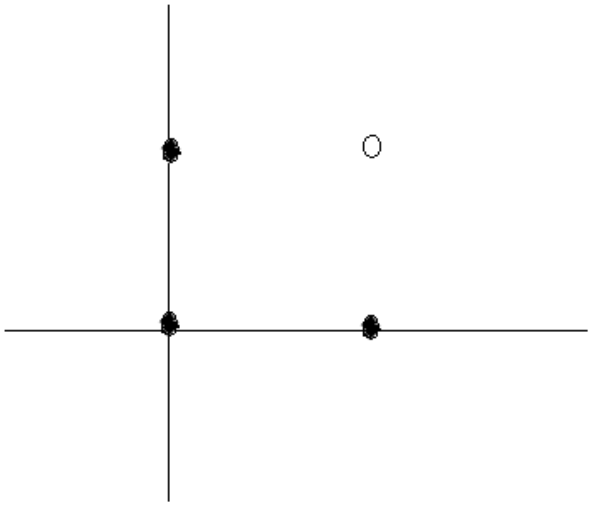
Logical AND problem

Hard Linear SVM



AND problem. Hard linear SVM.

The training set that belongs to class -1 is

$$\{X_1 = (0,0)^t, X_2 = (0,1)^t, X_3 = (1,0)^t\};$$

that which belongs to class $+1$ is $X_4 = (1,1)^t$.

KKT Conditions.

(1) $W = (w_1, w_2)^t = \sum_i \alpha_i y_i X_i = \begin{pmatrix} \alpha_4 - \alpha_3 \\ \alpha_4 - \alpha_2 \end{pmatrix}$

(2) $\Sigma \alpha_i y_i = 0 \Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = \alpha_4$

(3) $\alpha_i \geq 0, \forall i$. From geometry, we can infer that $\alpha_1 = 0. \alpha_2 = \alpha_3$.

(4) $\alpha_i \big(1 - y_i (W \cdot X_i + b)\big) = 0, \forall i$. We get 3 equations, solving which gives,

$$w_1 = 2, w_2 = 2, b = -3.$$

The decision boundary is given by $2x_1 + 2x_2 - 3 = 0$.

We get, $\alpha_1 = 0, \alpha_2 = \alpha_3 = 2, \alpha_4 = 4$.

Margin is $\frac{2}{\|W\|} = \frac{1}{\sqrt{2}}$.