DIGITAL IMAGE PROCESSING

Image Restoration : Session 1

Dr. Mrinmoy Ghorai

Indian Institute of Information Technology
Sri City, Andhra Pradesh

Today's Lecture

- Image Restoration
  - Noise Model

Image Restoration

- It is a process to recover an image that has been degraded by using a prior knowledge of the degradation phenomenon.

- Model the degradation and applying the inverse process in order to recover the original image.

- Image enhancement is largely a subjective process, while image restoration is mostly a objective process.

Image Restoration

A Model of Image Degradation/Restoration Process

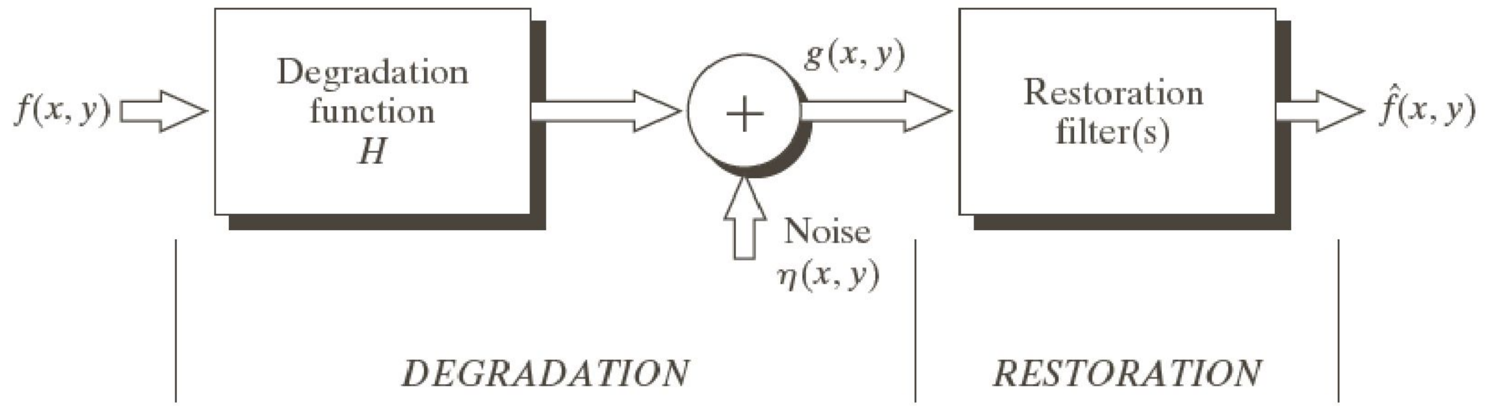


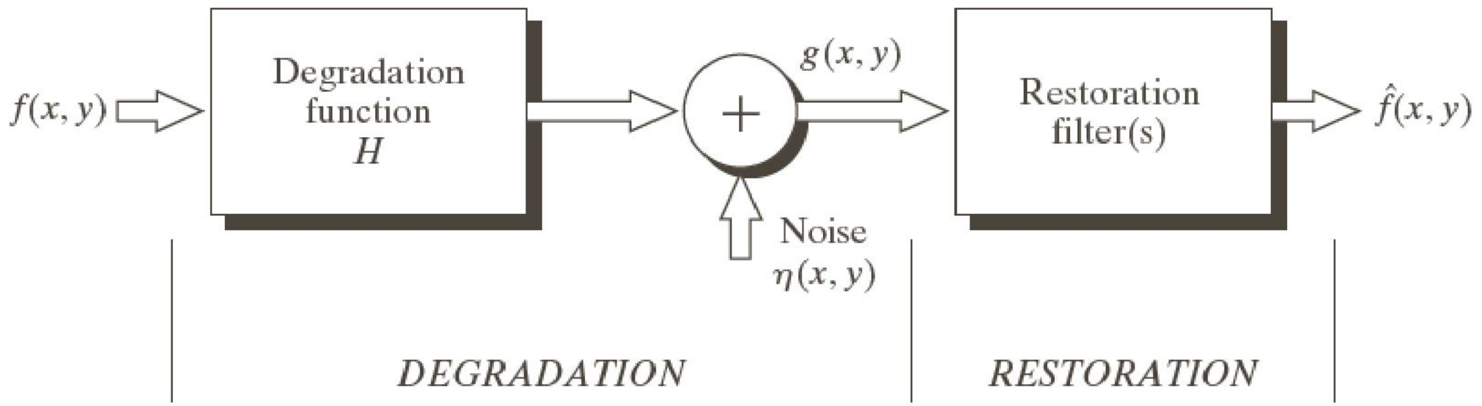
FIGURE 5.1 A model of the image degradation/ restoration process.

