DIGITAL IMAGE PROCESSING

Image Enhancement in Frequency Domain: Session 4

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Today's Lecture

- **Image Enhancement in Spatial Domain**
  - **Smoothing Filters**
  - **Sharpening Filters**

Image Enhancement in Frequency Domai

Smoothing Filters

- Edges and other sharp transitions (such as noise) contribute significantly to the high frequency content of Fourier transform.
- Smoothing (blurring) is achieved in frequency domain by attenuating a specified range of high frequency components.
- Three types of lowpass filters: Ideal, Butterworth and Gaussian filters
- These filters cover the range from very sharp (ideal) to very smooth (Gaussian) filter functions.

Image Enhancement in Frequency Domain

Smoothing Filters: **Ideal Lowpass Filters (ILPF)**

Ideal Lowpass Filters (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

Image Enhancement in Frequency Domain

Smoothing Filters: **ILPF**

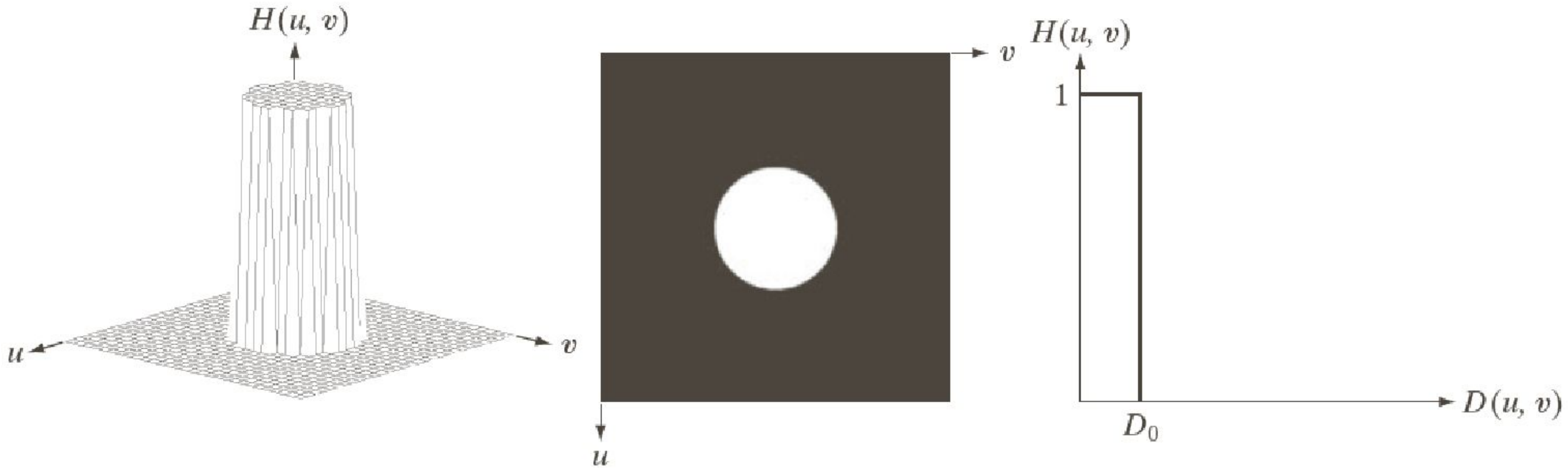

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

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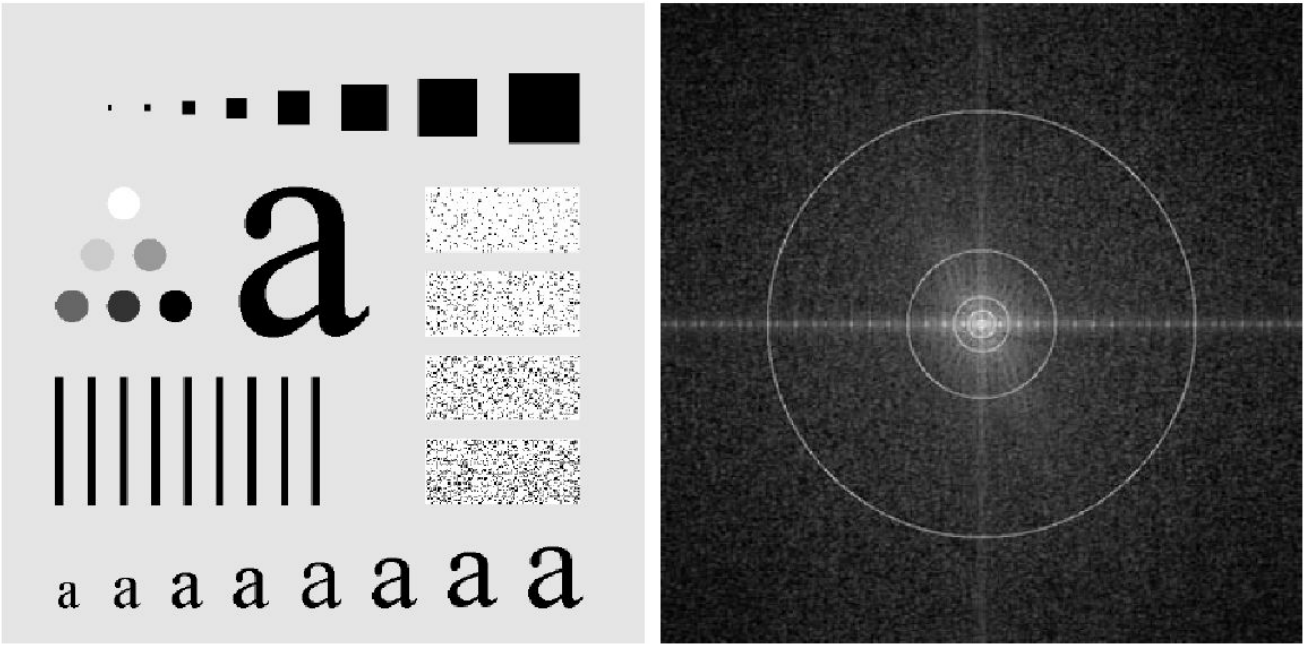
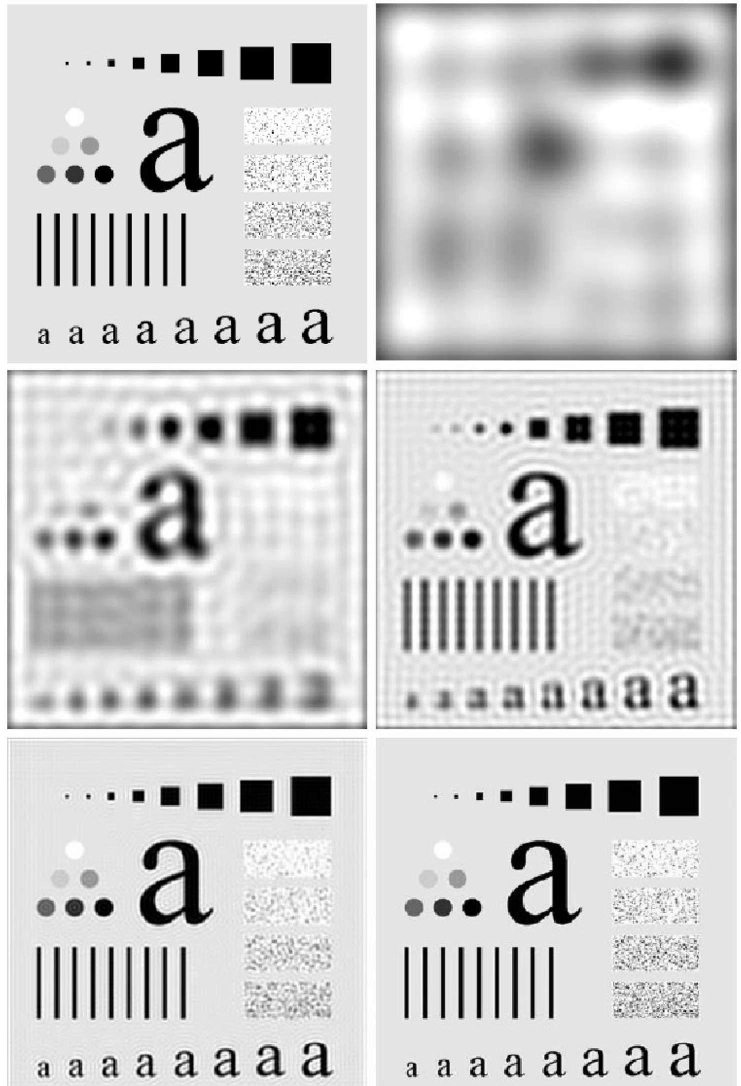
Smoothing Filters: **ILPF**



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.



a	b
c	d
e	f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

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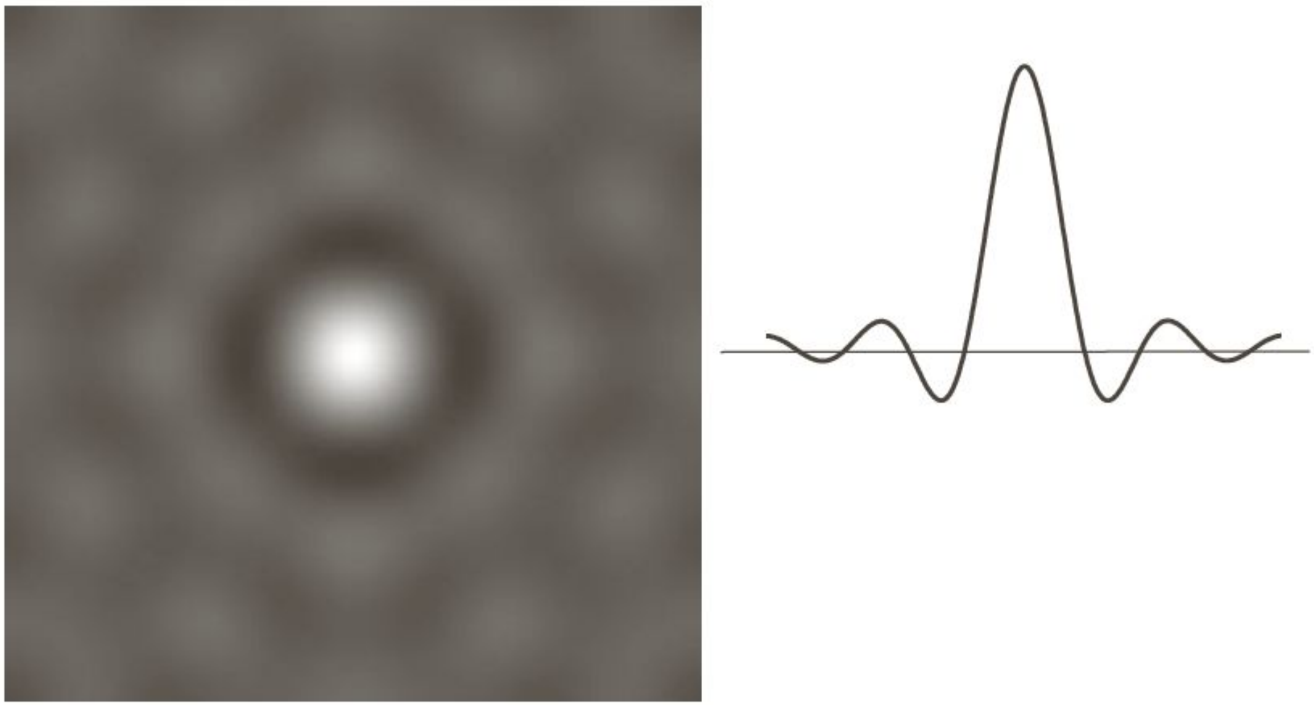
Smoothing Filters: **Spatial Representation of ILPF**



