

# Control Systems

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**Abstract**—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

## 1 POLAR PLOT

### 1.1 Introduction

## 2 BODE PLOT

### 2.1 Gain and Phase Margin

2.1. An aircraft roll control system can be represented by a block diagram shown in 2.1 with

$$G(s) = \frac{10K}{s(s+1)(s+5)} \quad (2.1.1)$$

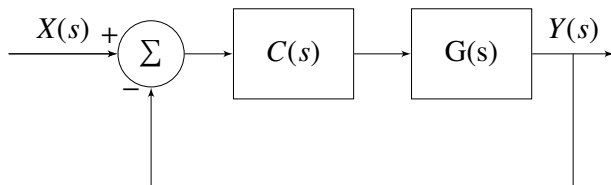


Fig. 2.1

Design a lag compensator  $C(s)$  for a  $60^\circ$  phase margin and an appropriate error constant of 5.

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**Solution:** For unity feedback we have Velocity error constant ( $K_v$ )

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad (2.1.2)$$

$$\lim_{s \rightarrow 0} \left( \frac{10K}{(s+1)(s+5)} \right) = 5 \quad (2.1.3)$$

$$\Rightarrow K = 2.5 \quad (2.1.4)$$

From Fig.2.2

Phase Margin =  $3.94^\circ$

Gain Crossover Frequency = 2.04 rad/s

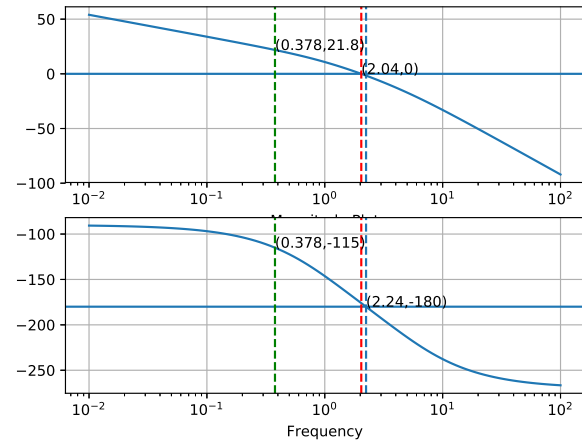


Fig. 2.2: Graph 1

2.2. Finding Lag Compensator ( $C(s)$ ) to yield a Phase margin of  $60^\circ$

**Solution:**

$$C(s) = \frac{1}{\beta} \frac{\left(s + \frac{1}{\tau}\right)}{\left(s + \frac{1}{\tau\beta}\right)} \quad (2.2.1)$$

Phase Margin required =  $60^\circ$

$$\angle C(j\omega) = -180^\circ + 60^\circ + 5^\circ \quad (2.2.2)$$

$$\angle C(j\omega) = -115^\circ \quad (2.2.3)$$

For phase lag network  $5^\circ$  is added.

To obtain graph in Fig.2.2 use the following code:

```
codes/ee18btech11048_1.py
```

code:

```
codes/ee18btech11048_2.py
```

From Fig.2.2

$$\angle C(j\omega) = -115^\circ \quad (2.2.4)$$

$$\Rightarrow \omega = 0.37 \text{ rad/s} \quad (2.2.5)$$

$$\Rightarrow \text{Gain} = 21.8 \text{ dB} \quad (2.2.6)$$

$$(2.2.7)$$

Calculating  $\beta$  :

$$-20 \log \frac{1}{\beta} = 21.8 \quad (2.2.8)$$

$$\Rightarrow \beta = 12.3 \quad (2.2.9)$$

Calculating  $\frac{1}{\tau}$  :

$$\frac{1}{\tau} = \frac{0.37}{10} \quad (2.2.10)$$

$$\Rightarrow \frac{1}{\tau} = 0.037 \quad (2.2.11)$$

$$C(s) = \frac{s + 0.037}{12.3s + 0.037} \quad (2.2.12)$$

2.3. Plotting the overall graph for  $C(s)G(s)$ .

Refer Fig 2.3

$$C(s)G(s) = \frac{s + 0.037}{12.3s + 0.037} \frac{25}{s(s+1)(s+5)} \quad (2.3.1)$$

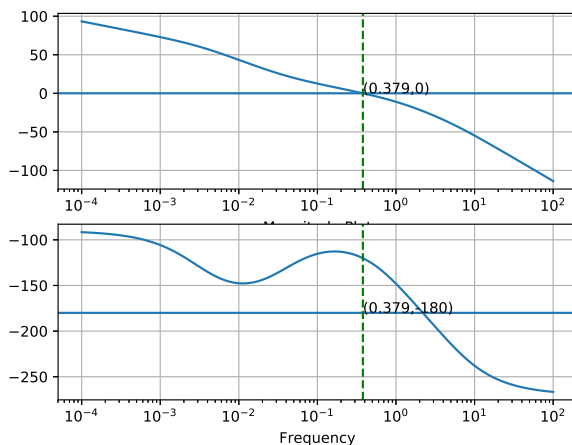


Fig. 2.3: Graph 2

Phase Margin =  $60^\circ$

To obtain graph in Fig.2.3 use the following