Static Program Analysis

Part 4 – flow sensitive analyses

http://cs.au.dk/~amoeller/spa/

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Agenda

- **Live variables analysis**
- Available expressions analysis
- Very busy expressions analysis
- Reaching definitions analysis
- Constant propagation analysis

Liveness analysis

- A variable is *live* at a program point if its current value may be read in the remaining execution

- This is clearly undecidable, but the property can be conservatively approximated

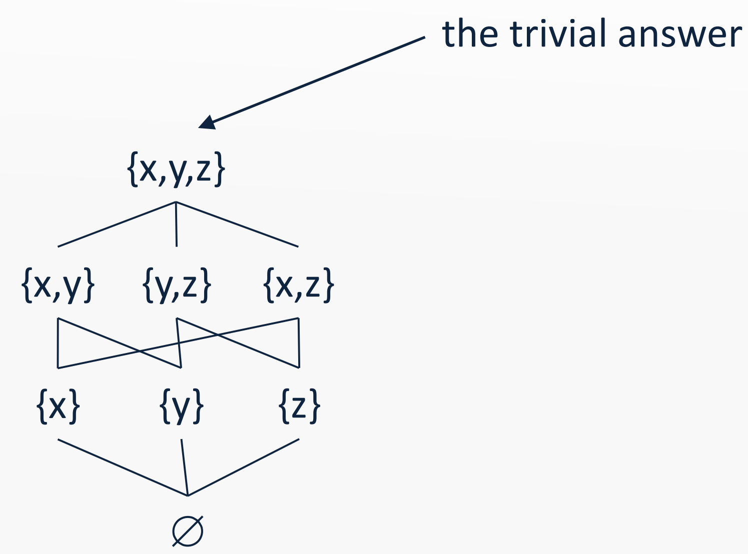
- The analysis must only answer *"dead"* if the variable is really dead
  - no need to store the values of dead variables

