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FUNCTIONS INTRODUCTION

ALGEBRA 2 MATH TEACHERS GUIDE

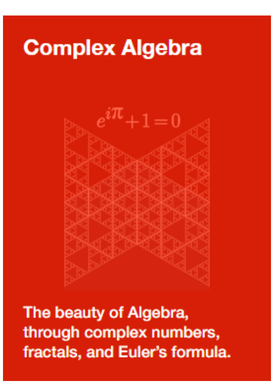
Task Introduction:
This exploration set aims to deepen students' understanding of functions. The problem set begins with identifying functions, evaluating functions, and determining domain and range. Then, students dive into a challenging series of writing and evaluating functions. The problems culminate with students creating functions for which $f(x) = -f(-x)$ and $f(x) = f(-x)$. (This activity assumes that students have not seen even and odd functions.)

Prerequisite Concepts:

- function notation
- domain
- range

Common Core Emphasis:
CCSS.MATH.CONTENT.HSF.IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
CCSS.MATH.CONTENT.HSF.IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
CCSS.MATH.CONTENT.HSF.BF.A.1 Write a function that describes a relationship between two quantities.

Follow Up: You can find more questions that relate to the problems in this activity in Brilliant's online course, Complex Algebra**.**



Course: Complex Algebra
Chapter: Functions and Transformations
Quiz: Functions Warmup
https://brilliant.org/practice/functions-warmup/

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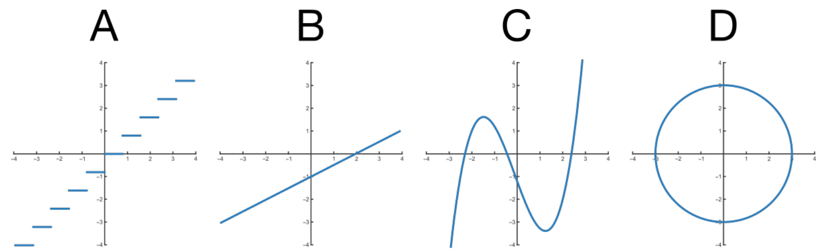
FUNCTIONS INTRODUCTION

Name: ____________________

Date: ____________________

SKILL PRACTICE

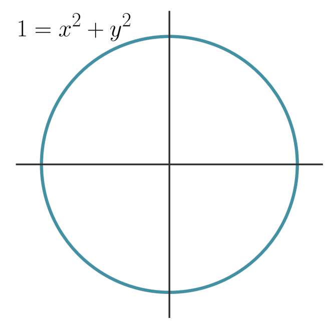
S1. Which graph does **not** represent a function?



◯ A ◯ B

◯ C ◯ D

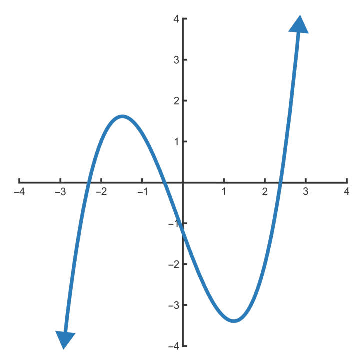
S2. What is the domain of the graph shown at right?



◯ [0,1] ◯ (-1,1)

◯ [-1,1] ◯ All real numbers

S3. Does the function shown in the graph at right have the same domain and range?



◯ Yes ◯ No

S4. If $f(x) = x^2 - 7x + 17$, for what value(s) of x does $f(x) = 5$?

