

# Control Systems

G V V Sharma\*

## CONTENTS

<b>1</b>	<b>Signal Flow Graph</b>	1
1.1	Mason's Gain Formula . . .	1
1.2	Matrix Formula . . . . .	1
<b>2</b>	<b>Bode Plot</b>	1
2.1	Introduction . . . . .	1
2.2	Phase . . . . .	1
<b>3</b>	<b>Second order System</b>	1
3.1	Damping . . . . .	1
3.2	Peak Overshoot . . . . .	1
3.3	Settling Time . . . . .	1
<b>4</b>	<b>Routh Hurwitz Criterion</b>	1
4.1	Routh Array . . . . .	1
4.2	Marginal Stability . . . . .	1
4.3	Stability . . . . .	1
<b>5</b>	<b>State-Space Model</b>	1
5.1	Controllability and Observability . . . . .	1
5.2	Second Order System . . . .	1
<b>6</b>	<b>Nyquist Plot</b>	1
6.1	Introduction . . . . .	1
<b>7</b>	<b>Compensators</b>	1
7.1	Phase Lead . . . . .	1
7.2	Lag Lead . . . . .	1
<b>8</b>	<b>Gain Margin</b>	1
8.1	Introduction . . . . .	1
8.2	Example . . . . .	1
<b>9</b>	<b>Phase Margin</b>	2
9.1	Intoduction . . . . .	2
<b>10</b>	<b>Oscillator</b>	2
10.1	Introduction . . . . .	2

## 11 Root Locus

2

*Abstract*—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

### 1 SIGNAL FLOW GRAPH

*1.1 Mason's Gain Formula*

*1.2 Matrix Formula*

### 2 BODE PLOT

*2.1 Introduction*

*2.2 Phase*

### 3 SECOND ORDER SYSTEM

*3.1 Damping*

*3.2 Peak Overshoot*

*3.3 Settling Time*

### 4 ROUTH HURWITZ CRITERION

*4.1 Routh Array*

*4.2 Marginal Stability*

*4.3 Stability*

### 5 STATE-SPACE MODEL

*5.1 Controllability and Observability*

*5.2 Second Order System*

### 6 NYQUIST PLOT

*6.1 Introduction*

### 7 COMPENSATORS

*7.1 Phase Lead*

*7.2 Lag Lead*

### 8 GAIN MARGIN

*8.1 Introduction*

*8.2 Example*

8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

margin and the phase margin.

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.1)$$

**Solution:** The system is defined as follows:

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.2)$$

Zeros	Poles
-	0
	-2
	-100

TABLE 8.1: Zeros and Poles

The magnitude and phase plot are as follows:  
Fig8.1

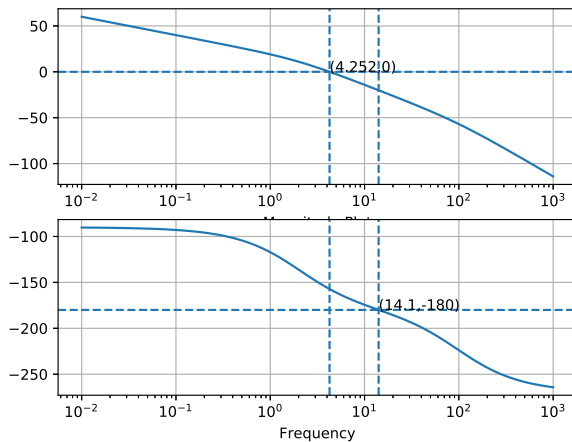


Fig. 8.1: Graphs

The python code to obtain the graphs:

```
codes/ee18btech11048.py
```

8.2. Finding the Phase Margin ( $PM$ ).

$$G(j\omega) = \frac{10}{j\omega(1 + 0.5j\omega)(1 + .01j\omega)} \quad (8.2.1)$$

$$PM = \angle G(j\omega_{gc}) + 180^\circ \quad (8.2.2)$$

where  $\omega_{gc}$  is frequency when gain = 1 .

**Solution:**

$$\frac{100}{\omega \sqrt{(0.5\omega)^2 + 1} \sqrt{(0.01\omega)^2 + 1}} = 1 \quad (8.2.3)$$

Solving Eq. (8.2.3) or from Fig 8.1 :

$$\Rightarrow \omega_{gc} = 4.25 \quad (8.2.4)$$

$$\angle G(j\omega_{gc}) = -157.2 \quad (8.2.5)$$

$$\Rightarrow PM = 22.8 \quad (8.2.6)$$

8.3. Finding the Gain Margin ( $GM$ )

$$GM = 0 - G(j\omega_{pc})db \quad (8.3.1)$$

where  $\omega_{pc}$  is frequency when phase =  $-180^\circ$

**Solution:**

$$\arctan(0) - \arctan\left(\frac{\omega}{0}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{100}\right) = -180^\circ \quad (8.3.2)$$

Solving Eq. (8.3.2) or from Fig 8.1 :

$$\Rightarrow \omega = 14.1 \quad (8.3.3)$$

$$-G(j\omega)db = -20.2 \quad (8.3.4)$$

$$\Rightarrow GM = 20.2db \quad (8.3.5)$$

## 9 PHASE MARGIN

### 9.1 Intoduction

## 10 OSCILLATOR

### 10.1 Introduction

## 11 ROOT LOCUS