a b

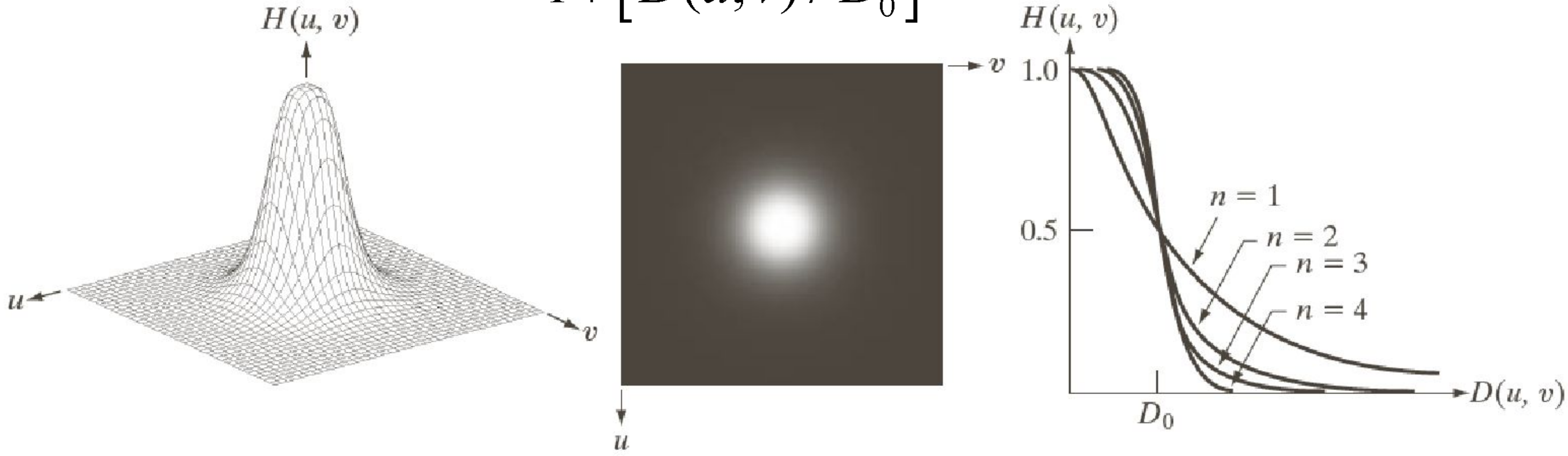
FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Image Enhancement in Frequency Domain

Smoothing Filters: **Butterworth Lowpass Filters (BLPF)**

Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u,v) = \frac{1}{1 + \left[D(u,v) / D_0 \right]^{2n}}$$



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

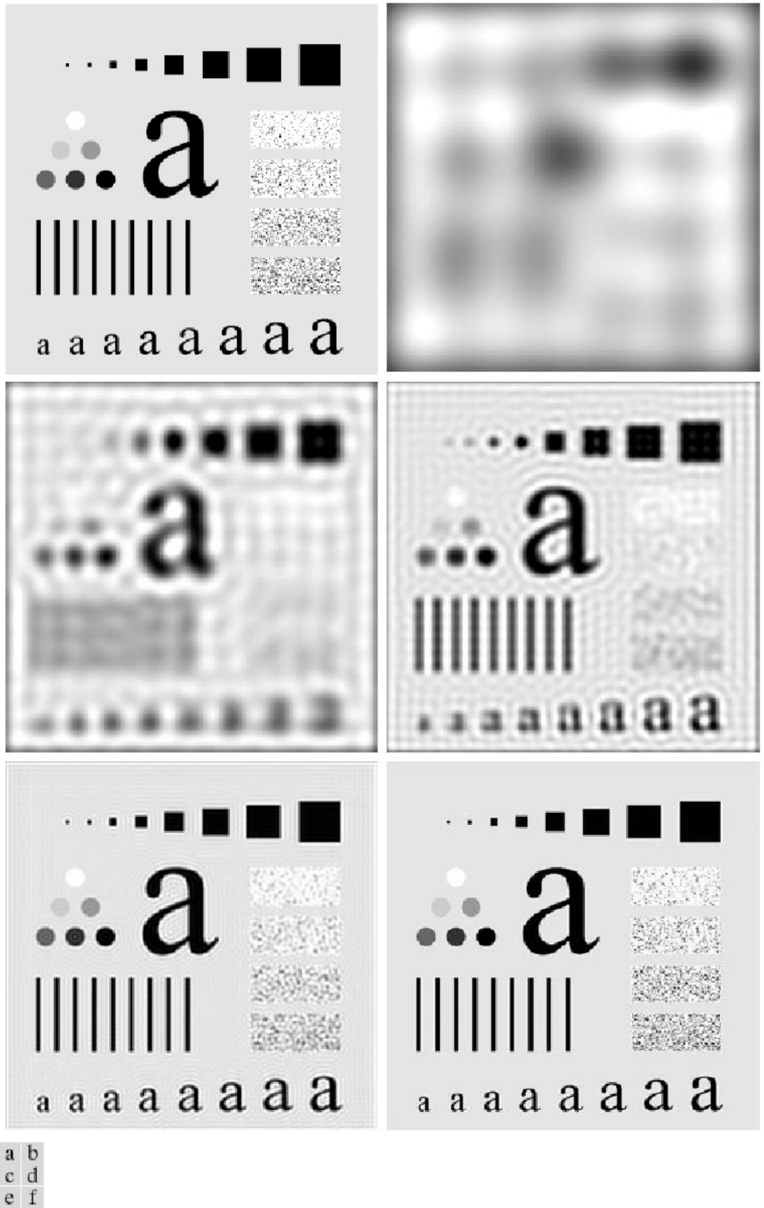


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

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Smoothing Filters: **Spatial Representation of BLPF**

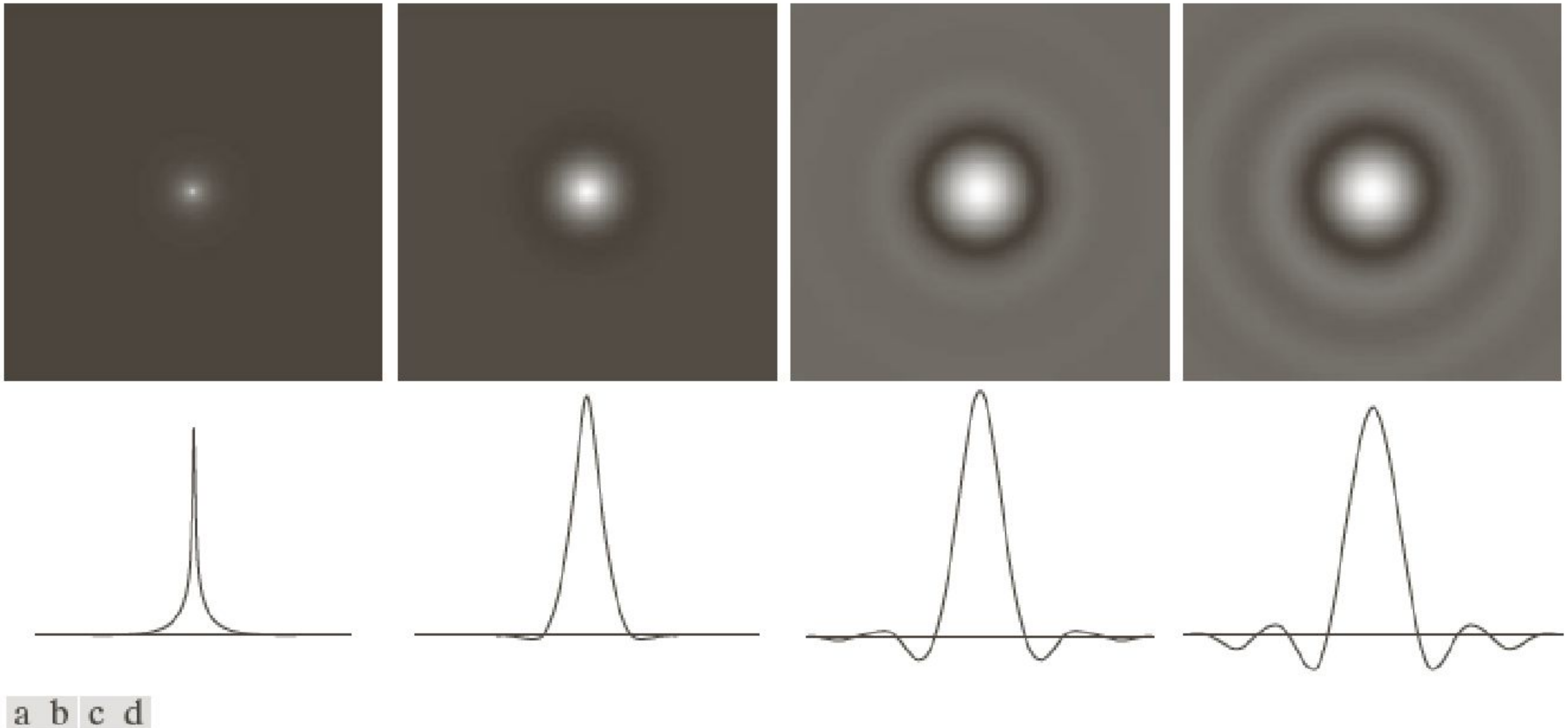

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

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Smoothing Filters: **Gaussian Lowpass Filters (GLPF)**