Your answer:

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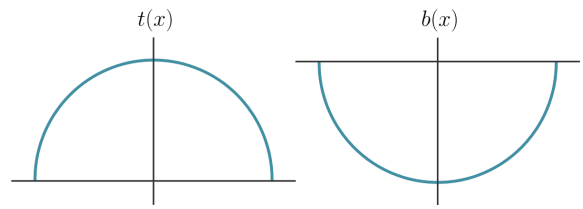
CHALLENGE QUESTIONS

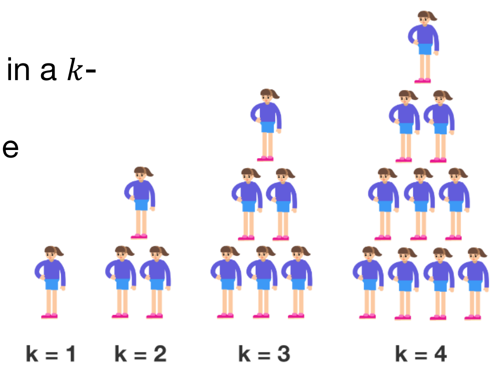
C1. Let $t(x)$ be the function defined on the domain [-1,1] for the **top half** of the unit circle, as shown below left. Let $b(x)$ be the function defined on the domain [-1,1] for the **bottom half** of the unit circle, asn shown below right.. Which of the following statements is **false**?



- ○ $b(x) = -t(x)$
- ○ $b(x) = t(-x)$
- ○ $\mathrm{t}(x) = t(-x)$
- ○ $t(x) = -b(x)$
- ○ $b(x) = b(-x)$

C2. This image shows the number of cheerleaders in a k-layer tower. For example, a 1-layer tower has 1 cheerleader. Write a function $C(k)$ that describes the number of cheerleaders in a k-layer tower.

Your answer:



C3. Consider two functions f and g such that $f(x) = x^2 - 5x + 10$ and $g(x-4) = f(2x+5)$. What is the value of $g(-3)$? (Hint: Start by finding the value of x for which $g(x - 4) = g(-3)$.)

Your answer:

THE CULMINATION

1. Sketch a graph of $\mathrm{y} = f(x)$ for which $f(x) = -f(-x)$.
2. Sketch a graph of $\mathrm{y} = f(x)$ for which $f(x) = f(-x)$.
3. Identify the domain and range of each function.

(Hint: If $f(-x) = f(x)$, then inputting opposite x values ouputs the same y values).

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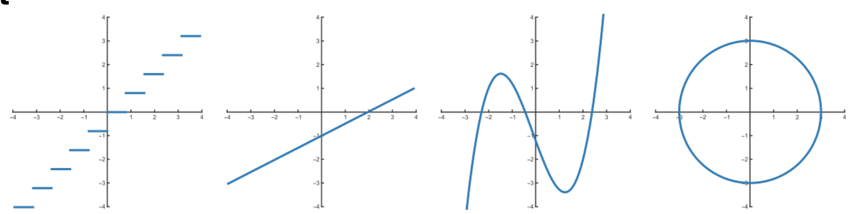
FUNCTIONS INTRODUCTION

SOLUTIONS

SKILL PRACTICE

S1. Which graph does **not** represent a function?

A B C D



A

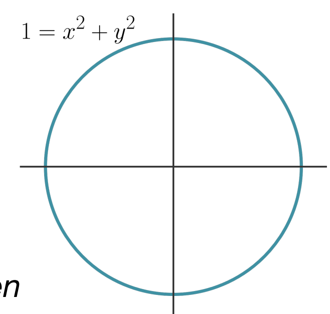
B

C

D

In a function, every input is paired with exactly one output. On the graph of a function, each x value can be matched with only one y value. Graph D, the circle, is not a function because many x values are paired with two y values.

S2. What is the domain of the graph shown at right?



[0,1]

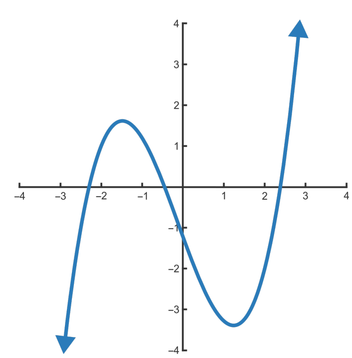
(-1,1)

[-1,1]

All real numbers

When we look at the graph, we see that the circle exists between x-values of -1 and 1. Therefore the possible input values, x, range from -1 to 1 including both -1 and 1 and the domain is [-1,1].

S3. Does the function shown in the graph at right have the same domain and range?



