Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1. Sketch the Bode Magnitude and Phase plot for the following system. Also compute the gain margin and the phase margin.

$$G(s) = \frac{100}{s(1+0.5s)(1+.01s)} \tag{8.1.1}$$

Solution: The system is defined as follows:

$$G(s) = \frac{10}{s(1+0.5s)(1+.01s)} \tag{8.1.2}$$

For the given system
poles = 0 , -2 , -100
zeros = none
The magnitude and phase plot are as follows: Fig 8.1

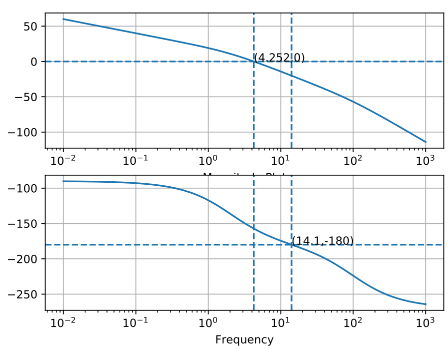

Fig. 8.1: a

The python code to obtain the graphs:

```
codes/ee18btech11048.py
```

8.2. Finding the Phase Margin

$$G(j\omega) = \frac{10}{j\omega(1+0.5j\omega)(1+.01j\omega)} \tag{8.2.1}$$

Phase Margin is $\angle G(\jmath\omega) + 180°$, where ω is frequency when gain = 1 .
Solution:

$$\frac{100}{\omega\sqrt{(0.5\omega)^2+1}\sqrt{(0.01\omega)^2+1}} = 1 \tag{8.2.2}$$

Solving (8.2.2) *or* from Fig 8.1 frequency at which gain = 1 ,is gain crossover frequency ω_{gc} or where the Magnitude plot value is *zero*.

$$\implies \omega_{gc} = 4.25 \tag{8.2.3}$$

$$\angle G\left(\jmath\omega_{gc}\right) = -157.2 \tag{8.2.4}$$

$$\implies PM = 22.8 \tag{8.2.5}$$

8.3. Finding the Gain Margin
Gain Margin is $0°-G(\jmath\omega)$ db , where $\omega_{pc}$is *phase crossover frequency*, frequency when phase = $-180°$
Solution:

$$\arctan(0) - \arctan\left(\frac{\omega}{0}\right) - \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{100}\right) = -180° \tag{8.3.1}$$

Solving (8.3.1) *or* from Fig 8.1 ω where the Phase plot value is *-180°*.

$$\implies \omega = 14.1 \tag{8.3.2}$$

$$-G(\jmath\omega)db = -20.2 \tag{8.3.3}$$

$$\implies GM = 20.2db \tag{8.3.4}$$

9 Phase Margin

9.1 Intoduction

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