Gaussian Lowpass Filters (GLPF) in two dimensions is given

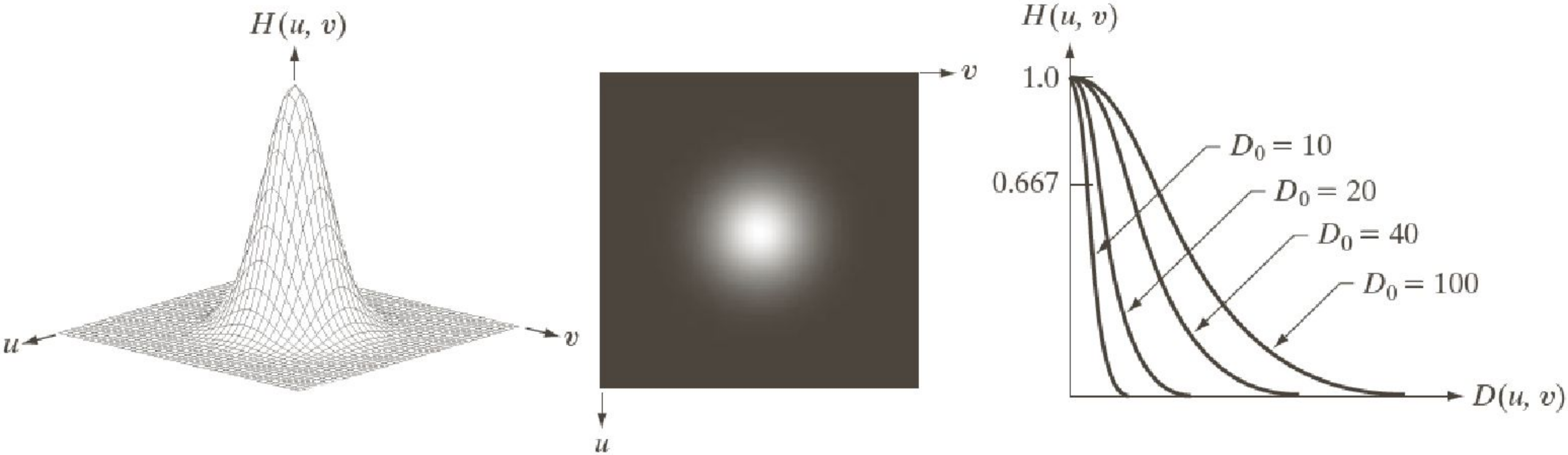
$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

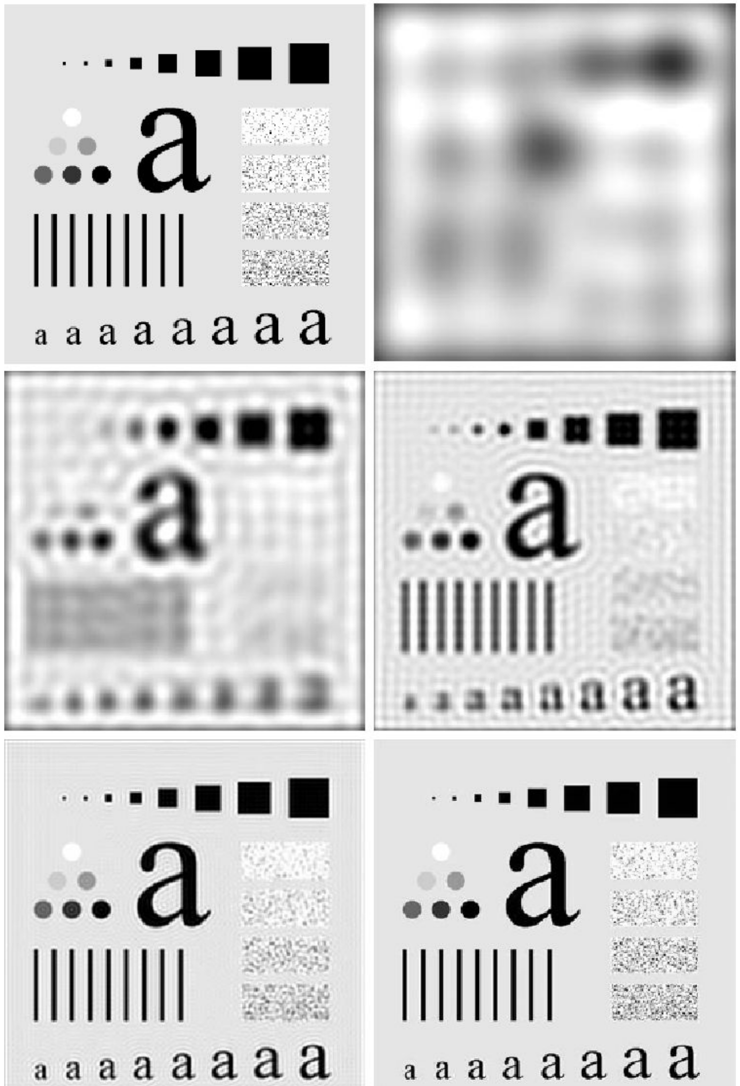
Image Enhancement in Frequency Domain

Smoothing Filters: **GLPF**



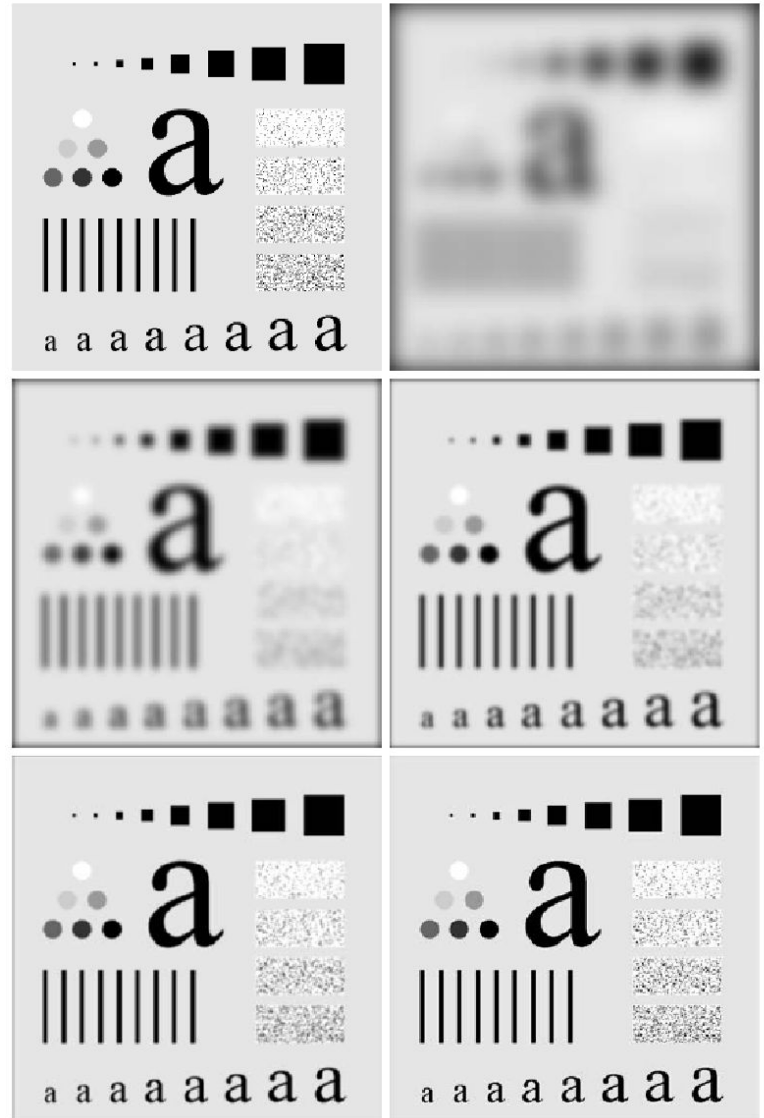
a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

