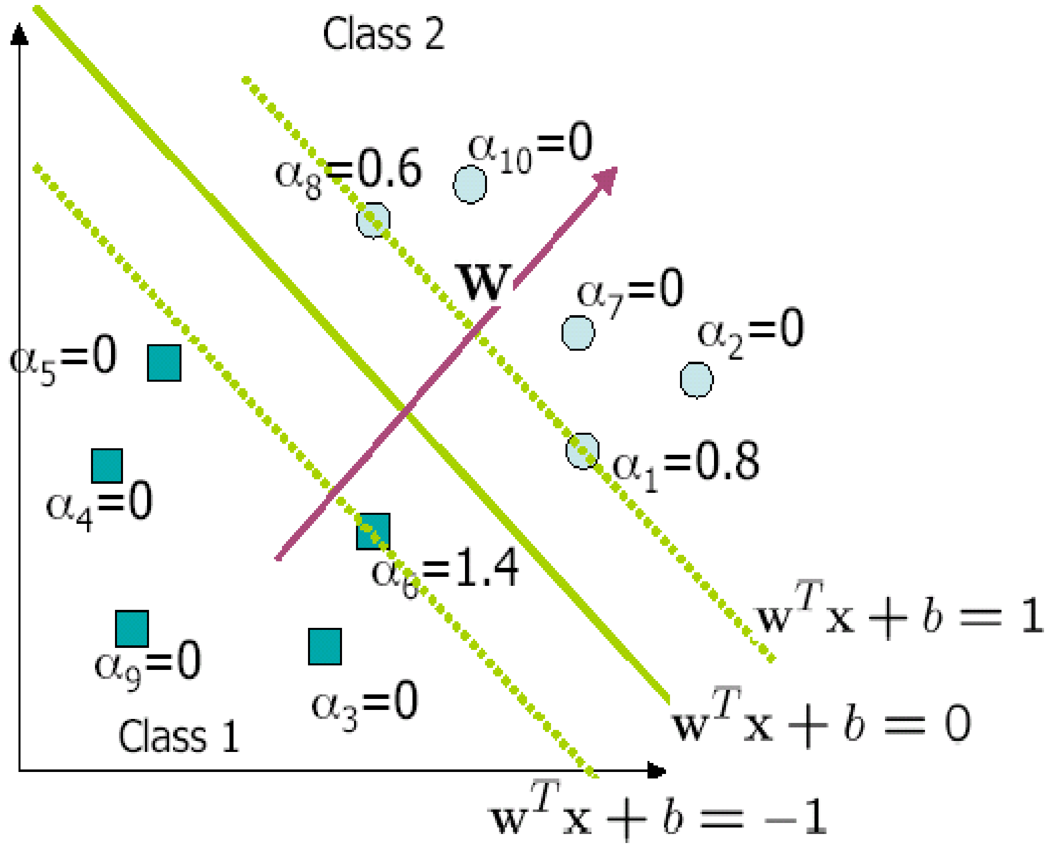


Class 2
α_{10}=0
α_8=0.6
$\mathbf{w}$
α_7=0
α_2=0
α_5=0
α_1=0.8
α_4=0
α_6=1.4
$\mathbf{w}^T\mathbf{x} + b = 1$
α_9=0
α_3=0
Class 1
$\mathbf{w}^T\mathbf{x} + b = 0$
$\mathbf{w}^T\mathbf{x} + b = -1$

XOR Problem

SVM Solution

$$X_1 = (1, 1)^t, y_1 = +1$$

$$X_2 = (-1, -1)^t, y_2 = +1$$

$$X_3 = (-1, 1)^t, y_3 = -1$$

$$X_4 = (1, -1)^t, y_4 = -1$$

$$k(X_i, X_j) = (X_i \cdot X_j + 1)^2$$

Wolfe Dual :

$$Max \quad L(\vec{\alpha}) = -\frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j k(X_i, X_j) + \sum_i \alpha_i$$

$$s.t. \qquad \sum_{i=1}^{n} \alpha_i y_i = 0, \quad \forall i$$

$$\alpha_i \geq 0, \qquad \forall i$$

From symmetry $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$

$$L = -16\alpha^2 + 4\alpha$$

$$\frac{dL}{d\alpha} = -32\alpha + 4$$

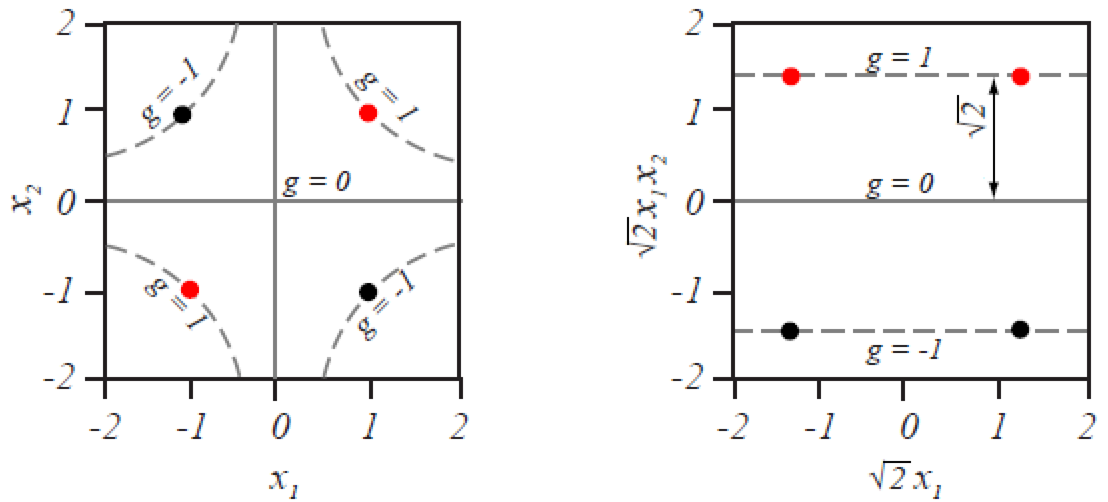
$$\frac{dL}{d\alpha} = 0 \quad \Rightarrow \quad \alpha = \frac{1}{8}$$

$$b = -\sum_i \alpha_i y_i k\left(X_i, X_j\right) + y_j \quad \text{for any } j, \; s.t. \; \alpha_j \neq 0$$

$$b = 0.$$

The Classifier, $g(X) = \sum_i \alpha_i y_i k\left(X_i, X \right) + b = 0$

$$g(X) = x_1 x_2 = 0.$$



The XOR problem in the original $x_1 - x_2$ feature space is shown at the left; the two red patterns are in category ω_1 and the two black ones in ω_2. These four training patterns $\mathbf{x}$ are mapped to a six-dimensional space by 1, $\sqrt{2}x_1$, $\sqrt{2}x_2$, $\sqrt{2}x_1x_2$, x_1^2 and x_2^2. In this space, the optimal hyperplane is found to be $g(x_1, x_2) = x_1x_2 = 0$ and the margin is $\sqrt{2}$. A two-dimensional projection of this space is shown at the right. The hyperplanes through the support vectors are $\sqrt{2}x_1x_2 = \pm 1$, and correspond to the hyperbolas $x_1x_2 = \pm 1$ in the original feature space, as shown.