- **Degradation Model**
  - *Degradation Function* H
  - *Additive noise* $\eta(x, y)$

Image Restoration

A Model of Image Degradation/Restoration Process

FIGURE 5.1
A model of the image degradation/ restoration process.



If *H* is a **linear, position invariant** process, then the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y),$$

where $h(x, y)$ is the spatial representation of the degradation function and the symbol "$*$" indicates evaluation.

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The model of the degraded image is given in the frequency domain

$$G(u,v) = H(u,v)F(u,v) + N(u,v),$$

where the terms in capital letters are the Fourier transforms of the corresponding terms in the previous equation.

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Noise Sources

- The principal sources of noise in digital images arise during **image acquisition and/or transmission**

✔ **Image acquisition:** Depends on the quality of the sensing elements

e.g., sensor temperature, light levels etc.

✔ **Transmission:** Due to interference in the channel used for transmission

e.g., lightning or other atmospheric disturbance in wireless network

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Noise Model 1

- **White noise**
  - The Fourier spectrum of noise is constant
  - The terminology “white” comes from the physical properties of white light, which contains nearly all frequencies in the visible spectrum in **equal proportions.**

- With the exception of spatially periodic noise, we assume
  - Noise is independent of spatial coordinates
  - Noise is uncorrelated with respect to the image itself

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Noise Model 2

- **Gaussian noise**

  Electronic circuit noise, sensor noise due to poor illumination and/or high temperature

- **Rayleigh noise**

  Range imaging

Gaussian Noise

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

where, z represents intensity

$\bar{z}$ is the mean (average) value of z

σ is the standard deviation

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Gaussian Noise

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

- 70% of its values will be in the range

$$\left[(\mu - \sigma), (\mu + \sigma)\right]$$

- 95% of its values will be in the range

$$\left[(\mu - 2\sigma), (\mu + 2\sigma)\right]$$

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Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b / 4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

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Noise Model 3

- **Erlang (gamma) noise**: Laser imaging
- **Exponential noise**: Laser imaging
- **Uniform noise**: Least descriptive; Basis for numerous random number generators
- **Impulse noise**: Quick transients, such as faulty switching

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Erlang (Gamma) Noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

Here *a>0* and *b* is an integer

The mean and variance of this density are given by

$$\bar{z} = b / a$$

$$\sigma^2 = b / a^2$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = 1/a$$

$$\sigma^2 = 1/a^2$$

Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for a} \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = (a+b)/2$$