A lattice for liveness

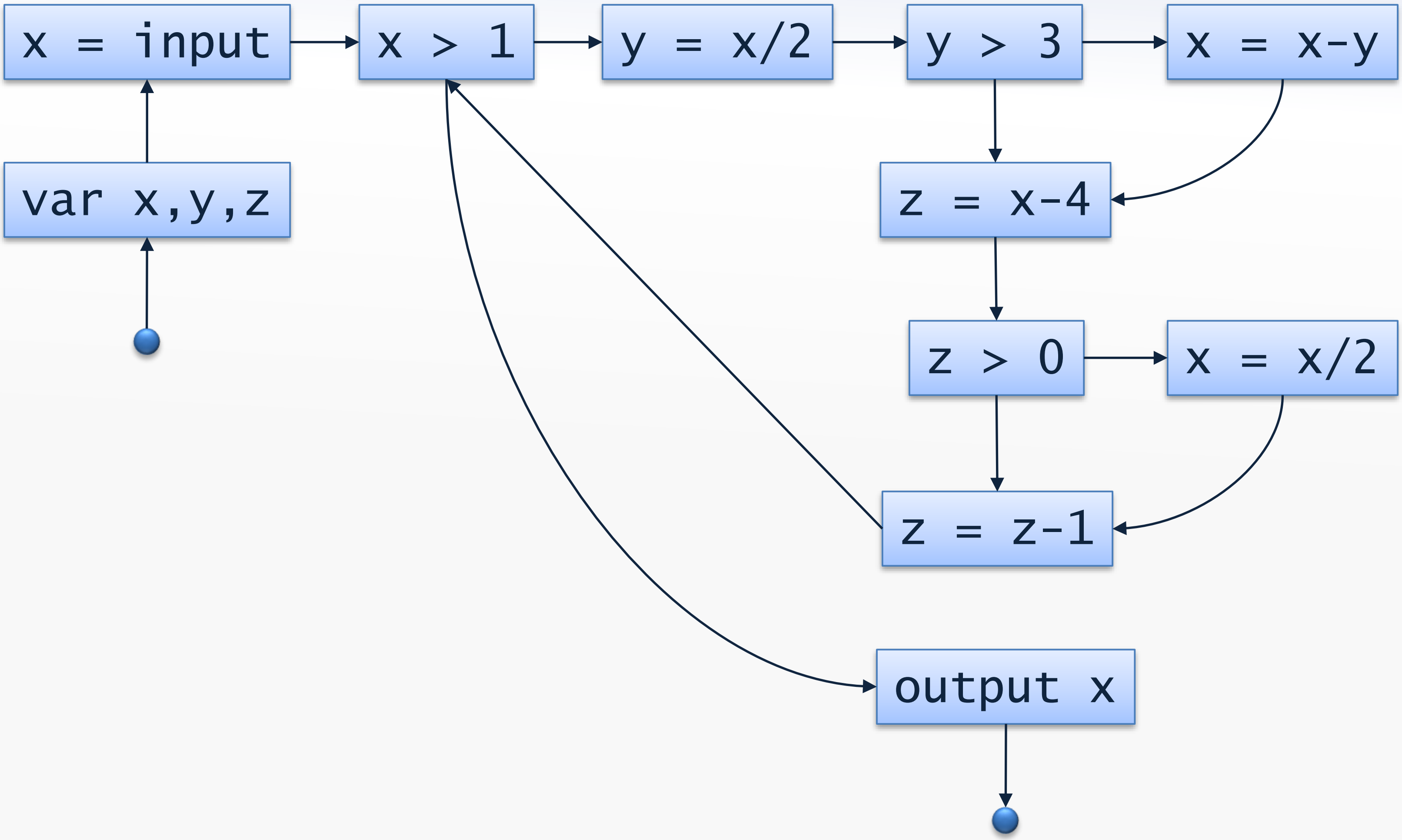
A subset lattice of program variables

```
var x,y,z;
x = input;
while (x>1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

$$L = (2^{\{x,y,z\}}, \subseteq)$$



The control flow graph



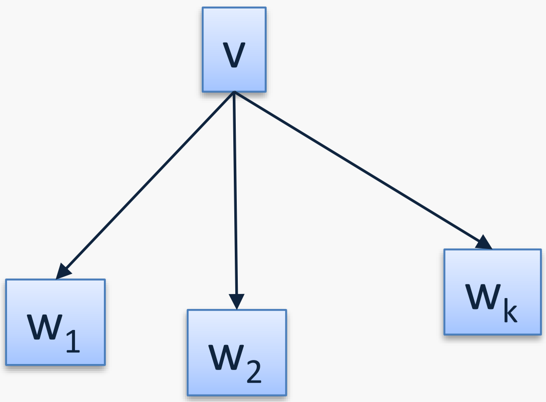
Setting up

- For every CFG node, v, we have a variable ⟦v⟧:
  - the subset of program variables that are live at the program point *before* v

- Since the analysis is conservative, the computed sets may be *too large*

- Auxiliary definition:

$$\mathit{JOIN}(v) = \bigcup_{w \in \mathit{succ}(v)} [\![w]\!]$$



Liveness constraints

- For the exit node:

  $[\![exit]\!] = \emptyset$

vars(E) = variables occurring in E

- For conditions and output:

  $[\![$ `if (E)` $]\!] = [\![$ `output` $E\,]\!] = JOIN(\mathrm{v}) \cup vars(E)$

- For assignments:

  $[\![\, x = E \,]\!] = JOIN(\mathrm{v}) \setminus \{x\} \cup vars(E)$

- For variable declarations:

  $[\![$ `var` x_1 `, ... ,` $x_n\,]\!] = JOIN(\mathrm{v}) \setminus \{x_1, \ldots, x_n\}$

- For all other nodes:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

right-hand sides are monotone since *JOIN* is monotone, and …

Generated constraints

$[\![\texttt{var x,y,z}]\!] = [\![\texttt{z=input}]\!] \setminus \{x,y,z\}$

$[\![\texttt{x=input}]\!] = [\![\texttt{x>1}]\!] \setminus \{x\}$

$[\![\texttt{x>1}]\!] = ([\![\texttt{y=x/2}]\!] \cup [\![\texttt{output x}]\!]) \cup \{x\}$

$[\![\texttt{y=x/2}]\!] = ([\![\texttt{y>3}]\!] \setminus \{y\}) \cup \{x\}$

$[\![\texttt{y>3}]\!] = [\![\texttt{x=x-y}]\!] \cup [\![\texttt{z=x-4}]\!] \cup \{y\}$

$[\![\texttt{x=x-y}]\!] = ([\![\texttt{z=x-4}]\!] \setminus \{x\}) \cup \{x,y\}$

$[\![\texttt{z=x-4}]\!] = ([\![\texttt{z>0}]\!] \setminus \{z\}) \cup \{x\}$

$[\![\texttt{z>0}]\!] = [\![\texttt{x=x/2}]\!] \cup [\![\texttt{z=z-1}]\!] \cup \{z\}$

$[\![\texttt{x=x/2}]\!] = ([\![\texttt{z=z-1}]\!] \setminus \{x\}) \cup \{x\}$

$[\![\texttt{z=z-1}]\!] = ([\![\texttt{x>1}]\!] \setminus \{z\}) \cup \{z\}$

$[\![\texttt{output x}]\!] = [\![\mathit{exit}]\!] \cup \{x\}$

$[\![\mathit{exit}]\!] = \varnothing$

Least solution

$[\![\mathit{entry}]\!] = \emptyset$
$[\![\texttt{var x,y,z}]\!] = \emptyset$
$[\![\texttt{x=input}]\!] = \emptyset$
$[\![\texttt{x>1}]\!] = \{x\}$
$[\![\texttt{y=x/2}]\!] = \{x\}$
$[\![\texttt{y>3}]\!] = \{x,y\}$
$[\![\texttt{x=x-y}]\!] = \{x,y\}$
$[\![\texttt{z=x-4}]\!] = \{x\}$

$[\![\texttt{z>0}]\!] = \{x,z\}$
$[\![\texttt{x=x/2}]\!] = \{x,z\}$
$[\![\texttt{z=z-1}]\!] = \{x,z\}$
$[\![\texttt{output x}]\!] = \{x\}$
$[\![\mathit{exit}]\!] = \emptyset$

Many non-trivial answers!

Optimizations

- Variables `y` and `z` are never simultaneously live

  ⇒ they can share the same variable location
- The value assigned in `z=z-1` is never read

  ⇒ the assignment can be skipped

```
var x,yz;
x = input;
while (x>1) {
  yz = x/2;
  if (yz>3) x = x-yz;
  yz = x-4;
  if (yz>0) x = x/2;
}
output x;
```

- better register allocation
- a few clock cycles saved

Time complexity (for the naive algorithm)

- With n CFG nodes and k variables:
  - the lattice L^n has height $k \cdot n$
  - so there are at most $k \cdot n$ iterations
- Subsets of Vars (the variables in the program) can be represented as bitvectors:
  - each element has size k
  - each $\cup$, $\setminus$, $=$ operation takes time $O(k)$
- Each iteration uses $O(n)$ bitvector operations:
  - so each iteration takes time $O(k \cdot n)$
- Total time complexity: $O(k^2 n^2)$

- Exercise: what is the complexity for the worklist algorithm?

Agenda

- Live variables analysis
- **Available expressions analysis**
- Very busy expressions analysis
- Reaching definitions analysis
- Constant propagation analysis

Available expressions analysis

- A (nontrivial) expression is *available* at a program point if its current value has already been computed earlier in the execution

- The approximation generally includes *too few* expressions
  - the analysis can only report *"available"* if the expression is definitely available
  - no need to re-compute available expressions (e.g. common subexpression elimination)

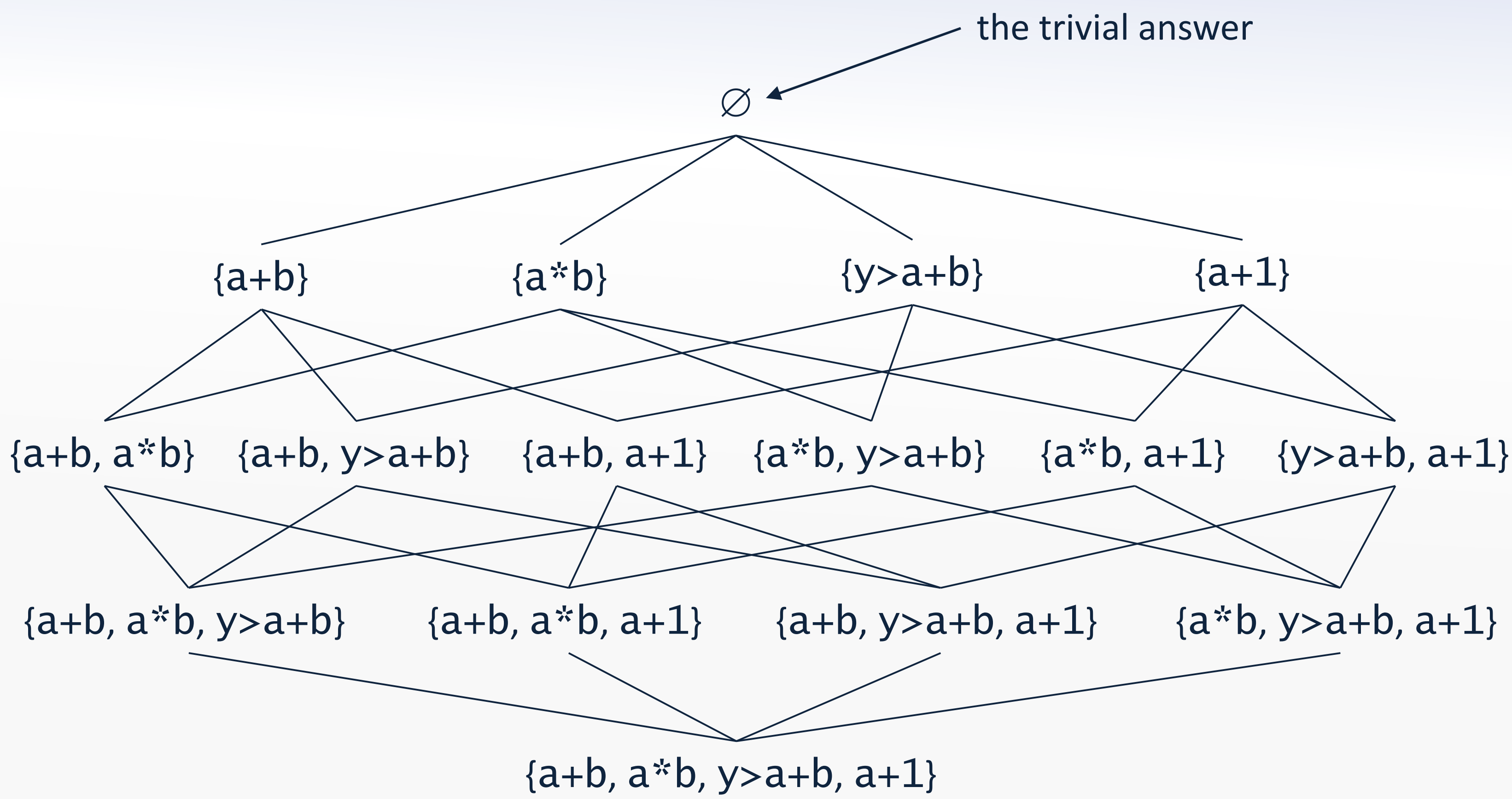
A lattice for available expressions

A reverse subset-lattice of nontrivial expressions

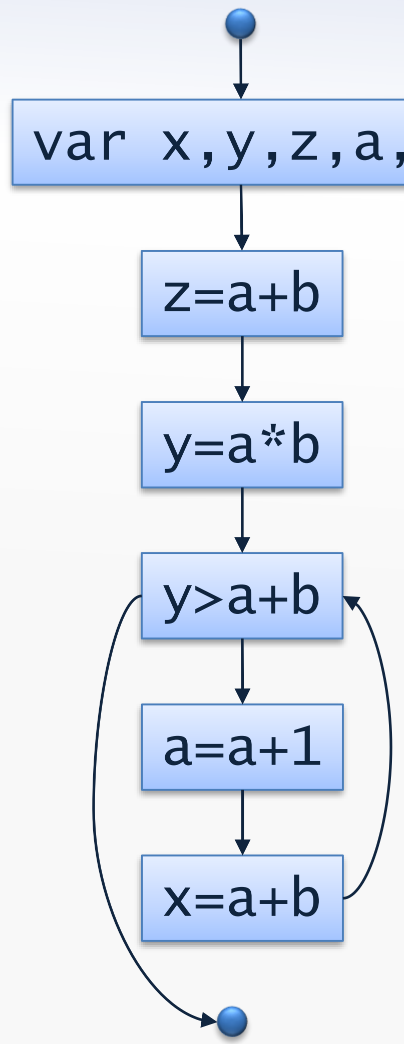
```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
  a = a+1;
  x = a+b;
}
```

$$L = (2^{\{a+b,\ a*b,\ y>a+b,\ a+1\}}, \supseteq)$$

Reverse subset lattice



The flow graph



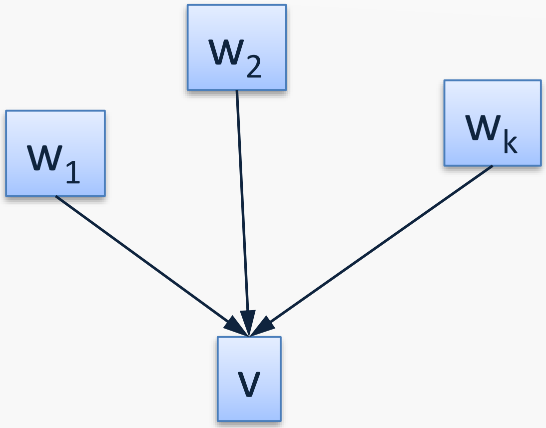
Setting up

- For every CFG node, v, we have a variable ⟦v⟧:
  - the subset of program variables that are available at the program point *after* v

- Since the analysis is conservative, the computed sets may be *too small*

- Auxiliary definition:

