Indian Institute of Technology Bombay

MA 105 Calculus

Solution to Short Quiz 6

by Adway Girish, D1-T3

Date: September 11, 2019 Day: Wednesday

Time: 2:00 PM - 2:05 PM Max. Marks: 5

Question. State whether the following statement is true or false. Justify your answer.

$$\int_0^{2\pi} e^{\sin x+\cos x}dx = \int_{\sqrt{2}-\pi}^{\sqrt{2}+\pi} e^{\sin x+\cos x}dx$$

(2 marks for correct alternative; 3 marks for correct justification)

Solution. The given statement is **true**. (2 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined as:

$$f(x) := e^{\sin x+\cos x} \ \forall \ x \in \mathbb{R}$$

which is continuous and is perioidic with period 2π. (1 mark) Now consider the function

$$F(x) := \int_x^{x+2\pi} f(t)dt = \int_0^{x+2\pi} f(t)dt - \int_0^x f(t)dt$$

By FTC (first part), F is differentiable and $F'(x) = f(x + 2\pi) - f(x) = 0$ for all $x \in \mathbb{R}$ (because f has period 2π), which means F is a constant function (by MVT). (1 mark) In the given equation, the LHS is $F(0)$ and the RHS is $F(\sqrt{2} - \pi)$, which are equal since F is constant. (1 mark)

$\therefore$ LHS = RHS, and hence the statement is true. $\square$

General observations:

1. The most common (major understatement) mistake was in directly using the property that the integral of a periodic function over an interval equal to its period remians constant. This is not given in the slides, and so, must be proven. (The proof is direct from FTC). Everyone who directly wrote has gotten only 3 marks (AS IN THE MARKING SCHEME GIVEN BY THE PROFESSORS).
2. Some students have taken $f(x) = \int_0^{2\pi} f(x)dx$ which is obviously a constant real number. They have then diiferentiated this f and gotten $f'(x) = e^{\sin x+\cos x}$ when $f'(x) = 0$ for all x because f is constant.
3. Some students have done some substitutions in the integral in the RHS and gotten the integral of some function not equal to $e^{\sin x+\cos x}$ from 0 to 2π, and then conculded that the statement is false. This is an incorrect conclusion because of obvious reasons- different functions may have the same value of the definite integral in the same interval.