$$\sigma^2 = (b-a)^2/12$$

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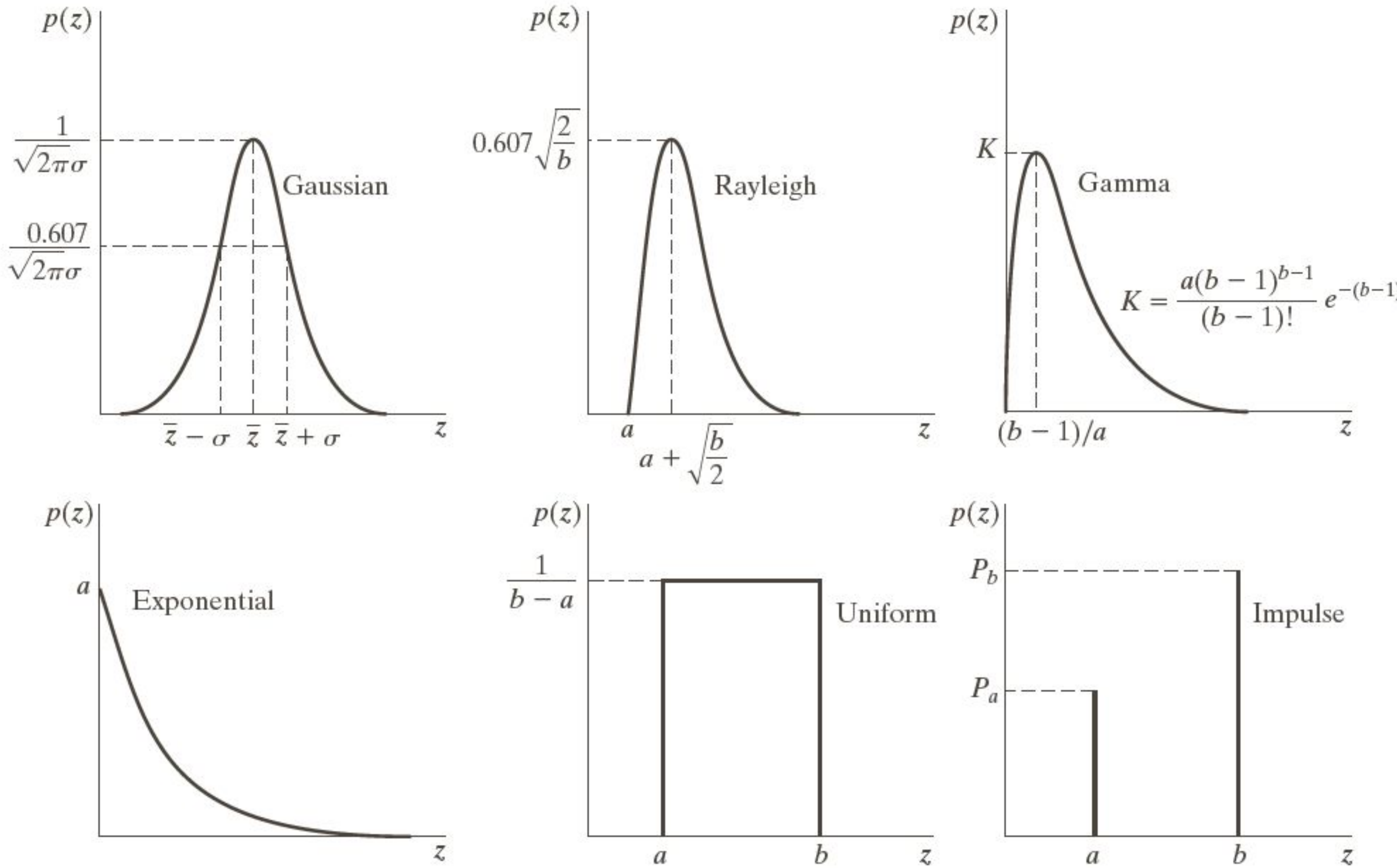
Impulse (Salt-and-Pepper) Noise

The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

if $b > a$, gray-level b will appear as a light dot, while level a will appear like a dark dot.

If either P_a or P_b is zero, the impulse noise is called *unipolar*



a b c
d e f

FIGURE 5.2 Some important probability density functions.

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Examples of Noise: Original Image

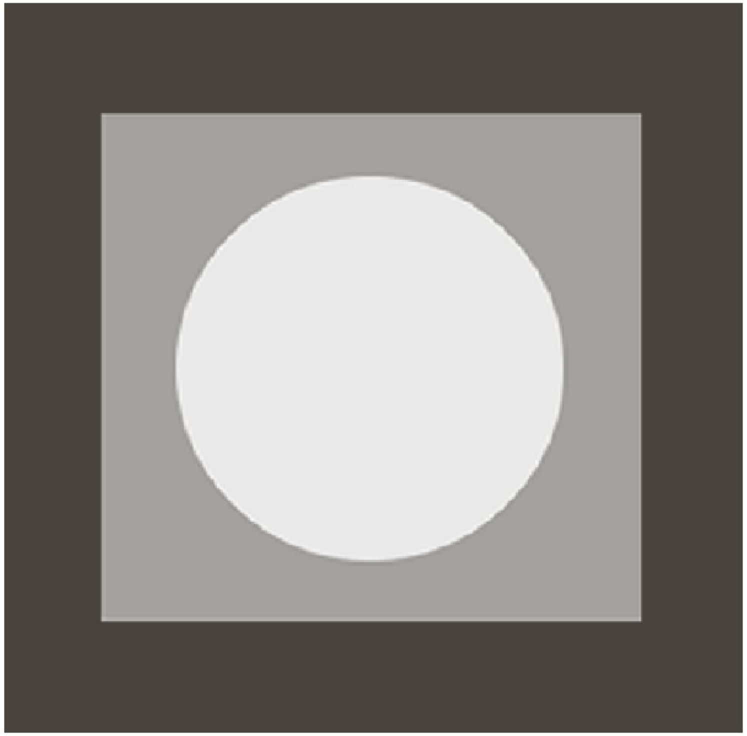

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

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Examples of Noise: Noisy Images