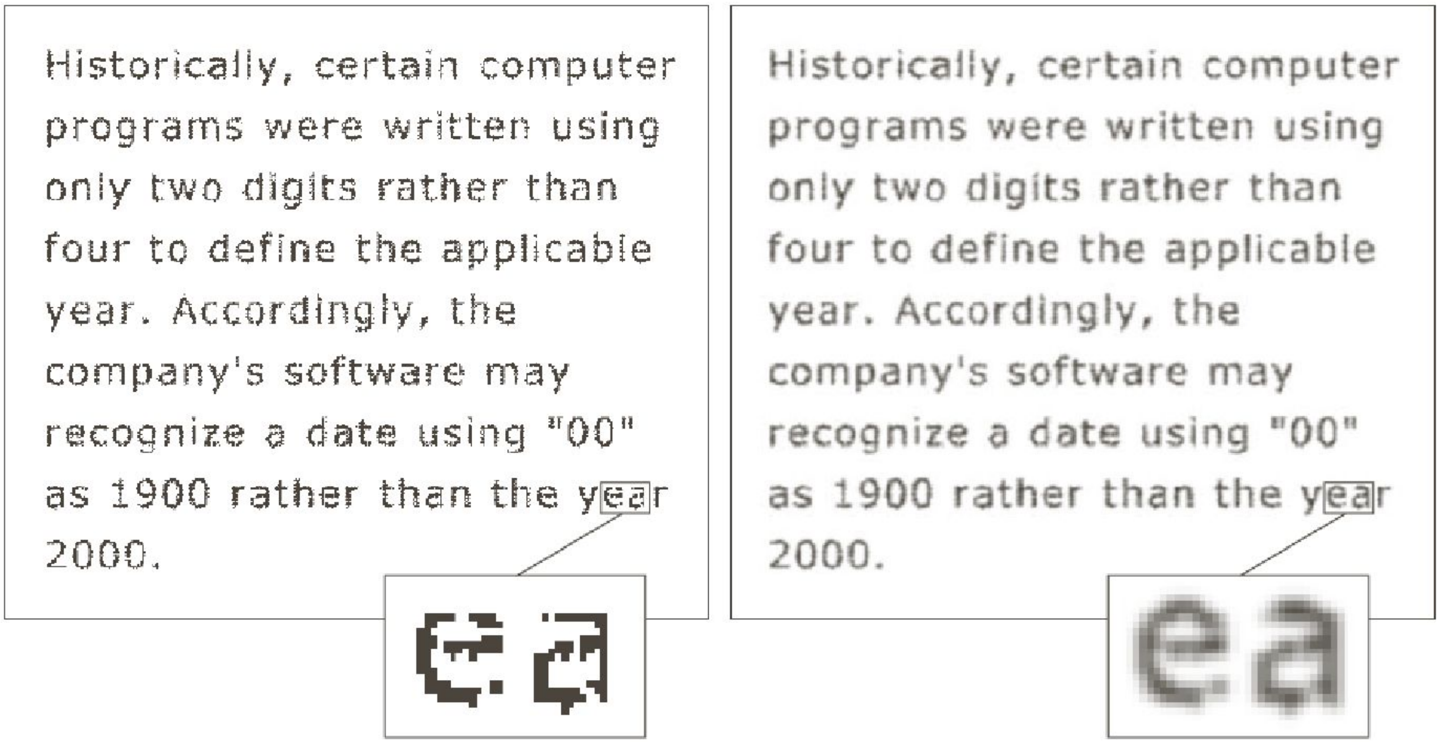


a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Image Enhancement in Frequency Domain

Smoothing Filters: **Example of smoothing by GLPF**



a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Image Enhancement in Frequency Domain

