

# Control Systems

G V V Sharma\*

## CONTENTS

|           |   |   |
|-----------|---|---|
| <b>1</b>  | <b>Signal Flow Graph</b>                    | 1 |
| 1.1       | Mason's Gain Formula . . .                  | 1 |
| 1.2       | Matrix Formula . . . . .                    | 1 |
| <b>2</b>  | <b>Bode Plot</b>                            | 1 |
| 2.1       | Introduction . . . . .                      | 1 |
| 2.2       | Phase . . . . .                             | 1 |
| <b>3</b>  | <b>Second order System</b>                  | 1 |
| 3.1       | Damping . . . . .                           | 1 |
| 3.2       | Peak Overshoot . . . . .                    | 1 |
| 3.3       | Settling Time . . . . .                     | 1 |
| <b>4</b>  | <b>Routh Hurwitz Criterion</b>              | 1 |
| 4.1       | Routh Array . . . . .                       | 1 |
| 4.2       | Marginal Stability . . . . .                | 1 |
| 4.3       | Stability . . . . .                         | 1 |
| <b>5</b>  | <b>State-Space Model</b>                    | 1 |
| 5.1       | Controllability and Observability . . . . . | 1 |
| 5.2       | Second Order System . . . .                 | 1 |
| <b>6</b>  | <b>Nyquist Plot</b>                         | 1 |
| 6.1       | Introduction . . . . .                      | 1 |
| <b>7</b>  | <b>Compensators</b>                         | 1 |
| 7.1       | Phase Lead . . . . .                        | 1 |
| 7.2       | Lag Lead . . . . .                          | 1 |
| <b>8</b>  | <b>Gain Margin</b>                          | 1 |
| 8.1       | Introduction . . . . .                      | 1 |
| 8.2       | Example . . . . .                           | 1 |
| <b>9</b>  | <b>Phase Margin</b>                         | 2 |
| 9.1       | Intoduction . . . . .                       | 2 |
| <b>10</b> | <b>Oscillator</b>                           | 2 |
| 10.1      | Introduction . . . . .                      | 2 |

## 11 Root Locus

2

*Abstract*—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

### 1 SIGNAL FLOW GRAPH

#### 1.1 Mason's Gain Formula

#### 1.2 Matrix Formula

### 2 BODE PLOT

#### 2.1 Introduction

#### 2.2 Phase

### 3 SECOND ORDER SYSTEM

#### 3.1 Damping

#### 3.2 Peak Overshoot

#### 3.3 Settling Time

### 4 ROUTH HURWITZ CRITERION

#### 4.1 Routh Array

#### 4.2 Marginal Stability

#### 4.3 Stability

### 5 STATE-SPACE MODEL

#### 5.1 Controllability and Observability

#### 5.2 Second Order System

### 6 NYQUIST PLOT

#### 6.1 Introduction

### 7 COMPENSATORS

#### 7.1 Phase Lead

#### 7.2 Lag Lead

### 8 GAIN MARGIN

#### 8.1 Introduction

#### 8.2 Example

8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

margin and the phase margin.

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.1)$$

**Solution:** The system is defined as follows:

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.2)$$

For the given system

poles = 0 , -2 , -100

zeros = none

The magnitude and phase plot are as follows:

Fig 8.1

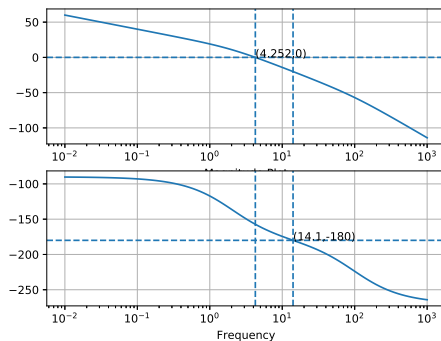


Fig. 8.1: a

The python code to obtain the graphs:

codes/ee18btech11048.py

## 8.2. Finding the Phase Margin

$$G(j\omega) = \frac{10}{j\omega(1 + 0.5j\omega)(1 + .01j\omega)} \quad (8.2.1)$$

*Phase Margin* is  $\angle G(j\omega) + 180^\circ$  , where  $\omega$  is frequency when gain = 1 .

**Solution:**

$$\frac{100}{\omega \sqrt{(0.5\omega)^2 + 1} \sqrt{(0.01\omega)^2 + 1}} = 1 \quad (8.2.2)$$

Solving (8.2.2) or from Fig 8.1 frequency at which gain = 1 ,is gain crossover frequency  $\omega_{gc}$  or where the Magnitude plot value is zero.

$$\Rightarrow \omega_{gc} = 4.25 \quad (8.2.3)$$

$$\angle G(j\omega_{gc}) = -157.2 \quad (8.2.4)$$

$$\Rightarrow PM = 22.8 \quad (8.2.5)$$

## 8.3. Finding the Gain Margin

*Gain Margin* is  $0^\circ - G(j\omega)$  db , where  $\omega_{pc}$  is *phase crossover frequency*, frequency when phase =  $-180^\circ$

**Solution:**

$$\arctan(0) - \arctan\left(\frac{\omega}{0}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{100}\right) = -180^\circ \quad (8.3.1)$$

Solving (8.3.1) or from Fig 8.1  $\omega$  where the Phase plot value is  $-180^\circ$ .

$$\Rightarrow \omega = 14.1 \quad (8.3.2)$$

$$-G(j\omega)db = -20.2 \quad (8.3.3)$$

$$\Rightarrow GM = 20.2db \quad (8.3.4)$$

## 9 PHASE MARGIN

### 9.1 Intoduction

## 10 OSCILLATOR

### 10.1 Introduction

## 11 ROOT LOCUS