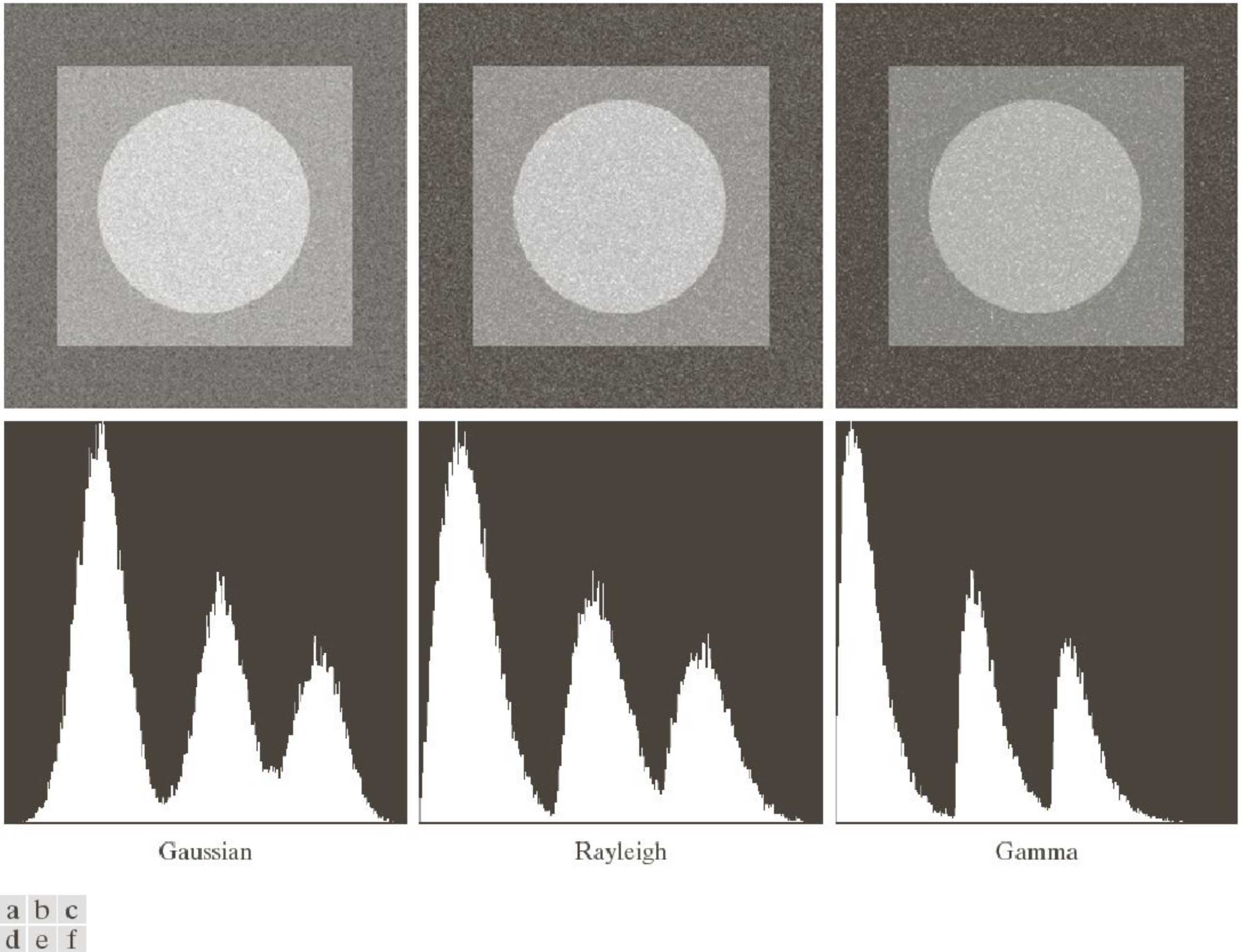

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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Examples of Noise: Noisy Images

