# Static Program Analysis

## Part 2 – type analysis and unification

http://cs.au.dk/~amoeller/spa/

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# Type errors

- Reasonable restrictions on operations:
  - arithmetic operators apply only to integers
  - comparisons apply only to like values
  - only integers can be input and output
  - conditions must be integers
  - only functions can be called
  - the * operator applies only to pointers
  - field lookup can only be performed on records

- Violations result in runtime errors

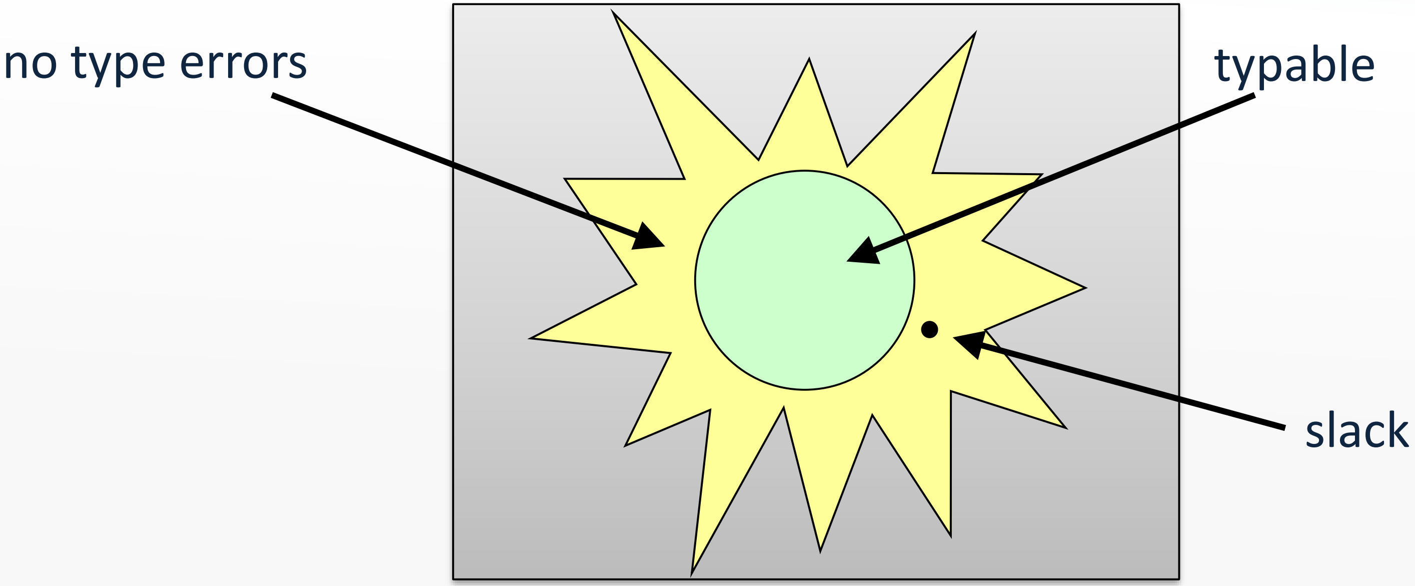
# Type checking

- Can type errors occur during runtime?
- This is interesting, hence instantly undecidable

- Instead, we use conservative approximation
  - a program is *typable* if it satisfies some *type constraints*
  - these are systematically derived from the syntax tree
  - if typable, then no runtime errors occur
  - but some programs will be unfairly rejected (*slack*)

- What we shall see next is the essence of the
  Damas–Hindley–Milner type inference technique,
  which forms the basis of the type systems of e.g. ML, OCaml, and Haskell

