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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

1.0.1. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is:

Solution: The Impulse response of the system is given by:

$$h(t) = tu(t) \quad (1.0.1.1)$$

Input:

$$x(t) = u(t-1) \quad (1.0.1.2)$$

The output will be given by:

$$y(t) = x(t) * h(t) \quad (1.0.1.3)$$

1.0.2. We find the output using Laplace Transformation.

In Laplace domain Convolution becomes Multiplication.

$$F(y(t)) = Fx(t)Fh(t) \quad (1.0.2.1)$$

$$\Rightarrow Y(s) = \frac{e^{-s}}{s} \frac{1}{s^2} \quad (1.0.2.2)$$

$$\Rightarrow Y(s) = \frac{e^{-s}}{s^3} \quad (1.0.2.3)$$

1.0.3. Since we want the output in time domain we find the inverse Laplace Transform using Contour Integration.

The Inverse Laplace Transform :

$$y(t) = \frac{1}{2i\pi} \oint_C Y(s) e^{st} ds \quad (1.0.3.1)$$

$$\Rightarrow y(t) = \frac{1}{2i\pi} 2i\pi \sum \text{Res} Y(s) e^{st} \quad (1.0.3.2)$$

1.0.4. Calculating the Residue.

$$\text{Res}_{s=s_o} = \frac{1}{(p-1)!} \lim_{s \rightarrow s_o} \frac{d^{p-1}}{dx^{p-1}} [(s-s_o)^p f(s)] \quad (1.0.4.1)$$

where p denotes the number of times the pole is repeated.

$$\text{Res} Y(s) e^{st} = \frac{1}{(3-1)!} \lim_{s \rightarrow 0} \frac{d^{3-1}}{dx^{3-1}} (s-0)^3 \frac{e^{s(t-1)}}{s^3} \quad (1.0.4.2)$$

$$\Rightarrow \text{Res} Y(s) e^{st} = \frac{1}{2!} \lim_{s \rightarrow 0} \frac{d^2}{dx^2} e^{s(t-1)} \quad (1.0.4.3)$$

$$\Rightarrow \text{Res} Y(s) e^{st} = \frac{1}{2} (t-1)^2 \quad (1.0.4.4)$$

$$\text{Hence, } y(t) = \frac{1}{2} (t-1)^2 \quad (1.0.4.5)$$

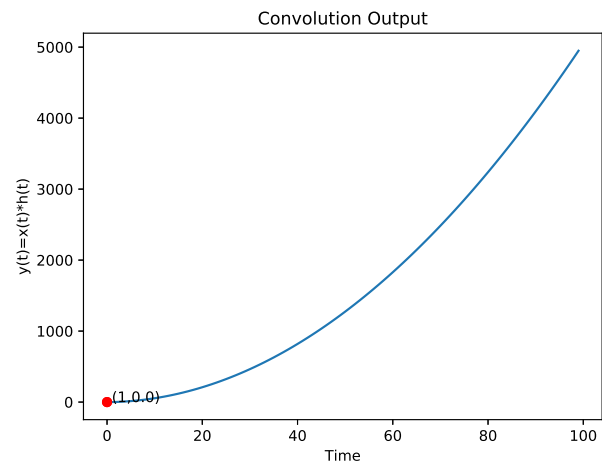


Fig. 1.0.4

The following code verifies the answer by performing convolution of the input and $h(t)$ to obtain $y(t)$ and plots the result .
codes/ee18btech11048.py

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT