

Control Systems

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11 Root Locus

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.2 Phase

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Peak Overshoot

3.3 Settling Time

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

6 NYQUIST PLOT

6.1 Introduction

7 COMPENSATORS

7.1 Phase Lead

7.2 Lag Lead

8 GAIN MARGIN

8.1 Introduction

8.2 Example

8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain

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margin and the phase margin.

$$G(s) = \frac{100}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.1)$$

Solution: The system is defined as follows:

$$G(s) = \frac{10}{s(1 + 0.5s)(1 + .01s)} \quad (8.1.2)$$

For the given system

poles = 0 , -2 , -100

zeros = none

The magnitude and phase plot are as follows:

Fig 8.1

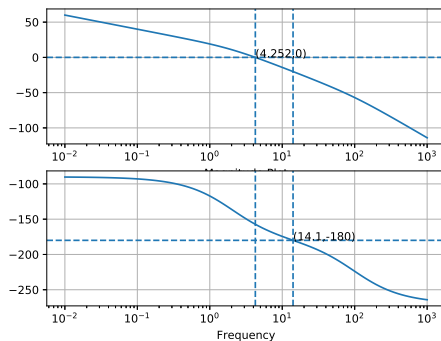


Fig. 8.1: a

The python code to obtain the graphs:

codes/ee18btech11048.py

8.2. Finding the Phase Margin

$$G(j\omega) = \frac{10}{j\omega(1 + 0.5j\omega)(1 + .01j\omega)} \quad (8.2.1)$$

Phase Margin is $\angle G(j\omega) + 180^\circ$, where ω is frequency when gain = 1 .

Solution:

$$\frac{100}{\omega \sqrt{(0.5\omega)^2 + 1} \sqrt{(0.01\omega)^2 + 1}} = 1 \quad (8.2.2)$$

Solving (8.2.2) or from Fig 8.1 frequency at which gain = 1 ,is gain crossover frequency ω_{gc} or where the Magnitude plot value is zero.

$$\Rightarrow \omega_{gc} = 4.25 \quad (8.2.3)$$

$$\angle G(j\omega_{gc}) = -157.2 \quad (8.2.4)$$

$$\Rightarrow PM = 22.8 \quad (8.2.5)$$

8.3. Finding the Gain Margin

Gain Margin is $0^\circ - G(j\omega)$ db , where ω_{pc} is *phase crossover frequency*, frequency when phase = -180°

Solution:

$$\arctan(0) - \arctan\left(\frac{\omega}{0}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{100}\right) = -180^\circ \quad (8.3.1)$$

Solving (8.3.1) or from Fig 8.1 ω where the Phase plot value is -180° .

$$\Rightarrow \omega = 14.1 \quad (8.3.2)$$

$$-G(j\omega)db = -20.2 \quad (8.3.3)$$

$$\Rightarrow GM = 20.2db \quad (8.3.4)$$

9 PHASE MARGIN

9.1 Intoduction

10 OSCILLATOR

10.1 Introduction

11 ROOT LOCUS