Compiler

Parsing: Top-Down

Implementing Parsers

- Two basic approaches
  - Top-down
  - Bottom-up
- Top-Down
  - Easier to understand and program manually
  - Supported by ANTLR
- Bottom-Up
  - Used by most other parser generators

Parsing Algorithms

- Top-down parser
  - LL(k)
  - Left-to-right scan of input
  - Leftmost derivation
  - k symbols of lookahead
- Bottom-up parser
  - LR(k)
  - Left-to-right scan of input
  - Rightmost derivation (in reverse)
  - k symbols of lookahead

Comparing Algorithms

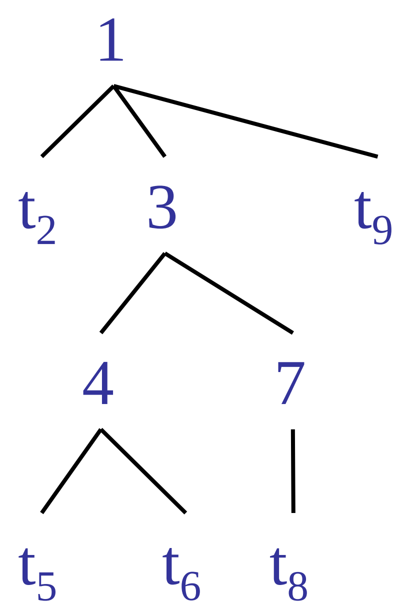
- Which approach to use?
  - LR(k) > LL(k)
- Usually one thinks of k=1
  - space(LL) = $O(n^k)$
  - LL(k+1) > LL(k)
  - LL(2) > LR(1)
- LR tolerates ambiguity in grammar
  - LL requires transformation
- LL naturally incorporates actions

Intro to Top-Down Parsing

- The parse tree is constructed
  - From the top
  - From left to right

- Terminals are seen in order of appearance in the token stream:

  $t_2 \ \ t_5 \ \ t_6 \ \ t_8 \ \ t_9$



Recursive Descent Parsing — An Example

- Consider the grammar

  $E \rightarrow T + E \mid T$

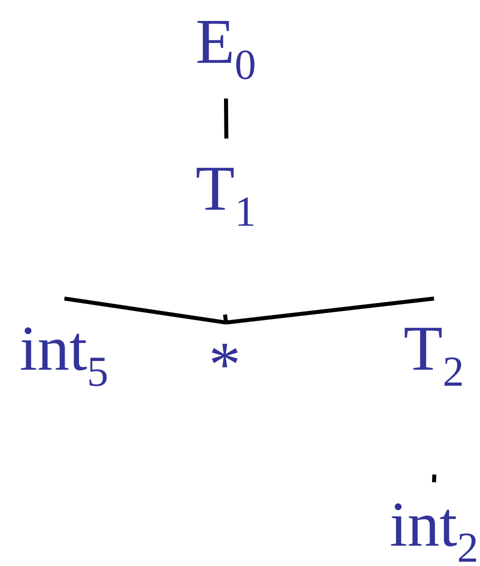
  $T \rightarrow int \mid int * T \mid (E)$

- Token stream is:
  $int_5 * int_2$
- Start with top-level non-terminal E
- Try the rules for E in order

- Try $E_0 \rightarrow T_1 + E_2$
- Then try a rule for $T_1 \rightarrow (E_3)$
  - But (does not match input token int_5
- Try $T_1 \rightarrow int$. Token matches.
  - But + after T_1 does not match input token *
- Try $T_1 \rightarrow int * T_2$
  - This will match but + after T_1 will be unmatched

Recursive Descent Parsing – An Example (Cont.)

- Has exhausted the choices for T_1
  - Backtrack to choice for E_0
- Try $E_0 \rightarrow T_1$
- Follow same steps as before for T_1
  - and succeed with $T_1 \rightarrow$ int * T_2 and $T_2 \rightarrow$ int



Recursive Descent Parsing – Preliminaries

- Let Token be the type of tokens
  - Special tokens INT, LPAREN, RPAREN, PLUS, TIMES

- Let the field next point to the next token

Recursive Descent Parser (2)

- Define boolean functions that check the token string for a match of
  - A given token terminal

```
boolean term(Token tok) {
  boolean result = next.equals(tok);
  next = nextToken(next);
  return result;
}
```

  - A given production of S (the n^{th})

boolean S_n() { … }

  - Any production of S:

```
boolean S() { … }
```

- These functions advance next

Recursive Descent Parser (3)

- For production $E \rightarrow T$

```
boolean E1() { return T(); }
```

- For production $E \rightarrow T + E$

```
boolean E2() { return T() && term(PLUS) && E(); }
```

