Assignment 1

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Download all python codes from

```
https://github.com/adyasa611/EE3250/blob/main/Assignment−1/Codes
```

and latex-tikz codes from

```
https://github.com/adyasa611/EE3250/blob/main/Assignment−1/Figures
```

1 Digital Filter

1.1 Download the sound file from

```
https://raw.githubusercontent.com/gadepall/EE1310/master/filter/codes/Sound_Noise.wav
```

1.2 Write the python code for removal of out of band noise and execute the code.
Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav')

#sampling frequency of Input signal
sampl_freq=fs

#order of the filter
order=4

#cutoff frquency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and denominator polynomials respectively
b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b, a, input_signal)
#output_signal = signal.lfilter(b, a, input_signal)

#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav', output_signal, fs)
```

2 Difference equation

2.1 Write the difference equation of above Digital filter obtained in problem 1.2.
Solution:

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{2.0.1}$$

$$y(n) - 2.52y(n-1) + 2.56y(n-2) - 1.206y(n-3) + 0.22013y(n-4) = 0.00345x(n) + 0.0138x(n-1) + 0.020725x(n-2) + 0.0138x(n-3) + 0.00345x(n-4) \tag{2.0.2}$$

2.2 Sketch x(n) and y(n).
Solution: The following code yields Fig. 2.2

```
Codes/plot_xn_yn.py
```

The filtered sound signal obtained through difference equation :

```
Codes/Sound_diff_eq.wav
```

3 Z-transform

3.1

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{3.0.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1} X(z) \tag{3.0.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{3.0.3}$$

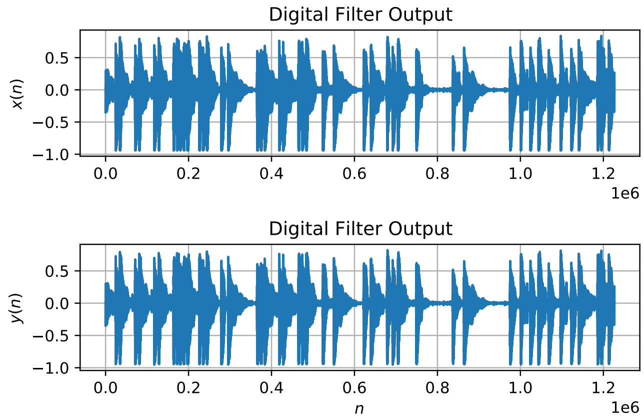

Fig. 2.2

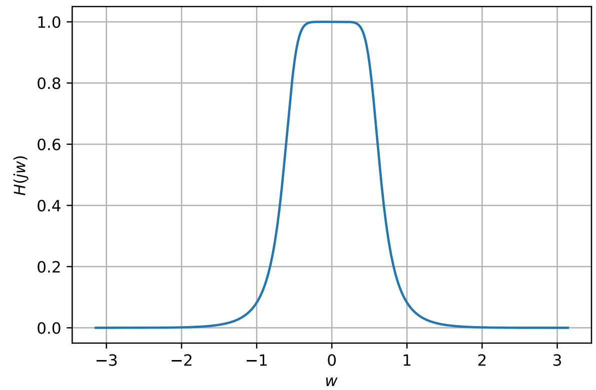

Fig. 3.3: $|H(e^{\jmath w})|$

Solution: From (3.0.1) we have,

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{3.0.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{3.0.5}$$

resulting in (3.0.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{3.0.6}$$

3.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{3.0.7}$$

from (2.0.2) assuming that the Z-transform is a linear operation.
Solution: Applying (3.0.6) in (2.0.2) we get,

$$H(z) = \frac{Y(z)}{H(z)}$$

$$= \frac{b[0] + b[1]z^{-1} + b[2]z^{-2} + b[3]z^{-3} + b[4]z^{-4}}{a[0] + a[1]z^{-1} + a[2]z^{-2} + a[3]z^{-3} + a[4]z^{-4}} \tag{3.0.8}$$

3.3 Let

$$H\left(e^{\jmath w}\right) = H\left(z = e^{\jmath w}\right). \tag{3.0.9}$$

Plot $|H(e^{\jmath w})|$.
Solution: The following code plots Fig. 3.3.

