Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. Figure 1.0.1 shows a feedback transconductance amplifier implemented using an op amp with open-loop gain μ , a very large input resistance, and an output resistance r_o. The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of the three resistances R_M, R_1, and R_2, and a proportional voltageV_f is fed back to the negative-input terminal of the op amp.

Find G,H and T. If the loop gain is large, find an approximate expression for T and state precisely the condition for which this applies.

Solution: The parameters given are shown in the TABLE.1.0.1:1 The equivalent circuit of the amplifier is in fig.1.0.1:2

Parameter	Value
input resistance	∞
output resistance	r_o
Input voltage	V_s
Output Voltage	V_o

TABLE 1.0.1: 1

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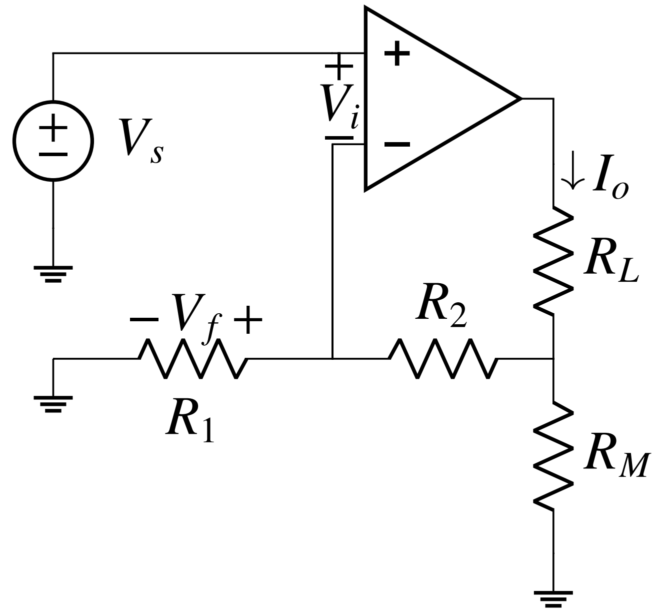

Fig. 1.0.1: 1 Original Circuit

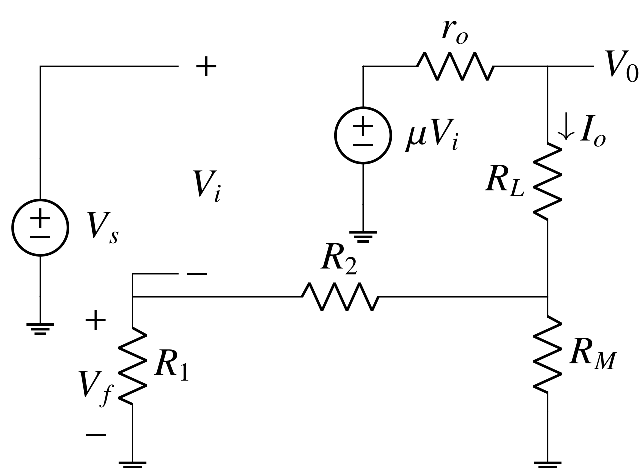

Fig. 1.0.1: 2 Equivalent Circuit

1.0.2. Calculating G
Solution:

$$G = \frac{I_o}{V_i} \tag{1.0.2.1}$$

$$\text{From fig 1.0.1:2} \tag{1.0.2.2}$$

$$\implies G = \mu \tag{1.0.2.3}$$

1.0.3. Calculating H
Solution:

$$H = \frac{V_f}{I_o} \tag{1.0.3.1}$$

From fig 1.0.1:2

$$V_f = R_1 I_o \frac{R_M}{R_M + R_1 + R_2} \tag{1.0.3.2}$$

$$\implies H = \frac{R_1 R_M}{R_1 + R_2 + R_M} \tag{1.0.3.3}$$

1.0.4. Calculating T
Solution:

$$T = \frac{G}{1 + GH} \tag{1.0.4.1}$$

From fig 1.0.1:2

$$T = \frac{\mu (R_1 + R_2 + R_M)}{R_1 + R_2 + R_M + \mu R_1 R_M} \tag{1.0.4.2}$$

Parameters	**Definition**	**For given circuit**
Open loop gain	G	μ
Feedback factor	H	$\frac{R_1 R_M}{R_1+R_2+R_M}$
Loop gain	GH	$\mu \frac{R_1 R_M}{R_1+R_2+R_M}$
Amount of feedback	1+GH	$1 + \frac{\mu R_1 R_M}{R_1+R_2+R_M}$
Closed loop gain	T	$\frac{\mu(R_1+R_2+R_M)}{R_1+R_2+R_M+\mu R_1 R_M}$

TABLE 1.0.4: 1

1.0.5. When Loop Gain is large
Solution:

$$GH \gg 1, \tag{1.0.5.1}$$

$$T \approx \frac{1}{H} = \frac{R_1 + R_2 + R_M}{R_1 R_M} \tag{1.0.5.2}$$

This is the key to designing a successful feedback system; if we can guarantee that $GH \gg 1$ for the frequencies that we are interested in, then the closed-loop gain will not be dependent on the details of the plant gain G. This is very useful, since in some cases the feedback function H can be implemented with a simple resistive divider, which can be cheap and accurate.