Production Efficiency with Hidden Trading: A Mechanism Design Approach

Brian C. Albrecht
University of Minnesota

V. V. Chari
University of Minnesota
FRB of Minneapolis

Adway De
University of Minnesota

Keyvan Eslami
Ryerson University

November 7, 2019

Revisit Production Efficiency in a Mirrleesian Economy

- Are optimal allocations on the PPF, i.e. production efficient?

- Should production inputs be taxed?

The Debate

- Classic public finance result: constrained optimal allocations are production efficient
  - Diamond and Mirrlees 1971; Atkinson and Stiglitz 1976; Saez 2004; Rothschild and Scheuer 2013
- Modern challenges: constrained optimal allocations are production inefficient
  - Naito 1999; Jacobs 2015; Gomes, Lozachmeur, and Pavan 2018
  - Robots: Guerreiro, Rebelo, and Teles 2018; Costinot and Werning 2018
- All of these papers take the set of tax instruments as given, even if non-linear
- Mirrleesian (Mechanism Design) Approach: derive optimal tax structure from primitives about the environment

Optimal Policy

- Maximizing social welfare generally requires production inefficiency
  - Create a wedge between the marginal products in two sectors to relax ICs
- Should tax production inputs to distort relative marginal products

Optimal Policy

- Maximizing social welfare generally requires production inefficiency
  - Create a wedge between the marginal products in two sectors to relax ICs

- Should tax production inputs to distort relative marginal products

Exceptions

- Linear production technology $\Rightarrow$ fixed MPs $\Rightarrow$ production efficiency

- Hidden trading $\Rightarrow$ equalized MPs across sectors $\Rightarrow$ production efficiency

Roadmap of Talk

Physical Environment

Observable Trading

Hidden Trading

Environment

- Two tasks/jobs, "managers" and "workers": $j \in \{m, w\}$
- Skill are distributed according to $H^j(\theta_i^j)$
- Two consumption goods: $k \in \{a, b\}$
- Allocation:
  1. Consumption: $\left\{ \left(c^a(\theta_i^j), c^b(\theta_i^j) \right) ; i \in \{1, 2\}, j \in \{m, w\} \right\}$
  2. Labor: $\left\{ \left(\ell^a(\theta_i^j), \ell^b(\theta_i^j) \right) ; i \in \{1, 2\}, j \in \{m, w\} \right\}$

Utility

- $u\left(c^a(\theta_i^j), c^b(\theta_i^j), \ell^a\left(\theta_i^j\right) + \ell^b\left(\theta_i^j\right)\right)$
- Weakly separable between consumption and labor
- Standard assumptions:
    - $u(\cdot)$ is C^2 over $(0, \infty)^3$
    - $u_k \equiv \partial u / \partial c^k > 0$
    - $u_\ell \equiv \partial u / \partial \ell < 0$
    - $u_{kk} \equiv \partial^2 u / \partial {c^k}^2 < 0$
    - $u_{\ell\ell} \equiv \partial^2 u / \partial \ell^2 > 0$

Production

- Effective labor: $\left(e_i^a(\theta_i^j), e_i^b(\theta_i^j)\right) = \left(\theta_i^j \ell^a(\theta_i^j), \theta_i^j \ell^b(\theta_i^j)\right)$
- Aggregate labor of type j in sector k: $L_j^k = \int_{\theta_i^j} \theta_i^j \ell^k(\theta_i^j) dH^j(\theta_i^j)$
- CRS Production in final good sector k: $y^k = F^k\left(L_w^k, L_m^k\right)$
- Resource Feasibility: $y^k = \int_{\theta_i^j} c^k(\theta_i^j) dH^j(\theta_i^j) \quad \forall k \in \{a, b\}$

- Marginal product of effective labor: $F_j^k\left(L_m^k, L_w^k\right) \equiv \frac{\partial F^k\left(L_m^k, L_w^k\right)}{\partial e_i^k}$.