```
Codes/dtft.py
```

4 Impulse Response

4.1 From the difference equation eq. 2.0.2. Sketch h(n)
Solution: When Input = $\delta(k)$, Output = $h(k)$.
We know that on shifting the input, the output will also shift.
Hence from eq.2.0.1, by substituting $x(n-k) = \delta(n-k)$, then $y(n-k) = h(n-k)$ for all values of k.
Code for the plot in Fig.4.1

```
Codes/hn_plot.py
```

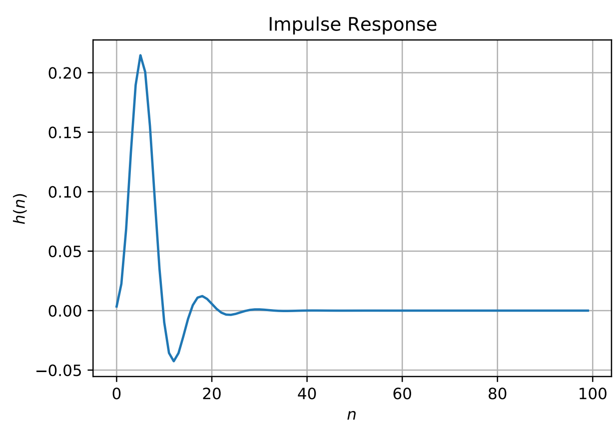

Fig. 4.1: $h(n)$

4.2 Check whether h(n) obtained is stable.
Solution:
We know that a system is stable if the following condition is satisfied. If the input is bounded, the output is bounded. This is known as BIBO stability.

From convolution formula,

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{4.0.1}$$

$$|y(n)| \leq \sum_{-\infty}^{\infty} |h(k)|\,|x(n-k)| \tag{4.0.2}$$

Let B_x be the maximum value x(n-k) can take as the audio input is bounded, then

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{4.0.3}$$

If

$$\sum_{-\infty}^{\infty} |h(k)| < \infty \tag{4.0.4}$$

Then

$$|y(n)| \leq B_y < \infty \tag{4.0.5}$$

Therefore to prove y(n) is bounded we need to prove h(n) is bounded.

$$\sum_{n=-\infty}^{n=-\infty} |h(n)| < \infty \tag{4.0.6}$$

The above equation can be re written as,

$$\sum_{n=-\infty}^{n=-\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \infty \tag{4.0.7}$$

$$\sum_{n=-\infty}^{n=-\infty} |h(n)| \left|z^{-n}\right|_{|z|=1} < \infty \tag{4.0.8}$$

From Triangle inequality,

$$\left|\sum_{n=-\infty}^{n=-\infty} h(n)z^{-n}\right|_{|z|=1} < \infty \tag{4.0.9}$$

$$\implies |H(n)|_{|z|=1} < \infty \tag{4.0.10}$$

From the equation (3.0.8) poles of the given transfer equation is:

$$\begin{aligned} z(approx) = 0.69382 \pm 0.41i, \\ 0.56617835 \pm 0.134423 \end{aligned} \tag{4.0.11}$$

From the above poles, we can see that that the ROC of the system is $|z| > \sqrt{0.69382^2 + 0.41^2}$. From the figure we can observe that ROC of the system includes unit circle $|z| = 1$.
The code for plotting the figure is:

Codes/plot_roc.py

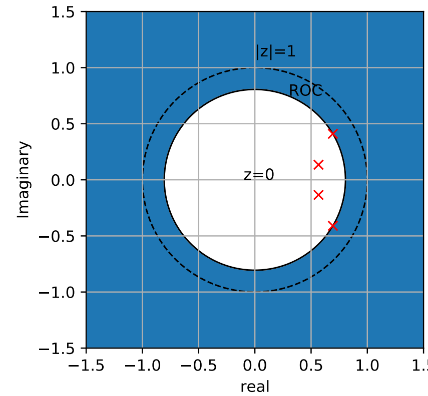

Fig. 4.2: $X(k) and H(k)$

This implies that the given IIR filter is stable, because h(n) is absolutely summable. Hence y(n) is bounded, and stable.

Verification from section 2:- Given bounded input x(n) (audio sample) and system difference equation 2.0.2 From 2.2 we can see that the maximum value of x(n) is 0.839 and minimum value is greater than -0.94171. For y(n) 2.2 we observe the maximum value is 0.822256 and minimum value is -0.953761 and it tends to zero after a certain interval. Therefore we can say that the system is BIBO stable.

4.3 Compute Filtered output using convolution formula using h(n) obtained in 4.1

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \tag{4.0.12}$$

Solution: The following code plots Fig. 4.3

Codes/yn_convolution.py

The filtered sound signal through convolution from this method is found in

Codes/Sound_convolution.wav

We can observe that the output obtained is same as y(n) obtained in Fig. 2.2

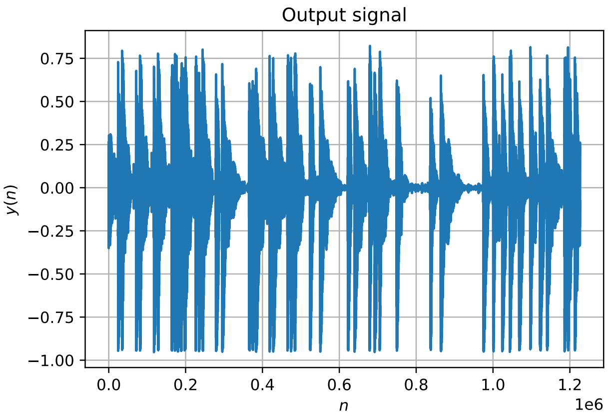

Fig. 4.3: $y(n)$ from the definition of convolution

5 FFT and IFFT

5.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{5.0.1}$$

and $H(k)$ using h(n).
Solution:
DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{5.0.2}$$

DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{5.0.3}$$

The following code plots FFT of $x(n)$,$y(n)$ and $h(n)$.