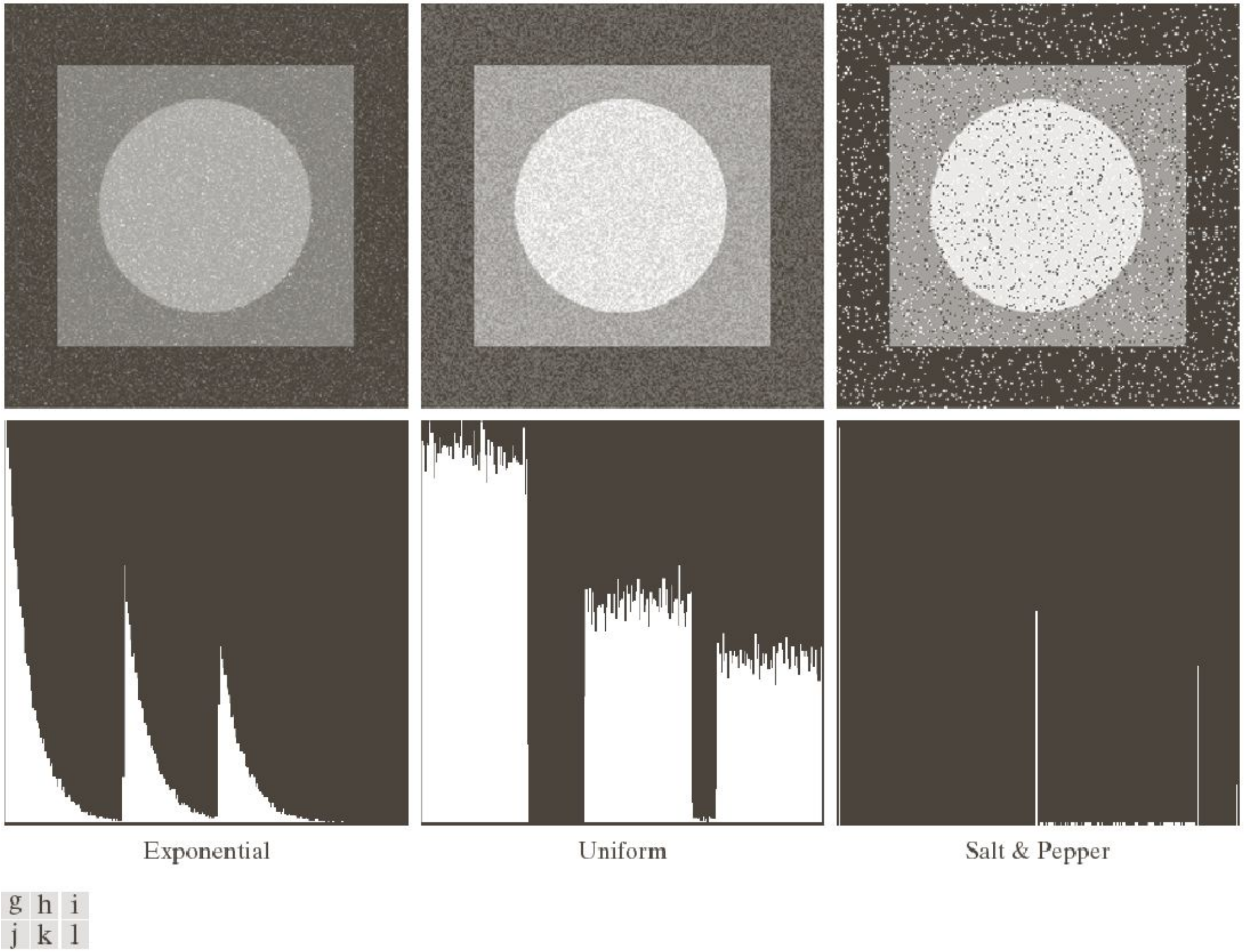

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

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Periodic Noise

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.

- It is a type of spatially dependent noise

- Periodic noise can be reduced significantly via frequency domain filtering

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Example of Periodic Noise

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

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Estimation of Noise Parameters