# Typability



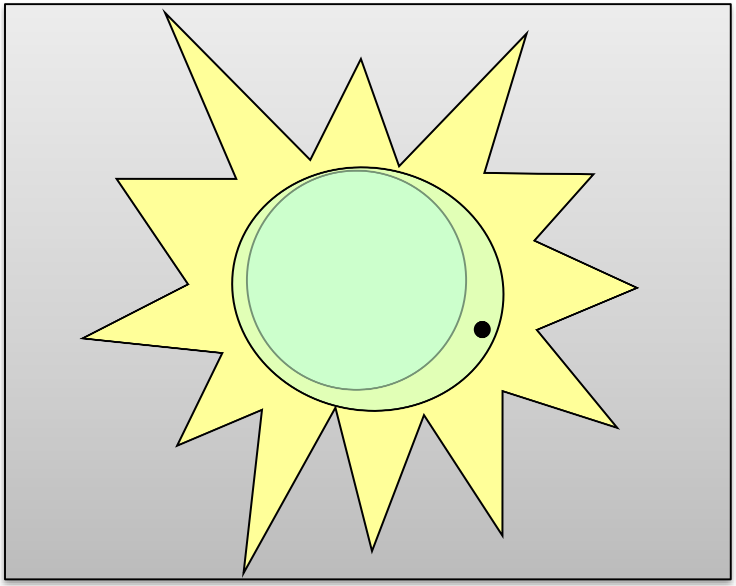
no type errors

typable

slack

# Fighting slack

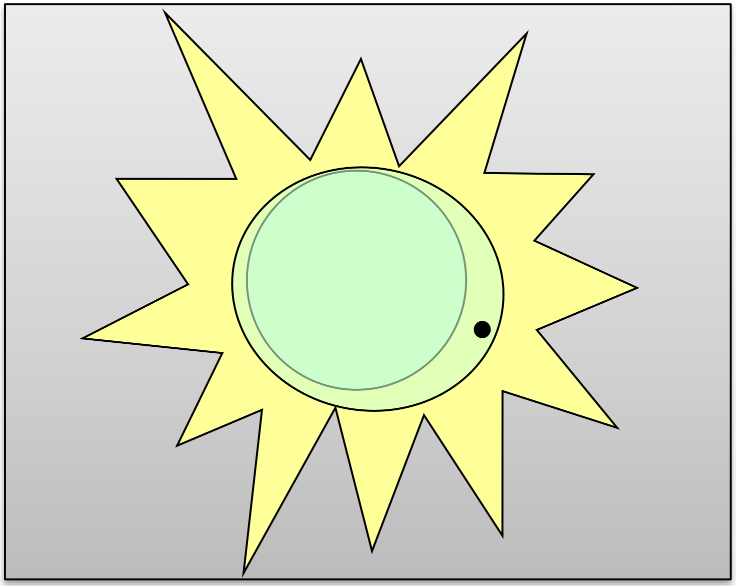
- Make the type checker a bit more clever:



- An eternal struggle

# Fighting slack

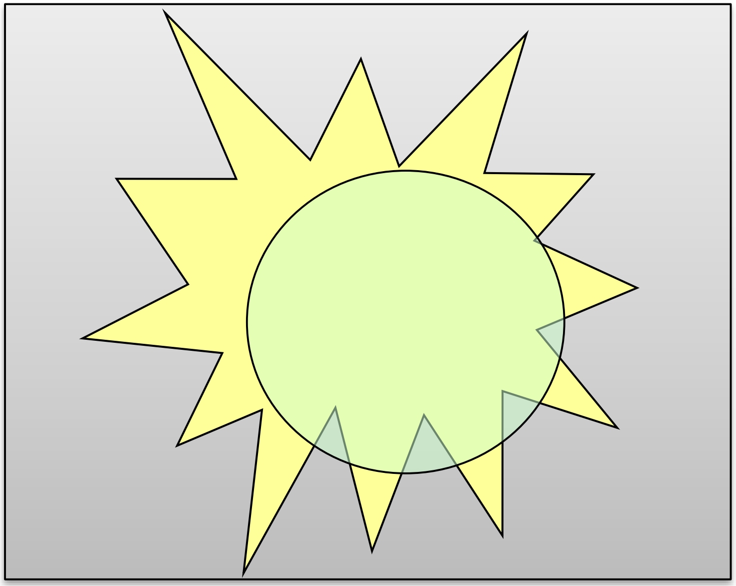
- Make the type checker a bit more clever:



- An eternal struggle
- And a great source of publications

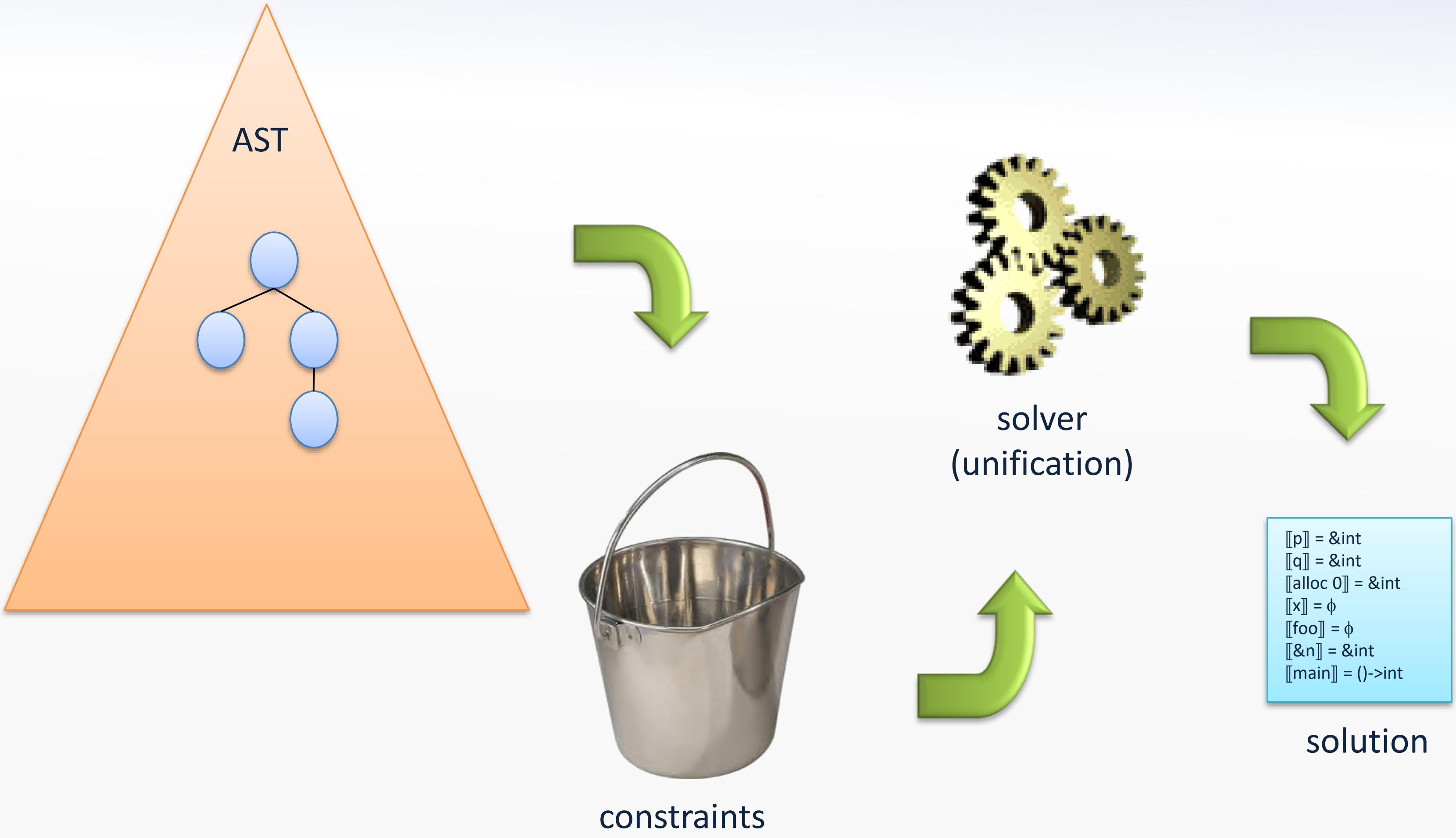
# Be careful out there

- The type checker may be unsound:



- Example: covariant arrays in Java
    - a deliberate pragmatic choice

# Generating and solving constraints



⟦p⟧ = &int
⟦q⟧ = &int
⟦alloc 0⟧ = &int
⟦x⟧ = ϕ
⟦foo⟧ = ϕ
⟦&n⟧ = &int
⟦main⟧ = ()->int

# Types

- Types describe the possible values:

```
τ → int
  | &τ
  | ( τ, …, τ ) -> τ
  | { X:τ, …, X:τ }
```

