Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain margin and the phase margin.

$$G(s) = \frac{10}{s(1+0.5s)(1+.01s)} \tag{8.1.1}$$

Solution: The system is defined as follows:

$$G(s) = \frac{10}{s(1+0.5s)(1+.01s)} \tag{8.1.2}$$

Zeros	Poles
-	0
	-2
	-100

TABLE 8.1: Zeros and Poles

The magnitude and phase plot are as follows: Fig8.1

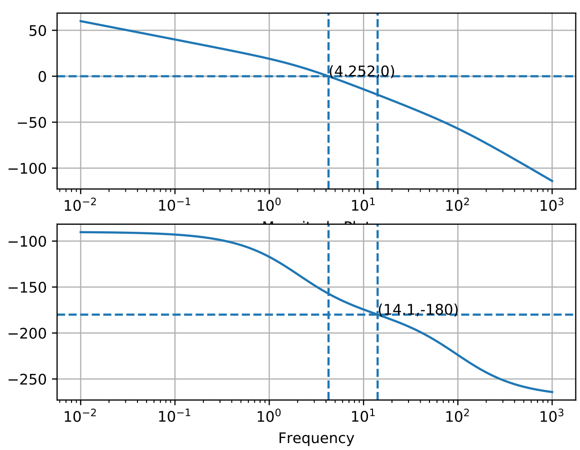

Fig. 8.1: Graphs

The python code to obtain the graphs:

```
codes/ee18btech11048.py
```

8.2. *Gain* and *Phase* of Transfer Function

$$G(j\omega) = \frac{10}{j\omega(1+0.5j\omega)(1+.01j\omega)} \tag{8.2.1}$$

Gain:

$$\frac{100}{\omega\sqrt{(0.5\omega)^2+1}\sqrt{(0.01\omega)^2+1}} \tag{8.2.2}$$

Phase:

$$\tan^{-1}(0) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) \tag{8.2.3}$$

8.3. Finding the Phase Margin(PM)

$$PM = \angle G(\jmath\omega_{gc}) + 180^\circ \tag{8.3.1}$$

$$\omega_{gc} = \text{Gain Crossover Frequency} \tag{8.3.2}$$

$$\text{At } \omega_{gc}\ |G(s)| = 1 \tag{8.3.3}$$

Solution:

$$\frac{100}{\omega_{gc}\sqrt{\left(0.5\omega_{gc}\right)^2+1}\sqrt{\left(0.01\omega_{gc}\right)^2+1}} = 1 \tag{8.3.4}$$

Solving Eq. (8.3.4) *or* from Fig 8.1 :

$$\implies \omega_{gc} = 4.25 \tag{8.3.5}$$

$$\angle G\left(\jmath\omega_{gc}\right) = -157.2 \tag{8.3.6}$$

$$\implies PM = 22.8 \tag{8.3.7}$$

8.4. Finding the Gain Margin (GM)

$$GM = 0 - G(\jmath\omega_{pc})db \tag{8.4.1}$$

$$\omega_{pc} = \text{Phase Crossover Frequency} \tag{8.4.2}$$

$$\text{At } \omega_{pc}\angle G(s) = -180^\circ \tag{8.4.3}$$

Solution:

$$\tan^{-1}(0) - \tan^{-1}\left(\frac{\omega_{pc}}{0}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{100}\right) = -180^\circ \tag{8.4.4}$$

Solving Eq. (8.4.4) *or* from Fig 8.1 :

$$\implies \omega_{pc} = 14.1 \tag{8.4.5}$$

$$-\text{G}(\jmath\omega_{pc})db = -20.2db \tag{8.4.6}$$

$$\implies GM = 20.2db \tag{8.4.7}$$

9 Phase Margin

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10 Oscillator

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