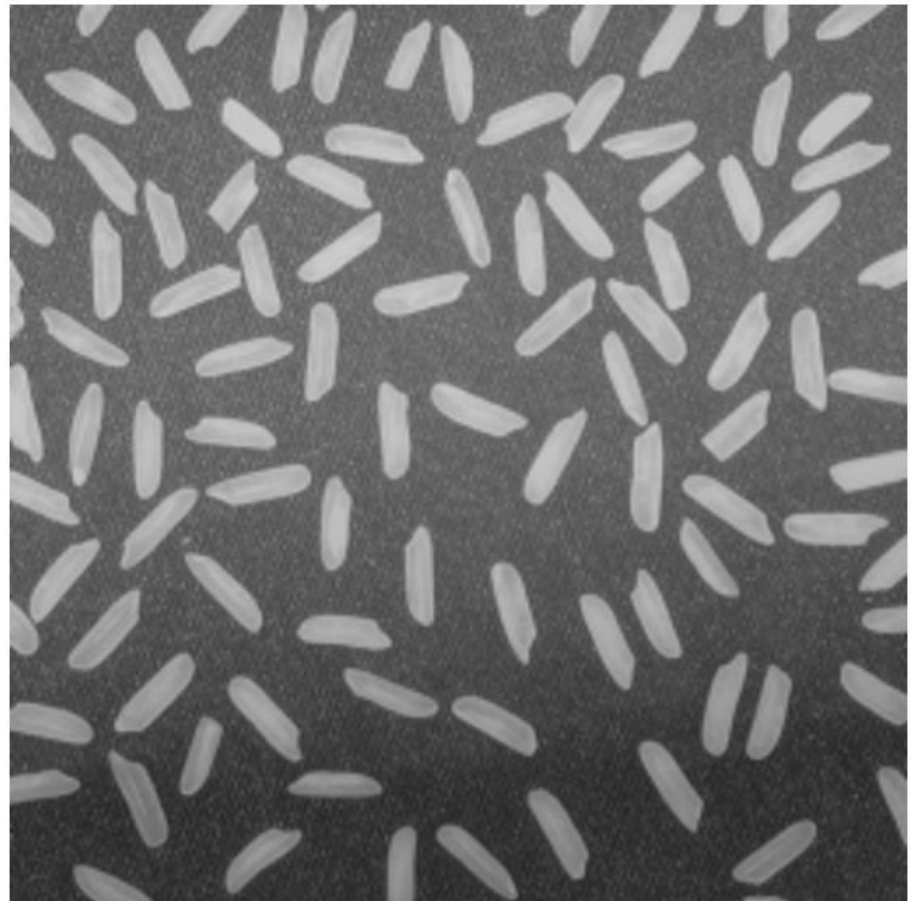
# Image Interpolation

## Example

original image



bicubic



# Relationship between Pixels

- Neighbourhood
- Adjacency
- Connectivity
- Paths
- Regions and

# Relationship between Pixels

**A. Neighbors** of a pixel  $p$  at coordinates  $(x, y)$

**a. 4-neighbors of  $p$** , denoted by  $\mathbf{N}_4(p)$ :

$(x - 1, y)$ ,  $(x + 1, y)$ ,  $(x, y - 1)$ , and  $(x, y + 1)$

**a. 4 diagonal neighbors of  $p$** , denoted by  $\mathbf{N}_D(p)$ :

$(x - 1, y - 1)$ ,  $(x + 1, y + 1)$ ,  $(x + 1, y - 1)$ , and  $(x - 1, y + 1)$

**a. 8 neighbors of  $p$** , denoted  $\mathbf{N}_8(p)$ :

$$\mathbf{N}_8(p) = \mathbf{N}_4(p) \cup \mathbf{N}_D(p)$$

# Relationship between Pixels

## A. Adjacency

Let  $V$  be the set of intensity values.

- a. **4-adjacency:** Two pixels  $p$  and  $q$  with values from  $V$  are 4-adjacent if  $q$  is in the set  $N_4(p)$ .
- b. **8-adjacency:** Two pixels  $p$  and  $q$  with values from  $V$  are 8-adjacent if  $q$  is in the set  $N_8(p)$ .