- For all productions of E (with backtracking)

```
boolean E() {
  Token save;
  save = next;
  if (E1()) return true;
  save = next;
  if (E2()) return true;
  return false;
}
```

Recursive Descent Parser (4)

- Functions for non-terminal T

```
boolean T1() {
   return term(LPAREN) && E() && term(RPAREN);
}
boolean T2() { return term(INT) && term(TIMES) && T(); }
boolean T3() { return term(INT); }

boolean T() {
   Token save;
   save = next;
   if (T1()) return true;
   save = next;
   if (T2()) return true;
   save = next;
   if (T3()) return true;
   return false;
}
```

Recursive Descent Parsing – Notes

- To start the parser
  - Initialize next to point to first token
  - Invoke E()
- Notice how this simulates our previous example

- Easy to implement by hand
- But does not always work …

When Recursive Descent Does Not Work

- Consider a production $S \to S\ a$

```
boolean S1() { return S() && term(a); }
boolean S() { return S1(); }
```

- S() will get into an infinite loop
- A *left-recursive grammar* has a non-terminal S

  $S \to^{+} S\alpha$ for some α

- Recursive descent does not work in such cases

Elimination of Left Recursion

- Consider the left-recursive grammar

$$S \rightarrow S\ \alpha \mid \beta$$

- S generates all strings starting with a β and followed by a number of α
- Can rewrite using right-recursion

$$S \rightarrow \beta\ S'$$

$$S' \rightarrow \alpha\ S' \mid \varepsilon$$

More Elimination of Left-Recursion

- In general

$$S \rightarrow S\ \alpha_1 \mid \ldots \mid S\ \alpha_n \mid \beta_1 \mid \ldots \mid \beta_m$$

- All strings derived from S start with one of $\beta_1,\ldots,\beta_m$ and continue with several instances of $\alpha_1,\ldots,\alpha_n$
- Rewrite as

$$S \rightarrow \beta_1\ S' \mid \ldots \mid \beta_m\ S'$$

$$S' \rightarrow \alpha_1\ S' \mid \ldots \mid \alpha_n\ S' \mid \varepsilon$$

General Left Recursion

- The grammar

$$S \rightarrow A\ \alpha \mid \delta$$

$$A \rightarrow S\ \beta$$

- is also left-recursive because

$$S \rightarrow^{+} S\ \beta\ \alpha$$

- This *indirect* left-recursion can also be eliminated
- See book for general algorithm

Summary of Recursive Descent Parsing

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient
- In practice, backtracking is eliminated by restricting the grammar

Predictive Parsers

- Like recursive-descent but parser can “predict” which production to use
  - By looking at the next few tokens (no backtracking)
- Predictive parsers accept LL(k) grammars
  - L means “left-to-right” scan of input
  - L means “leftmost derivation”
  - k means “predict based on k tokens of lookahead”
- In practice, LL(2) is used

LL(1) Languages

- In recursive-descent, for each non-terminal and input token there may be a choice of production
- LL(1) means that for each non-terminal and token there is only one production
- Can be specified via 2D tables
  - One dimension for current non-terminal to expand
  - One dimension for next token
  - A table entry contains one production

Predictive Parsing and Left Factoring

- Recall the grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow \text{int} \mid \text{int} * T \mid (E)$

- Hard to predict because
  - For T two productions start with int
  - For E it is not clear how to predict
- A grammar must be *left-factored* before use for predictive parsing

Left-Factoring Example

- Recall the grammar

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} \mid \text{int} * T \mid (E)$$

- Factor out *common prefixes* of productions

$$E \rightarrow T\,X$$

$$X \rightarrow + E \mid \varepsilon$$

$$T \rightarrow (E) \mid \text{int}\ Y$$

$$Y \rightarrow * T \mid \varepsilon$$

LL(1) Parsing Table Example

- Left-factored grammar

  $E \rightarrow T\ X$ $\quad$ $X \rightarrow +\ E \mid \varepsilon$

  $T \rightarrow (\ E\ ) \mid \text{int}\ Y$ $\quad$ $Y \rightarrow *\ T \mid \varepsilon$

- The LL(1) parsing table:

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Table Example (Cont.)

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

- Consider the [E, int] entry
  - “When current non-terminal is E and next input is int, use production $E \rightarrow T X$
  - This production generates an int in the first place

LL(1) Parsing Table Example (Cont.)

