BRILLIANT

FUNCTIONS - COMPOSITION

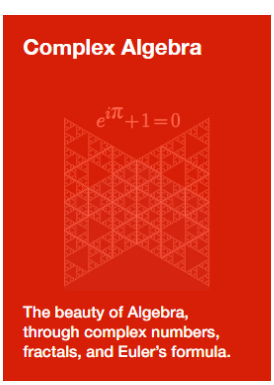
ALGEBRA 2 MATH TEACHERS GUIDE

Task Introduction:
These puzzle-like questions build upon students' intuitive understanding of compositions of functions. Students begin by working with single-step compositions before moving on to multi-step compositions. In the culmination, students explore possible compositions for which $f(g(x))$ and $g(f(x))$ both equal x.

Prerequisite Concepts:
- definition of a function and function notation
- function compositions and composition notation

Common Core Emphasis:
CCSS.MATH.CONTENT.HSF.BF.A.1.B Combine standard function types using arithmetic operations.
CCSS.MATH.CONTENT.HSF.BF.A.1.C Compose functions.

Follow Up: You can find more questions that relate to the problems in this activity in Brilliant's online course, Complex Algebra**.**



Course: Complex Algebra
Chapter: Functions and Transformations
Quiz: Composition
https://brilliant.org/practice/composition/

For a free subscription trial and to learn about group subscription pricing, email pricing@brilliant.org.

BRILLIANT

Name: ____________________

Date: ____________________

FUNCTIONS - COMPOSITION

SKILL PRACTICE

S1. x represents the original cost of an item at a store. The function $f(x)$ represents the cost of the item with a \$100 discount: $f(x) = x - 100$. The function $g(x)$ represents the cost of the item with a 15% discount: $g(x) = 0.85x$. Which combined function represents the cost when using both discounts and taking the 15% discount first?

- ○ $f(g(x)) = 0.85x - 100$
- ○ $g(f(x)) = 0.85(x - 100)$

S2. Suppose $f(x) = 2x$. For which function $g(x)$ will $(f \circ g)(x)$ equal $(g \circ f)(x)$?

- ○ $g(x) = 3x$
- ○ $g(x) = x^2$
- ○ $g(x) = 2x + 2$
- ○ It is true for any function $g(x)$

S3. If $f(x) = \frac{1}{x}$, what is the value of $(f \circ f)(0)$?

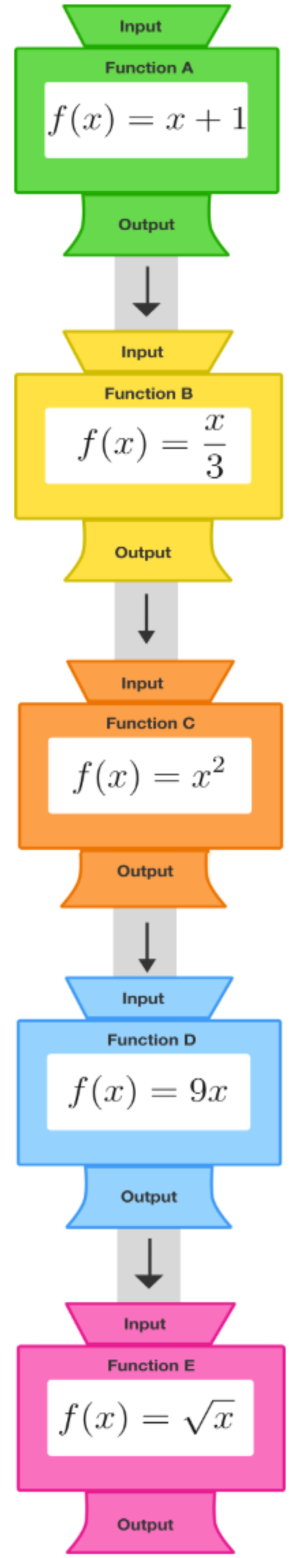
- ○ 0
- ○ ∞
- ○ 1
- ○ Undefined

S4. If f is a function such that $f(f(x)) = x^2 - 1$, what is the value of $f\left(f\left(f(f(2))\right)\right)$?