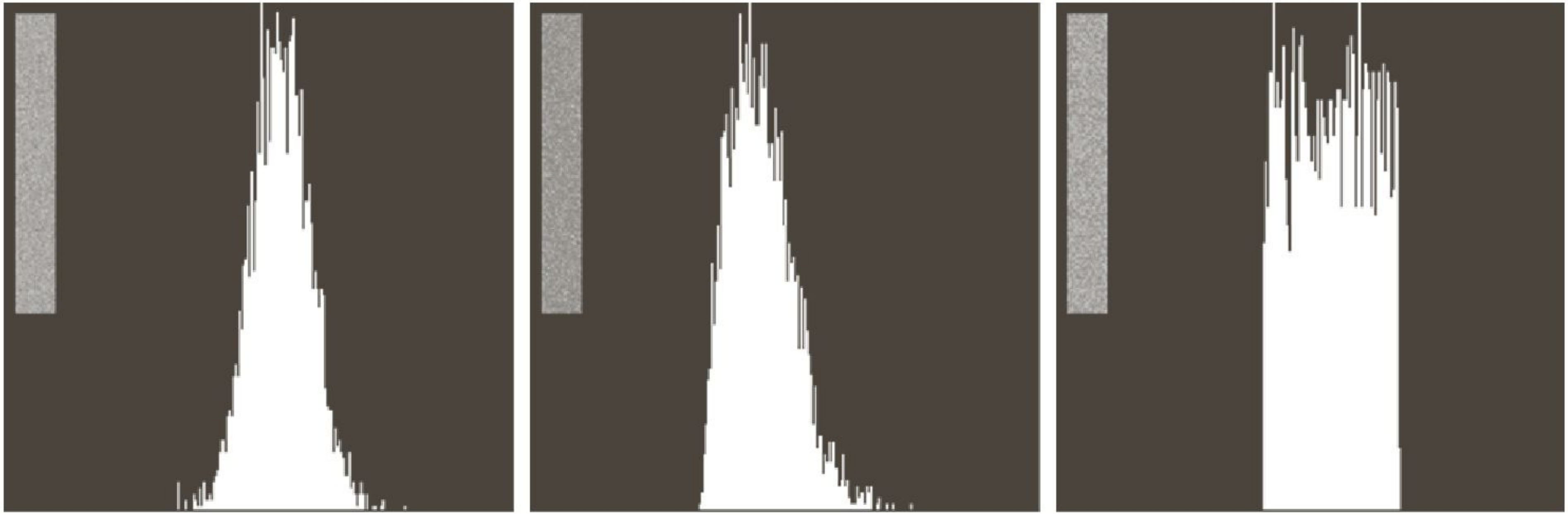

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

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Estimation of Noise Parameters

Consider a subimage denoted by S, and let $p_s(z_i)$, $i = 0,\ 1,\ \dots,\ L-1$, denote the probability estimates of the intensities of the pixels in S.

The mean and variance of the pixels in S:

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$

Restoration in the Presence of Noise Only

Spatial Filtering

Noise model without degradation

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u, v) = F(u, v) + N(u, v)$$

Spatial Filtering: Mean Filters(1)

Let S_{xy} represent the set of coordinates in a rectangle subimage window of size $m \times n$, centered at (x, y).

Arithmetic mean filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

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Spatial Filtering: Mean Filters(2)

Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Generally, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process

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Spatial Filtering: Mean Filters(3)

Harmonic mean filter

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

It works well for salt noise, but fails for pepper noise.
It does well also with other types of noise like Gaussian noise.

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Spatial Filtering: Mean Filters(4)

