

Control Systems

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11 Root Locus

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.2 Phase

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Peak Overshoot

3.3 Settling Time

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

6 NYQUIST PLOT

6.1 Introduction

7 COMPENSATORS

7.1 Phase Lead

7.2 Lag Lead

8 GAIN MARGIN

8.1 Introduction

8.2 Example

8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain

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margin and the phase margin.

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.1)$$

Solution: The system is defined as follows:

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.2)$$

Zeros	Poles
-	0
	-2
	-100

TABLE 8.1: Zeros and Poles

The magnitude and phase plot are as follows:
Fig8.1

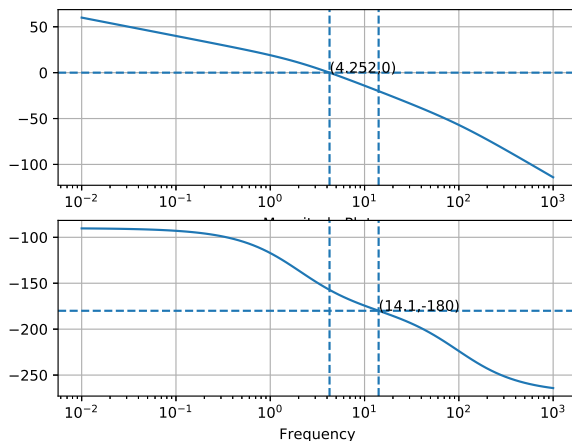


Fig. 8.1: Graphs

The python code to obtain the graphs:

codes/ee18btech11048.py

8.2. Gain and Phase of Transfer Function

$$G(j\omega) = \frac{10}{j\omega(1 + 0.5j\omega)(1 + .01j\omega)} \quad (8.2.1)$$

Gain:

$$\frac{100}{\omega \sqrt{(0.5\omega)^2 + 1} \sqrt{(0.01\omega)^2 + 1}} \quad (8.2.2)$$

Phase:

$$\tan^{-1}(0) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) \quad (8.2.3)$$

8.3. Finding the Phase Margin(PM)

$$PM = \angle G(j\omega_{gc}) + 180^\circ \quad (8.3.1)$$

$$\omega_{gc} = \text{Gain Crossover Frequency} \quad (8.3.2)$$

$$\text{At } \omega_{gc} |G(s)| = 1 \quad (8.3.3)$$

Solution:

$$\frac{100}{\omega_{gc} \sqrt{(0.5\omega_{gc})^2 + 1} \sqrt{(0.01\omega_{gc})^2 + 1}} = 1 \quad (8.3.4)$$

Solving Eq. (8.3.4) or from Fig 8.1 :

$$\Rightarrow \omega_{gc} = 4.25 \quad (8.3.5)$$

$$\angle G(j\omega_{gc}) = -157.2 \quad (8.3.6)$$

$$\Rightarrow PM = 22.8 \quad (8.3.7)$$

8.4. Finding the Gain Margin (GM)

$$GM = 0 - G(j\omega_{pc})db \quad (8.4.1)$$

$$\omega_{pc} = \text{Phase Crossover Frequency} \quad (8.4.2)$$

$$\text{At } \omega_{pc} \angle G(s) = -180^\circ \quad (8.4.3)$$

Solution:

$$\tan^{-1}(0) - \tan^{-1}\left(\frac{\omega_{pc}}{0}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{100}\right) = -180^\circ \quad (8.4.4)$$

Solving Eq. (8.4.4) or from Fig 8.1 :

$$\Rightarrow \omega_{pc} = 14.1 \quad (8.4.5)$$

$$-G(j\omega_{pc})db = -20.2db \quad (8.4.6)$$

$$\Rightarrow GM = 20.2db \quad (8.4.7)$$

9 PHASE MARGIN

9.1 Introduction

10 OSCILLATOR

10.1 Introduction

11 ROOT LOCUS