- ○ 3
- ○ 8
- ○ 31
- ○ 63

CHALLENGE QUESTIONS

C1. The image to the right shows a number crunching machine. Write and simplify a function, $m(x)$, that describes the net result that this number crunching machine has on each incoming value, x. (Assume that x is a positive number.)



Your answer:

C2. For three functions f, g and h, $f(x) = x - 3$ and $h(g(x)) = 6x + 4$. What is the value of x that satisfies $h(g(f(x))) = 52$?

Your answer:

THE CULMINATION

Identify three combinations of $f(x)$ and $g(x)$ for which $f(g(x)) = g(f(x)) = x$.

Your answer:

BRILLIANT

FUNCTIONS - COMPOSITION

SOLUTIONS

SKILL PRACTICE

S1. x represents the original cost of an item at a store. The function $f(x)$ represents the cost of the item with a \$100 discount: $f(x) = x - 100$. The function $g(x)$ represents the cost of the item with a 15% discount: $g(x) = 0.85x$. Which combined function represents the cost when using both discounts and taking the 15% discount first?

- [x] $f(g(x)) = 0.85x - 100$
- [ ] $g(f(x)) = 0.85(x - 100)$

The function $f(g(x)) = 0.85x - 100$ represents taking the 15% discount first because the price of the item, x, is multiplied by 0.85 before subtracting the \$100.

S2. Suppose $f(x) = 2x$. For which function $g(x)$ will $(f \circ g)(x)$ equal $(g \circ f)(x)$?

- [x] $g(x) = 3x$
- [ ] $g(x) = x^2$
- [ ] $g(x) = 2x + 2$
- [ ] It is true for any function $g(x)$

If $g(x) = 3x$, *then*

$$f(g(x)) = f(3x) = 2(3x) = 6x$$

and

$$g(f(x)) = g(2x) = 3(2x) = 6x.$$

Note that $g(x)$ *cannot be* x^2 *because then*

$$f(g(x)) = f(x^2) = 2(x^2) = 2x^2$$

and

$$g(f(x)) = g(2x) = (2x)^2 = 4x^2.$$

Also, $g(x)$ *cannot be* $2x + 2$ *because then*

$$f(g(x)) = f(2x + 2) = 2(2x + 2) = 4x + 4$$

and

$$g(f(x)) = g(2x) = 2(2x) + 2 = 4x + 2.$$

SOLUTIONS

SKILL PRACTICE

S3. If $f(x) = \frac{1}{x}$, what is the value of $(f \circ f)(0)$?

- 0
- ∞
- 1
- Undefined (selected)

$\frac{1}{x}$ can be evaluated with any real or even complex value for x, except the value $x = 0$. For that one input, the function is undefined. So, although $\frac{1}{\frac{1}{x}}$ simplifies to x, the restriction that x cannot equal 0 still affects the value of $\frac{1}{\frac{1}{x}} = x$ for all $x \neq 0$. For $x = 0$, $\frac{1}{\frac{1}{x}} \neq x$ because it is undefined at that value.

S4. If f is a function such that $f(f(x)) = x^2 - 1$, what is the value of $f\left(f\left(f(f(2))\right)\right)$?

- 3
- 8 (selected)
- 31
- 63

We have $f(f(2)) = 2^2 - 1 = 3$. Therefore, $f\left(f\left(f(f(2))\right)\right) = f(f(3)) = 3^2 - 1 = 8$.