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

- Consider the [Y,+] entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - Y can be followed by + only in a derivation in which $Y \to \varepsilon$

LL(1) Parsing Tables – Errors

- Blank entries indicate error situations
  - Consider the [E,*] entry
  - "There is no way to derive a string starting with * from non-terminal E"

Using Parsing Tables

- Method similar to recursive descent, except
  - For each non-terminal S
  - look at the next token a
  - choose the production shown at [S,a]
- use a stack to keep track of pending non-terminals
- reject when error state
- accept when end-of-input ($)

LL(1) Parsing Algorithm

```
initialize stack = <S, $> and next
repeat
  case stack of
    <X, rest> :
      if T[X,*next] = Y1…Yn
      then stack = <Y1…Yn, rest>;
      else  error();
    <t, rest> :
      if t.equals(next)
      then { next = nextToken(next);
             stack = <rest>; }
      else error();
until stack == < >
```

LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	T X
T X $	int * int $	int Y
int Y X $	int * int $	terminal
Y X $	* int $	* T
* T X $	* int $	terminal
T X $	int $	int Y
int Y X $	int $	terminal
Y X $	$	ε
X $	$	ε
$	$	ACCEPT

Constructing Parsing Tables

- LL(1) languages are those defined by a parsing table for the LL(1) algorithm
- No table entry can be multiply defined

- We want to generate parsing tables from CFG

Constructing Parsing Tables (Cont.)

- If $A \to \alpha$, where in the column of A does α go?
- In the column of t where t can start a string derived from α

$$\alpha \to^{*} t\ \beta$$

  - We say that $t \in \text{First}(\alpha)$
- In the column of t if α is ε and t can follow an A

$$S \to^{*} \beta\ A\ t\ \delta$$

  - We say $t \in \text{Follow}(A)$

Computing First Sets

Definition: $First(X) = \{ t \mid X \rightarrow^* t\alpha\} \cup \{\varepsilon \mid X \rightarrow^* \varepsilon\}$

Algorithm sketch (see book for details):

1. for all terminals t do $First(t) \leftarrow \{ t \}$
2. for each production $X \rightarrow \varepsilon$ do $First(X) \leftarrow \{ \varepsilon \}$
3. if $X \rightarrow A_1 \ldots A_n \alpha$ and $\varepsilon \in First(A_i)$, $1 \leq i \leq n$ do
   - add $First(\alpha)$ to $First(X)$
4. for each $X \rightarrow A_1 \ldots A_n$ s.t. $\varepsilon \in First(A_i)$, $1 \leq i \leq n$ do
   - add ε to $First(X)$
5. repeat steps 4 & 5 until no First set can be grown

First Sets – Example

- Recall the grammar

$E \to T\ X$ $\qquad$ $X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid \text{int}\ Y$ $\qquad$ $Y \to *\ T \mid \varepsilon$

- First sets

First(( ) = { (} $\qquad$ First(T) = {int, (}

First( )) = {) } $\qquad$ First(E) = {int, (}

First(int) = { int } $\qquad$ First(X) = {+, ε }

First(+) = { + } $\qquad$ First(Y) = {*, ε }

First(*) = { * }

Computing Follow Sets

- Definition:

$$\text{Follow}(X) = \{ t \mid S \to^{*} \beta\, X\, t\, \delta \}$$

- Intuition
  - If S is the start symbol then $\$ \in \text{Follow}(S)$
  - If $X \to A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and $\text{Follow}(X) \subseteq \text{Follow}(B)$
  - If $B \to^{*} \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$

Computing Follow Sets (Cont.)

Algorithm sketch:

1. Follow(S) $\leftarrow$ { \$ }
2. For each production $A \rightarrow \alpha X \beta$
   - add First(β) - {ε} to Follow(X)
3. For each $A \rightarrow \alpha X \beta$ where $\varepsilon \in$ First(β)
   - add Follow(A) to Follow(X)

- repeat step(s) until no Follow set grows

Follow Sets – Example

- Recall the grammar

  $E \rightarrow T\ X$ $\quad$ $X \rightarrow +\ E \mid \varepsilon$

  $T \rightarrow (\ E\ ) \mid \text{int}\ Y$ $\quad$ $Y \rightarrow *\ T \mid \varepsilon$

- Follow sets

  Follow(+) = { int, (}

  Follow(*) = { int, (}

  Follow(( ) = { int, (}

  Follow(E) = {), $}

  Follow(X) = {$,) }

  Follow(T) = {+,) , $}

  Follow( )) = {+,) , $}

  Follow(Y) = {+,) , $}

  Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G
- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $t \in First(\alpha)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$, for each $t \in Follow(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$ and $\$ \in Follow(A)$ do
    - $T[A, \$] = \alpha$

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous, left-recursive, not left-factored, …
- Most programming language grammars are not LL(1)
- There are tools that build LL(1) tables