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Assignment - 9

Q1) a) $a*b = \max(a,b)$

Commutative:

$a*b = \max(a,b)$

$b*a = \max(b,a) = \max(a,b)$

$\therefore$ It is Commutative

Associative

$(a*b)*c = \max(\max(a,b), c)$

$a*(b*c) = \max(a, \max(b,c))$

Final ans would be same

$\therefore$ It is associative

b) $a*b = \max(a, b+2)$

Commutative

$a*b = \max(a, b+2)$

$b*a = \max(b, a+2)$

$\therefore$ Not commutative

Associative

$(a*b)*c = \max(\max(a, b+2), c+2)$

$a*(b*c) = \max(a, \max(b+2, c+2))$

Final ans would be same

$\therefore$ It is associative

c) $a*b = a+2b$

$b*a = b+2a$

$\therefore$ Not commutative

$a*(b*c) = a*(b+2c)$

$= a+2b+4c$

$(a*b)*c = (a+2b)*c$

$= a+2b+2c$

$\therefore$ Not Associative

2) Let $S = \{0,1,2,3\}$

$x *_1 y = r$ $\qquad x + y = 4n + r$

$x *_2 y = u$ $\qquad xy = 5n + u$. Such that

$0 \le r, u < 3$ $\qquad m, n \in N$.

1) $*_1$ is an abelian group
2) $*_2$ is semigroup
3) $*_2$ is distributed over $*_1$

$0 *_1 0 = 0$, $0 *_1 1 = 1$, $0 *_1 2 = 2$, $0 *_1 3 = 3$

$\therefore$ 0 is identity and semigroup exists

$\therefore$ monoid

$2 *_1 3 = 2$, $3 *_1 2 = 1$

$\therefore$ Not commutative

$\therefore$ $*_1$ is not abelian group

$\therefore$ It is not a ring.

3) Consider a unity ring R

P.T. $(-1)a = -a$

$a + (-1)a = 1 \cdot a + (-1) \cdot a$

$= (1 + (-1)) \cdot a$

$= 0 \cdot a$

$= 0$

$\therefore$ Also $a - a = 0$

$\therefore$ $a + (-1)a = a - a$

$\therefore$ $\boxed{(-1)a = -a}$

$\therefore$ Hence Proved

Ans 4) $(A, *)$

$$a * b = a$$

i) Associative

$$(a * b) * c = a * c$$
$$= a$$

$$a * (b * c) = a * b$$
$$= a$$

$\therefore$ It is Associative

ii) $a * b = a$

$\cancel{b * a} =$ $b * a = b$

$\therefore$ Not commutative

Ans 5)

	α	β	γ	δ
α	α	β	γ	δ
β	β	α	δ	γ
γ	γ	δ	α	β
δ	δ	γ	β	α

$\alpha \cdot \alpha = \alpha$, $\alpha \cdot \beta = \beta$, $\alpha \cdot \gamma = \gamma$, $\alpha \cdot \delta = \delta$

$\therefore$ α is identity element

$\beta^2 = \alpha$, $\beta^3 = \beta^2 \beta = \alpha\beta = \beta$

$\beta^4 = \beta^2 \beta^2 = \alpha \cdot \alpha = \alpha$

$\therefore$ Not generator

$\gamma^2 = \alpha$, $\gamma^3 = \alpha\gamma = \gamma$, $\gamma^4 = \gamma^2\gamma^2 = \alpha \cdot \alpha = \alpha$

$\therefore$ Not generator

$\delta^2 = \beta$, $\cancel{\delta^3}$ $\delta^3 = \beta \cdot \delta = \gamma$, $\delta^4 = \beta \cdot \beta = \alpha$

$\therefore$ Not generator