Indian Institute of Technology Bombay

MA 105 Calculus

Solution to Short Quiz 12

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Date: October 30, 2019 Day: Wednesday
Time: 2:00 PM - 2:05 PM Max. Marks: 5

Question. State whether the following statement is true or false and justify your answer:

The vector field $\mathbf{F}(x, y) = (y, -x)$ on $\mathbb{R}^2$ is a gradient field.

(2 marks for correct alternative; 3 marks for correct justification)

Solution. The given statement is **false**.

For $\mathbf{F}$ to be a gradient field, its line integral over any closed, piecewise smooth path should be zero. Consider the path on the unit circle centered at the origin given by $(\cos t, \sin t), t \in [-\pi, \pi]$. The line integral of $\mathbf{F}$ along this path is given by

$$\int_{-\pi}^{\pi} (-\sin t, \cos t) \cdot (-\sin t, \cos t)\ dt = \int_{-\pi}^{\pi} dt = 2\pi \neq 0$$

Hence, $\mathbf{F}$ is NOT a gradient field (because it is not path independent). □

General observations:

1. Some students have done this by the cross-derivative test, which is also fine, but hardly anyone has mentioned that the vector field should be smooth to use this test. No marks have been deducted though.
2. Some have done by checking that the curl is non-zero, which is also fine, and have been given full marks.
3. Some students have written stuff like $\frac{\partial \mathbf{F}}{\partial x}$ and $\frac{\partial \mathbf{F}}{\partial y}$, which makes no sense because $\mathbf{F}$ is a vector field.
4. Some students have just said that it cannot be a gradient field without giving any justification, they have gotten no marks for justification. Duh.
