

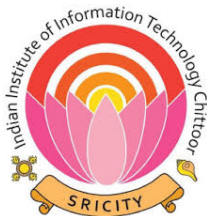
CO: Computer Organization

Day2

Indian Institute of Information Technology, Sri City

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<http://co-iiits.blogspot.in/>



Number Systems

- ▶ Representation of Integer Numbers
 - ▶ Signed Magnitude Representation
 - ▶ 1's Complement Representation
 - ▶ 2's Complement Representation
- ▶ Representation of Real Numbers
 - ▶ Fixed Point Representation
 - ▶ Floating Point Representation

Resolution is difference between two successive numbers.

Representation of Integer Numbers

Let $A = a_{n-1}a_{n-2}a_{n-3}\dots a_0$ is an n-bit binary number

if A is an **unsigned integer**, then value of A is : $\sum_{i=0}^{n-1}(2^i \times a_i)$.

if A is a **signed integer**:

▶ Signed Magnitude Representation:

- ▶ $A = \sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 0$
- ▶ $A = -\sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 1$

▶ 1's Complement Rep.: $A = -(2^{n-1} - 1) \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$

▶ 2's Complement Rep.: $A = -2^{n-1} \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$

Resolution: 1

Representation of Integer Numbers

Write all possible 4-bit numbers and write its equivalent value using the three rep.

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Binary Number	Signed Magnitude	1's Complement	2's Complement
0 0 0 0	+0	+0	+0
0 0 0 1	+1	+1	+1
0 0 1 0	+2	+2	+2
0 0 1 1	+3	+3	+3
0 1 0 0	+4	+4	+4
0 1 0 1	+5	+5	+5
0 1 1 0	+6	+6	+6
0 1 1 1	+7	+7	+7
1 0 0 0	-0	-7	-8
1 0 0 1	-1	-6	-7
1 0 1 0	-2	-5	-6
1 0 1 1	-3	-4	-5
1 1 0 0	-4	-3	-4
1 1 0 1	-5	-2	-3
1 1 1 0	-6	-1	-2
1 1 1 1	-7	-0	-1

Range of Numbers

Let $A = a_{n-1}a_{n-2}a_{n-3}\dots a_0$ is an n bit binary number

if A is an **unsigned integer**, then range of A is : 0 to $(2^n - 1)$.

if A is a **signed integer**:

- ▶ Signed Magnitude Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- ▶ 1's Complement Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- ▶ 2's Complement Rep., range of A is : -2^{n-1} to $(2^{n-1} - 1)$.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.

Let $A = 1\ 0\ 1\ 0$ is a 4-bit binary number,

Representation of A using 8-bits (i.e. B): $1\ 1\ 1\ 1\ 1\ 0\ 1\ 0$.

is $A=B$?

- ▶ In 2's Complement Rep.: Yes.
- ▶ In 1's Complement Rep.: Yes.
- ▶ In Signed Magnitude Rep.: No.

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- ▶ In 1's Complement Rep.: **Yes**.
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Real Numbers

❶ $(4.5)_{10} = (100.1)_2$

❷ $(8.25)_{10} = (1000.01)_2$

❸ $(16.125)_{10} = (10000.001)_2$

❹ $(0.875)_{10} = (0.111)_2$

❺ $(4.5)_{10} = (1.001)_2 \times 2^2$

❻ $(8.25)_{10} = (1.00001)_2 \times 2^3$

❼ $(16.125)_{10} = (1.0000001)_2 \times 2^4$

❽ $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**,
'xxxxxx' is a **Fraction/Mantissa**.

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IEEE 754

- ▶ The IEEE Standard for Floating-Point Arithmetic.



Single Precision N=32	1 bit	8 bits	23 bits	Bias Value: +127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value: +1023

Biased Exponent = True Exponent + Bias Value

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- ▶ Biased Exponent = $2 + 127 = 129 = 1000\ 0001$
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- ▶ Biased Exponent = True Exponent + Bias Value,
where $1 \leq \text{Biased Exponent} \leq (2^{\text{BiasedExponentBits}} - 2)$.
- ▶ Single Precision (N=32), $1 \leq \text{Biased Exponent} \leq 254$.
- ▶ Biased Exponent = 0,
 - ▶ Mantissa = ± 0 , then Value is ± 0 .
 - ▶ Mantissa $\neq 0$, then Value is **not a normalized number**.
- ▶ Biased Exponent = 255,
 - ▶ Mantissa = ± 0 , then Value is $\pm \infty$.
 - ▶ Mantissa $\neq 0$, then Value is **NAN**.
- ▶ Range of positive values: $[1.0 \times 2^{-126}, (2 - 2^{-23}) \times 2^{127}]$
- ▶ Range of negative values: $[-(2 - 2^{-23}) \times 2^{127}, -1.0 \times 2^{-126}]$
- ▶ Single Precision Number Resolution: $2^{-23} \times 2^{\text{TrueExponent}}$