AIML Assignment

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Question

Find the length of diameter of the circle which touches the X-axis at the point $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and passes through the point $B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Solution

As the circle touches X-axis at $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

X-axis is tangent for the circle

Equation of tangent at A:

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0$$

This gives equation of normal at A as:

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1$$

Equation of a circle :

$$X^T X - 2C^T X = R^2 - C^T C$$

where X is a vector, C represents center and R is radius of circle

Given that above circle passes through $A = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Substituting A and B in equation of circle,

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2C^T \begin{pmatrix} 2 \\ 3 \end{pmatrix} = R^2 - C^T C$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - 2C^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = R^2 - C^T C$$

On Simplification, we get

$$13 - 2C^T \begin{pmatrix} 2 \\ 3 \end{pmatrix} = R^2 - C^T C$$

$$1 - 2C^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = R^2 - C^T C$$

Subtracting the above eqautions, we get

$$12 - 2C^T \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$$

$$C^T \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 6$$

$$\begin{pmatrix} 1 & 3 \end{pmatrix} C = 6 ....(1)$$

A normal passes through centre of a circle
Hence C passes through equation

$$\begin{pmatrix} 1 & 0 \end{pmatrix} C = 1....(2)$$

From (1) and (2),

$$C = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 \\ 5/3 \end{pmatrix}$$

Radius of circle :
$$R = ||C - A||$$
$$R = ||\begin{pmatrix} 1 \\ 5/3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}||$$
$$R = ||\begin{pmatrix} 0 \\ 5/3 \end{pmatrix}||$$
which gives R = 5/3

Diameter d = 2R

Thus, diameter of required circle is 10/3

Figure