Smoothing Filters: **Example of smoothing by GLPF**

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

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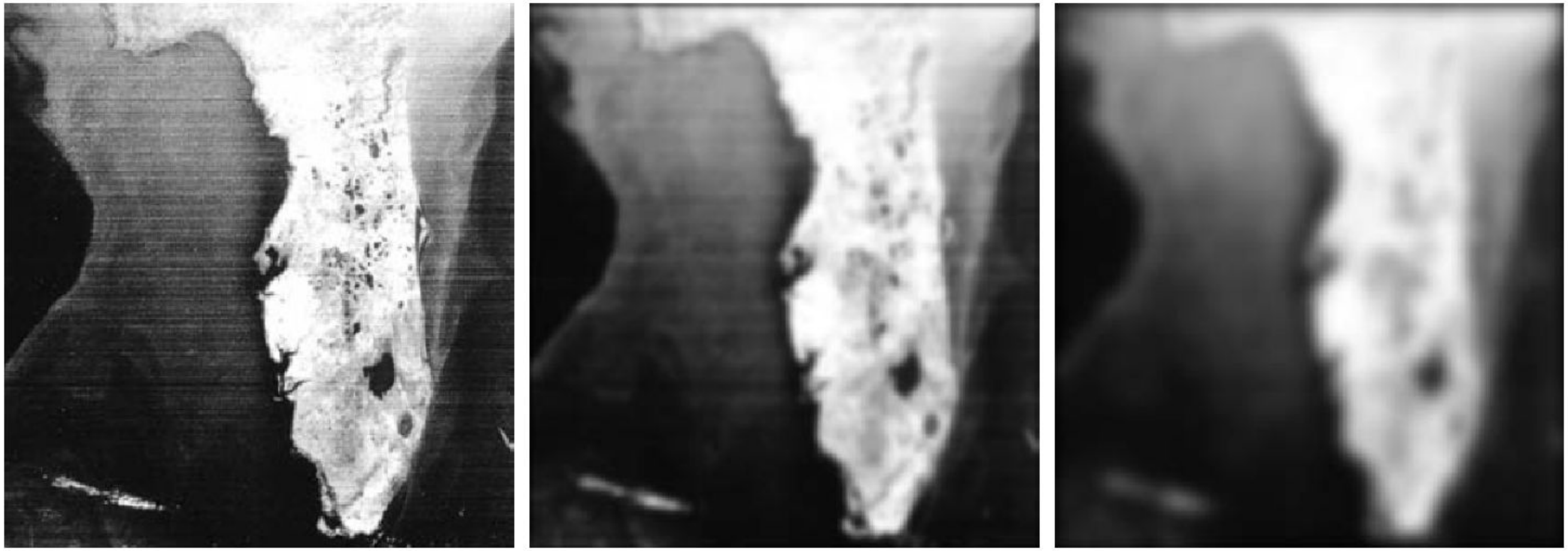
Smoothing Filters: **Example of smoothing by GLPF**



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

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Sharpening Filters

- Image sharpening can be achieved in the frequency domain by highpass filtering.

- It attenuates low-frequency components without disturbing high-frequency information in the Fourier transform.

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Sharpening Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

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Sharpening Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