$$JOIN(v) = \bigcap_{w \in pred(v)} [\![w]\!]$$



Auxiliary functions

- The function $X{\downarrow}x$ removes all expressions from X that contain a reference to the variable *x*

- The function *exps*(*E*) is defined as:
  - $exps(intconst) = \emptyset$
  - $exps(x) = \emptyset$
  - $exps(\texttt{input}) = \emptyset$
  - $exps(E_1\ op\ E_2) = \{E_1\ op\ E_2\} \cup exps(E_1) \cup exps(E_2)$
    but don't include expressions containing `input`

Availability constraints

- For the *entry* node:

  $[\![entry]\!] = \emptyset$

- For conditions and output:

  $[\![\ \texttt{if (}E\texttt{)}\ ]\!] = [\![\ \texttt{output}\ E\ ]\!] = JOIN(\mathrm{v}) \cup exps(E)$

- For assignments:

  $[\![\ x = E\ ]\!] = (JOIN(\mathrm{v}) \cup exps(E))\!\downarrow\! x$

- For any other node v:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

Generated constraints

$[\![entry]\!] = \emptyset$

$[\![$`var x,y,z,a,b`$]\!] = [\![entry]\!]$

$[\![\mathrm{z{=}a{+}b}]\!] = exps(\mathrm{a{+}b}){\downarrow}\mathrm{z}$

$[\![\mathrm{y{=}a{*}b}]\!] = ([\![\mathrm{z{=}a{+}b}]\!] \cup exps(\mathrm{a{*}b})){\downarrow}\mathrm{y}$

$[\![\mathrm{y{>}a{+}b}]\!] = ([\![\mathrm{y{=}a{*}b}]\!] \cap [\![\mathrm{x{=}a{+}b}]\!]) \cup exps(\mathrm{y{>}a{+}b})$

$[\![\mathrm{a{=}a{+}1}]\!] = ([\![\mathrm{y{>}a{+}b}]\!] \cup exps(\mathrm{a{+}1})){\downarrow}\mathrm{a}$

$[\![\mathrm{x{=}a{+}b}]\!] = ([\![\mathrm{a{=}a{+}1}]\!] \cup exps(\mathrm{a{+}b})){\downarrow}\mathrm{x}$

$[\![exit]\!] = [\![\mathrm{y{>}a{+}b}]\!]$

Least solution

$[\![\mathit{entry}]\!] = \emptyset$

$[\![$`var x,y,z,a,b`$]\!] = \emptyset$

$[\![$z=a+b$]\!]$ = {a+b}

$[\![$y=a*b$]\!]$ = {a+b, a*b}

$[\![$y>a+b$]\!]$ = {a+b, y>a+b}

$[\![$a=a+1$]\!] = \emptyset$

$[\![$x=a+b$]\!]$ = {a+b}

$[\![\mathit{exit}]\!]$ = {a+b}

Again, many nontrivial answers!

Optimizations

- We notice that a+b is available before the loop
- The program can be optimized (slightly):