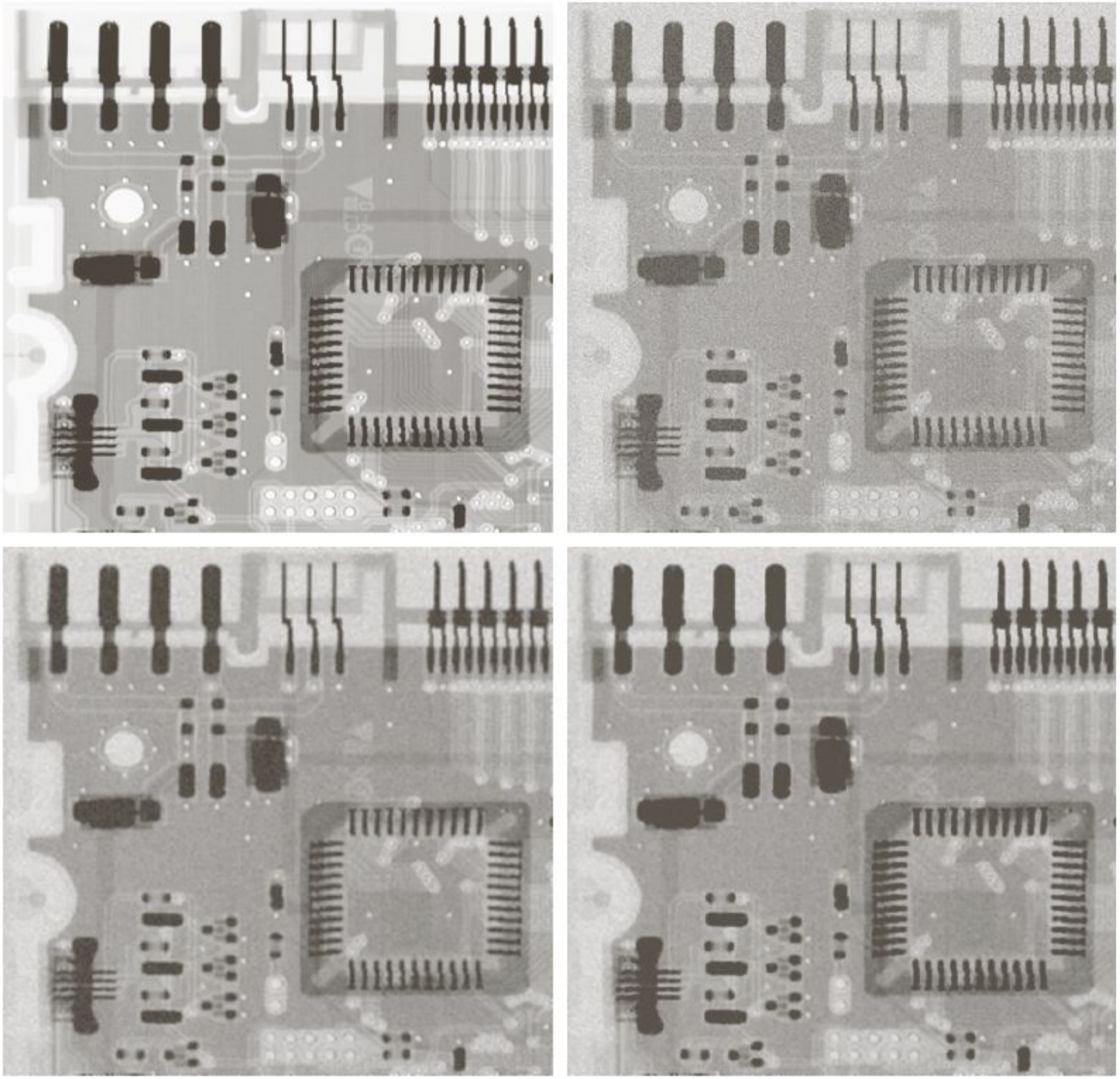
Contraharmonic mean filter

$$\hat{f}(x, y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the order of the filter.

It is well suited for reducing the effects of salt-and-pepper noise. Q>0 for pepper noise and Q<0 for salt noise.

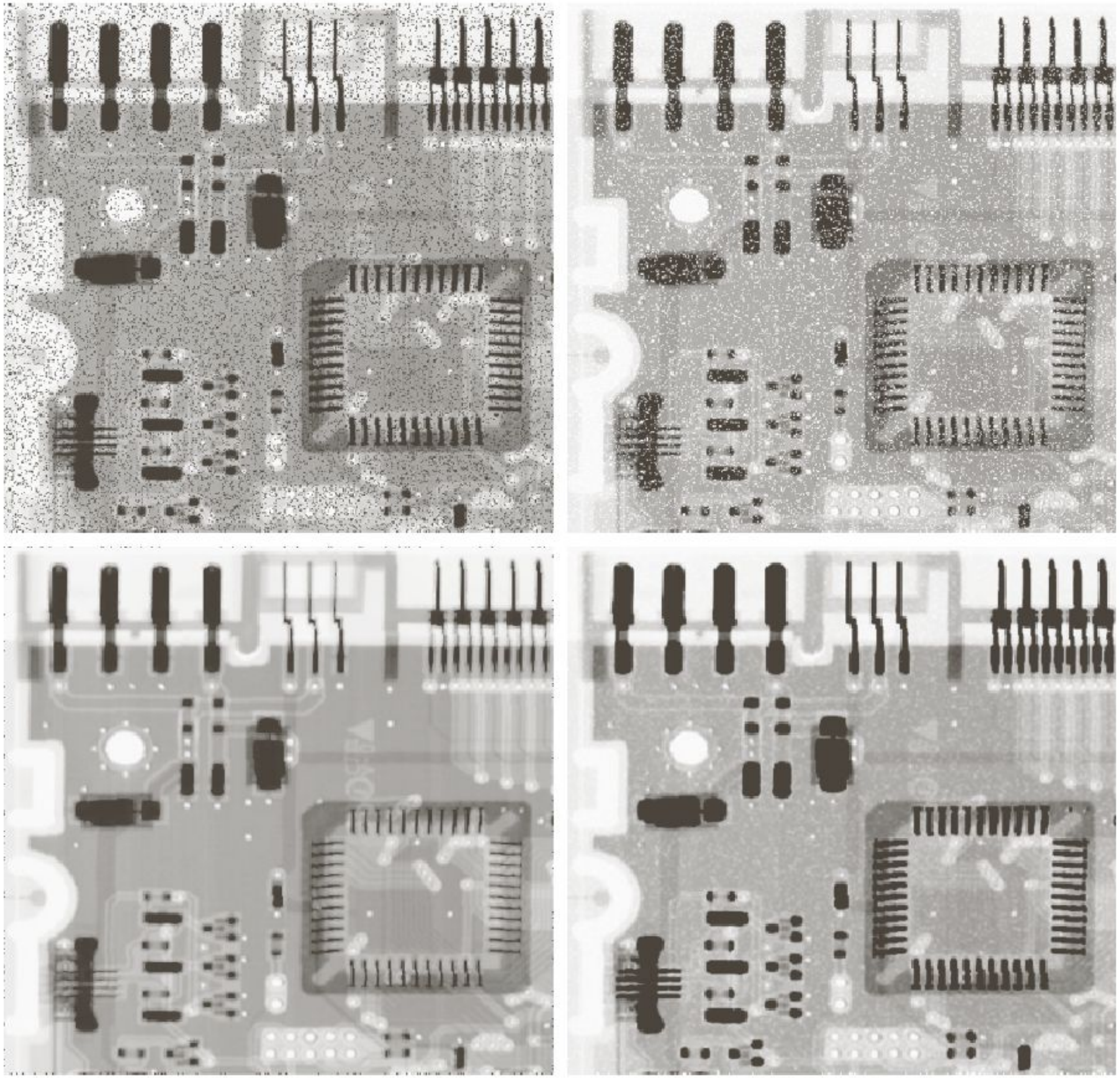
Spatial Filtering: Example(1)