# Relationship between Pixels

**c. m-adjacency:** Two pixels  $p$  and  $q$  with values from  $V$  are m-adjacent if

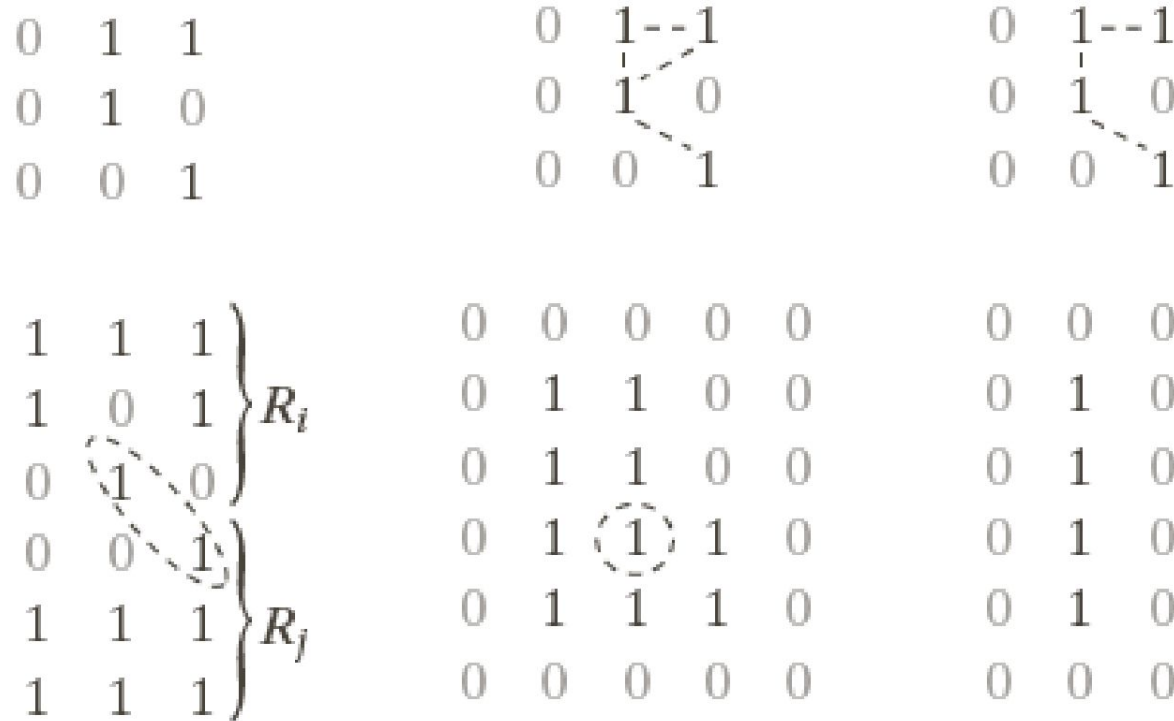
- i.  $q$  is in the set  $N_4(p)$ , or
- ii.  $q$  is in the set  $N_D(p)$  and the set  $N_4(p) \cap N_4(q)$  has no pixels whose values are from  $V$ .

# Relationship between Pixels

## A. Path

- a. A (digital) *path* (or curve) from pixel  $p$  with coordinates  $(x_0, y_0)$  to pixel  $q$  with coordinates  $(x_n, y_n)$  is a sequence of distinct pixels with coordinates  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ ; where  $(x_i, y_i)$  and  $(x_{i-1}, y_{i-1})$  are adjacent for  $1 \leq i \leq n$ .
- b. Here  $n$  is the *length* of the path.
- c. If  $(x_0, y_0) = (x_n, y_n)$ , the path is *closed* path.
- d. We can define 4-, 8-, and m-paths based on the type of adjacency used.

# Relationship between Pixels



a	b	c
d	e	f

**FIGURE 2.25** (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c)  $m$ -adjacency. (d) Two regions that are adjacent if 8-adjacency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.

# Relationship between Pixels

## Connected in $S$

Let  $S$  represent a subset of pixels in an image. Two pixels  $p$  with coordinates  $(x_0, y_0)$  and  $q$  with coordinates  $(x_n, y_n)$  are said to be **connected in  $S$**  if there exists a path

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n),$$

where  $\forall i, 0 \leq i \leq n, (x_i, y_i) \in S$



# Relationship between Pixels

Let  $S$  represent a subset of pixels in an image

- For every pixel  $p$  in  $S$ , the set of pixels in  $S$  that are connected to  $p$  is called a *connected component* of  $S$ .
- If  $S$  has only one connected component, then  $S$  is called *connected set*.
- We call  $R$  a *region* of the image if  $R$  is a connected set.
- Two regions,  $R_i$  and  $R_j$  are said to be *adjacent* if their union forms a connected set.
- Regions that are not to be adjacent are said to be *disjoint*.

# Relationship between Pixels

## □ *Boundary (or border)*

- The *boundary* of the region  $R$  is the set of pixels in the region that have one or more neighbors that are not in  $R$ .
- If  $R$  happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

## □ *Foreground and background*

- An image contains  $K$  disjoint regions,  $R_k, k = 1, 2, \dots, K$ . Let  $R_u$  denote the union of all the  $K$  regions, and let  $(R_u)^c$  denote its complement. All the points in  $R_u$  is called *foreground*; All the points in  $(R_u)^c$  is called *background*.

# Relationship between Pixels

## Distance Measure

- Given pixels  $p$ ,  $q$  and  $z$  with coordinates  $(x, y)$ ,  $(s, t)$ ,  $(u, v)$  respectively, the distance function  $D$  has following properties:
  - a.  $D(p, q) \geq 0$       $[D(p, q) = 0, \text{ iff } p = q]$
  - b.  $D(p, q) = D(q, p)$
  - c.  $D(p, z) \leq D(p, q) + D(q, z)$

# Relationship between Pixels

## Distance Measure

The following are the different Distance measures:

- Euclidean Distance :

$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

- City Block Distance:

$$D_4(p, q) = |x-s| + |y-t|$$

- Chess Board Distance:

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

# Next Class



- **Digital Image Fundamentals**
  - Mathematical Operations in DIP

**Thank you:  
Question?**