- These describe integers, pointers, functions, and records

- Types are *terms* generated by this grammar
  - example: `(int,&int) -> &&int`

# Type constraints

- We generate type constraints from an AST:
  - all constraints are equalities
  - they can be solved using a unification algorithm

- Type variables:
  - for each identifier declaration *X* we have the variable $[\![X]\!]$
  - for each non-identifier expression *E* we have the variable $[\![E]\!]$

- Recall that all identifiers are unique
- The expression *E* denotes an AST node, not syntax

- (Possible extensions: polymorphism, subtyping, …)

# Generating constraints (1/3)

| | |
|---|---|
| $I$: | $\llbracket I \rrbracket = \texttt{int}$ |
| $E_1\ op\ E_2$: | $\llbracket E_1 \rrbracket = \llbracket E_2 \rrbracket = \llbracket E_1\ op\ E_2 \rrbracket = \texttt{int}$ |
| $E_1$ == $E_2$: | $\llbracket E_1 \rrbracket = \llbracket E_2 \rrbracket \wedge \llbracket E_1 \texttt{==} E_2 \rrbracket = \texttt{int}$ |
| `input`: | $\llbracket \texttt{input} \rrbracket = \texttt{int}$ |
| $X = E$: | $\llbracket X \rrbracket = \llbracket E \rrbracket$ |
| `output` $E$: | $\llbracket E \rrbracket = \texttt{int}$ |
| `if` ($E$) {$S$}: | $\llbracket E \rrbracket = \texttt{int}$ |
| `if` ($E$) {$S_1$} `else` {$S_2$}: | $\llbracket E \rrbracket = \texttt{int}$ |
| `while` ($E$) {$S$}: | $\llbracket E \rrbracket = \texttt{int}$ |

# Generating constraints (2/3)

| | | |
|---|---|---|
| $X(X_1,\dots,X_n)\{\ \dots\ \texttt{return}\ E;\ \}$: | | |
| | $\llbracket X \rrbracket = (\llbracket X_1 \rrbracket, \dots, \llbracket X_n \rrbracket)\ \texttt{->}\ \llbracket E \rrbracket$ | |
| $E(E_1, \dots, E_n)$: | | |
| | $\llbracket E \rrbracket = (\llbracket E_1 \rrbracket, \dots, \llbracket E_n \rrbracket)\ \texttt{->}\ \llbracket E(E_1, \dots, E_n) \rrbracket$ | |
| $\texttt{alloc}\ E$: | $\llbracket \texttt{alloc}\ E \rrbracket = \& \llbracket E \rrbracket$ | |
| $\& X$: | $\llbracket \& X \rrbracket = \& \llbracket X \rrbracket$ | |
| $\texttt{null}$: | $\llbracket \texttt{null} \rrbracket = \& \alpha$ | (each $\alpha$ is a fresh type variable) |
| $*E$: | $\llbracket E \rrbracket = \& \llbracket *E \rrbracket$ | |
| $*X = E$: | $\llbracket X \rrbracket = \& \llbracket E \rrbracket$ | |

# Generating constraints (3/3)

~~$\{X_1 : E_1 , \ldots , X_n : E_n\}$:~~

~~$\llbracket \{X_1 : E_1 , \ldots , X_n : E_n\} \rrbracket = \{ X_1 : \llbracket E_1 \rrbracket , \ldots , X_n : \llbracket E_n \rrbracket \}$~~

~~$E . X$: $\llbracket E \rrbracket = \{ \ldots , X : \llbracket E . X \rrbracket , \ldots \}$~~

This is the idea, but not directly expressible in our language of types

# Generating constraints (3/3)

Let $\{f_1, f_2, \ldots, f_m\}$ be the set of field names that appear in the program

$\{X_1 : E_1,\ \ldots,\ X_n : E_n\}$: $[\![\{X_1 : E_1,\ \ldots,\ X_n : E_n\}]\!] = \{f_1 : \gamma_1,\ \ldots,\ f_m : \gamma_m\}$

$$\text{where } \gamma_i = \begin{cases} [\![E_1]\!] \text{ if } f_i = X_j \text{ for some } j \\ \alpha_i \quad \text{otherwise} \end{cases}$$