```
var x,y,x,a,b,aplusb;
aplusb = a+b;
z = aplusb;
y = a*b;
while (y > aplusb) {
  a = a+1;
  aplusb = a+b;
  x = aplusb;
}
```

Agenda

- Live variables analysis
- Available expressions analysis
- **Very busy expressions analysis**
- Reaching definitions analysis
- Constant propagation analysis

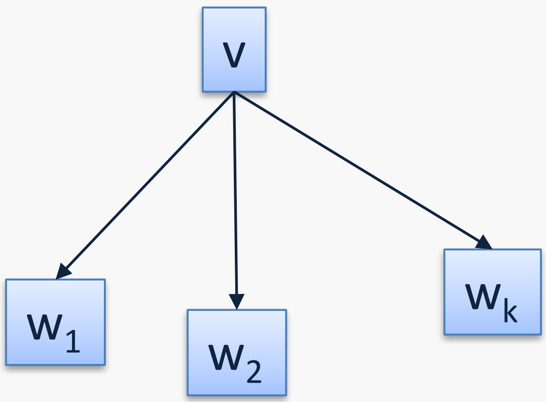
Very busy expressions analysis

- A (nontrivial) expression is *very busy* if it will definitely be evaluated before its value changes
- The approximation generally includes *too few* expressions
  - the answer "*very busy*" must be the true one
  - very busy expressions may be pre-computed (e.g. loop hoisting)
- Same lattice as for available expressions

Setting up

- For every CFG node, v, we have a variable ⟦v⟧:
  - the subset of program variables that are very busy at the program point *before* v
- Since the analysis is conservative, the computed sets may be *too small*
- Auxiliary definition:

$$\mathit{JOIN}(v) = \bigcap_{w \in \mathit{succ}(v)} [\![w]\!]$$



Very busy constraints

- For the *exit* node:

  $[\![exit]\!] = \emptyset$

- For conditions and output:

  $[\![\,\texttt{if (}E\texttt{)}\,]\!] = [\![\,\texttt{output}\ E\,]\!] = JOIN(\mathrm{v}) \cup exps(E)$

- For assignments:

  $[\![\ x = E\ ]\!] = JOIN(\mathrm{v}){\downarrow}x \cup exps(E)$

- For all other nodes:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

An example program

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

The analysis shows that `a*b` is very busy

Code hoisting

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