```
Codes/xn_hn_yn_fft.py
```

Magnitude and Phase plots obtained through above code is

5.2 From

$$Y(k) = X(k)H(k) \tag{5.0.4}$$

Compute

$$y(n) \triangleq \sum_{k=0}^{N-1} Y(k)e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{5.0.5}$$

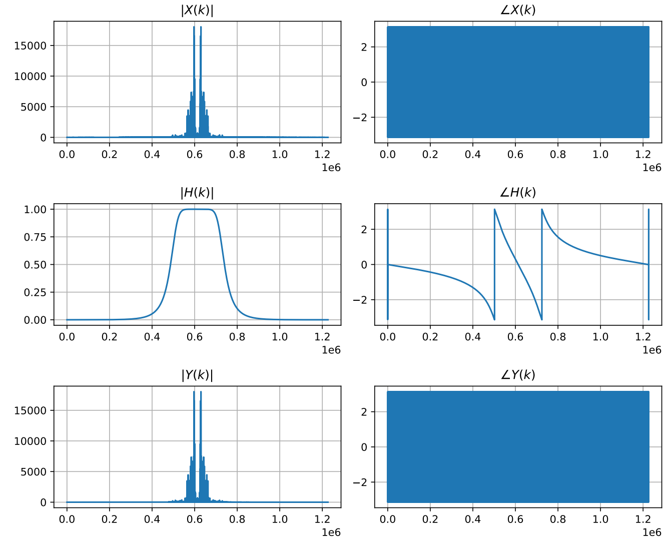

Fig. 5.1: $X(k), H(k) and Y(k)$

Solution: The following code plots Fig.5.2

```
Codes/yn_fft.py
```

The IFFT of Y(k) gives the following plot.
The filtered sound signal from this method is

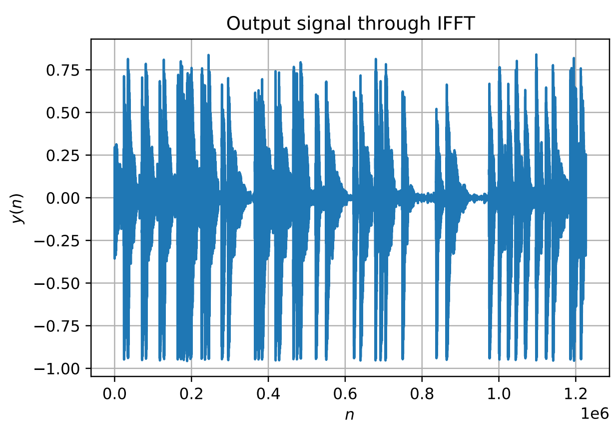

Fig. 5.2: $y(n)$

found in

```
Codes/Sound_fft.wav
```

This plot is same as that observed in Fig.2.2