

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

1 FEEDBACK CIRCUITS

1.0.1. Figure 1.0.1 shows a feedback transconductance amplifier implemented using an op amp with open-loop gain μ , a very large input resistance, and an output resistance r_o . The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of the three resistances R_M , R_1 , and R_2 , and a proportional voltage V_f is fed back to the negative-input terminal of the op amp.

Find G, H and T. If the loop gain is large, find an approximate expression for T and state precisely the condition for which this applies.

Solution: The parameters given are shown in the TABLE.1.0.1:1 The equivalent circuit of

Parameter	Value
input resistance	∞
output resistance	r_o
Input voltage	V_s
Output Voltage	V_o

TABLE 1.0.1: 1

the amplifier is in fig.1.0.1:2

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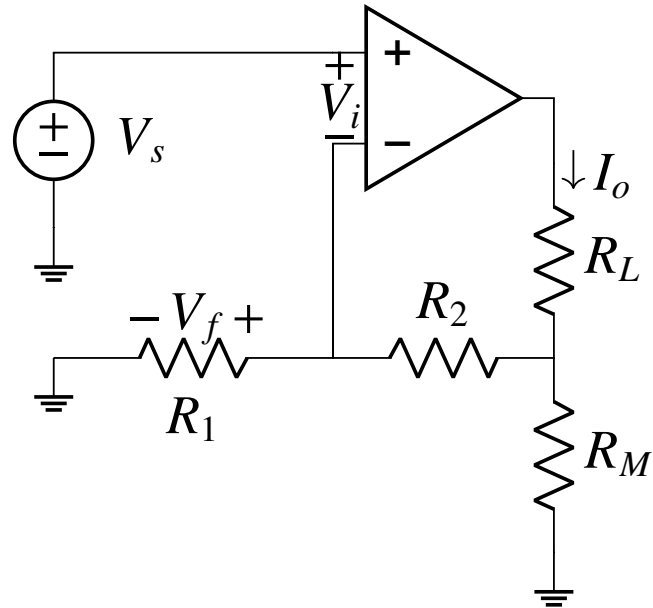


Fig. 1.0.1: 1 Original Circuit

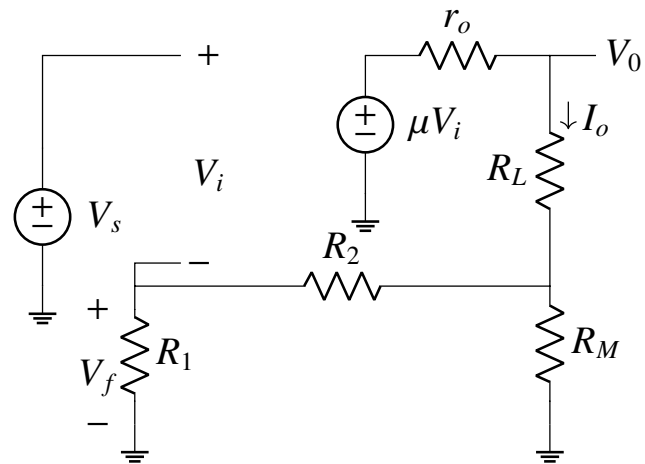


Fig. 1.0.1: 2 Equivalent Circuit

1.0.2. Calculating G

Solution:

$$G = \frac{I_o}{V_i} \quad (1.0.2.1)$$

$$\text{From fig 1.0.1:2} \quad (1.0.2.2)$$

$$\Rightarrow G = \mu \quad (1.0.2.3)$$

1.0.3. Calculating H

Solution:

$$H = \frac{V_f}{I_o} \quad (1.0.3.1)$$

$$(1.0.3.2)$$

From the equivalent circuit in fig 1.0.1:2

$$V_f = R_1 I_o \frac{R_M}{R_M + R_1 + R_2} \quad (1.0.3.3)$$

$$\Rightarrow H = \frac{R_1 R_M}{R_1 + R_2 + R_M} \quad (1.0.3.4)$$

1.0.4. Calculating T

Solution:

$$T = \frac{G}{1 + GH} \quad (1.0.4.1)$$

$$(1.0.4.2)$$

From the equivalent circuit in fig 1.0.1:2

$$T = \frac{\mu(R_1 + R_2 + R_M)}{R_1 + R_2 + R_M + \mu R_1 R_M} \quad (1.0.4.3)$$

Parameters	Definition	For given circuit
Open loop gain	G	μ
Feedback factor	H	$\frac{R_1 R_M}{R_1 + R_2 + R_M}$
Loop gain	GH	$\mu \frac{R_1 R_M}{R_1 + R_2 + R_M}$
Amount of feedback	1+GH	$1 + \frac{\mu R_1 R_M}{R_1 + R_2 + R_M}$
Closed loop gain	T	$\frac{\mu(R_1 + R_2 + R_M)}{R_1 + R_2 + R_M + \mu R_1 R_M}$

TABLE 1.0.4: 1

1.0.5. When Loop Gain is large

Solution:

$$GH \gg 1, \quad (1.0.5.1)$$

$$T \approx \frac{1}{H} = \frac{R_1 + R_2 + R_M}{R_1 R_M} \quad (1.0.5.2)$$

This is the key to designing a successful feedback system; if we can guarantee that $GH \gg 1$ for the frequencies that we are interested in, then the closed-loop gain will not be dependent on the details of the plant gain G. This is

very useful, since in some cases the feedback function H can be implemented with a simple resistive divider, which can be cheap and accurate.