- An allocation is production efficient if there exists no alternative with
  - Same labor supplied by each type
  - At least as much production of each good
  - Strictly more production of one good

- An interior allocation is production efficient iff the allocation is on the PPF in each sector and

$$MRT_{w,m}^a = \frac{F_w^a\left(L_m^a, L_w^a\right)}{F_m^a\left(L_m^a, L_w^a\right)} = \frac{F_w^b\left(L_m^b, L_w^b\right)}{F_m^b\left(L_m^b, L_w^b\right)} = MRT_{w,m}^b.$$

- In general, we will focus on cases where $F_j^k\left(L_m^k, L_w^k\right)$ is endogenous

- Drop $\left(L_m^k, L_w^k\right)$ for notational convenience

Hidden Labor and Task

- The planner can only observe incremental output in each sector:

$$z^k(\theta_i^j) = F_j^k \theta_i^j \ell^k(\theta_i^j)$$

- The planner cannot observe
  1. Productivity: Mirrlees (1971)
  2. Task: Naito (1999), Saez (2004), and Rothschild and Scheuer (2013)

Roadmap of Talk

Physical Environment

Observable Trading

Hidden Trading

Planner's Problem

$$\max_{\left\{\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j),L_i^k\right\}} \int_{\theta_i^j} u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) dH^j(\theta_i^j)$$

$$\text{s.t.} \quad u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) \geq u\left(\boldsymbol{c}(\theta_{i'}^j), \frac{\theta_{i'}^j}{\theta_i^j}\left[\ell^a(\theta_{i'}^j) + \ell^b(\theta_{i'}^j)\right]\right),$$

$$\forall \theta_i^j, \theta_{i'}^j, i \qquad \text{(Within-Task ICs)}$$

$$u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) \geq u\left(\boldsymbol{c}(\theta_{i'}^{-j}), \frac{\theta_{i'}^{-j}}{\theta_i^j}\left[\frac{F_{-j}^a}{F_j^a}\ell^a(\theta_{i'}^{-j}) + \frac{F_{-j}^b}{F_j^b}\ell^b(\theta_{i'}^{-j})\right]\right),$$

$$\forall \theta_i^j, \theta_{i'}^{-j}, i \qquad \text{(Across-Task ICs)}$$

Resource Feasibility

Proposition: Production Inefficiency (Naito 1999; Costinot and Werning 2018)

For a generic production function, the solution to the planner's problem is production inefficient.

- Proof: Take first order conditions
- Each sector's MRT shows up in different FOCS
- The difference in the multiplier on IC in different sectors shows up as a wedge between MRTs
- By changing the relative MPs between sectors, the planner relaxes the incentive constraint

Roadmap of Talk

Physical Environment

Observable Trading

Hidden Trading

Hidden Trading

- Suppose there exists hidden trading after production, but before returning to the planner

Hidden Trading

- Suppose there exists hidden trading after production, but before returning to the planner
- Hidden trading in the goods market means that there is a price p for good b

Hidden Trading

- Suppose there exists hidden trading after production, but before returning to the planner
- Hidden trading in the goods market means that there is a price p for good b
- For people to work in each sector (like planner wants), it must be that value of marginal products are equalized

$$F^a_w(L^a_w, L^a_m) = pF^b_w(L^b_w, L^b_m) \quad \text{and} \quad F^a_m(L^a_w, L^a_m) = pF^b_m(L^b_w, L^b_m)$$

Hidden Trading

- Suppose there exists hidden trading after production, but before returning to the planner
- Hidden trading in the goods market means that there is a price p for good b
- For people to work in each sector (like planner wants), it must be that value of marginal products are equalized

$$F^a_w(L^a_w, L^a_m) = pF^b_w(L^b_w, L^b_m) \quad \text{and} \quad F^a_m(L^a_w, L^a_m) = pF^b_m(L^b_w, L^b_m)$$