$E\,.\,X$: $[\![E]\!] = \{f_1 : \gamma_1,\ \ldots,\ f_m : \gamma_m\}$

$$\text{where } \gamma_i = \begin{cases} [\![E\,.\,X]\!] \text{ if } f_i = X \\ \alpha_i \quad \text{otherwise} \end{cases}$$

# Exercise

```
main() {
   var x, y, z;
   x = input;
   y = alloc 8;
   *y = x;
   z = *y;
   return x;
}
```

- Generate and solve the constraints
- Then try with `y = alloc 8` replaced by `y = 42`

# General terms

Constructor symbols:

- 0-ary: a, b, c
- 1-ary: d, e
- 2-ary: f, g, h
- 3-ary: i, j, k

Ex: int

Ex: &τ

Ex: $(\tau_1,\tau_2)$->$\tau_3$

Terms:

- a
- d(a)
- h(a,g(d(a),b))

Terms with variables:

- f(X,b)
- h(X,g(Y,Z))

# The unification problem

- An equality between two terms with variables:

  k(X,b,Y) = k(f(Y,Z),Z,d(Z))

- A solution (a unifier) is an assignment from variables to closed terms that makes both sides equal:

  X = f(d(b),b)

  Y = d(b)

  Z = b

Implicit constraint for term equality:
$c(t_1,\ldots,t_k) = c(t_1',\ldots,t_k') \Rightarrow t_i = t_i'$ for all *i*

# Unification errors

- Constructor error:

  d(X) = e(X)

- Arity error:

  a = a(X)

# The linear unification algorithm

- Paterson and Wegman (1978)
- In time O(*n*):
  - finds a most general unifier
  - or decides that none exists

- Can be used as a back-end for type checking

- ... but only for finite terms

# Recursive data structures

The program

```
var p;
p = alloc null;
*p = p;
```

creates these constraints

$\llbracket \texttt{null} \rrbracket = \&\alpha$

$\llbracket \texttt{alloc null} \rrbracket = \&\llbracket \texttt{null} \rrbracket$

$\llbracket \texttt{p} \rrbracket = \&\llbracket \texttt{alloc null} \rrbracket$

$\llbracket \texttt{p} \rrbracket = \&\llbracket \texttt{p} \rrbracket$

which have this “recursive solution” for p:

$\llbracket \mathrm{p} \rrbracket = \alpha$ where $\alpha = \&\alpha$

# Regular terms

- Infinite but (eventually) repeating:
  - e(e(e(e(e(e(…))))))
  - d(a,d(a,d(a, …)))
  - f(f(f(f(…),f(…)),f(f(…),f(…))),f(f(f(…),f(…)),f(f(…),f(…))))

- Only finitely many *different* subtrees

- A non-regular term:
  - f(a,f(d(a),f(d(d(a)),f(d(d(d(a))),…))))

# Regular unification

- Huet (1976)
- The unification problem for regular terms can be solved in O($n \cdot A(n)$) using a union-find algorithm

- $A(n)$ is the inverse Ackermann function:
  - smallest $k$ such that $n \leq$ Ack($k$,$k$)
  - this is never bigger than 5 for any real value of $n$

- See the TIP implementation...

# Union-Find

```
makeset(x) {
    x.parent := x
    x.rank := 0
}
```

```
find(x) {
    if x.parent != x
        x.parent := find(x.parent)
    return x.parent
}
```

```
union(x, y) {
    xr := find(x)
    yr := find(y)
    if xr = yr
        return
    if xr.rank < yr.rank
        xr.parent := yr
    else
        yr.parent := xr
        if xr.rank > yr.rank
            xr.rank := xr.rank + 1
}
```

# Union-Find (simplified)

```
makeset(x) {
    x.parent := x
}
```

```
find(x) {
    if x.parent != x
        x.parent := find(x.parent)
    return x.parent
}
```

```
union(x, y) {
    xr := find(x)
    yr := find(y)
    if xr = yr
        return
    xr.parent := yr
}
```

Implement ‘unify’ procedure using union and find to unify terms…

# Implementation strategy

- Representation of the different kinds of types (plus type variables)
- Map from AST nodes to types
- Union-Find
- Traverse AST, generate constraints, unify on the fly
  - report type error if unification fails
  - when unifying a type variable with e.g. a function type, it is useful to pick the function type as representative
  - for outputting solution, assign names to type variables (that are roots), and be careful about recursive types

# The complicated function

```
foo(p,x) {
  var f,q;
  if (*p==0) {
    f=1;
  } else {
    q = alloc 0;
    *q = (*p)-1;
    f=(*p)*(x(q,x));
  }
  return f;
}
```

```
main() {
  var n;
  n = input;
  return foo(&n,foo);
}
```

# Generated constraints

```
⟦foo⟧ = (⟦p⟧,⟦x⟧)->⟦f⟧
⟦*p⟧ = int
⟦1⟧ = int
⟦p⟧ = &⟦*p⟧
⟦alloc 0⟧ = &⟦0⟧
⟦q⟧ = &⟦*q⟧
⟦f⟧ = ⟦(*p)*(x(q,x))⟧
⟦x(q,x)⟧ = int
⟦input⟧ = int
⟦n⟧ = ⟦input⟧
⟦foo⟧ =(⟦&n⟧,⟦foo⟧)->⟦foo(&n,foo)⟧
```

```
⟦*p==0⟧ = int
⟦f⟧ = ⟦1⟧
⟦0⟧ = int
⟦q⟧ = ⟦alloc 0⟧
⟦q⟧ = &⟦(*p)-1⟧
⟦*p⟧ = int
⟦(*p)*(x(q,x))⟧ = int
⟦x⟧ = (⟦q⟧,⟦x⟧)->⟦x(q,x)⟧
⟦main⟧ = ()->⟦foo(&n,foo)⟧
⟦&n⟧ = &⟦n⟧
⟦(*p)-1⟧ = int
⟦*p⟧ = ⟦0⟧
```

# Solutions

$[\![\texttt{p}]\!] = \texttt{\&int}$

$[\![\texttt{q}]\!] = \texttt{\&int}$

$[\![\texttt{alloc 0}]\!] = \texttt{\&int}$

$[\![\texttt{x}]\!] = \phi$

$[\![\texttt{foo}]\!] = \phi$

$[\![\texttt{\&n}]\!] = \texttt{\&int}$

$[\![\texttt{main}]\!] = \texttt{()->int}$

NO TYPE ERRORS

Here, ϕ is the regular type that is the unfolding of

$$\phi = (\texttt{\&int},\phi)\texttt{->int}$$

which can also be written $\phi = \mu\,\alpha.(\texttt{\&int},\alpha)\texttt{->int}$

All other variables are assigned `int`

# Infinitely many solutions

The function

```
poly(x) {
  return *x;
}
```

has type (&α)->α for any type α

(which is not expressible in our current type language)

# Recursive and polymorphic types

- Extra notation for recursive and polymorphic types:

$$
\begin{aligned}
\tau \rightarrow\ & \ldots \\
\mid\ & \mu\, \alpha.\ \tau \\
\mid\ & \alpha
\end{aligned}
$$

(not very useful unless we also add
polymorphic expansion at calls,
but that makes complexity exponential,
or even undecidable…)

- Types are (finite) terms generated by this grammar
- $\mu\, \alpha.\ \tau$ is the (potentially recursive) type $\tau$ where occurrences of $\alpha$ represent $\tau$ itself
- $\alpha$ is a type variable (implicitly universally quantified if not bound by an enclosing $\mu$)

# Slack

```
bar(g,x) {
  var r;
  if (x==0) {
    r=g;
  } else {
    r=bar(2,0);
  }
  return r+1;
}

main() {
   return bar(null,1)
}
```

This never causes a type error – but is not typable:

$$\mathtt{int} = \llbracket \mathtt{r} \rrbracket = \llbracket \mathtt{g} \rrbracket = \&\alpha$$

# Other errors

- Not all errors are type errors:
  - dereference of `null` pointers
  - reading of uninitialized variables
  - division by zero
  - escaping stack cells

  (why not?)

```
baz()  {
  var x;
  return &x;
}

main() {
  var p;
  p=baz();
  *p=1;
  return *p;
}
```

- Other kinds of static analysis may catch these