a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

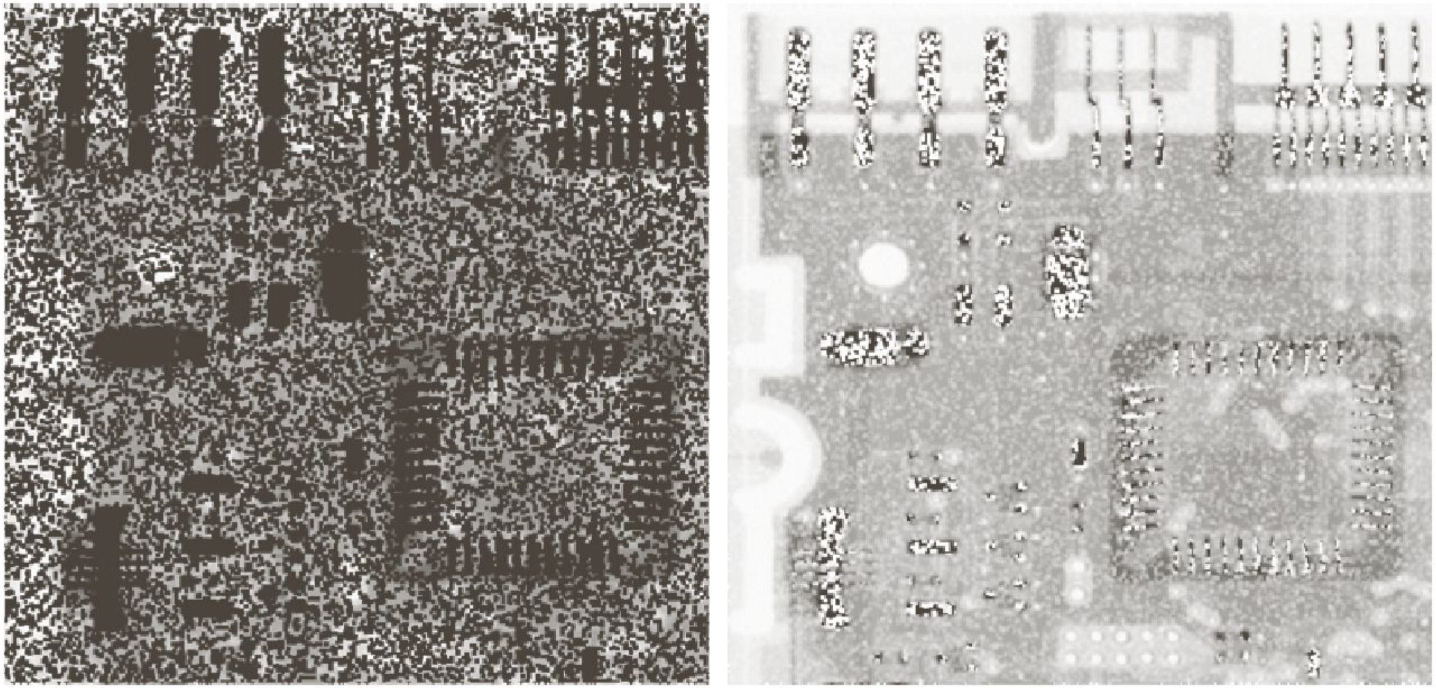
Spatial Filtering: Example(2)



a	b
c	d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Spatial Filtering: Example(3)



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Next Class

- Image Restoration
  - More Filters

Thank you:
Question?