- Any incentive feasible allocation for the planner has same restriction

Planner's Problem with Hidden Trading

$$\max_{\left\{\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j),L_i^k\right\}} \quad \int_{\theta_i^j} u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) dH^j(\theta_i^j)$$

$$\text{s.t.} \quad u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) \geq u\left(\boldsymbol{c}(\theta_{i'}^j), \frac{\theta_{i'}^j}{\theta_i^j}\left[\ell^a(\theta_{i'}^j)+\ell^b(\theta_{i'}^j)\right]\right),$$

$$\forall \theta_i^j, \theta_{i'}^j, i \qquad \text{(Within-Task ICs)}$$

$$u\left(\boldsymbol{c}(\theta_i^j),\boldsymbol{\ell}(\theta_i^j)\right) \geq u\left(\boldsymbol{c}(\theta_{i'}^{-j}), \frac{F_{-j}^a\theta_{i'}^{-j}}{F_j^a\theta_i^j}\left[\ell^a(\theta_{i'}^{-j})+\ell^b(\theta_{i'}^{-j})\right]\right),$$

$$\forall \theta_i^j, \theta_{i'}^{-j}, i \qquad \text{(Across-Task ICs)}$$

Resource Feasibility

Proposition: Production Efficiency (Diamond and Mirrlees 1971)

With hidden trading, all optimal allocations are production efficient.

- Proof:

$$\frac{F_w^a\left(L_m^a, L_w^a\right)}{F_m^a\left(L_m^a, L_w^a\right)} = \frac{pF_w^b\left(L_m^b, L_w^b\right)}{pF_m^b\left(L_m^b, L_w^b\right)}$$

- Implementation requires only a single income tax

- Identical result if hidden trading in intermediate goods

Conclusion: What’s the Difference?

- Now able to pin down mathematical mechanism for production (in)efficiency
- No need to mention things like “the long-run” (Saez 2004)
- With observable trading, both MRTs show up in incentive constraints
- $\Rightarrow$ Reason to distort MRTs $\Rightarrow$ Production inefficiency

Conclusion: What's the Difference?

- Now able to pin down mathematical mechanism for production (in)efficiency
- No need to mention things like "the long-run" (Saez 2004)
- With observable trading, both MRTs show up in incentive constraints
- $\Rightarrow$ Reason to distort MRTs $\Rightarrow$ Production inefficiency
- With hidden trading, one MRT drops out of incentive constraints
- $\Rightarrow$ Production efficiency

Conclusion: What's the Difference?

- Now able to pin down mathematical mechanism for production (in)efficiency
- No need to mention things like "the long-run" (Saez 2004)
- With observable trading, both MRTs show up in incentive constraints
- $\Rightarrow$ Reason to distort MRTs $\Rightarrow$ Production inefficiency
- With hidden trading, one MRT drops out of incentive constraints
- $\Rightarrow$ Production efficiency
- Question: When else do MRTs drop out of incentive constraints?

References I

Atkinson, Anthony B., and Joseph E. Stiglitz. 1976. "The Design of Tax Structure: Direct versus Indirect Taxation." *Journal of Public Economics* 6 (1): 55–75.

Costinot, Arnaud, and Ivan Werning. 2018. "Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation." Cambridge, MA. http://www.nber.org/papers/w25103.

Diamond, Peter A., and James A. Mirrlees. 1971. "Optimal Taxation and Public Production I: Production Efficiency." *The American Economic Review* 61 (1): 8–27. doi:10.2307/2296446.

Gomes, Renato, Jean-Marie Lozachmeur, and Alessandro Pavan. 2018. "Differential Taxation and Occupational Choice." *Review of Economic Studies* 85 (1): 511–557. doi:10.1093/restud/rdx022.

References II

Guerreiro, Joao, Sergio Rebelo, and Pedro Teles. 2018. "Should Robots be Taxed?" Cambridge, MA.

Jacobs, Bas. 2015. "Optimal Inefficient Production." `http://people.few.eur.nl/bjacobs/dmpet.pdf`.

Naito, Hisahiro. 1999. "Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency." *Journal of Public Economics* 71 (2): 165–188.

Rothschild, Casey, and Florian Scheuer. 2013. "Redistributive Taxation in the Roy Model." *Quarterly Journal of Economics* 128 (2): 623–668. doi:`10.1093/qje/qjs076`.

References III

Saez, Emmanuel. 2004. “Direct or indirect tax instruments for redistribution: Short-run versus long-run.” *Journal of Public Economics* 88 (3-4): 503–518. doi:10.1016/S0047-2727(02)00222-0.