Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. Figure 1.0.1 shows a feedback transconductance amplifier implemented using an op amp with open-loop gain μ , a very large input resistance, and an output resistance r_o. The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of the three resistances R_M, R_1, and R_2, and a proportional voltageV_f is fed back to the negative-input terminal of the op amp.

Find G,H and T. If the loop gain is large, find an approximate expression for T and state precisely the condition for which this applies.

Solution: The parameters given are shown in the TABLE.1.0.1:1 The equivalent circuit of

Parameter	Value
input resistance	∞
output resistance	r_o
Input voltage	V_s
Output Voltage	V_o

TABLE 1.0.1: 1

the amplifier is in fig.1.0.1:2

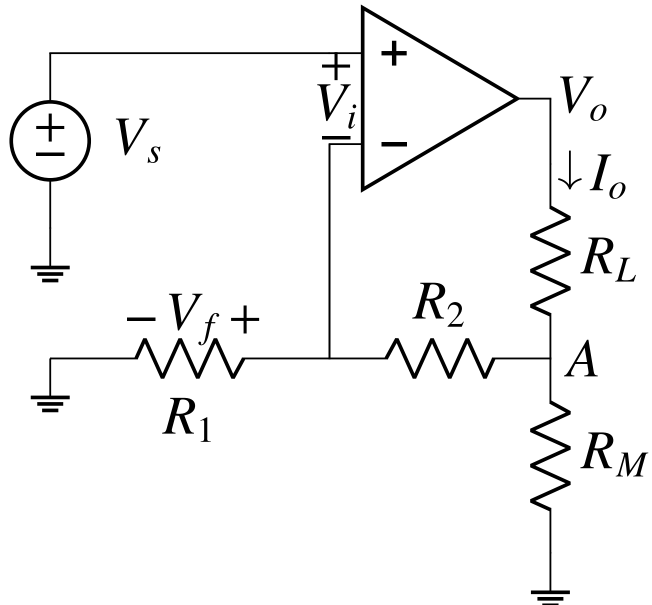

Fig. 1.0.1: 1 Original Circuit

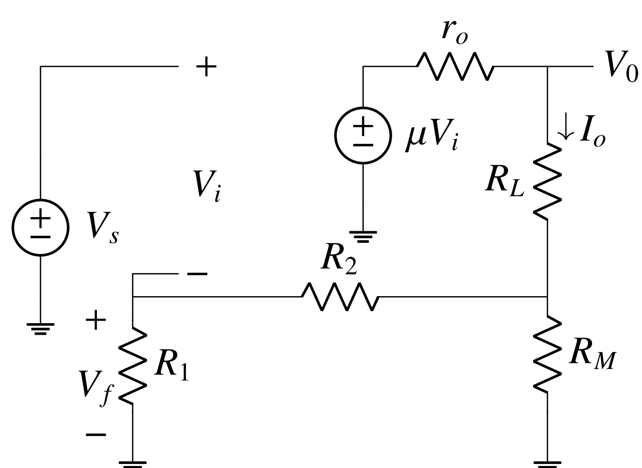

Fig. 1.0.1: 2 Equivalent Circuit

1.0.2. Calculating G
Solution:

$$G = \frac{I_o}{V_i} \tag{1.0.2.1}$$

$$\text{From fig 1.0.1:2} \tag{1.0.2.2}$$

$$\implies G = \mu \tag{1.0.2.3}$$

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1.0.3. Calculating H
Solution:

$$H = \frac{V_f}{I_o} \tag{1.0.3.1}$$

From fig 1.0.1:2

$$V_f = R_1 I_o \frac{R_M}{R_M + R_1 + R_2} \tag{1.0.3.2}$$

$$\implies H = \frac{R_1 R_M}{R_1 + R_2 + R_M} \tag{1.0.3.3}$$

1.0.4. Calculating T
Solution:

$$T = \frac{I_o}{V_s} \tag{1.0.4.1}$$

$$T = \frac{G}{1 + GH} \tag{1.0.4.2}$$

From fig 1.0.1:2

$$T = \frac{\mu (R_1 + R_2 + R_M)}{R_1 + R_2 + R_M + \mu R_1 R_M} \tag{1.0.4.3}$$

Parameters	Definition	For given circuit
Open loop gain	G	μ
Feedback factor	H	$\frac{R_1 R_M}{R_1+R_2+R_M}$
Loop gain	GH	$\mu\frac{R_1 R_M}{R_1+R_2+R_M}$
Amount of feedback	1+GH	$1+\frac{\mu R_1 R_M}{R_1+R_2+R_M}$
Closed loop gain	T	$\frac{\mu(R_1+R_2+R_M)}{R_1+R_2+R_M+\mu R_1 R_M}$

TABLE 1.0.4: 1

1.0.5. When Loop Gain is large
Solution:

$$GH \gg 1, \tag{1.0.5.1}$$

$$T \approx \frac{1}{H} = \frac{R_1 + R_2 + R_M}{R_1 R_M} \tag{1.0.5.2}$$

This is the key to designing a successful feedback system; if we can guarantee that $GH \gg 1$ for the frequencies that we are interested in, then the closed-loop gain will not be dependent on the details of the plant gain G. This is very useful, since in some cases the feedback function H can be implemented with a simple resistive divider, which can be cheap and accurate.

1.0.6. Example
We need to calculate V_o for the parameters in TABLE 1.0.6:1
Solution: From Fig1.0.1

Parameter	Value
R_1	1000Ω
R_2	1000Ω
R_L	1000Ω
R_M	1000Ω
V_s	$1V$

TABLE 1.0.6: 1

$$V_o - V_A = I_o R_L \tag{1.0.6.1}$$

$$V_A = I_o (R_M \parallel (R_1 + R_2)) \tag{1.0.6.2}$$

$$\implies V_o = I_o (R_L + (R_M \parallel (R_1 + R_2))) \tag{1.0.6.3}$$

Dividing both sides by V_s

$$\frac{V_o}{V_s} = \frac{I_o}{V_s} (R_L + (R_M \parallel (R_1 + R_2))) \tag{1.0.6.4}$$

From equation 1.0.4.1 and 1.0.6.4

$$\frac{V_o}{V_s} = T (R_L + (R_M \parallel (R_1 + R_2))) \tag{1.0.6.5}$$

From values in table1.0.6

$$H = \frac{(1000)(1000)}{1000 + 1000 + 1000} \tag{1.0.6.6}$$

$$\implies H = \frac{1000}{3} \tag{1.0.6.7}$$

For an op amp:

$$G \in (20000, 200000) \tag{1.0.6.8}$$

So, from equation 1.0.5.1 and Table1.0.6

$$T \approx \frac{1}{H} \tag{1.0.6.9}$$

$$T = \frac{3}{1000} \tag{1.0.6.10}$$

Hence,

$$V_o = 5V_s \tag{1.0.6.11}$$

$$\implies V_o = 5V \tag{1.0.6.12}$$