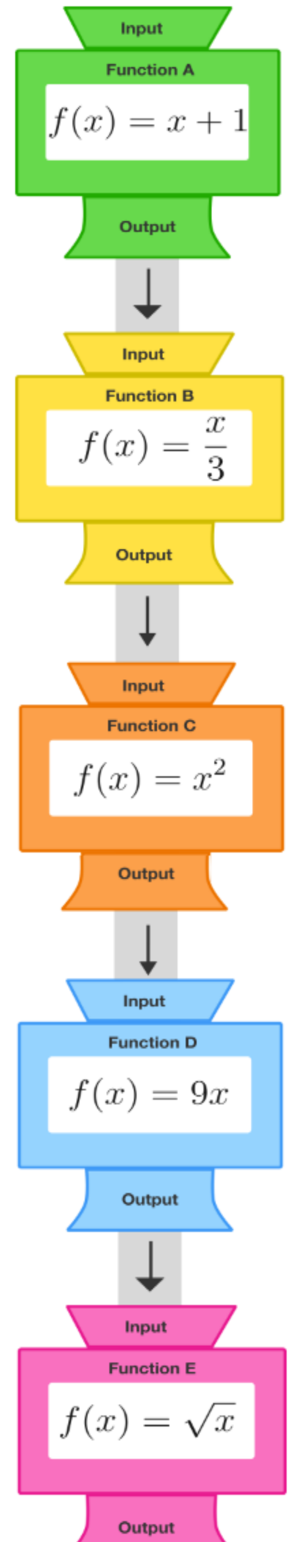
BRILLIANT

FUNCTIONS - COMPOSITION

SOLUTIONS

CHALLENGE QUESTIONS

C1. The image to the right shows a number crunching machine. Write and simplify a function, $m(x)$, that describes the net result that this number crunching machine has on each incoming value, x. (Assume that x is a positive number.)



Answer:

$m(x) = x + 1$

Writing out how this function machine behaves with each input gives

$$\sqrt{\left(\frac{x+1}{3}\right)^2 (9)}\,,$$

which can be rewritten as

$$\sqrt{\left(\frac{(x+1)^2}{9}\right)(9)}\,.$$

The 9s cancel out, as do the remaining square and square root, leaving $x+1$. Therefore, $m(x) = x + 1$.

C2. For three functions f, g and h, $f(x) = x - 3$ and $h(g(x)) = 6x + 4$. What is the value of x that satisfies $h(g(f(x))) = 52$?

Answer:

11

Using the properties of composite functions, we have

$$\begin{aligned} h\big(g(f(x))\big) &= 52 \\ h\big(g(x-3)\big) &= 52 \\ 6(x-3)+4 &= 52 \\ 6x &= 66 \\ x &= 11. \end{aligned}$$

SOLUTIONS

DISCUSSING THE CULMINATION QUESTION

Identify three combinations of $f(x)$ and $g(x)$ for which $f(g(x)) = g(f(x)) = x$.

Answer:

Possible examples include:
$f(x) = x$ and $g(x) = x$
$f(x) = x + 1$ *and* $g(x) = x - 1$
$f(x) = x^2$ *and* $g(x) = \sqrt{x}$, *if* $x \geq 0$
$f(x) = 2x$ *and* $g(x) = \frac{1}{2}x$

Possible solutions include any two functions that reverse the actions of one another. These functions are called ***inverses.***

If $f(x) = x$ *and* $g(x) = x$, *then* $f\big(g(x)\big) = x$ *and* $g\big(f(x)\big) = x$.

If $f(x) = x + 1$ *and* $g(x) = x - 1$, *then* $f\big(g(x)\big) = (x-1) + 1 = x$ *and* $g\big(f(x)\big) = (x+1) - 1 = x$.

If $f(x) = x^2$ *and* $g(x) = \sqrt{x}$, *then* $f\big(g(x)\big) = (\sqrt{x})^2 = x$ *and* $g\big(f(x)\big) = \sqrt{x^2} = x$.

If $f(x) = 2x$ *and* $g(x) = \frac{1}{2}x$, *then* $f\big(g(x)\big) = 2\left(\frac{1}{2}x\right) = x$ *and* $g\big(f(x)\big) = \frac{1}{2}(2x) = x$.