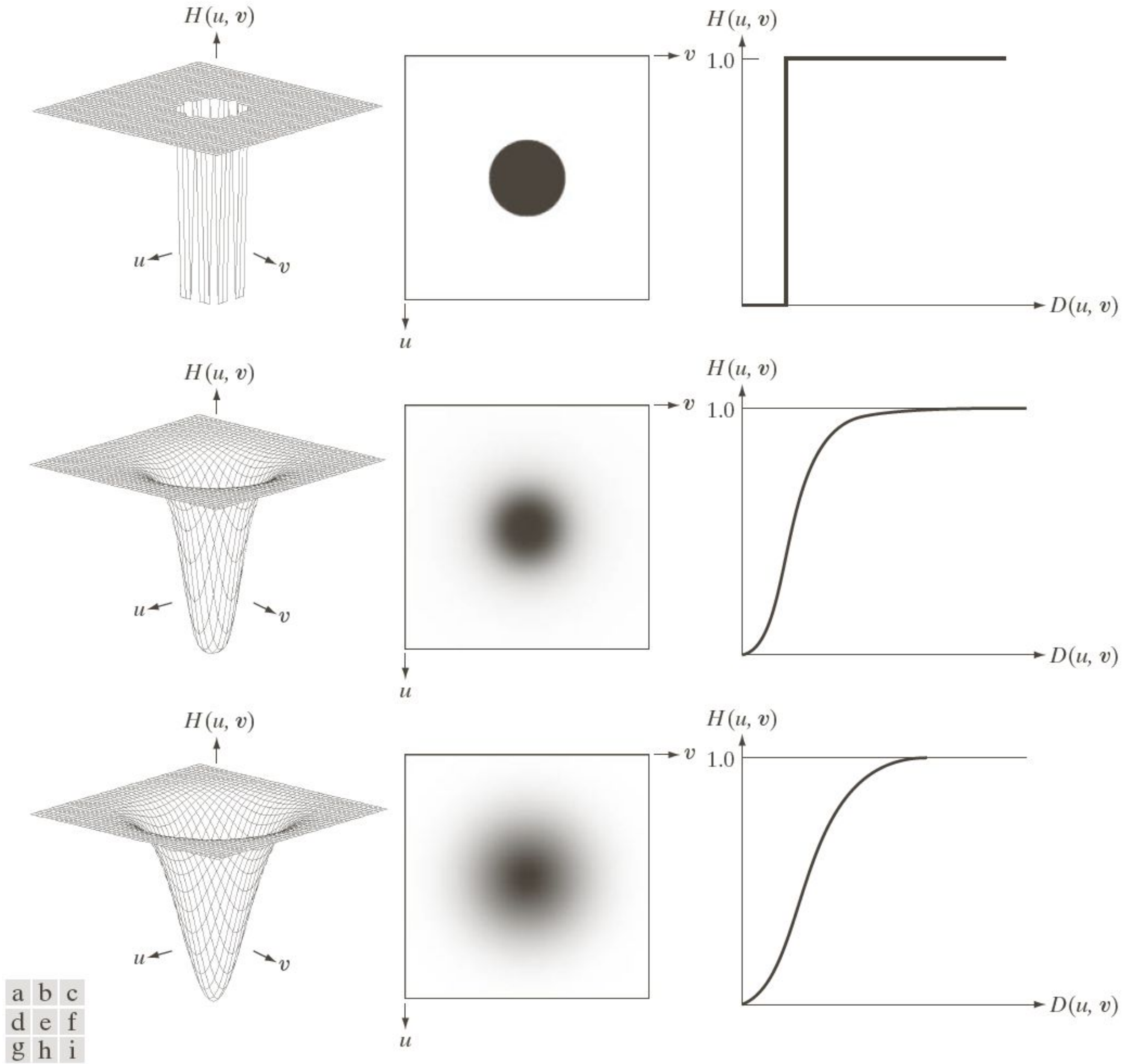

FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

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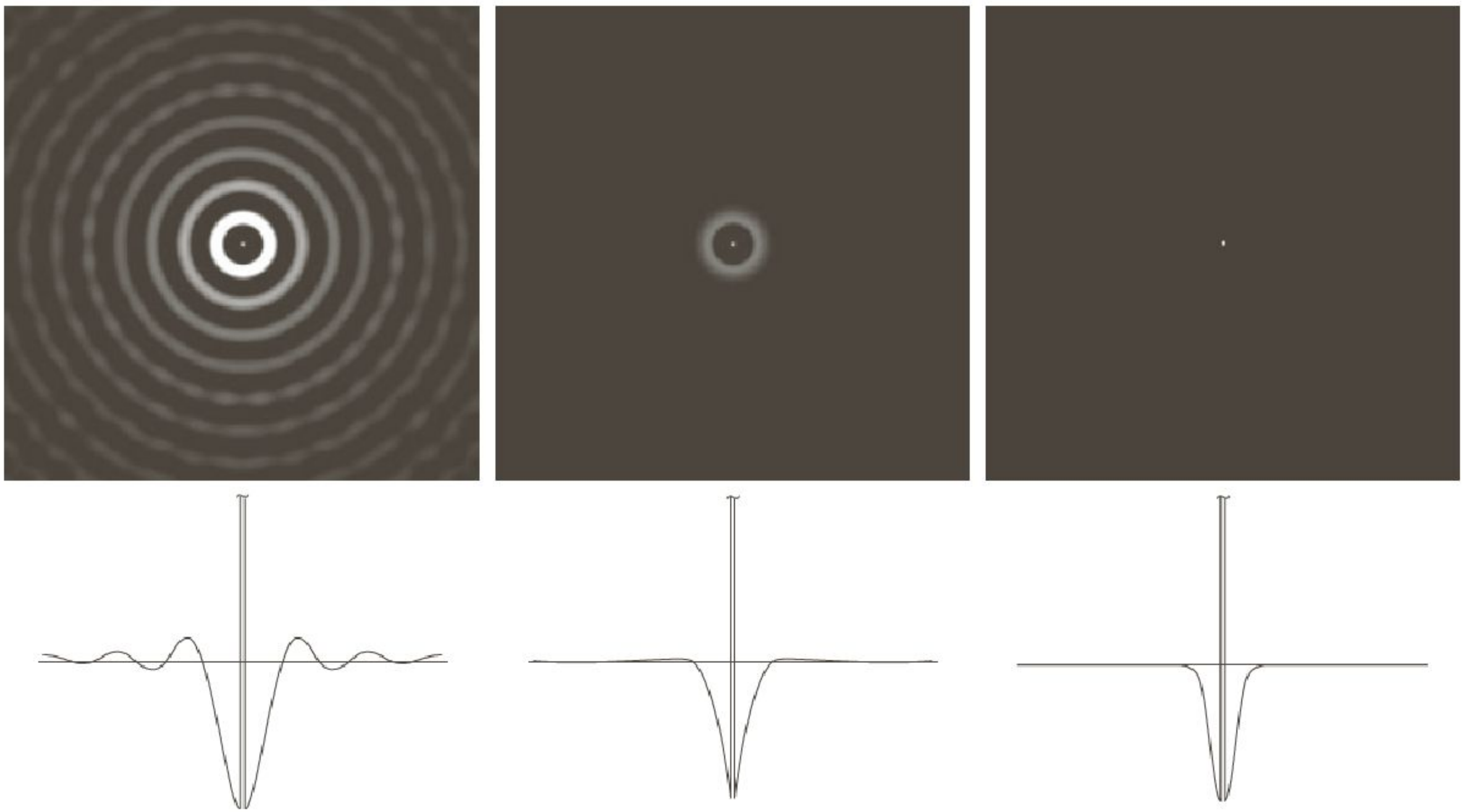
Sharpening Filters: **Spatial Representation**



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

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Sharpening Filters: **Result of IHPF**

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

Image Enhancement in Frequency Domain

Sharpening Filters: **Result of BHPF**

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Image Enhancement in Frequency Domain

Sharpening Filters: **Result of GHPF**

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

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Laplacian in Frequency Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

$$H(u,v) = -4\pi^2\left[(u - P/2)^2 + (v - Q/2)^2\right]$$
$$= -4\pi^2 D^2(u,v)$$

The Laplacian image

$$\nabla^2 f(x,y) = \Im^{-1}\{H(u,v)F(u,v)\}$$

Enhancement is obtained

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y) \qquad c = -1$$

Image Enhancement in Frequency Domain

Laplacian in Frequency Domain

The enhanced image

$$
\begin{aligned}
g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$

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Laplacian in Frequency Domain

a b

FIGURE 4.58
(a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Next Class

- Image Restoration

Thank you:
Question?