Yes

No

The graph travels infinitely left and right, indicating a domain of all real numbers. The graph also travels infinitely up and down, indicating a range of all real numbers. Therefore, the function has the same domain and range.

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SOLUTIONS

S4. If $f(x) = x^2 - 7x + 17$, for what value(s) of x does $f(x) = 5$?

Answer:

4,3

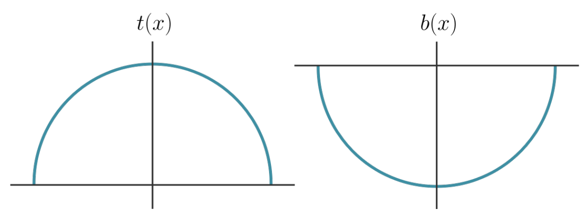
We can find the solution for x by substituting 5 for $f(x)$ and solving:

$$x^2 - 7x + 17 = 5$$
$$x^2 - 7x + 12 = 0$$
$$(x-4)(x-3) = 0$$
$$x = 4, x = 3.$$

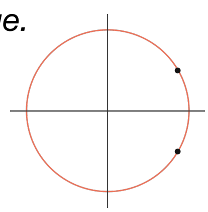
CHALLENGE QUESTIONS

C1. Let $t(x)$ be the function defined on the domain [-1,1] for the **top half** of the unit circle, as shown below left. Let $b(x)$ be the function defined on the domain [-1,1] for the **bottom half** of the unit circle, as shown below right.. Which of the following statements is **false**?

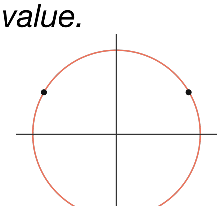
- ○ $b(x) = -t(x)$
- ● $b(x) = t(-x)$
- ○ $t(x) = t(-x)$
- ○ $t(x) = -b(x)$
- ○ $b(x) = b(-x)$



$b(x) = -t(x)$ and $t(x) = -b(x)$ because for any given x value in $b(x), t(x)$ will produce the exact opposite y value.

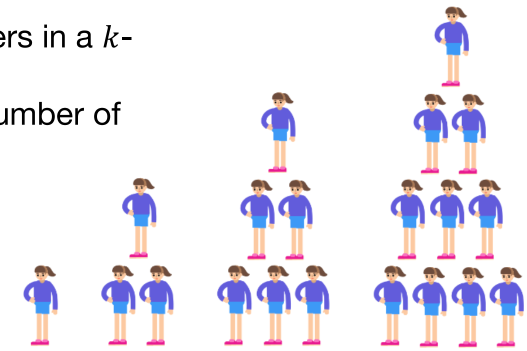


$b(x) = b(-x)$ and $t(x) = t(-x)$ because in either function, any x value and its opposite will produce the same y value.



$b(x) \neq t(-x)$ because all of the output, or y, values in $b(x)$ are negative and both $t(x)$ and $t(-x)$ yield only positive outputs.

C2. This image shows the number of cheerleaders in a k-layer tower. For example, a 1-layer tower has 1 cheerleader. What function $C(k)$ describes the number of cheerleaders in a k-layer tower?

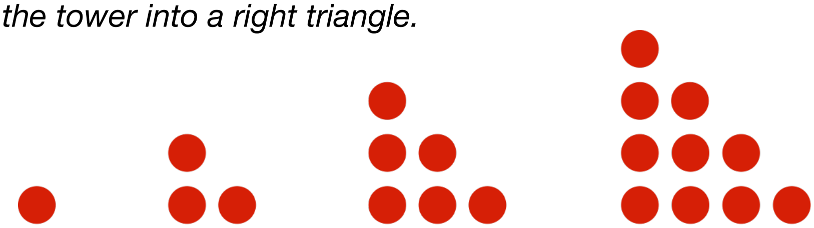


k = 1 k = 2 k = 3 k = 4

Answer:

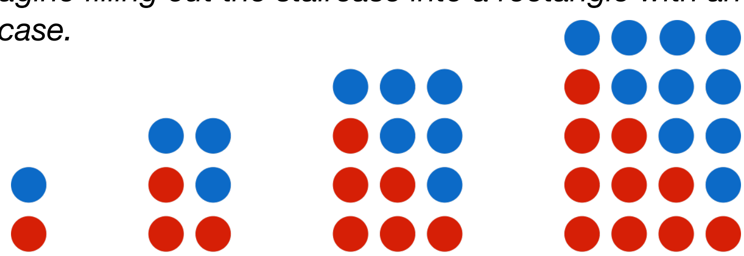
$$\boxed{C(k) = \frac{k(k+1)}{2}}$$

Let's use dots instead of cheerleaders to help us visualize a solution. For any tower, we can shift the tower into a right triangle.



k = 1 k = 2 k = 3 k = 4

Now, let's imagine filling out the staircase into a rectangle with an identical but inverted staircase.



k = 1 k = 2 k = 3 k = 4

Notice that each rectangle has a base of k and a height of $k+1$. Therefore, the area of each rectangle, or the total number of dots in each rectangle, is $k(k+1)$. The number of cheerleaders, represented by the red dots, is exactly half of the total number of dots, or $\frac{k(k+1)}{2}$.

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SOLUTIONS

C3. Consider two functions f and g such that $f(x) = x^2 - 5x + 10$ and $g(x-4) = f(2x+5)$. What is the value of $g(-3)$? (Hint: Start by finding the value of x for which $g(x - 4) = g(-3)$.)

Answer:

24

If $x - 4 = -3$, then $x = 1$. Substituting 1 for x, we have

$$g(-3) = g(1-4) = f(2(1)+5) = f(7).$$

Substituting 7 for x into $f(x)$, we have

$$f(7) = 7^2 - 5(7) + 10 = 24.$$

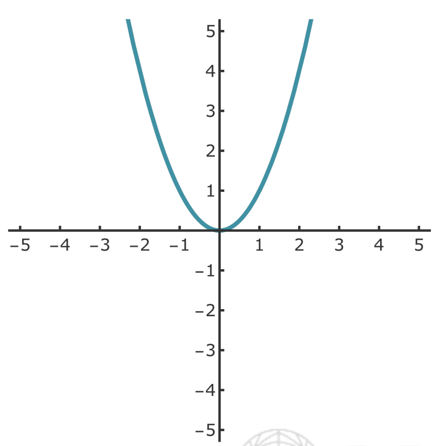
DISCUSSING THE CULMINATION QUESTION

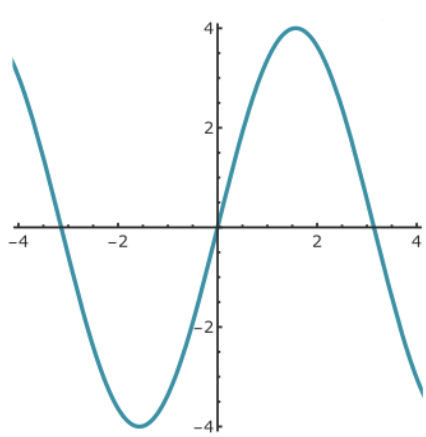
1. Sketch a graph of $y = f(x)$ for which $f(x) = -f(-x)$.
2. Sketch a graph of $y = f(x)$ for which $f(x) = f(-x)$.
3. Identify the domain and range of each function.

(Hint: If $f(-x) = f(x)$, then inputting opposite x values ouputs the same y values).

If $f(x) = f(-x)$, then inputting an opposite x-value outputs the same y-value. Therefore, if (x, y) is on the graph of $f(x)$, then $(-x, y)$ is also on the graph. One possible example is shown at left below. This example has a domain of all real numbers and a range of $y \geq 0$.

f $f(x) = -f(-x)$, then inputting an opposite x-value outputs the opposite y-value. Therefore, if (x, y) is on the graph of $f(x)$, then $(-x, -y)$ is also on the graph. One possible example is shown at right below. This example has a domain and range of all real numbers.





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