

# Homework 5

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## Problem 1

a

*# The probability of not finding a defective bulb in the first 5 trials and finding one on the 6th trial:*

```
#P(defective on 6th trial) = (1 - p)^5
#P(defective on 6th trial) = (1 - 1/25)^5 * 1/25
#P(defective on 6th trial) = (24/25)^5 * 1/25
#P(defective on 6th trial) = 0.018
```

b

```
#P(at least 4 trials) = 1 - (1 - p)^3
#P(at least 4 trials) = 1 - (1 - 1/25)^3 = 0.007
```

c

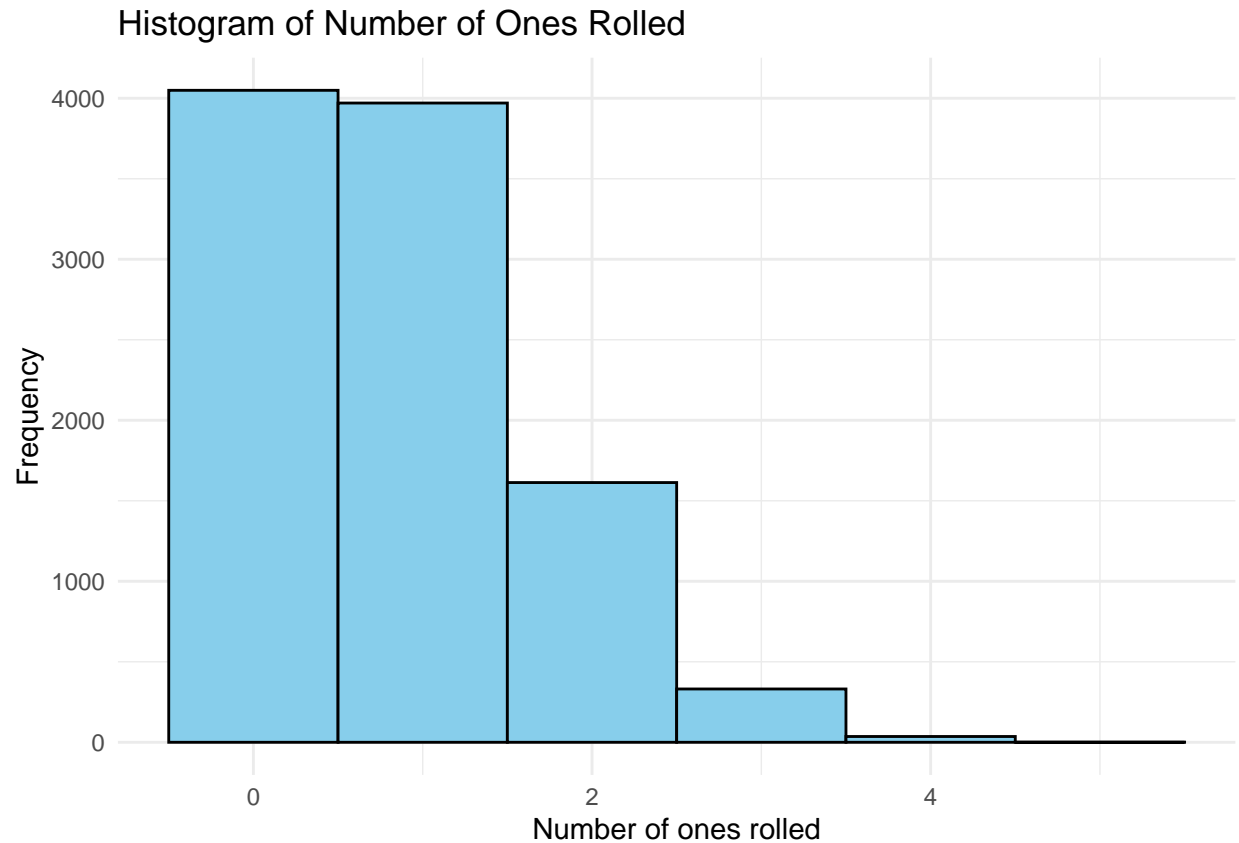
```
#P(at most 10 trials) = 1 - (1 - p)^10
#P(at most 10 trials) = 1 - (1 - 1/25)^10 = 0.118
```

## Problem 2

```
# Set seed for reproducibility
set.seed(20220707)

# Simulate rolling 5 fair six-sided dice 10,000 times
rolls <- replicate(10000, sum(sample(1:6, 5, replace = TRUE) == 1))

# Plot histogram
library(ggplot2)
ggplot(data = data.frame(rolls), aes(x = rolls)) +
  geom_histogram(binwidth = 1, fill = "skyblue", color = "black") +
  labs(x = "Number of ones rolled", y = "Frequency", title = "Histogram of Number of Ones Rolled") +
  theme_minimal()
```



```
# Calculate sample mean and sample variance
sample_mean <- mean(rolls)
sample_variance <- var(rolls)

# Print sample mean and sample variance rounded to 3 decimal places
print(paste("Sample mean:", round(sample_mean, 3)))
```

```
## [1] "Sample mean: 0.834"
```

```
print(paste("Sample variance:", round(sample_variance, 3)))
```

```
## [1] "Sample variance: 0.705"
```

### Problem 3

```
# Average rate of cars passing per hour
lambda <- 180

# Convert average rate to rate per minute
lambda_minute <- lambda / 60

# Calculate the probability of congestion (more than 5 cars passing in any one minute) using the Poisson
probability_congestion <- 1 - ppois(5, lambda_minute)
```

```

# Round the probability to 3 decimal places
probability_congestion <- round(probability_congestion, 3)

# Print the probability
print(paste("Probability of congestion:", probability_congestion))

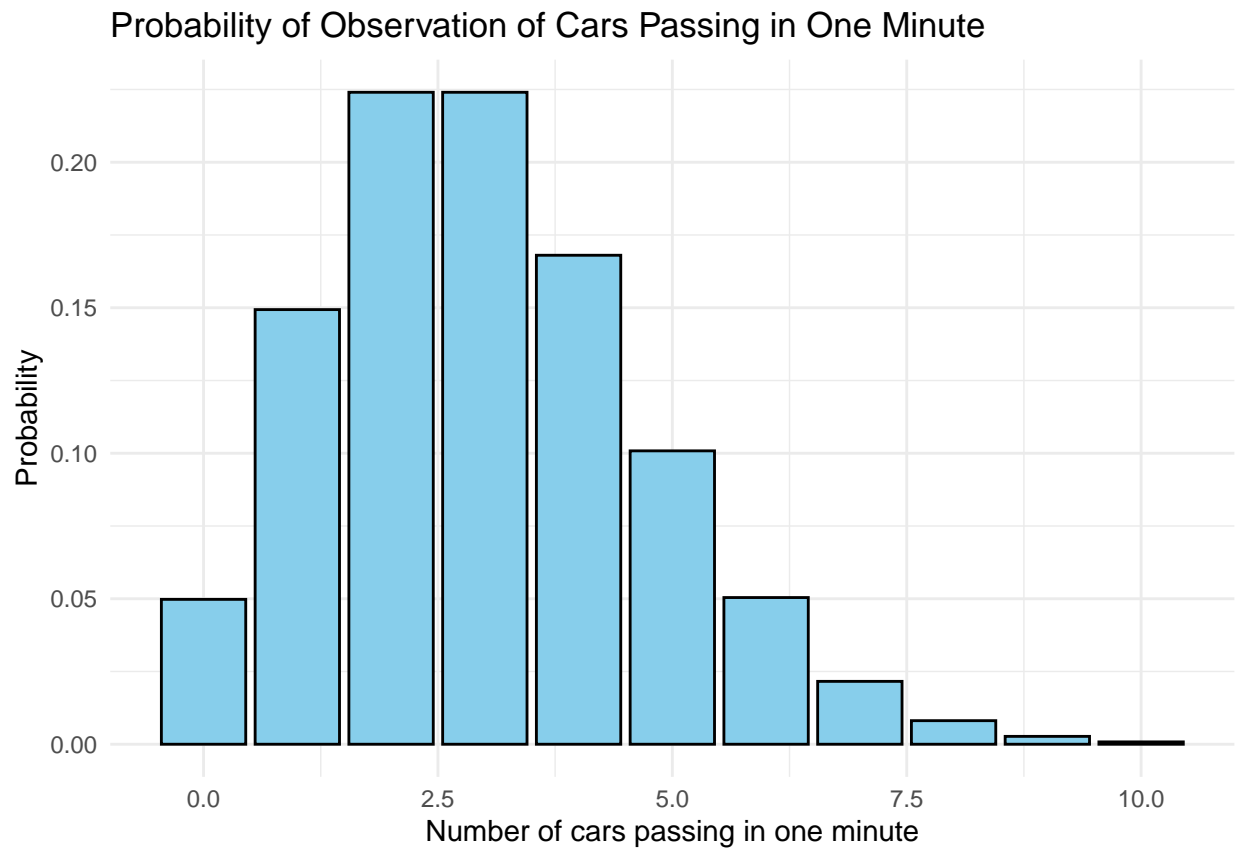
## [1] "Probability of congestion: 0.084"

# Generate x values (number of cars passing in any one minute)
x <- 0:10

# Calculate the corresponding probabilities using the Poisson distribution
probabilities <- dpois(x, lambda_minute)

# Plot bar chart
library(ggplot2)
ggplot(data = data.frame(x = x, probabilities = probabilities), aes(x = x, y = probabilities)) +
  geom_bar(stat = "identity", fill = "skyblue", color = "black") +
  labs(x = "Number of cars passing in one minute", y = "Probability", title = "Probability of Observation of Cars Passing in One Minute") +
  theme_minimal()

```



Problem 4

```
#Probability of scoring 585 or less:  $Z = (585 - 500)/100 = 0.85$   
#  $Z = 0.85$  corresponds to 0.802
```

```
#ZLower q =  $qnorm(0.25, 0, 1) = -0.675$   
#ZMedium =  $qnorm(0.50, 0, 1) = 0$   
#ZUpper =  $qnorm(0.75, 0, 1) = 0.675$ 
```

#### Problem 5

```
#P(Red)=0.05 (probability of the cab being red)  
#P(Green)=0.95 (probability of the cab being green)  
#P(Red/Red)=0.8 (probability of the witness saying the cab was red if it was red)  
#P(Green/Green)=0.8 (probability of the witness saying the cab was green if green)  
  
# $P(Red/Red) = (P(Red/Red) * P(Red)) / P(Red) = 0.8$ 
```