```
var x,a,b,atimesb;
x = input;
a = x-1;
b = x-2;
atimesb = a*b;
while (x > 0) {
  output atimesb-x;
  x = x-1;
}
output atimesb;
```

Agenda

- Live variables analysis
- Available expressions analysis
- Very busy expressions analysis
- **Reaching definitions analysis**
- Constant propagation analysis

Reaching definitions analysis

- The *reaching definitions* for a program point are those assignments that may define the current values of variables

- The conservative approximation may include *too many* possible assignments

A lattice for reaching definitions

The subset lattice of assignments

$L = (2^{\{x=input,\ y=x/2,\ x=x-y,\ z=x-4,\ x=x/2,\ z=z-1\}}, \subseteq)$

```
var x,y,z;
x = input;
while (x > 1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

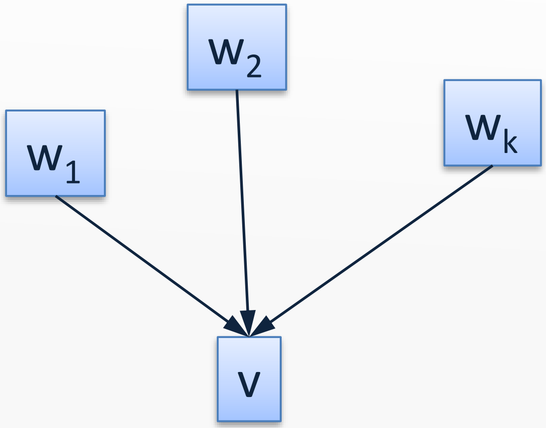
Reaching definitions constraints

- For assignments:

$$[\![x = E]\!] = JOIN(\mathrm{v}){\downarrow}x \cup \{ x = E \}$$

- For all other nodes:

$$[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$$



- Auxiliary definition:

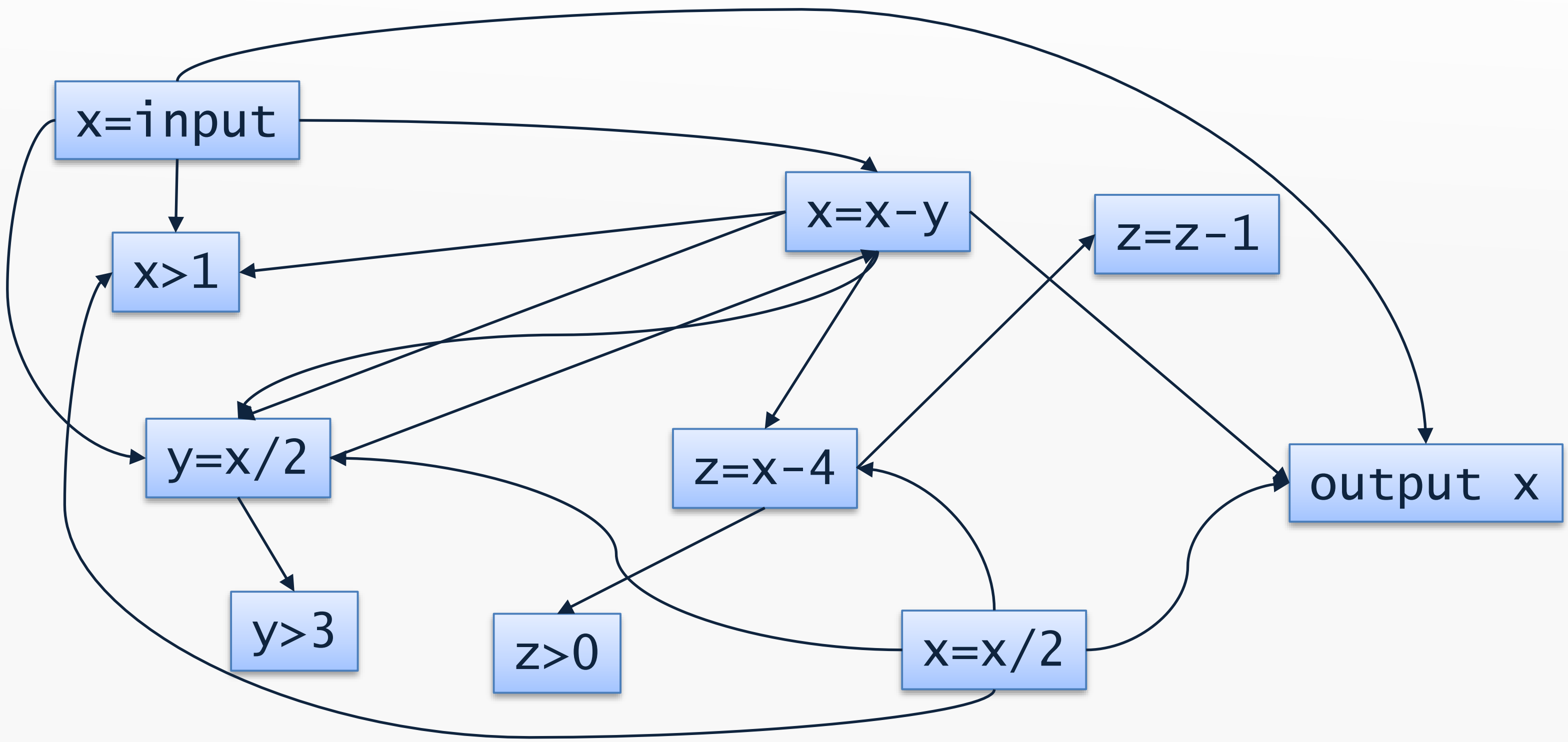
$$JOIN(\mathrm{v}) = \bigcup_{\mathrm{w} \in pred(\mathrm{v})} [\![\mathrm{w}]\!]$$

- The function $X{\downarrow}x$ removes assignments to x from X

Def-use graph

Reaching definitions define the def-use graph:

- like a CFG but with edges from *def* to *use* nodes
- basis for *dead code elimination* and *code motion*



Forward vs. backward

- A *forward* analysis:
  - computes information about the *past* behavior
  - examples: available expressions, reaching definitions
- A *backward* analysis:
  - computes information about the *future* behavior
  - examples: liveness, very busy expressions

May vs. must

- A *may* analysis:
  – describes information that is *possibly* true
  – an *over*-approximation
  – examples: liveness, reaching definitions

- A *must* analysis:
  – describes information that is *definitely* true
  – an *under*-approximation
  – examples: available expressions, very busy expressions

Classifying analyses

	forward	backward
may	example: reaching definitions $[\![v]\!]$ describes state after v JOIN(v) = $\bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcup_{w \in pred(v)} [\![w]\!]$	example: liveness $[\![v]\!]$ describes state before v JOIN(v) = $\bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcup_{w \in succ(v)} [\![w]\!]$
must	example: available expressions $[\![v]\!]$ describes state after v JOIN(v) = $\bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcap_{w \in pred(v)} [\![w]\!]$	example: very busy expressions $[\![v]\!]$ describes state before v JOIN(v) = $\bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcap_{w \in succ(v)} [\![w]\!]$

Initialized variables analysis

- Compute for each program point those variables that have *definitely* been initialized in the *past*
- (Called *definite assignment* analysis in Java and C#)
- $\Rightarrow$ *forward must analysis*
- Reverse subset lattice of all variables

$$JOIN(v) = \bigcap_{w \in pred(v)} [\![w]\!]$$

- For assignments: $[\![\ x = E\ ]\!] = JOIN(v) \cup \{x\}$
- For all others: $[\![v]\!] = JOIN(v)$

Agenda

- Live variables analysis
- Available expressions analysis
- Very busy expressions analysis
- Reaching definitions analysis
- **Constant propagation analysis**

Constant propagation optimization

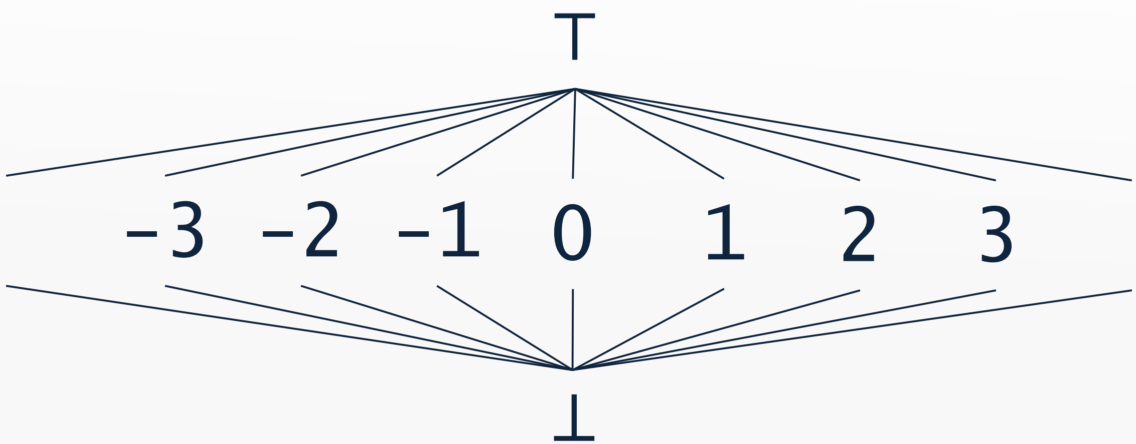
```
var x,y,z;
x = 27;
y = input,
z = 2*x+y;
if (x<0) { y=z-3; } else { y=12 }
output y;
```

```
var x,y,z;
x = 27;
y = input;
z = 54+y;
if (0) { y=z-3; } else { y=12 }
output y;
```

```
var y;
y = input;
output 12;
```

Constant propagation analysis

- Determine variables with a constant value
- Flat lattice:



Constraints for constant propagation

- Essentially as for the Sign analysis…

- Abstract operator for addition:

$$\overline{+}(n,m) = \begin{cases} \bot & \text{if } n=\bot \vee m=\bot \\ \top & \text{else if } n=\top \vee m=\top \\ n+m & \text{otherwise} \end{cases}$$